I. Vibration in Time

Consider the background coordinates $t, x$ for the flat space-time as observed in an inertial frame $O$. Assume there exists a plane wave with matter that has vibrations in time relative to the background coordinate frame, $t$. We will define a plane wave’s amplitude for vibration in time, $T$, as the maximum difference between the time of matter inside the wave, $t_j$, and the external time, $t$. Therefore, if matter inside the plane wave carries a clock measuring its internal time, an inertial observer outside will see the matter’s clock running at a different rate. The vibrations of matter in time can be written as

$$t_j = t + T \sin(k \cdot x - \omega t) = t + \text{Re}(\zeta^+_t),$$

where

$$\zeta^+_t = -iT \sqrt{k^2 - \omega^2},$$

and the amplitude of the plane wave

$$\zeta^+_t = \frac{\partial \phi^+_t}{\partial t}.$$

We can further define a plane wave $\phi^+_t = T \sqrt{k^2 - \omega^2} \sin(k \cdot x - \omega t)$, such that

$$\phi^+_t = \frac{\partial \phi^+_t}{\partial t}.$$

Let us investigate the properties of a system in a cube with volume $V$ that can have multiple particles with mass $m$ vibrating in time. We make the following ansätze

$$\phi^+_n = \omega_n \frac{m}{2V} e^{-i\omega_n t},$$

$$\partial \phi^+_n = \frac{\partial \phi^+_n}{\partial t} = \omega_n \frac{m}{2V} e^{-i\omega_n t}.$$

The wave $\phi^+_n$ satisfies the equation of motion:

$$\partial_t \partial_t \phi^+_n + \omega_n^2 \phi^+_n = 0.$$  

II. Hamiltonian Density

The corresponding Hamiltonian density is

$$H^+_n = \frac{\text{Re}(\partial \phi^+_n) (\partial \phi^+_n) + (\nabla \phi^+_n) \cdot (\nabla \phi^+_n) + \omega_n^2 \phi^+_n}{V}.$$  

The energy inside volume $V$ is $E = m \omega_0 T^2 T_0$ of a simple harmonic oscillating system in proper time. Energy $E$ shall correspond to certain energy intrinsic to matter. However, we have only consider matter with mass $m$ in this simple harmonic oscillating system with no other force field. Here, we will consider this energy as the internal energy of mass.

Allowing matter to have an additional degree of freedom to vibrate in time, we show that such system has the same properties of a bosonic field. The temporal vibrations are physical quantities introduced to restore symmetry between time and space in the matter field. The spacetime outside a particle with oscillation in time also satisfies the Schwarzschild field solution as shown in Part 2 of this paper.

III. Proper Time Oscillator

Taking the energy of the harmonic oscillator as the internal mass-energy of matter, it can only be observed as the energy of mass $m$ which is on shell. For a single particle system,

$$E = m \omega_0^2 T^2 T_0,$$

or

$$\omega_0^2 T^2 T_0 = 1.$$  

Therefore, a particle with mass $m$ has oscillation in proper time with amplitude $|\Theta|t = 1/\omega_0$. The amplitude of this oscillation is unique. The internal time $t_j$ of the point mass’s internal clock is

$$t_j(t) = t - \frac{\sin(\omega_0 t)}{\omega_0}.$$  

IV. Field Quantization

For a many-particle system,

$$\omega_n^2 T^2 T_0 = n.$$  

Taking the point mass as a particle (antiparticle) with de Broglie’s mass/energy $(m = \omega_0)$,

$$H_0^{\pm} = \frac{n \omega_0}{V}.$$  

The energy in the plane wave with proper time oscillations is quantized with $n = 0, 1, 2, ...$ Instead of considering $\phi^+_n$, let us consider a plane wave $\phi^+_n$ which is normalized in volume $V$ when $n = 1$,

$$\phi^+_n = \gamma^{-1/2} \phi^+_n,$$

where $\gamma = (1 - |\nu|^2)^{-1/2} = T^2/\omega_0$. Replace $\phi^+_n$ with $\phi^+_n$ in Eq. (9).

The energy in this plane wave is quantized with $n$ particles (antiparticles) in volume $V$. We can obtain a real scalar field by superposition of plane waves,

$$\phi(x) = \sum_k \phi_k^+(x) + \phi_k^-(x),$$

$$= \sum_k (2V \gamma)^{-1/2} (\omega_n T^2 T_0)^{-1/2} + \omega_n T^2 T_0 \phi_k^0 e^{ikx}.$$  

It satisfies the Klein-Gordon equation. The transition to quantum field can be done via canonical quantization with $a_k = \omega_n T^2 T_0$ as the annihilation operator and $\phi_k = \omega_n T^2 T_0$ as the creation operator.

Therefore, the real scalar field has the same properties of a bosonic field. Although not shown in here, it is straightforward to show in non-relativistic limit that the system satisfies Schrodinger equation and has the properties of a quantum wave.

Experimenting with Neutrino?

Neutrino is the lightest known elementary particle. The neutrino oscillation length is in the order of kilometer. The energy in experiments are in the order of GeV. Neutrino can be an interesting candidate that can be used to study the possible effects of the temporal oscillation.
I. Fourier Decomposition of the Proper Time Oscillation

From Part 1 poster, time at the location of a point mass is driven by its energy to oscillate. Taking this proper time oscillation as a part of the spacetime geometry, its differential can be extracted from the assumed flat spacetime at spatial infinity. The spacetime around it shall be curved. By neglecting all quantum effects and assuming the particle as a classical object that can remain stationary in space, the temporal vibrations at and around the particle are

\[ \dot{t}(t, x) = -\frac{\partial t}{\partial x} \left( \frac{\Pi(x) \sin(\omega t)}{\omega} \right) = t + \zeta(t, x), \]

where the wave velocity can have effects on the proper time oscillator with infinite amplitude as discussed in Section I. We therefore arrive at the following equation:

\[ \dot{t}(t, x) = t - \frac{\Pi(x) \sin(\omega t)}{\omega} \]

As inertial observers with their clocks synchronized, the moving length of the rod is, \( L = \text{constant} \) under time translation as discussed in Section I. We therefore arrive at the following equation:

\[ L(t, x) = \text{constant} \]

Hence, the moving length of the rod is, \( L(t, x) = \text{constant} \) under time translation as discussed in Section I. We therefore arrive at the following equation:

\[ L(t, x) = \text{constant} \]

### II. Shell with Fictitious Oscillations

Instead of considering the shell with infinitesimal radius, we will first study a thin shell with finite radius \( \hat{r} \). On this shell, we introduce radial oscillations:

\[ \dot{r}(t, x) = \frac{\partial r}{\partial x} \left( \frac{\Pi(x) \sin(\omega t)}{\omega} \right) = \hat{r} + \zeta(\hat{r}, t), \]

\[ \dot{\zeta}(\hat{r}, t) = -\frac{\partial \zeta(\hat{r}, t)}{\partial x} \left( \frac{\Pi(x) \sin(\omega t)}{\omega} \right) = \frac{\Pi(x) \sin(\omega t)}{\omega} \]

where \( \zeta(\hat{r}, t) \) is the fictitious radial oscillation around \( \hat{r} \).

### III. Measurements on the Shell with Fictitious Oscillations

At \( t = t_m = \pi/(2\omega) \), the fictitious displacement and instantaneous velocity can have effects on \( \hat{r} \). The combined effects shall remain constant. Although the effects of the fictitious displacement are not yet defined, we can obtain the spacetime geometrical properties at \( r = \hat{r} \) when only a fictitious velocity with \( |v| < 1 \).

\[ \dot{r}(t, x) = \frac{\partial r}{\partial x} \left( \frac{\Pi(x) \sin(\omega t)}{\omega} \right) = \hat{r} + \zeta(\hat{r}, t), \]

\[ \dot{\zeta}(\hat{r}, t) = -\frac{\partial \zeta(\hat{r}, t)}{\partial x} \left( \frac{\Pi(x) \sin(\omega t)}{\omega} \right) = \frac{\Pi(x) \sin(\omega t)}{\omega} \]

\[ \Pi(x) = 0 \text{ if } |x| < \epsilon/2, \]

\[ \Pi(x) = 1 \text{ if } |x| > \epsilon/2. \]

\[ \Pi(r) \text{ denotes the derivative of } \Pi(r) \text{ with respect to } r, \text{ such that} \]

\[ \Pi'(r) = 0 \text{ if } r \neq \epsilon/2, \]

\[ \Pi'(r) = -\infty \text{ if } r = \epsilon/2. \]

The radial vibrations are on an infinitesimally thin spherical shell with radius \( \hat{r} \to 0 \). An observer \( \hat{O} \) on this timelike hypersurface is stationary relative to an inertial observer \( O \) at spatial infinity. The moving length of the rod is, \( L \approx 2\hat{r} \).

\[ L(t, x) = 2\hat{r} \]

\[ \dot{L}(t, x) = \text{constant} \]

\[ L(t, x) = \text{constant} \]

IV. Schwarzschild field

As geometrical properties, both \( r_f \) and \( r_f \) from Eqs. (12) and (13) have effects on \( \hat{O} \). Their summation is a constant under time translation as discussed in Section II. Therefore, we can define a constant,

\[ I = \hat{r}_f^2 \theta^2 + (r_f^2)^2 = \mathbb{R}^2 \]

such that Eq. (12) becomes,

\[ \begin{bmatrix} \dot{t} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} I \end{bmatrix} \begin{bmatrix} \dot{t} \\ \dot{r} \end{bmatrix}. \]

According to Eqs. (18) and (23). The measurements in the temporal and radial directions in frames \( O \) and \( \hat{O} \) have different scales, i.e.

\[ g_{11}(\hat{r}) = \hat{e}_1 \cdot \hat{e}_1 = (1 - \hat{I}) \hat{e}_1 \cdot \hat{e}_1 = 1 - \hat{I}, \]

\[ g_{11}(\hat{r}) = \hat{e}_1 \cdot \hat{e}_1 = (1 - \hat{I})^{-1} \hat{e}_1 \cdot \hat{e}_1 = - (1 - \hat{I})^{-1}, \]

\[ g_{11}(\hat{r}) = \hat{e}_1 \cdot \hat{e}_1 = 0, \]

\[ \text{where } \hat{e}_1 \cdot \hat{e}_1 = 1, \hat{e}_1 \cdot \hat{e}_1 = -1, \text{and } \hat{e}_1 \cdot \hat{e}_1 = 0. \]

Therefore, the line element at \( r = \hat{r} \) is,

\[ ds^2 = [1 - \hat{I} \dot{r}^2 - (1 - \hat{I}) \dot{r}^2 - \hat{r}^2 d\hat{r}^2]. \]

Setting \( I = 2m/\hat{r}^2 \) and \( m = \mathbb{R}^2 \), the vacuum spacetime \( v' \) outside \( \Sigma \) is the Schwarzschild spacetime. Applying Birkhoff theorem, \( \Sigma \) can be contracted. As long as the equivalent mass \( m \) of the shell is remaining constant during this contraction, the metric of the external field will not be affected.

The amplitude of the radial oscillation is,

\[ \hat{R} = \sqrt{4/(\pi m)}. \]

This amplitude \( \hat{R} \) and the related curvature tensors derived are well defined until the shell is contracted to \( r = 0 \). At this point, the shell is infinitely small but has infinitely large amplitude, \( \hat{R} \to \infty \).

This is the same fictitious radial oscillations around the proper time oscillator with infinite amplitude as shown in Section I. We therefore arrive at the following result:

**Key Result**

By neglecting all quantum effects and assuming the particle as a classical object that can remain stationary in space, the space-time geometry resulting from the proper time oscillator can mimic the Schwarzschild gravitational field.