Simulation Research on the Time-Varying Meshing Stiffness and Vibration Response of Micro-Cracks in Gears under Variable Tooth Shape Parameters

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Abstract: The gear is one of the important parts of a rotary gearbox. Once catastrophic gear failure occurs, it will cause a great threat to production and life safety. The crack is an important failure factor causing changes in time-varying stiffness and vibration response. It is difficult to effectively identify the vibration response and meshing stiffness changes when there is a fine crack in the gear. Therefore, it is of great importance to improve the accuracy of meshing stiffness calculation and dynamic simulations under micro-cracks. Investigations of meshing stiffness and the vibration response of a gearbox is almost all about fixed gear shape parameters. However, the actual production process of gear system needs to change gear shape parameters. In this paper, the meshing stiffness and vibration response of the dynamic simulation signals of gear teeth with different crack depths at different tooth shape parameters (the pressure angle, the modulus, and the tooth number) were calculated, respectively. The influence of cracks on the vibration response was investigated by the fault detection indicators, the Root Mean Square (RMS), the kurtosis, and the crest factor. The result shows that when the pressure angle and modulus change, the vibration response changes erratically. However, when the tooth numbers change, the vibration response changes regularly. The results could be a guide for choosing gears in different shape parameters when system stability is the aim.

Keywords: meshing stiffness; vibration response; pressure angle; modulus; tooth number

1. Introduction

The gearbox is the key unit of modern machinery and has been widely used in all kinds of engineering situations. Due to the complex structure and tough running environments of large-scale machinery, key gearbox components emerge easily to local defects, which brings about fatal accidents and generates major disruptions [1,2]. Therefore, timely detection of faults residing in rolling bearings and gears are valuable for ensuring the safe and stable operation of mechanical equipment [3–6]. Meshing stiffness is the ability of the teeth to resist deformation during meshing, and it is the key factor of gear vibration response. Therefore, it is of great significance to study the time-varying meshing stiffness. In order to better study the various working conditions of rotating equipment, it is necessary to study its dynamic modeling. Scholars have also done a lot of research in the field of dynamics simulation. When calculating the meshing stiffness, the analytical method is widely used due to its efficiency and accuracy when comparing with FEM (finite element method). Zhang [7] studied the meshing stiffness when accounting for loading and stressing distribution on helical gears; the results matched well with the Finite Element Analysis (FEA) method. Howard [8] used the FEA to calculate the
torsional mesh stiffness of gears in mesh when there is a crack in the tooth root. Mohammed [9] used a 2D FEM model to calculate the meshing stiffness in different crack depths. In this model, the actual meshing force was replaced by the uniformly distributed load. By drawing the boundary of the root stress, the effect of tooth thickness could be calculated, and then the meshing stiffness could be derived. The accuracy of the 2D FEM method was proven by the photoelastic experiment method by Yogesh and Anand [10]. The 3D FEM method was established by Chaari [11]. By putting a uniformly distributed load in the tooth profile, the displacement in the meshing direction was extracted, and the meshing stiffness could then be derived by calculating the ratio between the meshing force and the displacement. Above all, the 2D FEM method is a widely used method, matches well with the experimental result, and could be the right reference for the analytical method. Cui et al. [12] analyzed the dynamic model signal by their method, and the results were consistent with the experimental signals, which proves the method is effective.

Analytical methods are widely used methods due to the high efficiency when compared with FEM methods. The Silurian method was proposed by Silurian [13]. In this method, the gear profile is simplified by regarding the gear as a complex of a trapezoid and a rectangular, so it is far away from the actual gear profile. The relationship between meshing stiffness and potential energy was derived by Yang and Lin [14]. On the basis of Yang and Lin, a time-varying meshing stiffness calculation method that accounts for the Hertzian, bending, shear and axial compressive energies was proposed by Tian [15], but the fill-foundation stiffness has not been considered. The theoretically precise calculation method of fillet foundation deflection, which was verified by FEM, was originally proposed by Sainsot [16]. Then, the effect of different types of cracks on the dynamic responses was investigated by Chen [17] by taking the fillet-foundation stiffness into account. On this basis, Cui et al. [18] proposed a quantitative trend fault diagnosis method of a coupling gearbox based on Sparsogram and Lempel-Ziv. The performance is much better on the quantitative diagnosis of cracked components. Nevertheless, the tooth profile between the base circle and the root circle had rarely been considered. To solve this problem, an improved mesh stiffness calculating method was proposed by Wan [19] by considering the misaligned behavior between the base circle and the gear root circle; however, the expression of equations should be changed by the actual tooth numbers. A step-by-step fuzzy diagnostic method based on frequency domain symptom extraction and trivalent logic fuzzy diagnosis (TLFD) theory was proposed by Song et al. [20]. This method can generate membership functions of each state based on the possibility theory. Next, an improved method, which is in reference to the cutting process of a gear tooth, was proposed by Ma [21,22] to calculate precisely the stiffness between the root circle and the base circle. However, this method for calculating the involute part of the gear profile is the same as [15]. In this paper, based on the rolling angle of cutting tools, a ‘universal equation of gear profile’ is introduced to investigate the changing behaviors of meshing stiffness and vibration response of a gear system under different depths and angles of cracks in the tooth root.

For the construction of the dynamic model, from one degree of freedom (DOF) to the multi-degree of freedom dynamic model considering overall the input/output torque, gear pair, bearing, damping, friction, and manufacturing error, many scholars have done a lot of work [23–25]. The earlier dynamic model was developed by Özgüven and Houser [26]. The earlier peeling off dynamic model was established by Velex and Berthe [27], and the relationship between peeling off and the vibration response was investigated. By studying the influence of the tip between the meshing gear pair caused by foil on the meshing stiffness, the dynamic model considering the DOF in the torsion direction was proposed by Theodossiades [28]. A dynamic model with four DOFs was proposed by Ma and Chen [29] to investigate the dynamic behavior of a gearbox. Cui et al. [30] investigated the influence of the angle-changing crack on the vibration response both in the time domain and in the frequency domain. Mohammed [31] put the 6, 8, and 12 DOF dynamic system together to compare the different dynamic responses generated from the different systems. Wang et al. [32] proposed a new feature enhancement method. The result shows that the increasing tendency of the signal of different fault sizes generated
could not be affected by a different dynamic model. A two-stage gear dynamic system of 26-dof considering the displacement in the torsional and vertical direction was established by Jia [8]. In that model, the fault detection indicators was also used for a vibration tendency investigation. In this paper, a 6-DOF system was used for dynamic investigation.

The paper is mainly arranged as the following steps: In part 2, the calculation method of meshing stiffness under a universal equation of a gear profile and the dynamic model for the vibration response is introduced, and also the meshing stiffness method is verified by the FEM method. In part 3, the meshing stiffness and vibration response for cracked gears in different gear shape parameters are investigated. In part 5, the conclusion for the paper is introduced.

2. Meshing Stiffness and Vibration Response for Cracked Gears in Different Shape Parameters

2.1. Meshing Stiffness Calculation under a Universal Equation of Gear Profile

Figure 1 shows the process of cutting a tooth: $P_2y$, which is the vertical axis of $x_2P_2y$, is rolling along the profile of the pitch circle.

Any point on the tooth cutter in the coordinate system $x_2P_2y$ is defined as $M'(x_2,y_2)$, whose rolling angle is $\varphi$. The point $M(x,y)$, which is known as the corresponding point to $M'$, is in the coordinate system $xOy$. The normal tooth profile at point $M'$—$M'N$ is bound to intersect with $P_2y$ at $N$. When $P_2y$ reaches point $N$, $M'$ will be bound to coincide with point $M$. Next, by transforming the coordinates of $M'$ into the coordinate system $xOy$, the coordinate of $M$ will be obtained. The meaning of each parameter of the gear tooth profile is listed in Table 1. Afterward, the universal equation of the gear profile can be used to calculate the meshing stiffness. The detailed process of calculating meshing stiffness has already been presented in [33]:

\[
x = (r - x_2) \cos \varphi + (r_p - y_2) \sin \varphi - r_f \cos \frac{\pi}{n} \tag{1}
\]

\[
y = (r - x_2) \sin \varphi - (r_p - y_2) \cos \varphi \tag{2}
\]

\[
\varphi = \frac{P_2N}{r} \tag{3}
\]
2.1.1. Shear Stiffness

The excessive curve formula of the gear tooth root can be calculated by putting coordinate \((x_2, y_2)\) on the tooth nose into Equation (1):

\[
\begin{bmatrix}
    x \\
    y
\end{bmatrix} = \begin{bmatrix}
(r - x_c - \rho_0 \cos \gamma) \cos \varphi + (x_c \tan \gamma + \rho_0 \sin \gamma) \sin \varphi - r_f \cos \frac{\pi}{N} \\
(r - x_c - \rho_0 \cos \gamma) \sin \varphi - (x_c \tan \gamma + \rho_0 \sin \gamma) \cos \varphi
\end{bmatrix}
\]

The involute formula of the gear tooth profile can be known by projecting the points on the linear gear profile into the coordinate system \(xOy\):

\[
\begin{bmatrix}
    x \\
    y
\end{bmatrix} = \begin{bmatrix}
[r - \frac{1}{2}(rp \cos \gamma) \cos \varphi + (rp - y_w) \cos^2 \alpha_0 \sin \varphi - r_f \cos(\pi/N)] \\
[r - \frac{1}{2}(rp - y_w) \sin 2\alpha_0] \sin \varphi - (rp - y_w) \cos^2 \alpha_0 \cos \varphi
\end{bmatrix}
\]

### Table 1. The meaning of each parameter of the gear tooth profile.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>The Main Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>the radius of the reference circle</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>the angle between the center line of the tooth and NO</td>
</tr>
<tr>
<td>(x_2)</td>
<td>the abscissa that M projected onto in frame (x_2P_2)</td>
</tr>
<tr>
<td>(y_2)</td>
<td>the ordinate that M projected onto in frame (x_2P_2)</td>
</tr>
<tr>
<td>(r_f)</td>
<td>root circle radius of gear</td>
</tr>
<tr>
<td>(N)</td>
<td>number of teeth of pinion</td>
</tr>
<tr>
<td>(x_c)</td>
<td>the abscissa of the hobbing cutter point</td>
</tr>
<tr>
<td>(\rho_0)</td>
<td>the radius of the hobbing cutter point</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>the angle between the normal of the tip outline</td>
</tr>
<tr>
<td>(y_w)</td>
<td>passing through the center of the tip and (Px_2)</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>half the width of the alveoli</td>
</tr>
<tr>
<td>(\phi)</td>
<td>the pressure angle</td>
</tr>
</tbody>
</table>

The excessive curve formula of the gear tooth root can be expressed as:

\[
\frac{F^2}{2k_{sg}} = U_{sg} = \int_{0}^{x_c} \frac{1.2r^2_k}{2G_xsg} \, dx_g
\]

A healthy model is proposed in Figure 2, and the parameters for the transition part of the curve are listed in Table 2. The shear stiffness on the excessive curve can be expressed as:

![Figure 2. A standard gear model for calculating mesh stiffness.](image)
Table 2. Parameters for the transition part of the curve.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>The Main Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>the meshing force on the tooth curve</td>
</tr>
<tr>
<td>( F_b )</td>
<td>the y-direction component of meshing ( F )</td>
</tr>
<tr>
<td>( F_a )</td>
<td>the x-direction component of meshing ( F )</td>
</tr>
<tr>
<td>( I_{xg} )</td>
<td>rotational inertia of gear on the excessive part curve</td>
</tr>
<tr>
<td>( d_G )</td>
<td>distance between the ending point on the excessive curve and tooth root</td>
</tr>
<tr>
<td>( h_G )</td>
<td>distance between the end point of excessive curve and tooth midline</td>
</tr>
<tr>
<td>( h_{xg} )</td>
<td>the distance between the tooth midline and the contact point on the excessive curve</td>
</tr>
<tr>
<td>( x_g )</td>
<td>the distance between the tooth root and points on the excessive curve in x-direction</td>
</tr>
<tr>
<td>( \phi_f )</td>
<td>rolling angle at the tooth root</td>
</tr>
<tr>
<td>( \phi_{1} )</td>
<td>the rolling angle corresponding to the excessive curve ending point</td>
</tr>
</tbody>
</table>

Then,

\[
\frac{1}{k_{xg}} = \int_{\phi_f}^{\phi_{1}} \frac{1.2(1 + \nu) \cos^2 \alpha_1}{ELh_{xg}} x'_g \, d\phi + \int_{\phi_{1}}^{\phi_{1}} \frac{-1.2(1 + \nu) \cos^2 \alpha_1}{ELh_{xj}} x'_j \, d\phi
\]  

The shear stiffness on the involute curve can be expressed as:

\[
\frac{F^2}{2k_{sj}} = U_{sj} = \int_{d_{c}}^{d} \frac{1.2F_b^2}{2GA_{sj}} \, dx_j
\]

Then,

\[
\frac{1}{k_{sj}} = \int_{\phi_f}^{\phi_{1}} \frac{-1.2(1 + \nu) \cos^2 \alpha_1}{ELh_{xj}} x'_j \, d\phi
\]

where

\( h_{sj} \) is the distance between the contact point on the involute part and tooth midline, \( x_j \) is the distance between the tooth root and points on the involute part in the \( x \) direction, and \( \phi_1 \) is the rolling angle corresponding to the contact point.

The shear stiffness under the universal equation of a gear profile could be expressed as:

\[
\frac{1}{k_{sx}} = \frac{1}{k_{sg}} + \frac{1}{k_{sj}} = \int_{\phi_f}^{\phi_{1}} \frac{1.2(1 + \nu) \cos^2 \alpha_1}{ELh_{xg}} x'_g \, d\phi + \int_{\phi_{1}}^{\phi_{1}} \frac{-1.2(1 + \nu) \cos^2 \alpha_1}{ELh_{xj}} x'_j \, d\phi
\]

2.1.2. Bending and Compressive Stiffness

The bending stiffness under the universal equation of a gear profile can be expressed as:

\[
\frac{1}{k_{bx}} = \frac{1}{k_{bg}} + \frac{1}{k_{bj}} = \int_{\phi_f}^{\phi_{1}} \frac{3(d_G \cos \alpha_1 + h_G \sin \alpha_1 - x_g \cos \alpha_1)^2}{2ELh_{xg}} x'_g \, d\phi + \int_{\phi_{1}}^{\phi_{1}} \frac{-3(d \cos \alpha_1 + h \sin \alpha_1 - x_j \cos \alpha_1)^2}{2ELh_{xj}} x'_j \, d\phi
\]

The compressive stiffness under the universal equation of a gear profile can be expressed as:

\[
\frac{1}{k_{ax}} = \frac{1}{k_{ag}} + \frac{1}{k_{aj}} = \int_{\phi_f}^{\phi_{1}} \frac{\sin^2 \alpha_1}{2ELh_{xg}} x'_g \, d\phi + \int_{\phi_{1}}^{\phi_{1}} \frac{-\sin^2 \alpha_1}{2ELh_{xj}} x'_j \, d\phi
\]

2.1.3. Meshing Stiffness of a Gear Pair

The meshing stiffness of a gear pair can be expressed by considering the meshing condition:

\[
\frac{1}{k_{tx}} = \frac{1}{k_{tr}} + \frac{1}{k_{tx1}} + \frac{1}{k_{ax1}} + \frac{1}{k_{f1}} + \frac{1}{k_{tx2}} + \frac{1}{k_{ax2}} + \frac{1}{k_{f2}}
\]
where

\[ I \text{ gear of the gear pair, } 2 \text{ is the pinion of the gear pair, } k_h \text{ is the Hertz stiffness [11], and } k_{f1}/k_{f2} \text{ is the fillet-foundation stiffness of the gear/pinion [13].} \]

To verify the accuracy of the universal equation of gear profile, the calculated meshing stiffness was compared with the FEM method, as shown in Figures 3 and 4. Both figures show the change in time-varying mesh stiffness (TVMS) with the change in the rotation angle. Figure 3 is the meshing stiffness calculated by FEM, and Figure 4 is the meshing stiffness by the universal equation of the gear profile. The max error of the universal equation of the gear profile was 2.15% when compared to the FEM, and the veracity of the universal equation of the gear profile has been proven.

![Figure 3](image1)

**Figure 3.** Meshing stiffness by the finite element method (FEM).

![Figure 4](image2)

**Figure 4.** Meshing stiffness by the universal equation of a gear profile.

2.2. The Construction of the Dynamic Model

The vibration response of the gear system could be obtained in the dynamic model, as shown in Figure 5, and the main parameters of the gear transmission system are listed in Tables 3 and 4.
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The vibration response of the gear system could be obtained in the dynamic model, as shown in Figure 5, and the main parameters of the gear transmission system are listed in Tables 3 and 4.

Differential equations are established in simulation modeling. The xy-direction motion equations are shown as follows:

\[
\begin{bmatrix}
M_1 & M_2 \\
M_1 & M_2
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2
\end{bmatrix} +
\begin{bmatrix}
-K_{x1}\dot{x}_1 - C_{x1}\dot{x}_1 + f_1 \\
-K_{x2}\dot{x}_2 - C_{x2}\dot{x}_2 + f_2 \\
-K_{y1}\dot{y}_1 - C_{y1}\dot{y}_1 + N \\
-K_{y2}\dot{y}_2 - C_{y2}\dot{y}_2 + N
\end{bmatrix} =
\begin{bmatrix}
F_d \\
0
\end{bmatrix}
\tag{14}
\]

The rotary motion equations are shown as follows:

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix} =
\begin{bmatrix}
R_{y1}N + T_P + M_1 \\
M_2 - K_{y2}N - T_g
\end{bmatrix}
\tag{16}
\]

Figure 5. Dynamic model for a gearbox.

Table 3. Main parameters of the gear pair.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus E</td>
<td>2.068 × 10^3 pa</td>
</tr>
<tr>
<td>Tooth number N_1, N_2</td>
<td>25, 30</td>
</tr>
<tr>
<td>Module M</td>
<td>2 mm</td>
</tr>
<tr>
<td>Output torque T_y</td>
<td>60 Nm</td>
</tr>
<tr>
<td>Mass of pinion M_1</td>
<td>0.3083 kg</td>
</tr>
<tr>
<td>Contact ratio C_y</td>
<td>1.63</td>
</tr>
<tr>
<td>Rotary inertia of driven gear I_2</td>
<td>2 × 10^-4 kg m^2</td>
</tr>
</tbody>
</table>

Table 4. Main parameters of the transmission system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meshing frequency f_m</td>
<td>1000 Hz</td>
</tr>
<tr>
<td>Torsional stiffness of bearings K_x1</td>
<td>K_y1 = K_x2 = 6.56 × 10^8 Nm/rad</td>
</tr>
<tr>
<td>Damping coefficient of bearings C_x1</td>
<td>C_y1 = C_x2 = 1.8 × 10^3 Nms/rad</td>
</tr>
<tr>
<td>Damping coefficient of meshing teeth C_t</td>
<td>67 ns/m</td>
</tr>
<tr>
<td>Coefficient of friction u</td>
<td>0.06</td>
</tr>
</tbody>
</table>
where
\[ N = K_t(R_{b1} \dot{\theta}_1 - R_{b2} \dot{\theta}_2 - y_1 + y_2) + C_t(R_{b1} \dot{\theta}_1 - R_{b2} \dot{\theta}_2 - y_1 + y_2), \]  
(18)

The \( f_1 \) is the friction force of the pinion. It can be expressed as:
\[ f_1 = uN_1 \]  
(19)

The \( f_2 \) is the friction force of the gear. It can be expressed as:
\[ f_2 = -uN_2 \]  
(20)

where
\( N_1 \) is the meshing force of the first tooth pair, and \( N_2 \) is the meshing force of the second tooth pair.

3. Meshing Stiffness and Vibration Response for Cracked Gears in Different Gear Shape Parameters

3.1. Meshing Stiffness and Vibration Response for Cracked Gears in Different Pressure Angles

For calculating the meshing stiffness, most work has been based on fixed gear shape parameters. However, gear shape parameters for an actual manufacture process need to change. Different pressure angles (14.5°, 20°, and 25°) were taken into consideration for the meshing stiffness and vibration response investigation in this part.

The installation of the meshing gear needs high precision. However, in the actual machining process, the axis of the meshing gear is usually unparalleled due to the manufacturing error and the limited skill of installation. Then, a non-uniform meshing force will be operating on the gear profile, causing crack propagation along the tooth width with non-uniform depths. In this paper, the non-uniform depth of the crack was also introduced for the investigation of meshing stiffness and vibration response. The expression of the crack depth is shown below.

\[ q_w = q \sqrt{\frac{w - w_c}{w}}, \quad w_c \in [0 - w] \]  
(21)

where
\( q \) is the initial crack depth and \( w \) is the width of the gear.

Since changing the different pressure angles could not affect the length of the tooth, the depths of the crack in different pressure angles were set the same (0.3 mm, 0.6 mm, 0.9 mm, 1.2 mm, 1.5 mm and 1.8 mm). The crack direction was set as 63.3° with the central line of the tooth, as shown in Figure 6 [33].

The meshing stiffness in different pressure angles of different crack depths is shown in Figures 7–9. The meshing stiffness tended to increase with the increase of pressure angle, but tended to decrease with the angle rotating in each signal and double tooth interval.

![Figure 6. Gear tooth microcrack model.](image-url)
The meshing stiffness tended to increase with the increase of pressure angle, but tended to decrease with the angle rotating in each signal and double tooth interval.

Figure 6. Gear tooth microcrack model.

Figure 7. Meshing stiffness of the 14.5° pressure angle.

Figure 8. Meshing stiffness of the 20° pressure angle.

Figure 9. Meshing stiffness of the 25° pressure angle.

Then, the vibration signal could be obtained by the dynamic model presented in Section 2.2, as shown in Figures 10–12. To more clearly express the effect of a crack on the vibration response, the method in which the healthy vibration signals are to be removed from the original signal was used in this paper for the vibration tendency investigation. The residual signals in the different crack depths with different pressure angles are shown below. From the residual, we can see that the residual signal under the pressure angle 20° had the lowest amplitude.

Figure 10. Vibration response for the cracked gear under the 14.5° pressure angle.
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**Figure 9.** Meshing stiffness of the 25° pressure angle.

**Figure 10.** Vibration response for the cracked gear under the 14.5° pressure angle.

**Figure 11.** Vibration response for the cracked gear under the 20° pressure angle.

**Figure 12.** Vibration response for the cracked gear under the 25° pressure angle.

To see the tendency clearly, the dealing result of Root Mean Square (RMS), the kurtosis and the crest factor (CF) are shown in Figures 13–15. The calculated method of the fault detection indicators is shown from Equations (22)–(24). The kurtosis had an unstable tendency in a small crack, and the CF not only had an unstable tendency in a small crack but also had an uncertain tendency in a large crack. Moreover, the value of CF turned out to be the largest before the 1.2-mm crack under the pressure angle 20° when compared with the value of CF under the other two pressure angles. It does not meet with the conclusion that the residual signal under the pressure angle 20° has the lowest amplitude. However, the RMS had a monotonically increasing tendency with an increase in crack depth. Then, we chose RMS as the best fault detection indicator of the three for the investigation of vibration response. From the RMS result, we see that with an increasing tendency of the pressure, the vibration response of the gearbox changed erratically. The peak of vibration response whose pressure angle was 20° turned out to be the smallest peak of vibration response whose modulus was 14.5°, and 25° turned out to be the largest. The fault detection indicators included mainly the RMS, the kurtosis, and the crest factor. Explanations of the fault detection indicators are shown below.
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1. The RMS is the mean square value. The calculated method is as follows:

$$RMS = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (x(n) - \bar{x})^2}$$

(22)

where

$N$ is the length of the original data, $x(n)$ is the original data, and $\bar{x}$ is the arithmetic square root of the original data.

2. The kurtosis describes the distribution character of the vibration signal. The calculated method is as follows:

$$Kurtosis = \frac{N \sum_{n=1}^{N} [(x(n) - \bar{x})^2]^2}{\left[ \sum_{n=1}^{N} (x(n) - \bar{x})^2 \right]^2}$$

(23)
3. The crest factor is the ratio between the max value of the vibration signal and RMS, and it is expressed as follow:

\[ CF = \frac{\text{MAX}|x(n)|}{\text{RMS}} \]  

(24)

3.2. Meshing Stiffness and Vibration Response for Cracked Gears in Different Modulus

In this part, the moduli were set as 1.5 mm, 2.5 mm, and 4 mm; the crack direction was also set as 63.3° with the central line of the tooth. The different moduli caused a big difference in gear shape. The cracks in this paper were set as below. When the modulus was 1.5 mm, the crack was set as q0 = 0, q1 = 0.25, q2 = 0.5, q3 = 0.75, q4 = 1, q5 = 1.25, and q6 = 1.5 mm. When the modulus was 2.5 mm, the crack was set as q0 = 0, q1 = 0.4, q2 = 0.8, q3 = 1.2, q4 = 1.6, q5 = 2.0, and q6 = 2.4 mm. When the modulus was 4 mm, the crack was set as q0 = 0, q1 = 0.65, q2 = 1.3, q3 = 1.95, q4 = 2.6, q5 = 3.25, and q6 = 3.9 mm. The crack depth ended up being 50% of the root length of the arc. The different meshing stiffnesses under different moduli are shown in Figures 16–18. The meshing stiffnesses of different moduli were nearly the same.

![Figure 16. Meshing stiffness for the cracked gear under the 1.5 mm modulus.](image1)

![Figure 17. Meshing stiffness for the cracked gear under the 2.5 mm modulus.](image2)
Figure 18. Meshing stiffness for the cracked gear under the 4 mm modulus.

The dynamic response generated by the change of meshing stiffness in different moduli could be obtained in the dynamic system, and the residual signals are shown in Figures 19–21. The result of RMS is shown in Figure 22. From the result, we can see that with the increasing tendency of the modulus, the vibration response of the gearbox changed erratically. The peak of the vibration response whose modulus was 1.5 mm turned out to be the smallest.

Figure 19. Vibration response for the cracked gear under the 1.5 mm modulus.
Figure 20. Vibration response for the cracked gear under the 2.5 mm modulus.

Figure 21. Vibration response for the cracked gear under the 4 mm modulus.

Figure 22. RMS for the vibration response under the changing modulus.
3.3. Meshing Stiffness and Vibration Response for Cracked Gears in Different Tooth Numbers

The tooth numbers in this part were set as 20/20, 40/40, and 60/60 (the number of teeth of a pair of meshing gears is, respectively, 20 and 20, 40 and 40, and 60 and 60), and the direction of the crack was also set as 63.3° with the central line of the tooth. Since the modulus was fixed and the tooth length was the same in different tooth numbers, the crack depths were set the same as 0.34 mm, 0.68 mm, 1.02 mm, 1.36 mm, 1.70 mm, and 2.04 mm. The meshing stiffnesses of the different tooth numbers are shown in Figures 23–25. The mesh stiffness increases with the number of teeth. The difference of mesh stiffness between the number of teeth 40/40 and the number of teeth 20/20 is much larger than that between the number of teeth 60/60 and the number of teeth 40/40.

![Figure 23](image_url)

**Figure 23.** Meshing stiffness for the cracked gear under tooth number 20/20.

![Figure 24](image_url)

**Figure 24.** Meshing stiffness for the cracked gear under tooth number 40/40.
Figure 25. Meshing stiffness of cracked gears with 60/60 teeth.

The residual signal of different tooth numbers is shown from Figures 26–28, and the dealing result of RMS is shown in Figure 29. The residual signal amplitude of different number of teeth decreases with the increase of number of teeth. The peak of vibration response whose tooth number was 60/60 turned out to be the smallest.

Figure 26. Vibration response of cracked gear at gear number 20/20.
4. Discussion

In this paper, the effects of crack size, modulus, and gear tooth number on the time-varying meshing stiffness and vibration of gears are discussed. By changing one of the variables, the variation
law of meshing stiffness and vibration was studied. It was found that when the pressure angle of the gear was 20°, the influence of the crack on the vibration response was the smallest. The influence of modulus on the vibration response was relatively weak. The larger the number of teeth, the smaller the influence of the crack on the vibration response. In order to prolong the service life of gears, this paper provides a theoretical basis for the selection of relevant parameters in the design of gear structures.

However, the influence of the number of teeth on the meshing stiffness and vibration response was analyzed qualitatively. A quantitative study of the number of teeth on the meshing stiffness and vibration response will be further analyzed in the future. In addition, the experiment will be conducted to verify the accuracy of the analysis results.

5. Conclusions

The meshing stiffness and vibration response of a gear with different shape parameters and different cracks were investigated in this paper. All the cracks were set at 50% of the root length of the arc. The conclusions are as follows.

1. The meshing stiffness decreases with the increase of cracks, while the vibration response increases with the increase of cracks.

2. At the same crack level, the meshing stiffness tends to increase with an increase in the pressure angle, but tends to decrease with the angle rotating in each signal and double tooth interval; the vibration response under the pressure angle 20° turns out to have the lowest amplitude.

3. At the same crack level, the meshing stiffness is nearly the same under different modulus, and the vibration response under the modulus of 1.5 mm is the lowest.

4. On the same crack level, the meshing stiffness increases with an increase in tooth number; the amplitudes of the vibration response in different tooth numbers decrease with an increasing tooth number.

5. For the vibration investigation, the RMS turns out to be a better fault detection indicator than the kurtosis and crest factor for its monotonically increasing tendency with an increase of fault severity.

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References


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