Synchronization Full-Order Terminal Sliding Mode Control for an Uncertain 3-DOF Planar Parallel Robotic Manipulator

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Featured Application: The proposed control method could be applied to not only the joint position tracking control for parallel robotic manipulators but also other mechanical systems such as the position synchronization control of multiple motion axes, mobile robotics, and serial robotic manipulators.

Abstract: The control of a parallel robotic manipulator with uncertain dynamics is a noteworthy challenge due to the complicated dynamic model; multi-closed-loop chains; and singularities. This study develops a Synchronization Full-Order Terminal Sliding Mode Control (S-FOTSMC) for a 3-DOF planar parallel robotic manipulator with uncertain dynamics. First, to achieve faster convergence of position error and synchronization error variables with minimum values at the same time, a Synchronization Full-Order Terminal Sliding Mode Surface (S-FOTSMS) is constructed in the cross-coupling error’s state space. Next; an integral of the switching control term is applied; that means, a continuous control term is extended for rejecting the effects of chattering. Finally, an SFOTSMC is designed to guarantee that sliding mode motion will occur. Consequently, the stability and the robustness of the proposed method are secured with high-performance irrespective of the influences of uncertain terms in the robot system. The simulation performances show the effectiveness of our proposed system for position tracking control of a 3-DOF planar parallel robotic manipulator.

Keywords: synchronization control; cross-coupling error; Full-Order Terminal Sliding Mode; parallel robot manipulator

1. Introduction

Robot manipulators have contributed endless roles in both the industrial automation systems and applied research fields. Recently, studies on parallel robots have attracted a lot of interest in the research community. This is because parallel manipulators have numerous highlighted benefits such as high speed, high stiffness, high-accuracy positioning, high payload capacity and low moving inertia. Parallel manipulators have widely applied in real applications, including precise manufacturing, medical science, humanoid robots, space exploration equipment, and automobile simulators [1]. However, a drawback of parallel robotic manipulators is their limited workspace, especially compared with serial manipulators. Moreover, parallel robotic manipulators are saddled with a complicated dynamic model, singularities, and the forward kinematic problem. Therefore, many field-specific trials are needed to thoroughly handle these disadvantages of parallel robotic manipulators. The path tracking
control is then considered for a representative analysis to confirm the effectiveness of the proposed control systems.

As reported in the literature, several control methods have been successfully applied to control robot manipulators. Outstanding examples, such as the proportional-integral-derivative (PID) controller [2], Nonlinear PD synchronized control [3], the adaptive switching learning PD control method (ASL-PD) [4], or [5] were reported to control the motion of parallel manipulators. However, these control systems cannot always obtain a prescribed performance due to the presence of external disturbances and uncertainties. Next, numerous advanced control algorithms were introduced, such as the computed torque controllers (CTC) [6–8], adaptive controllers [9–12], the model-based iterative learning controller [13], and sliding mode control (SMC) [14–25]. The common property of those controllers is that only local feedback information of each joint will be provided to the control loop of each private actuator without any feedback information from other joints. Therefore, errors generated by external disturbances and dynamic uncertainties in the control loop of each actuator are fixed by this control loop, while other errors do not respond. Additionally, the end-effector trajectory of a parallel robot is driven by all actuators synchronously. For this reason, all joints of the parallel robot should be synchronously driven to improve performance.

Recently, studies on the synchronization control of parallel robots have been widely performed. The synchronization controller considers kinematic coupling among the active joints of a parallel robot. Consequently, the tracking performance of the end-effector is enhanced significantly. The synchronization control algorithm was first reported in Reference [26]. This control method has extensively enhanced in References [27,28] and tracking control for parallel robots [5,29–32].

Nonsingular Fast Terminal Sliding mode control (NFTSMC) is one of the most powerful approaches to solving nonlinear systems with uncertain dynamic terms. NFTSMC well handles the disadvantages of both conventional SMC and conventional terminal sliding mode control (TSMC), such as singularity, undefined time convergence, chattering, and slow convergence. NFTSMC has been successfully applied to serial manipulators and parallel manipulators [18,33–40]. With NFTSMC, the system states not only achieve fast convergence but also avoids the singularity problem. However, NFTSMC also does not consider kinematic coupling among the active joints of a parallel robot to design the control system. Furthermore, the control input signals of NFTSMC still have chattering by using high-frequency control law. To remove chattering, some effective algorithms were suggested. For example, the Boundary Layer Approach (BLA) [22,41,42], High-Order Sliding Mode Control (HOSMC) [43–46], Full-Order Sliding Mode Control (FOSMC) [47–50], and Fuzzy-Sliding Mode Control (F-SMC) [51–54]. Among these controllers, FOSMC not only effectively removes chattering, but also inherits the advantages of NFTSMC, such as non-singularity, fast convergence, robustness against uncertain terms, and high accuracy.

Consequently, the main goal of our study is to propose a new S-FOTSMC for a 3-DOF planar parallel robotic manipulator with uncertain dynamics which has the following benefits: (1) inherits the advantages of the synchronization control and FOTSMC in terms of robustness, fast convergence of position and synchronization errors, singularity removal, and high tracking accuracy; (2) the accuracy in the trajectory tracking control was further improved; (3) all joints of the parallel robot are synchronously driven; (4) the control input system is smooth with minimum chattering.

The rest of this study is arranged as follows. The problem formulations are depicted in Section 2. Section 3 describes the proposed method synthesis. Following Section 3, the proposed control controller is utilized to allow the trajectory tracking control simulation for a 3-DOF parallel robot in Section 4. After its trajectory tracking performance is compared with those of the SMC, the S-SMC and NFTSMC, Section 5 summarizes some highlighted conclusions.
2. Problem Formulations

2.1. Dynamic Model of Parallel Robot Manipulator

The general dynamic equation for a 3-DOF parallel robotic manipulator is depicted as in References [55,56]:

\[ M_a \ddot{\theta}_a + Q_a \dot{\theta}_a + f_{ra} + \delta_a = \tau_a \]  \hspace{1cm} (1)

where \( \theta_a = [\theta_{a1}, \theta_{a2}, \theta_{a3}]^T \), \( \dot{\theta}_a = [\dot{\theta}_{a1}, \dot{\theta}_{a2}, \dot{\theta}_{a3}]^T \), and \( \ddot{\theta}_a = [\ddot{\theta}_{a1}, \ddot{\theta}_{a2}, \ddot{\theta}_{a3}]^T \) are the position vector, velocity vector, and acceleration vector at each active joint, respectively. \( M_a = M_a + \Delta M_a \in \mathbb{R}^{3 \times 3} \) represents the real inertia matrix and \( Q_a = \hat{Q}_a + \Delta Q_a \in \mathbb{R}^{3 \times 3} \) represents the real Coriolis and centrifugal force matrix. \( \hat{M}_a \in \mathbb{R}^{3 \times 3} \) represents the estimated inertia matrix and \( \hat{Q}_a \in \mathbb{R}^{3 \times 3} \) represents the estimated Coriolis and centrifugal force matrix. \( f_{ra} \in \mathbb{R}^{3 \times 1} \) and \( \delta_a \in \mathbb{R}^{3 \times 1} \) are the friction vector and disturbance vector at the active joints, respectively. \( \Delta M_a \in \mathbb{R}^{3 \times 3} \) and \( \Delta Q_a \in \mathbb{R}^{3 \times 3} \) are the errors of the real dynamic model. The detail elements of \( M_a, Q_a, f_{ra} \) and \( \delta_a \) can be found in Reference [55].

Consequently, the real dynamic equation of the 3-DOF parallel robot manipulator is achieved as

\[ \hat{M}_a \ddot{\theta}_a + \hat{Q}_a \dot{\theta}_a + \Delta U_a = \tau_a \]  \hspace{1cm} (2)

The vector of the lumped uncertain terms \( \Delta U_a \) from Equation (2) is stated as follows:

\[ \Delta U_a = \Delta M_a \ddot{\theta}_a + \Delta Q_a \dot{\theta}_a + f_{ra} + \delta_a \]  \hspace{1cm} (3)

Accordingly, the robot dynamic Equation (1) is rewritten with the following expression:

\[
\begin{align*}
\ddot{\theta}_a &= \hat{M}_a^{-1}[\tau_a - \hat{Q}_a \dot{\theta}_a] - \hat{M}_a^{-1}[\Delta U_a] \\
&= \hat{M}_a^{-1}[\tau_a - \hat{Q}_a \dot{\theta}_a] - \Xi(\theta_a, \Delta U_a)
\end{align*}
\]  \hspace{1cm} (4)

where \( \Xi(\theta_a, \Delta U_a) = \hat{M}_a^{-1} \Delta U_a \).

The control objective of this research is to further increase the precision in the path tracking control for an uncertain 3-DOF parallel robotic manipulator (Equations (1) or (4)) such that the position tracking errors and synchronization errors can approach zero at the same time, regardless of the influences of uncertain dynamics in the robotic system.

In order to obtain this control objective, the cross-coupling errors will be used to design the sliding surfaces and develop the proposed control system.

**Assumption 1.** The lumped uncertain terms are a limited function, which needs to satisfy the constrained condition:

\[ \left| \Xi(\theta_a, \Delta U_a) \right| \leq G \]  \hspace{1cm} (5)

where \( G \) is a known positive constant, that is, the first-order differentiable.

2.2. Definition of Synchronization Error Cross-Coupling Error

In synchronization control, there are two error types, and they are called the synchronization error and cross-coupling error. These error types are determined as in Reference [57].

The error of the position tracking on each active joint is calculated as follows:

\[ e_a = \theta_{da}(t) - \theta_a(t) \]  \hspace{1cm} (6)

where \( \theta_{da}(t) \in \mathbb{R}^{3 \times 1} \) denotes the angle of the desired position on each active joint.
In the synchronization matter of the error tracking, the position error of \( e_{a1}(t) \) not only achieves zero it but also tunes the motion relationship among multiple active joints during the tracking operation:

\[
e_{a1}(t) = e_{a2}(t) = e_{a3}(t)
\]  

where \( e_{a1}(t) \) is the position error at the \( i \)th joint.

The synchronization errors are defined in the case of three active joints as follows:

\[
\begin{align*}
e^e_1(t) &= e_{a1}(t) - e_{a2}(t) \\
ne^e_2(t) &= e_{a2}(t) - e_{a3}(t) \\
ne^e_3(t) &= e_{a3}(t) - e_{a1}(t)
\end{align*}
\]  

where, the vector of synchronization error is \( e^e = \begin{bmatrix} e^e_1(t), e^e_2(t), e^e_3(t) \end{bmatrix}^T \). Therefore, the goal of synchronization error can be achieved if \( e^e_i(t) = 0 \) for all active joints.

The cross-coupling error was defined as follows:

\[
\begin{align*}
\epsilon^e_{a1} &= e_{a1} + \lambda \int_0^t (e^e_1 - e^e_3) \, dv \\
\epsilon^e_{a2} &= e_{a2} + \lambda \int_0^t (e^e_2 - e^e_1) \, dv \\
\epsilon^e_{a3} &= e_{a3} + \lambda \int_0^t (e^e_3 - e^e_2) \, dv
\end{align*}
\]  

where \( \lambda \) is the positive constant, \( v \) is the variable from time zero, and the vector of cross-coupling errors is \( \epsilon^e_a = \begin{bmatrix} \epsilon^e_{a1}, \epsilon^e_{a2}, \epsilon^e_{a3} \end{bmatrix} \).

We define vectors of \( \Delta e^e = \begin{bmatrix} e^e_1 - e^e_3, e^e_2 - e^e_1, e^e_3 - e^e_2 \end{bmatrix}^T \) and \( \Delta \epsilon^e = \begin{bmatrix} \epsilon^e_1 - \epsilon^e_3, \epsilon^e_2 - \epsilon^e_1, \epsilon^e_3 - \epsilon^e_2 \end{bmatrix}^T \).

Then, the first and second derivative of the cross-coupling error can be computed as follows:

\[
\begin{align*}
\dot{\epsilon}^e_a &= \dot{e}_a + \lambda \Delta e^e \\
\ddot{\epsilon}^e_a &= \ddot{e}_a + \lambda \Delta \epsilon^e
\end{align*}
\]  

3. Design Procedure of Control Scheme

In this section, a novel control algorithm is proposed for the parallel robot as follows.

3.1. Design of S-FOTSMS

To synchronously achieve fast convergence of both the position error and synchronization error along with the robustness rejects the effects of the lumped uncertain terms, the S-FOTSMS is designed by using the cross-coupling error (Equation (9)) as follows:

\[
S_{S-FOTSMS} = \dot{e}^e_a + \gamma_1 |e^e_a| \gamma_1 \text{sgn}(e^e_a) + \Gamma_1 |e^e_a| \gamma_1 \text{sgn}(e^e_a)
\]  

where \( S_{S-FOTSMS} = \begin{bmatrix} S_{S-FOTSMS1}, \ldots, S_{S-FOTSMS3} \end{bmatrix}^T \in \mathbb{R}^{3x1} \) are the S-FOTSMS, \( \gamma_1, \gamma_2, \Gamma_1, \Gamma_2 \) are the positive constants, which are chosen as in References [47,58,59]. \( \Gamma_1 \) and \( \Gamma_2 \) can be selected such that the polynomial \( p^2 + \Gamma_2 p + \Gamma_1 \) is Hurwitz, i.e., the eigenvalue of the polynomial are all in the left-half side of the complex plane. \( \gamma_1 \) and \( \gamma_2 \) can be determined based on the following conditions: \( 0 < \gamma_1 < 1, \) and \( \gamma_2 = \frac{2\gamma_1}{1-\gamma_1} \).

From the second derivative of the cross-coupling error in Equations (11) and (12), we obtain

\[
S_{S-FOTSMS} = \dot{e}^e_a + \lambda \Delta \epsilon^e + \Gamma_2 |e^e_a| \gamma_2 \text{sgn}(e^e_a) + \Gamma_1 |e^e_a| \gamma_1 \text{sgn}(e^e_a)
\]
With the robot dynamic Equation (4), Equation (13) gives
\[
S_{S\text{-FOTSMC}} = \ddot{\theta}_{da}(t) - \dot{M}_a^{-1}[\tau_a - \dot{Q}_a \dot{\theta}_a] + \Xi(\theta_a, \Delta U_a) \\
+ \lambda \Delta e^a + \Gamma_2 [e^{\alpha}]^T \text{sgn}(e^{\alpha}) + \Gamma_1 [e^{\alpha}]^T \text{sgn}(e^{\alpha})
\]
(14)

\[
\begin{align*}
\text{Theorem 1.} & \quad \text{Consider a 3-DOF parallel robot manipulator (4). If the proposed torque commands are designed} \\
& \quad \text{for the robot (1) as Equations (15)–(17), then the position error synchronization error speedily converge to zero at} \\
& \quad \text{the same time. That means the system (4) is guaranteed to have stability.}
\end{align*}
\]

3.2. Design of S-FOTSMC

The following proposed controller is designed to attain the desired tracking performance:
\[
\tau_a = \dot{M}_a (\tau_{eq} + \tau_{sw})
\]
(15)

The equivalent control term is designed as follows:
\[
\tau_{eq} = \ddot{\theta}_{da}(t) + \dot{M}_a^{-1} \dot{Q}_a \dot{\theta}_a + \lambda \Delta e^a + \Gamma_2 [e^{\alpha}]^T \text{sgn}(e^{\alpha}) + \Gamma_1 [e^{\alpha}]^T \text{sgn}(e^{\alpha})
\]
(16)

Additionally, the switching control term is designed as follows:
\[
\tau_{sw} = (G + g) \text{sgn}(S_{S\text{-FOTSMC}})
\]
(17)

where \( g \) is a positive constant.

Consequently, the following theorem is created to be stable proof.

\textbf{Theorem 1.} Consider a 3-DOF parallel robot manipulator (4). If the proposed torque commands are designed for the robot (1) as Equations (15)–(17), then the position error synchronization error speedily converge to zero at the same time. That means the system (4) is guaranteed to have stability.

\textbf{Proof.} Adopting control inputs (15)–(17) to Equation (14) gives
\[
S_{S\text{-FOTSMC}} = -\tau_{sw} + \Xi(\theta_a, \Delta U_a)
\]
(18)

Taking time derivative of Equation (18) yields:
\[
\dot{S}_{S\text{-FOTSMC}} = -\dot{\tau}_{sw} + \dot{\Xi}(\theta_a, \Delta U_a)
\]
(19)

The following Lyapunov function is defined with the following expression:
\[
V = \frac{S_{S\text{-FOTSMC}}^T S_{S\text{-FOTSMC}}}{2}
\]
(20)

With the result of Equation (19), the time derivative of Equation (20) gives
\[
\dot{V} = S_{S\text{-FOTSMC}}^T (-\dot{\tau}_{sw} + \dot{\Xi}(\theta_a, \Delta U_a))
\]
\[
= S_{S\text{-FOTSMC}}^T \left( -(G + g) \text{sgn}(S_{S\text{-FOTSMC}}) + \dot{\Xi}(\theta_a, \Delta U_a) \right)
\]
\[
= \left( -G S_{S\text{-FOTSMC}} \right) + \dot{\Xi}(\theta_a, \Delta U_a) S_{S\text{-FOTSMC}}
\]
\[
\leq -g|S_{S\text{-FOTSMC}}| + |\dot{\Xi}(\theta_a, \Delta U_a)| S_{S\text{-FOTSMC}}
\]
(21)

From Equation (21), \( g \) is assigned to be greater than zero. Therefore, \( \dot{V} \) will be negative. Consequently, according to the Lyapunov theory [60], it is verified that both position error and synchronization error synchronously approach zero under control system (15)–(17) irrespective of the influences of the uncertain terms in the robot system. \( \square \)
Remark 1. In the design of the control laws (15)–(17), the design procedure is based on the assumption that the bounded value of $G$ in Assumption 1 can be obtained in advance. However, this parameter is difficult to be obtained in advance in practical engineering applications. In the next work, we use a neural network or an adaptive control law to estimate the bounded value of $G$ in Assumption 1. Therefore, our controller can provide high performance without the requirement of the bounded value of $G$ in Assumption 1.

Remark 2. The proposed control method could be applied to not only the joint position tracking control for parallel Robot. manipulators but also other mechanical systems such as the position synchronization control of multiple motion axes, mobile Robot.s, serial Robot. manipulators.

4. Numerical Simulation Studies

To confirm the effectiveness of the suggested algorithm, the simulated examples are performed for a 3-DOF planar parallel robot on SOLIDWORKS and the SimMechanics of MATLAB. First, The SOLIDWORKS software was used to construct the parallel manipulator model which has the 3-D computer-aided design (CAD) type. Where each mechanical element of the robot is separately built then linked by the joints. Second, the SimMechanics link plug-in was used to export an XML file of the robot mode. Then, this XML file is imported to Simulink. Therefore, the CAD assembly is of geometry files and linked the main body in SimMechanics. Third, sensors, joint actuators were set up to the mechanical system, and external disturbance and fictions were applied to test robustness. Finally, the torque commands from the control method block were applied to this robot system. The kinematic illustration of the robotic system and its 3D CAD model are shown in Figures 1 and 2, respectively. The robot parameters are stated in Table 1 and the selection parameters of the control algorithms are stated in Table 2. We used MATLAB/SIMULINK software for all numerical simulation studies, solver ode3 (Bogacki-Shampine), and the sampling time was set to $10^{-3}$ s.

To analyze the effectiveness of the suggested approach, the approach was applied to a prescribed trajectory tracking control for a 3-DOF planar parallel robotic manipulator, and its tracking results were compared with those of SMC, synchronization SMC (S-SMC), and NFTSMC. These control methods for comparison have been briefly stated in Appendices A–C.

Figure 1. The kinematic illustration of the robotic system.
The simulations were carried out with respect to 2 cases when the parallel manipulator tracks a circular trajectory and a linear trajectory.

**Case 1.** The desired trajectory of the end-effector for the position tracking when the parallel manipulator tracks a circular trajectory:

\[
\begin{align*}
  x &= 0.49 + 0.03 \cos(\pi t/3) \\
  y &= 0.37 + 0.03 \sin(\pi t/3) \\
  \phi &= \pi/2
\end{align*}
\]  

(22)

We assume that the effect of the friction force on the passive joints is much smaller than that on the active joints. Thus, in order to simplify the dynamic model, only the friction forces on the active joints are considered. The friction models of the system, including the viscous friction and the Coulomb friction torques, are only assumed to test the robustness of the control system. Because it is difficult to exactly calculate these friction terms, in this paper, we do not measure the physical value of friction. Therefore, the following friction forces at each active joint were modeled:

**Figure 2.** The 3D CAD model of the parallel manipulator.
\[
\begin{align*}
    f_{r1} &= (5/2)\dot{\theta}_{r1} + (1/2)\text{sgn}(\dot{\theta}_{r1}) \\
    f_{r2} &= (5/2)\dot{\theta}_{r1} + (1/2)\text{sgn}(\dot{\theta}_{r2}) \\
    f_{r3} &= (5/2)\dot{\theta}_{r1} + (1/2)\text{sgn}(\dot{\theta}_{r3})
\end{align*}
\]  

Figures 3–6 show the trajectory position tracking, the tracking errors of the end-effector in the X-direction and in the Y-direction, and comparison of tracking errors in active joint space, respectively. Synchronization errors and cross-coupling errors in active joint space are shown in Figure 7. The end-effector of the manipulator has the initial position as (0.5284, 0.3681) and this end-effector is driven to follow a circular path. It should be noted that the initial positions of the robotic system were selected according to the workspace of the robotic system. That means these initial positions must be satisfied inside the workspace. From Figures 3–6 it is seen that the tracking path generated by SMC has the biggest discrepancy compared to the prescribed path; it has the worse tracking result among the four methods. S-SMC and NFTSMC provide better results than SMC but NFTSMC provides better performance and faster error convergence than both S-SMC and SMC. It is noteworthy that the proposed system is developed by applying the combination of synchronization control and FOTSNC. Accordingly, the tracking path generated by the proposed system has the smallest discrepancy compared to the prescribed path, and it has the fastest convergence rate to the prescribed path among the four control methods. From Figures 4–6, it is observed that the tracking errors generated by S-SMC are smaller than the tracking errors generated by SMC. However, S-SMC provides a worse tracking error than NFTSMC. Especially since the proposed system provides the smallest tracking errors compared to SMC, NFTSMC and S-SMC. The tracking accuracy under the proposed method has been significantly improved in comparison with NFTSMC and S-SMC.

The control input signals for all control types, including SMC, S-SMC, NFTSMC, and the suggested method are illustrated in Figure 8. From Figure 8, it is clear that SMC, S-SMC, and NFTSMC provide a continuous control signal because these three methods use a BLA to eliminate chattering. However, it is seen that the chattering has not been completely removed in the torque input of the NFTSMC. Additionally, the use of a BLA leads to the reduce robustness and accuracy of the control method. While the suggested method uses an integral of the switching control term to reject chattering without an accuracy reduction.

![Position Tracking](image_url)

**Figure 3.** The desired trajectory and real trajectory of the end-effector when the parallel manipulator tracks a circular trajectory.
Figure 4. The tracking errors of the end-effector in the X-direction.

Figure 5. The tracking errors of the end-effector in the Y-direction.
Figure 6. The tracking errors in the active joint space (a) at Joint 1, (b) at Joint 2, and (c) at Joint 3.

Figure 7. The Synchronization Error and Cross-Coupling Error in the active joint space (a) at Joint 1, (b) at Joint 2, and (c) at Joint 3.
NFTSMC provide a continuous control signal because these three methods use a BLA to eliminate chattering. However, it is seen that the chattering has not been completely removed in the torque input of the NFTSMC. Additionally, the use of a BLA leads to the reduction in robustness and accuracy of the control method. While the suggested method uses an integral of the switching control term to reject chattering without an accuracy reduction.

Figure 8. The control input signals: (a) SMC, (b) S-SMC, (c) NFTSMC, and (d) S-FOTSMC.

Case 2. When the parallel manipulator tracks a linear trajectory.

Figure 9 shows the results of tracking a linear trajectory. From Figure 9, it is seen that the tracking path generated by the proposed system has the smallest discrepancy compared to the prescribed path, and it has the fastest convergence rate to the prescribed path among the four control methods.

Figure 9. The desired trajectory and real trajectory of the end-effector when the parallel manipulator tracks a linear trajectory.

Therefore, it is concluded that the suggested method is highly effective for a 3-DOF parallel manipulator in tracking control. The effects of uncertainties and external disturbances have been completely compensated by using the suggested method.
Remark 3. Through simulation studies comparison among those of SMC, S-SMC, NFTSMC, S-FOTSMC, the experimental results performance comparison could be expected to show the effectiveness viability of our proposed scheme. In the next work, the authors will apply the proposed control method to the practical Robot system compare them with other state-of-the-art control algorithms to demonstrate the effectiveness of this control method.

5. Conclusions

This work proposed the S-FOTSMC for a 3-DOF planar parallel robot manipulator with an uncertainty dynamic. First, to achieve faster convergence of position and synchronization error variables with minimum values at the same time, the S-FOTSMS is formed in the cross-coupling error’s state space. Next, an integral of the switching control term is applied, that means a continuous control term is extended for rejecting the effects of chattering. Finally, an S-FOTSMC is designed to guarantee that the position tracking errors and synchronization error synchronously attain a zero value. Therefore, the stability and the robustness of the proposed method are secured with high-performance irrespective of the influences of uncertain terms in the robot system. From the simulated performance, it is concluded that the suggested controller is highly effective for a 3-DOF planar parallel robotic manipulator in trajectory tracking control. The effects of uncertainties and external disturbances have been fully compensated by the suggested system.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Design of the SMC

The SMC was suggested in References [61,62], which was stated as follows.

We select the sliding manifold and take its time derivative as follows:

\[ \dot{S}_{SMC} = \dot{e}_a + z e_a \]
\[ = \dot{\theta}_a(t) - \dot{\theta}_{Ra}(t) \] and
\[ \dot{S}_{SMC} = \ddot{e}_a + z \ddot{e}_a \]
\[ = \ddot{\theta}_a(t) - \ddot{\theta}_{Ra}(t) \] (A1)

where \( z \) is a positive constant, \( e_a = \theta_a(t) - \theta_{da}(t) \) are the path tracking errors, \( \theta_{da}(t) \) are the prescribed trajectories, \( \dot{\theta}_{Ra} = \dot{\theta}_{da} - z \dot{e}_a \) is the reference velocity vector, and \( \ddot{\theta}_{Ra} = \ddot{\theta}_{da} - z \ddot{e}_a \) is the reference acceleration vector.

Substituting the robotic Equation (4) into Equation (A1) yields:

\[ \dot{S}_{SMC} = \hat{M}_a^{-1} \left[ \tau_a - \dot{Q}_a \dot{\theta}_a \right] - \Delta(\theta_a, \tau_a) - \ddot{\theta}_{Ra}(t) \] (A2)

Then, to attain the desired tracking performance for parallel robot manipulator (1), the torque command was proposed as in References [61,62]:

\[ \tau_a = \hat{M}_a \ddot{\theta}_{Ra}(t) + \dot{Q}_a \dot{\theta}_{Ra} - (G + g) \text{sgn}(S_{SMC}) \] (A3)

The BLA was used to eliminate the chattering in the torque signal. Accordingly, the torque command of Equation (A3) becomes

\[ \tau_a = \hat{M}_a \ddot{\theta}_{Ra}(t) + \dot{Q}_a \dot{\theta}_{Ra} - (G + g) \text{sat}(S_{SMC}/u) \] (A4)
where \( G \) and \( g \) are positive constants, \( \text{sat}(S_{\text{SMC}}/u) \) is a saturation vector [62], and \( u \) is the boundary layer thicknesses.

**Appendix B**

**Design of the S-SMC**

Define the synchronization sliding manifold and its time derivative as follows:

\[
S_{\text{S-SMC}} = e_a^\alpha + ze^\alpha_a \quad \text{and} \quad \dot{S}_{\text{S-SMC}} = \dot{e}_a^\alpha + \dot{z}e^\alpha_a
\]

(A5)

where \( z \) is a positive constant, \( e^\alpha_a \) is the cross-coupling error in Equation (9), \( \theta_{Ra} \) is the prescribed trajectory, \( \dot{\theta}_{Ra} = \dot{\theta}_{Ra} \) is the reference velocity vector, and \( \dot{\theta}_{Ra} = \dot{\theta}_{Ra} \) is the reference acceleration vector. \( \Delta e^\alpha \) and \( \Delta e^\alpha \) were defined in Equation (9).

Adding the robotic Equation (4) into Equation (A5) gives

\[
\dot{S}_{\text{S-SMC}} = \ddot{M}_a \dot{\theta}_{Ra}(t) + \dot{Q}_a \dot{\theta}_{Ra} - K_S S_{\text{S-SMC}} - (G + g) \text{sgn}(S_{\text{S-SMC}})
\]

(A6)

Then, to attain the desired tracking performance for the parallel robot manipulator (1), the torque command is designed as in Reference [63]:

\[
\tau_a = \ddot{M}_a \dot{\theta}_{Ra}(t) + \dot{Q}_a \dot{\theta}_{Ra} - K_S S_{\text{S-SMC}} - (G + g) \text{sat}(S_{\text{S-SMC}}/u)
\]

(A7)

The BLA was applied to replace the discontinuous term in the torque command. Hence, the torque command of Equation (A7) becomes

\[
\tau_a = \ddot{M}_a \dot{\theta}_{Ra}(t) + \dot{Q}_a \dot{\theta}_{Ra} - K_S S_{\text{S-SMC}} - (G + g) \text{sat}(S_{\text{S-SMC}}/u)
\]

(A8)

where \( G, g \) and \( K \) are positive constants, \( \text{sat}(S_{\text{S-SMC}}/u) \) is a saturation vector [62], and \( u \) are the boundary layer thicknesses.

**Appendix C**

**Design Nonsingular Fast Terminal Sliding Mode Control**

The nonsingular fast terminal sliding mode surface is designed as in reference [64]

\[
S_{\text{NFTSMC}} = \dot{e}_a + \Gamma_1 e^\alpha_a + \Gamma_2 e^\alpha_a
\]

(A9)

where \( S_{\text{NFTSMC}} = [S_{\text{NFTSMC}_1}, \ldots, S_{\text{NFTSMC}_3}]^T \in \mathbb{R}^{3 \times 1} \) are the sliding variables, \( e_a = \theta_{Ra} - \theta(t) \) are the position tracking errors, \( \Gamma_1, \Gamma_2, \gamma_1, \gamma_2 \) are the positive constants and \( 1 < \gamma_1 < 2, \gamma_2 > \gamma_1 \), which are chosen as in Reference [64]. In addition, \( e^\alpha_a^{[\gamma_1]} \) and \( e^\alpha_a^{[\gamma_2]} \) are defined as in

\[
e^\alpha_a^{[\gamma_i]} = |e_a|^{\gamma_i} |e_a| e^\alpha_a \quad \text{and} \quad e^\alpha_a^{[\gamma_i]} = |e_a|^{\gamma_i} |e_a| e^\alpha_a
\]

(A10)

in which \( \text{sgn}[e_a] = \begin{cases} 1 & \text{if } e_a > 0 \\ -1 & \text{if } e_a < 0 \\ 0 & \text{if } e_a = 0 \end{cases} \).

Taking time derivative of Equation (A9) yields

\[
\dot{S}_{\text{NFTSMC}} = \ddot{e}_a + \Gamma_1 \gamma_1 |e_a|^{\gamma_1 - 1} \dot{e}_a + \Gamma_2 \gamma_2 |e_a|^{\gamma_2 - 1} \dot{e}_a
\]

(A11)
Adding the robotic Equation (4) into Equation (A11) gives
\[
\dot{S}_{\text{NFTSMC}} = \ddot{\theta}_{da}(t) - \ddot{M}_a^{-1}\left[\tau_a - \dot{\dot{Q}}_a \dot{\theta}_a\right] + \Delta(\theta_a, \tau_a) \\
+ \Gamma_1 \gamma_1 |\epsilon_a|^{\gamma_1-1} \dot{\epsilon}_a + \Gamma_2 \gamma_2 |\epsilon_a|^{\gamma_2-1} \dot{\epsilon}_a 
\]
(A12)

The following proposed controller is designed to obtain the desired performance:
\[
\tau_a = M_a(\tau_{eq} + \tau_{sw}) 
\]
(A13)

The equivalent control term is designed as follows:
\[
\tau_{eq} = \ddot{\theta}_{da}(t) + \ddot{M}_a^{-1}\dot{Q}_a \dot{\theta}_a + \Gamma_1 \gamma_1 |\epsilon_a|^{\gamma_1-1} \dot{\epsilon}_a + \Gamma_2 \gamma_2 |\epsilon_a|^{\gamma_2-1} \dot{\epsilon}_a 
\]
(A14)

Additionally, the switching control term is designed as follows:
\[
\tau_{sw} = (G + g) \text{sgn}(S_{\text{NFTSMC}}) 
\]
(A15)

The BLA was used to eliminate the chattering in the torque input. Consequently, the torque command of Equation (A13) becomes
\[
\tau_a = M_a \left( \ddot{\theta}_{da}(t) + \ddot{M}_a^{-1}\dot{Q}_a \dot{\theta}_a + \Gamma_1 \gamma_1 |\epsilon_a|^{\gamma_1-1} \dot{\epsilon}_a + \Gamma_2 \gamma_2 |\epsilon_a|^{\gamma_2-1} \dot{\epsilon}_a + \text{Gsat}(S_{\text{NFTSMC}} / u) \right) 
\]
(A16)

where \(G\) and \(g\) are positive constants, \(\text{sat}(S_{\text{NFTSMC}} / u)\) is a saturation vector [62], and \(u\) are the boundary layer thicknesses.

References
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