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Coupling Efficiency of a Partially Coherent Radially Polarized Vortex Beam into a Single-Mode Fiber

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Abstract: We study the problem of coupling partially coherent radially polarized (PCRP) vortex beams into a single-mode optical fiber. Using the well-known concept of the cross-spectral density (CSD) matrix, we derive a general expression for the coupling efficiency of the partially coherent beam into a single-mode fiber. We adopt PCRP vortex beams for incident beams and use our general results to discuss the effects of the coherence, topological charge, and wavelength on the coupling efficiency of an optical beam focused onto a single-mode fiber with a lens. Our results should be useful for any application that requires coupling of partially coherent beams into optical fibers.

Keywords: coupling efficiency; partially coherent; vortex beam; single-mode fiber

1. Introduction

Optical fiber, as the fundamental optical components of a coupling process, have been widely applied in various fields, such as networking [1,2], ladar [3,4], imaging [5], free-space optical communications [6,7], and biomedical optics [8–10]. In the above applications, the input beam must be coupled into a single-mode or multi-modes fiber for propagation and detection before being amplified. Therefore, the coupling efficiency of the input light beam into optical fiber plays an important role. One can improve the signal-to-noise ratio and save energy through increasing the coupled light beam. During the past decades, many theoretical methods and experimental techniques were used to improve the coupling efficiency, such as a fiber array [11], cylindrical glass fiber [12], wedge-shaped fiber endface [13], combination lens [14], chemically etched self-centered diffracting element [15], and a microlens [16].

The degree of freedom of the input light beam, such as phase, amplitude, coherence, and polarization, is one of the key factors when analyzing the coupling efficiency. For example, the coupling efficiency of the plane-wave to fiber was studied as the input light in detail, the influences caused by the misalignments of the fibers and mode’s field distribution are considered [17]. Ruilier and Cassaing discussed the coupling efficiency when the stellar is coupled into the single mode fiber [18]. Wheeler and Schmidt developed analytic equations that describe the mean and normalized variance of the coupling efficiency of the Gaussian Schell-model (GSM) beams into single-mode optical fibers [19]. The light sources in the above studies were scalar beams and did not consider the vector beams. In 2009, the polarization state of the beam was first taken into account by Salem and Agrawal. They studied the problem of coupling an electromagnetic beam of any state of coherence and polarization into a multimode optical fiber. The coherence and polarization properties of the electromagnetic Gaussian Schell-model (EGSM) beams are discussed in detail in Refs. [20,21]. In 2012, Zhao and others
demonstrated experimentally the procedure of coupling of an EGSM beam into a single-mode optical fiber. The results showed that the coupling efficiency depends closely on the coherence and polarization properties of the EGSM beam, which is consistent with Salem and Agrawal’s theoretical prediction [22]. Furthermore, in the past several years, due to the partially coherent radially polarized (PCRP) beam having an advantage over a linearly polarized partially coherent beam for reducing turbulence-induced scintillation, which is useful in free-space optical communications, the PCRP beam was introduced in theory and generated in experiment [23−25]. The propagation properties of a PCRP beam are quite different from those of a radially polarized beam. On the other hand, it is well known that any beam with a helical phase factor, called a vortex beam, possesses an optical vortex and carries orbital angular momentum (OAM). Thus, there are growing interests in focusing partially coherent radially polarized vortex beams [26−29]. However, to the best of our knowledge, there are no papers dealing with the backpropagation technique from B to A and is given by:

\[ W_{\text{backprop}}(p_1, \theta_1, \rho_2, \theta_2) = W_{\text{forward}}(p_1, \theta_1, \rho_2, \theta_2) \exp(-im\theta_1 + im\theta_2) \]

where \( m \) is the topological charge. According to the coupling theory introduced in Ref. [28], the field distribution of the fiber mode can be approximated by a Gaussian function [30,31]:

\[ F_j = \sqrt{\frac{2}{\pi}} \frac{\omega_j^{-1}}{\omega_j} \exp\left(-\frac{\rho_j^2}{\omega_j^2}\right) \quad (j = x, y) \]

where \( \rho_j \) is a transverse position vector in the fiber plane B and \( \omega_j \) is a measure of the mode width. Furthermore, the field distribution of the mode at the aperture plane A is found by using the backpropagation technique from B to A and is given by:
\[ F_{jA} = \sqrt{\frac{2}{\pi}} \omega_{jA}^{-1} \exp \left( -\frac{\rho^2}{\omega_{jA}^2} \right) \]  

(4)

where \( \omega_{jA} = \lambda f / \pi \omega_j \) is the effective mode width at the aperture plane, \( \lambda \) is the wavelength of the incident, and \( f \) is the focal length of the coupling lens. We can get the coupling efficiency as:

\[ \eta_c = \frac{\langle P_c \rangle}{\langle P_a \rangle} \]  

(5)

where \( \langle P_c \rangle \) is the power coupled into the fiber and \( \langle P_a \rangle \) is the power in the aperture plane. They can be expressed as:

\begin{align*}
P_c &= \int_D W_j(\rho_1, \rho_2, \omega) F_{jA}(\rho_1) F_{jA}(\rho_2) \times \exp \left( -\frac{\rho_1^2 + \rho_2^2}{W^2} \right) \, d^2 \rho_1 \, d^2 \rho_2 \\
&= \int_D W_j(\rho, \omega) \times \exp \left( -\frac{\rho^2}{W^2} \right) \, d^2 \rho
\end{align*}

(6)

where \( D \) denotes a hard aperture of diameter, \( W^2 = D^2 / 8 \), and \( NA = D / 2f \) [32].

For PCRP vortex beams, we obtain the following expression for the power coupled into the single fiber with \( P_{c,\text{even}} \) \((m \text{ is even number})\) and \( P_{c,\text{odd}} \) \((m \text{ is odd number})\):

\begin{align*}
P_{c,\text{even}} &= \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m-1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{A_y^{m-1}+2s+1} \left( \frac{m-1}{2} \right)! \left( \frac{m-1}{2}+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m+1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{A_y^{m+1}+2s+1} \left( \frac{m+1}{2} \right)! \left( \frac{m+1}{2}+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m+1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{A_y^{m+1}+2s+1} \left( \frac{m+1}{2} \right)! \left( \frac{m+1}{2}+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m-1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{A_y^{m-1}+2s+1} \left( \frac{m-1}{2} \right)! \left( \frac{m-1}{2}+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m+1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m+1} \frac{1}{A_y^{m+1}+2s+1} \left( \frac{m+1}{2} \right)! \left( \frac{m+1}{2}+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m-1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m+1} \frac{1}{A_y^{m-1}+2s+1} \left( \frac{m+1}{2} \right)! \left( \frac{m+1}{2}+1 \right)! \right]
\end{align*}

(7)

\begin{align*}
P_{c,\text{odd}} &= \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m-1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{(2A_x)^{m-1}+2s+1} \left( |m-1|+2s+1 \right)! \left( |m-1|+2s+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m+1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m-1} \frac{1}{(2A_x)^{m+1}+2s+1} \left( |m+1|+2s+1 \right)! \left( |m+1|+2s+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m+1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m+1} \frac{1}{(2A_x)^{m+1}+2s+1} \left( |m+1|+2s+1 \right)! \left( |m+1|+2s+1 \right)! \right] \\
&+ \frac{\pi}{2 \omega_0^2 w_{yA}^2} \left[ \sum_{s=0}^{\infty} \frac{\Gamma\left(\frac{1}{2}\right)^2}{\Gamma\left(\frac{m-1}{2}+s+1\right)^2} \left( \frac{1}{2w_0^2} \right)^{m+1} \frac{1}{(2A_x)^{m-1}+2s+1} \left( |m-1|+2s+1 \right)! \left( |m-1|+2s+1 \right)! \right]
\end{align*}

(8)

where \( A_x = 1/\omega_0^2 + 1/(2\Delta_0^2) + 1/w_{yA}^2 + 1/D^2 \), and \( A_y = 1/\omega_0^2 + 1/(2\Delta_0^2) + 1/w_{yA}^2 + 1/D^2 \).

Figure 1. Illustrating notation related to the coupling of a beam into an optical fiber.
The power of the input PCRP vortex beams at the aperture plane is

$$P_a = \frac{\pi}{4a_0^2} \left( \frac{1}{\omega_0^2} + \frac{1}{W^2} \right)^{-2}$$  \hspace{1cm} (9)

3. Results and Discussion

In this section, with the help of the formulas obtained in Section 2, we will study the coupling efficiency of PCRP vortex beams for different topological charges, coherence length, and wavelength. First, we carefully chose the values of $\omega_0 (j = x, y)$ in Equation (3) to match with the core radius of the fiber because the theoretical model is only valid for the single-mode fiber. Let us recall the definition of the normalized frequency of the fiber [33]

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2}$$  \hspace{1cm} (10)

where $a$ is the radius of the fiber core, $\lambda$ is the wavelength of the incident light. $n_1$ and $n_2$ are the peak values of the refractive index of the fiber core and the refractive index of the fiber cladding, respectively. It is known that when the value of $V$ is smaller than 2.405, the fiber belongs to the single-mode fiber. Assuming that the fiber is weakly guiding with the refractive index profile, the last term in Equation (9) can be written as $\sqrt{n_1^2 - n_2^2} \approx n_1 \Delta$ with $\Delta = (n_1 - n_2) / n_1$ as a good approximation. To ensure the single-mode fiber, the parameter is chosen to be $a = 5 \mu m$, $n_1 = 1.44$, $\Delta = 0.0034$, and $\lambda = 1550 \text{nm}$. On substituting these parameters into Equation (9), we obtained a value of $V$ of 2.4, smaller than 2.405. Under this condition, we found the radius of the mode field of the fundamental propagating mode in the fiber by using the empirical formula to be $\omega_j = 5.15 \mu m (j = x, y)$. Therefore, in the following numerical examples, the value of $\omega_j$ was kept fixed at 5.15 $\mu m$ throughout the paper. If the focal length of the lens in plane $A$ is 10 $cm$, the radius $\omega_{jA} (j = x, y)$ of the mode field was calculated to be 9.6 $mm$.

Figure 2 shows the intensity distribution of PCRP vortex beams in plane $B$ with different spatial coherence width and topological charge. The propagation properties of PCRP vortex beams through a paraxial ABCD optical system have been studied in Ref. [28]. Figure 2 shows that the focusing properties of a PCRP vortex beam are closely affected by its initial spatial coherence and topological charge. The beam spot got larger with increasing topological charge from $m = 1$ to $3$, but not $m = 0$. This is because the PCRP vortex beam became the PCRP beam for $m = 0$, which has a ring-shaped beam profile. The PCRP vortex beam had a dark hollow beam spot (see Figure 2h) with large coherence. When the spatial coherence was decreased, a flat-topped beam spot was formed (see Figure 2d). With the further decrease of the spatial coherence, a focused Gaussian beam spot was formed.

![Figure 2](attachment:image.png)

**Figure 2.** Distribution of the intensity of PCRP vortex beams in plane $B$ for different coherence length and topological charge.
Figure 3 shows that the coupling efficiency varied with NA for different values of $\delta_0$ under different topological charges. In Figure 3, one can find that the coupling efficiency increased at first and then decreased with increasing NA, and there was a peak value of the coupling efficiency, which means there was a best match between the incident beam and the distribution of the mode field. We also confirm that the position of the peak (i.e., the value of NA) was different for different coherence length and topological charge. We could adjust the spatial coherence to improve the coupling efficiency, and an optimum value was found. Obviously, the result of Figure 3b is different to other results. We found that when PCRP vortex beams had $m = 1$, the coupling efficiency increased with the increasing of the spatial coherence width of the beam. We can explain this phenomenon using the property of PCRP vortex beams (see Figure 2), where the PCRP vortex beams with $m = 1$ have a similar Gauss distribution beam profile.

Based the analysis above, we know that there is a best choice (peak value of the coupling efficiency) of the value of coherence and NA for different topological charge. Next, we simulated how the density plot of the coupling efficiency varied with NA and initial spatial coherence under different topological charges in Figure 4. In Figure 4, the numbers in brackets are the peak value of the coupling efficiency and corresponding value of the NA and spatial coherence width. One can clearly see the distribution of the coupling efficiency in the whole region under different topological charges. One can also find out clearly the peak of the coupling efficiency under different topological charges and the corresponding value of the coherence length and NA. Therefore, for the suitable value of $\delta_0$ and $m$, we could obtain the maximum coupling efficiency.
with the increasing of NA and wavelength. Moreover, from Figure 5b, we could also find that the coupling efficiency shows the same change rule as Figure 5d; it increases at first and then decreased with increasing NA, and there was a peak value of the coupling efficiency. However, the coupling efficiency increased with decreasing wavelength for smaller NA (for example, NA < 0.08), and for larger NA (for example, NA > 0.16), the coupling efficiency increased with the increasing wavelength. However, for larger topological charges (for example, m > 3, we did not plot it in here), we found that the coupling efficiency showed the same change rule as Figure 5d; it increases with the increasing of NA and wavelength. Moreover, from Figure 5b, we could also find that the coupling efficiency showed some different change rule due to the different properties of PCRP vortex beams with m = 1.

Figure 4. The density plot distribution of how the coupling efficiency of the symmetric beam with \( \omega_0 = 10 \text{ mm} \) and \( \lambda = 1550 \text{ nm} \) varies with numerical aperture and initial spatial coherence under different topological charges.

Figure 5 shows the coupling efficiency of the beam versus NA for different wavelengths \( \lambda \) under different topological charges. From Figure 5, we can find that all the coupling efficiencies showed the similar change rule for smaller topological charges (see Figure 5a–c); the coupling efficiency increased at first and then decreased with increasing NA, and there was a peak value of the coupling efficiency. However, the coupling efficiency increased with decreasing wavelength for smaller NA (for example, NA < 0.08), and for larger NA (for example, NA > 0.16), the coupling efficiency increased with the increasing wavelength. However, for larger topological charges (for example m > 3, we did not plot it in here), we found that the coupling efficiency shows the same change rule as Figure 5d; it increases with the increasing of NA and wavelength. Moreover, from Figure 5b, we could also find that the coupling efficiency showed some different change rule due to the different properties of PCRP vortex beams with m = 1.

Figure 5. The coupling efficiency of the beam with \( \delta_0 = 5 \text{ mm} \), \( \omega_0 = 10 \text{ mm} \) versus numerical aperture for different wavelengths \( \lambda \) under different topological charges.
4. Conclusions

In this paper, we derived a general expression for the coupling efficiency of PCRP vortex beams into a single-mode optical fiber in terms of its CSD matrix. We were able to derive an analytical expression for the coupling efficiency, and the field distribution of the fundamental mode of an optical fiber is approximated. We studied the effects of the coherence, topological charge and wavelength of the PCRP vortex beams, and NA on the coupling efficiency through some numerical simulation. The results showed that the coupling efficiency was significantly affected by the numerical aperture and parameters of the incident beam, such as topological charge, initial spatial coherence, and wavelength, and we also determined the best choice for the PCRP vortex beams with different topological charge. Our results will help users to determine for themselves which initial beam parameters and NA provides optimal results for their applications. It should be useful for any application requiring coupling of a partially coherent radially polarized vortex beam into optical fibers.

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References


