Stochastic Electrodynamics: The Closest Classical Approximation to Quantum Theory

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Abstract: Stochastic electrodynamics is the classical electrodynamic theory of interacting point charges which includes random classical radiation with a Lorentz-invariant spectrum whose scale is set by Planck's constant. Here, we give a cursory overview of the basic ideas of stochastic electrodynamics, of the successes of the theory, and of its connections to quantum theory.

1. Introduction

1.1. Two Regimes: Classical and Quantum

Physical theory operates in two different regimes. The familiar macroscopic word is classical, dealing with waves and particles which experience friction and can be described in terms of continuously changing forces, energies, and angular momentum. On the other hand, the microscopic world of atomic and molecular physics is usually described quantum mechanically with intrinsic randomness, wave–particle duality, entanglement, and discrete values for energy and angular momentum. Occasionally, frictionless phenomena (appropriate for the quantum world) intrude into the classical macroscopic world in the form of persistent currents in superconducting wires or superfluids, which flow over gravitational potential barriers. Furthermore, phenomena such as the blackbody radiation spectrum and the decrease of specific heats at low temperatures do not seem to follow classical descriptions given by traditional classical statistical mechanics. The currently-accepted descriptions of these intruding phenomena are traced back to quantum behavior for the constituents at the microscopic level. Accordingly, the question arises as to the location of the "microscopic" level. At what level of interaction for energy or angular momentum are continuous ideas of classical physics no longer applicable? Where is the quantum–classical boundary?

1.2. Boundary Based Upon Planck's Constant

According to the elementary theory taught commonly at present [1], the boundary between classical and quantum physics is given by the appearance of Planck's constant h or $\hbar = h/(2\pi)$. If the action variable J for a system is of the order of $\hbar$, then the system must be described within quantum theory. Thus, the angular momentum of an atom or molecule must come in discrete values involving $\hbar$. If an oxygen molecule has a vibrational energy $\mathcal{E} = \omega J$, then the energy must come in discrete multiples $J = (n + 1/2)\hbar$, $n = 0, 1, 2, ...$ In instructional university physics, the transition between classical and quantum physics is given by the presence of Planck's constant: Planck's constant does not appear within traditional classical physics, and Planck's constant is the crucial constant entering the quantum domain [2].

1.3. Stochastic Electrodynamics Reopens the Boundary Question

Stochastic electrodynamics (SED) is a theory of classical electrodynamics where the homogeneous (source-free) boundary condition on Maxwell's equations is taken to include random classical radiation with a Lorentz-invariant spectrum which is supposed to exist in all spacetime [3–8]. The scale of this Lorentz-invariant random radiation is set by Planck's constant $\hbar$. Thus, stochastic electrodynamics is a classical theory which incorporates Planck's constant. However, if we allow stochastic electrodynamics as a theory of classical physics, then we have Planck's constant included within both classical and quantum theories. Clearly, the traditional classical–quantum distinction favored within main-steam academic circles is no longer valid. Indeed, the classical theory of stochastic electrodynamics can describe accurately some of the phenomena (such as blackbody radiation, the decrease of specific heats at low temperatures, and the absence of atomic collapse) that are currently claimed to have explanations only in terms of quantum theory. Thus, the question of the boundary between classical and quantum theories has to be reconsidered, and not just in some sophisticated "quantum" effect of decoherence. Stochastic electrodynamics (which contains Planck's constant) goes beyond the results of traditional classical theory (which does not include Planck's constant) and comes closer to giving the results of quantum theory. Indeed, stochastic electrodynamics is at present the closest classical approximation to quantum theory. The unanswered question remains: "How many of the phenomena which are currently regarded as requiring quantum explanations can actually be described by the classical theory of stochastic electrodynamics?"

2. Foundations of Stochastic Electrodynamics

2.1. Theoretical Assumptions

Stochastic electrodynamics is a relativistic theory of point charges and electromagnetic fields based on three fundamental ideas. (1) The electromagnetic fields satisfy Maxwell's equations with sources given by point charged particles. (2) The particles experience forces due to electromagnetic fields and move according to Newton's second law with the force given by the Lorentz force law, $d\mathbf{p}/dt = e\mathbf{E} + e\mathbf{v} \times \mathbf{B}/\mathbf{c}$. (3) The solution of the source-free Maxwell equations is given by classical electromagnetic zero-point radiation; namely, the Lorentz-invariant spectrum of random classical radiation with an energy per normal mode $\mathcal{E}(\omega) = \hbar\omega/2$. The basis for these assumptions comes from experimental work in macroscopic electrodynamics (which forms the foundations for much of our electrical technology) together with the inference of random classical zero-point radiation which arises from the experiments on Casimir forces between conducting parallel plates.

2.2. Inference from Measurements on the Casimir Effect

The presence of classical electromagnetic zero-point radiation is inferred from Casimir force measurements [9] at low temperature. The Casimir effect is a force between uncharged, parallel conducting plates of area A and separation d. Within classical theory, the force between the plates arises from the ambient radiation which surrounds the plates. At high temperatures T, the (attractive) force F between the plates goes as $F = -[\zeta(3)/(4\pi)]k_B TA/d^3$, (where k_B is Boltzmann's constant) associated with the thermal radiation surrounding the plates corresponding to the Rayleigh–Jeans spectrum of thermal radiation with an energy $\mathcal{E}(\omega) = k_B T$ per normal mode at high temperature. If continued to zero temperature $T \to 0$, this formula predicts that the force between the plates vanishes. However, experimental measurements of the Casimir force at low temperatures indicate that the force between the plates does not vanish at low temperature but rather goes over to the temperature-independent form $F = -[const \times \pi^2 c/120]A/d^4$. The non-zero temperature-independent force holding at low temperatures indicates the presence of random classical radiation even at the absolute zero of temperature. Furthermore, the d^{-4}-functional form for the force indicates that the spectrum of the random radiation must be given by $\mathcal{E}(\omega) = const \times \omega$. If we choose the value of the constant *const* so as to fit the experimental data, we find that the value corresponds to a familiar

constant, $const = \hbar/2$. Thus, the natural inference in classical physics is that at low temperatures the world contains random classical radiation with an energy spectrum $\mathcal{E}(\omega) = \hbar\omega/2$ per normal mode, corresponding to classical electromagnetic zero-point radiation.

2.3. Change in the Classical Outlook Compared with Current Classical Physics

The foundational ideas of currently-accepted classical physics assume that as the ambient temperature decreases toward absolute zero, all ambient radiation vanishes. This is the same erroneous assumption as was made a century ago by the physicists in the early years of the 20th century. H.A. Lorentz, who developed traditional classical electron theory, makes explicit this assumption of vanishing ambient random radiation at the absolute zero of temperature [10]. Indeed, this assumption seems so natural to most physicists, that it is rarely questioned. Because traditional classical physics lacks the idea of classical zero-point radiation, the description of phenomena at the microscopic level presents an impossible challenge. Historically, the challenge was met only by turning to entirely different "quantum" ideas.

The presence of classical electromagnetic zero-point radiation will have a dramatic effects for microscopic systems when the energy changes associated with the effects of zero-point radiation are large compared with other energies in the situation. On the other hand, for macroscopic phenomena involving large electromagnetic fields, the energy changes associated with the presence of zero-point radiation or thermal radiation can be ignored as insignificantly small. However, for microscopic phenomena at the atomic scale, the zero point energy $\mathcal{E}(\omega) = \hbar\omega/2$ can be comparable to the atomic energies of the problem, and so becomes vitally important to an understanding of the phenomena. Stochastic electrodynamics is a classical theory involving the same continuous ideas on both the macroscopic and microscopic scales, and so is completely different from a quantum theory with its discrete values for the action variable, energy, and angular momentum. Stochastic electrodynamics also involves a boundary between macroscopic phenomena (where the contributions of zero-point radiation are generally so small as to be irrelevant) and microscopic phenomena (where the contributions of zero-point radiation may be vital). However, the boundary is clearly defined in terms of energy scales with no change in concepts as one crosses the boundary.

3. Successes of Stochastic Electrodynamics

3.1. Charged Harmonic Oscillator in Zero-Point Radiation

The spectrum of classical zero-point radiation can be inferred from the Casimir effect. Indeed, the presence of classical zero-point radiation accounts completely for the experimentally-measured forces associated with the Casimir effect. However, the presence of classical zero-point radiation will influence any other classical system. The easiest system to consider is a charged harmonic oscillator because the equations of motion are linear and so can be solved completely in the usual dipole approximation for the driving radiation [3–8].

A small dipole oscillator which is immersed in random classical radiation will have an equation of motion given by $m\ddot{x} = -kx + m\tau\dddot{x} + eE_x(0,t)$, where the particle of charge e and mass m is attached to a spring of spring-constant k. The damping constant τ is given by $\tau = 2e^2/(3mc^3)$ and the random electric field E_x is evaluated at the center of the dipole. The random classical radiation drives the harmonic oscillator into random oscillation with an average oscillator energy which is the same as the random energy in the electromagnetic field at the same frequency as that of the oscillator. Indeed, this calculation was carried out for random classical radiation by Max Planck in the last years of the 19th century [11]. However, Planck thought in terms of thermal radiation at non-zero temperature, and did not consider the possibility of random classical radiation at zero temperature.

If one takes the limit as the charge e becomes small (or indeed e goes to zero), the oscillator mechanical motion becomes uncoupled from the random radiation. Sometimes one speaks of the mechanical oscillator as behaving according to "stochastic mechanics". Indeed, if the random radiation

takes on the Planck thermal spectrum (including zero-point radiation) before taking the small-e limit, then the average values for position x^n and for momentum p^n for any power n are identical between classical and quantum theory at every temperature $T \geq 0$. In addition, the average value of the oscillator energy is the same between classical and quantum theories.

Both classical and quantum theories incorporate randomness, but the theories are very different. The theories part company when considering the fluctuations of the energy or the average values of $x^m p^n$ involving products of positions and momenta. Stochastic electrodynamics is a classical theory with stochastic fluctuations in the random driving radiation, and the harmonic oscillator (for $e > 0$) behaves as a random classical system; the energy of the oscillator changes continuously as it absorbs energy from the random radiation and radiates away energy according the the rules of Newton's laws and Maxwell's equations [12]. On the other hand, the quantum oscillator behaves as a system that is never found in familiar classical theory; at zero temperature, the quantum oscillator energy is absolutely fixed at $\mathcal{E}(\omega) = \hbar\omega/2$, with no fluctuations whatsoever, while the position and momentum indeed fluctuate. At all temperatures (including zero temperature) and for charge $e > 0$, the classical oscillator is exchanging energy continuously with the random radiation, both emitting radiation continuously on acceleration and absorbing energy continuously from the random radiation. At non-zero temperature, the quantum oscillator is exchanging energy with the radiation field in only fixed amounts of energy, in "quanta" of magnitude $\hbar\omega$. At zero temperature, the quantum oscillator does not exchange energy at all; it does not radiate away energy and it does not absorb energy.

3.2. Harmonic Oscillators Used to Describe Natural Phenomena

Harmonic oscillators are used to describe many aspects of natural phenomena. Thus, solids are often modeled as lattices of molecules described as harmonic oscillators; interacting molecules are often modeled by interacting harmonic oscillators. Because of the agreement at equilibrium of the average energies of the classical and quantum oscillators at all temperatures, there is complete agreement between the results of classical and quantum calculations involving charged harmonic oscillators [13]. Thus, the decrease of specific heats at low temperatures can be accounted for in both the classical and quantum theories. In addition, van der Waals forces between molecules modeled as harmonic oscillators (both unretarded van der Waals forces and retarded forces) are completely in agreement between classical and quantum systems. Furthermore, the diamagnetic behavior of molecules modeled as three-dimensional harmonic oscillator systems is in complete agreement between classical and quantum theories.

3.3. Absence of Atomic Collapse

In 1911, when Rutherford's scattering experiments with alpha particles suggested that atoms contained a small positively charged nucleus, the physicists of the period felt that they confronted the "problem of atomic collapse". According to classical electromagnetic theory, an accelerating charge will radiate away electromagnetic energy. Thus, an accelerating electron in a Coulomb orbit around a small nucleus must be losing energy continuously through radiation. Since the physicists of that period saw no source of radiation for the electron, they anticipated that according to classical theory, the electron must collapse into the nucleus in a very short time. In order for an atom to be stable, some part of the classical description had to change. Bohr's suggestion was that the rules of classical physics should be changed over to new "quantum" rules; the quantum rules claimed that (contrary to classical electromagnetic theory) in certain "stationary states", the accelerating electrons did not radiate. The ad hoc rules prevented atomic collapse.

What the physicists of 1911 did not consider, was the possibility of classical zero-point radiation. If classical electromagnetic zero-point energy is present, then there is a possibility that the electron will absorb energy from the random zero-point radiation while losing energy as radiation while accelerating. The situation would be entirely analogous to that for a harmonic oscillator where (in the classical description) energy was being both absorbed and emitted. Unfortunately, the classical

calculations involving a charged particle in a Coulomb potential are vastly more difficult than the linear equations involving the harmonic oscillator. However, it is possible (though difficult) to do numerical calculations. Numerical calculations for a non-relativistic charged electron in a Coulomb potential were first carried out by Cole and Zou [14] in 2003, with results strongly suggesting that classical zero-point radiation indeed provided the basis for understanding an average behavior for the electron in a Coulomb potential; the probability distribution in position for the classical electron in zero-point radiation agreed fairly closely with the quantum-mechanical ground state distribution. This successful calculation was challenged in 2015 by Nieuwenhuisen and Liska [15,16] who claimed that, according to their more powerful calculations, the electron indeed did not fall into the nucleus but rather was ejected, ionizing the atom, because the electron picked up too much energy from the zero-point energy. Both sets of calculations remove the atomic collapse problem that troubled physicists at the beginning of the 20th century. However, the discrepancies between the two calculations indicate that more work on the self-ionization aspect of this problem is needed.

3.4. Importance of Relativity

The physicists who introduced quantum theory at the beginning of the 20th century missed two aspects of classical physics which today are considered crucial for understanding natural phenomena. One aspect is the presence of classical electromagnetic zero-point radiation; this aspect is explicitly incorporated into stochastic electrodynamics. The second aspect is the importance of special relativity.

Relativity is important for stochastic electrodynamics because classical electrodynamics is a relativistic theory. Indeed, the spectrum of classical electromagnetic zero-point radiation is Lorentz invariant. It turns out that there are two simple (at least approximately) relativistic systems. A (relativistic) point charge in a Coulomb potential when considered within classical electromagnetic theory is fully relativistic. A simple harmonic oscillator system of small amplitude when considered within classical electrodynamics is relativistic in the approximation of small amplitude, so that the speed of the harmonic oscillator particle is small compared to the speed of light c.

Relativity is important for understanding the hydrogen atom within stochastic electrodynamics. It was pointed out in 1982 by Claverie and Soto [17] that an analytic treatment of a non-relativistic charged particle in a Coulomb potential in the presence of zero-point radiation led to self-ionization through the plunging particle orbits. However, the plunging particle orbits are exactly the ones which are strongly modified by relativity [18]. Thus, the conclusion of self-ionizing behavior based upon non-relativistic analysis is suspect. In addition to the situation for the classical hydrogen atom, it turns out that relativistic behavior is vitally important for understanding the blackbody problem within classical physics.

3.5. Planck Spectrum of Blackbody Radiation

Planck first introduced his constant h in connection with the energy per normal mode for an oscillator located in Wien's suggestion for the blackbody radiation spectrum. Subsequently, Planck was able to extract the constant which we call "Boltzmann's constant", $k_B = R/N_A$ (where R is the gas constant and N_A is Avogadro's number), so that the thermal part of his experimentally-based interpolation-guess for the blackbody spectrum corresponded to an energy per normal mode $\mathcal{E}(\omega) = \hbar\omega/[\exp(\hbar\omega/k_B T) - 1]$. Although Planck turned to statistical arguments in order to give a derivation of his interpolation-guess, other physicists of the period tried to derive the blackbody radiation spectrum from purely classical arguments. However, none of these physicists included the idea of classical zero-point radiation, and so all arrived at the Rayleigh–Jeans spectrum corresponding to an energy $\mathcal{E}(\omega) = k_B T$ for each radiation normal mode. Indeed, Rayleigh and Jeans considered classical statistical mechanics in deriving this radiation spectrum for thermal radiation.

The presence of classical electromagnetic zero-point radiation modifies all the classical arguments advanced by the physicists in the first quarter of the 20th century. If zero-radiation is present, then traditional classical statistical mechanics with its equipartition theorem is no longer valid.

The equipartition theorem suggests that an oscillator has zero energy at the absolute zero of temperature; however, this result is contradicted within stochastic electrodynamics since zero-point radiation gives a non-zero energy to an oscillator at zero temperature. Significantly, several of the arguments advanced by the physicists of the early 20th century can be easily modified when zero-point radiation is introduced, and the modified arguments [19] lead directly to Planck's spectrum including zero-point energy, corresponding to an energy per normal mode $\mathcal{E}(\omega) = \hbar\omega/[\exp(\hbar\omega/k_BT) - 1] + \hbar\omega/2 = (\hbar\omega/2)\coth[\hbar\omega/(2k_BT)]$.

However, there remain a set of classical scattering calculations (using non-relativistic mechanical systems as scatterers) which lead to the Rayleigh–Jeans spectrum. These non-relativistic scattering calculations are indeed valid, but only at long wavelengths where relativity is not important and where the Rayleigh–Jeans spectrum is an appropriate approximation. Classical zero-point radiation is Lorentz-invariant, and it is unchanged under scattering only if a relativistic scatterer is considered. Recently, a relativistic scattering calculation for zero-point radiation has been carried out, and indeed detailed balance is found [20]. Only classical analyses that are valid relativistic calculations can lead to the Planck spectrum with zero-point radiation.

Further evidence for the connection between relativity and the Planck spectrum is found in calculations which are usually associated with general relativity. The correlation functions found in a Rindler frame (an accelerating relativistic frame) accelerating through zero-point radiation are associated with the Planck spectrum. Indeed, conformal transformations in a Rindler frame can carry zero-point radiation into thermal radiation at non-zero temperature [21].

4. Phenomena Suggestively Connected to Stochastic Electrodynamics

4.1. Particle Diffraction

It is often stated that the wave-like interference pattern found when microscopic particles pass through two slits represents the essence of quantum behavior. At present, this phenomenon has not been calculated within stochastic electrodynamics. However, stochastic electrodynamics can provide a suggestive description of the experimental observations. Whereas the traditional description views the microscopic particles as moving in empty space when passing through the slits, this is not the appropriate description within stochastic electrodynamics. If zero-point radiation is present, all particles will experience random forces due to the zero-point radiation. In addition, the slits through which the particles must pass will modify the correlation functions associated with the zero-point radiation. Thus, with the stochastic electrodynamic description, the microscopic particles are experiencing random forces due to zero-point radiation, and the correlation functions for the zero-point radiation reflect the information about the slits through which the microscopic particles will pass. Thus, in the stochastic electrodynamic view, it is reasonable that the pattern of the microscopic particles after passing through the screen reflects the random forces experienced by the particles due to the zero-point radiation. Indeed, we know that the average van der Waals forces between molecules and conducting walls are exactly the same in classical and quantum theories, even for the long-range van der Waals forces involving electromagnetic radiation. Perhaps this classical–quantum agreement persists when dealing with the forces when there is relative motion of a particle relative to the wall with slits. Furthermore, at higher temperatures where the electromagnetic radiation correlation functions should change according to the ideas of stochastic electrodynamics, the particle interference pattern should change accordingly. At present, the required calculations which can test the qualitative descriptions arising in stochastic electrodynamics have not been carried out for particles passing through slits.

4.2. Excited States and Spectral Lines

The appearance of sharp spectral lines associated with transitions between excited atomic states presents another basic aspect of nature for which stochastic electrodynamics has only a qualitative

explanation. The harmonic oscillator is a system allowing easy calculations within both classical and quantum theories. However, stochastic electrodynamics has no excited states for the harmonic oscillator. At non-zero temperatures, the classical oscillator has a continuous distribution of energies above the average zero-point energy. In contrast, the quantum oscillator has discrete excited states which are occupied with a Boltzmann probability distribution, giving exactly the same average energy distribution as is found for the classical oscillator in the Planck spectrum of random classical radiation. However, the selection rules for quantum dipole transitions between the excited states are such as to give radiation emission at only the fundamental frequency ω of the harmonic oscillator. Indeed, in the (point) dipole approximation, an excited classical or quantum oscillator radiates energy at only the oscillator frequency ω, and the decay rate is exactly the same between the classical and quantum descriptions.

For a harmonic oscillator, the appearance of the discrete quantum excited states requires that the analysis goes beyond the dipole approximation over to forbidden transitions, corresponding to an expansion of the electromagnetic field as $E_x(x,t) = E_x(0,t) + x[\partial_{x'}E(x',t)]_{x'=0} + ...$ However, such an expansion beyond the dipole approximation can also be taken for the treatment of the classical harmonic oscillator in stochastic electrodynamics, and corresponds to the introduction of parametric resonances for the oscillator. Indeed, Huang and Batelaan [22] showed that, when a pulse of light is incident upon a collection of classical or quantum oscillators, the absorption of the light is resonant when the carrier frequency of the light corresponds to the harmonics of the oscillator frequency ω, namely frequencies $n\omega$, $n = 1, 2, 3, ...$ Both the classical and the quantum descriptions absorb the same amounts of radiation energy at the same frequencies. These sharp absorption lines occur because of parametric resonance for the classical oscillator in stochastic electrodynamics, and because of quantum excited states for the quantum oscillator. This is a fascinating result and may point the direction toward understanding more about atomic spectra in terms of classical physics. At the moment, there is no analysis of radiation emission for the classical hydrogen atom within stochastic electrodynamics, although Cole and Zou [23–25] found striking subharmonic resonances.

4.3. Photon Behavior

The idea of photons was introduced by Einstein in the early years of the 20th century. In 1909, Einstein's analysis of the fluctuations in Planck's spectrum of blackbody radiation claimed that there were two aspects for the fluctuations. One aspect of the fluctuations agreed with classical wave theory and the other aspect was associated with particle-like behavior with particle energy $\hbar\omega$. Crucially, Einstein's analysis did not include the possibility of classical electromagnetic zero-point radiation. Indeed, when classical electromagnetic zero-point radiation is introduced, Einstein's analysis immediately gives a classical analysis for the Planck spectrum including zero-point radiation [19]. Furthermore, Einstein's fluctuation analysis can be completely understood in terms of purely classical physics where the particle-like "photon" fluctuations are understood in terms of the interference between the thermal contribution and the zero-point radiation contribution, which is already present when the thermal contribution is added.

The photon behavior involving atomic radiative decay presents a more complicated problem for stochastic electrodynamics. Thus, a classical dipole oscillator radiates away its energy in a continuous wave of radiation which spreads out smoothly in all directions of space. In contrast, the experimental data from X-ray emission suggest that the radiation is absorbed in a specific direction and that the atom recoils in the opposite direction from which the radiation was absorbed. This localized absorption and associated recoil is currently described within a quantum photon model that claims the radiation from an atom is not emitted as a smooth classical wave emerging in all directions but rather is emitted as a localized photon moving in a specific direction.

Within a qualitative analysis, the directed emission aspect remains a possibility for stochastic electrodynamics. What the traditional quantum explanation omits is the presence of random zero-point radiation that interferes with the radiation emitted by the atom. It is this interference that gives the

possibility of directed behavior, similar to how interference was crucial for a classical understanding of the blackbody fluctuations discussed by Einstein. Indeed, a complete classical calculation has been carried out for the much simpler situation of a pulsed current sheet located in a standing wave of varying phase [26]. If the surface-current pulse occurs in empty space, then the emitted radiation spreads out symmetrically on both sides of the current sheet. However, in the presence of a standing wave, the interference between the radiation emitted by the surface-current pulse and the standing wave leads to radiation energy that is directed to one side or the other while the emitting surface experiences a recoil force in the opposite direction. This complete classical calculation suggests a basis for understanding directed-emission behavior in terms of the interference with the classical electromagnetic zero-point radiation which must be present in the space according to stochastic electrodynamics.

4.4. Superfluid Behavior

In the quantum literature, it is suggested that the superfluidity of liquid helium below the lambda point arises due to the high zero-point energy of the helium atoms. Since stochastic electrodynamics includes classical zero-point radiation, which induces zero-point energy in all systems interacting with electromagnetic radiation, we expect that stochastic electrodynamics might be able to describe superfluidity from a classical viewpoint. Indeed, there is already suggestive work in this direction. Back in 1910, Einstein and Hopf noted that a moving classical particle, which contained an internal classical harmonic-oscillator interacting with random classical radiation, would experience a random walk in velocity space; the random impulses are delivered to the oscillator by the fluctuating radiation and damping is provided by the average velocity-dependent force due to the motion of the harmonic oscillator through the random radiation. Indeed, Einstein and Hopf used this model to derive the Rayleigh–Jeans radiation spectrum for blackbody radiation. What Einstein and Hopf did not anticipate was the possibility of classical electromagnetic zero-point radiation. When classical zero-point radiation is introduced (as in stochastic electrodynamics), then an equilibrium situation at zero temperature requires that there must be an additional damping force because zero-point radiation will provide random forces but does not give any velocity-dependent damping. The additional damping is associated with the radiation emission when the particle is accelerated at the walls of the confining box. A reanalysis of the Einstein–Hopf model led to a derivation of Planck's blackbody radiation spectrum including zero-point radiation [5]. At high temperatures T, the velocity-dependent damping is dominant and the particle in the confining (one-dimensional) box has an average kinetic energy $k_B T/2$. However, as the temperature T decreases toward absolute zero, the velocity-dependent damping force (which depends upon the thermal part of the random radiation) decreases steadily, whereas the the random impulse due to both the thermal contribution and the zero-point contribution goes over to a temperature-independent random impulse. Zero-point radiation is Lorentz invariant and cannot give rise to velocity-dependent forces. Thus, at sufficiently low temperatures, the moving particle experiences a random walk in velocity with only the accelerations at the walls providing an energy loss which will lead to equilibrium. The change from a velocity-dependent damping at high temperatures over to an acceleration-dependent damping at the walls allows the possibility that the particle behavior will be quite different at high and low temperatures. Indeed, the expected low-temperature behavior has frictionless aspects, which are usually associated with superfluidity [27].

5. Connections Between Classical and Quantum Theories

Traditional classical physics does not contain Planck's constant, whereas quantum physics does contain $\hbar$. At the present time, all the textbooks of modern physics present the dichotomy between classical and quantum physics in terms of the presence of this constant. Some physicists would like to keep the situation this way for pedagogical reasons. Within this traditional distinction between the theories, there is a concern for the "infamous boundary" between the classical and quantum theories which invoke differing theoretical concepts.

The theory of stochastic electrodynamics upsets this facile distinction between classical and quantum theories. Stochastic electrodynamics is a purely classical theory that includes randomness and contains Planck's constant. All the concepts of stochastic electrodynamics are classically-recognized ideas, and the boundary between traditional classical physics and stochastic electrodynamics involves simply how prominent a role is played by the random classical zero-point radiation.

How much of nature can be described by stochastic electrodynamics? The answer to this question is still not known. In contrast to the enthusiastic contributions of thousands of physicists to quantum theory, the predictions of stochastic electrodynamics have been calculated by a tiny group of scientists. Thus, we cannot yet answer confidently as to how much of nature requires a quantum description, or where the boundary lies between classical and quantum physics. At present, all that we can report with assurance is that stochastic electrodynamics appears to be the closest classical theory to quantum theory, and that stochastic electrodynamics can describe far more of nature at the microscopic scale than can traditional classical physics.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Eisberg, R.; Resnick, R. *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*, 2nd ed.; Wiley: New York, NY, USA, 1985.
2. Aspelmeyer, M.; Calarco, T.; Eisert, J.; Schmidt-Kaler, F. (Eds.) *Handbook of Quantum Information*; Springer: Berlin/Heidelberg, Germany, 2014.
3. Marshall, T.W. Random electrodynamics. *Proc. R. Soc.* **1963**, *A276*, 475–491.
4. Marshall, T.W. Stochastic Electrodynamics. *Proc. Camb. Philos. Soc.* **1965**, *61*, 537–546. [CrossRef]
5. Boyer, T.H. Derivation of the Blackbody Radiation Spectrum without Quantum Assumptions. *Phys. Rev.* **1969**, *182*, 1374–1383. [CrossRef]
6. Boyer, T.H. Random electrodynamics: The theory of classical electrodynamics with classical electormagnetic zero-point radiation. *Phys. Rev. D* **1975**, *11*, 790–808. [CrossRef]
7. de la Pena, L.; Cetto, A.M. *The Quantum Dice—An Introduction to Stochastic Electrodynamics*; Kluwer Academic: Dordrecht, The Netherlands, 1996.
8. de la Pena, L.; Cetto, A.M.; Hernandez, A. *The Emerging Quantum: The Physics Behind Quantum Mechanics*; Springer: New York, NY, USA, 2015.
9. Lamoreaux, S.K. Resource letter CF-1: Casimir Force. *Am. J. Phys.* **1999**, *67*, 850–861. [CrossRef]
10. Lorentz, H.A. *The Theory of Electrons*; Dover: New York, NY, USA, 1952.
11. Planck, M. *The Theory of Heat Radiation*; Dover: New York, NY, USA, 1959.
12. Huang, W.; Batelaan, H. Dynamics Underlying the Gaussian Distribution of the Classical Harmonic Oscillator in Zero-Point Radiation. *J. Comput. Methods Phys.* **2013**, *2013*, 308538. [CrossRef]
13. Boyer, T.H. General connection between random electrodynamics and quantum electrodynamics for free electromagnetic fields and for dipole oscillator systems. *Phys. Rev. D* **1975**, *11*, 809–830. [CrossRef]
14. Cole, D.C.; Zou, Y. Quantum Mechanical Ground State of Hydrogen Obtained from Classical Electrodynamics. *Phys. Lett. A* **2003**, *317*, 14–20. [CrossRef]
15. Nieuwenhuizen, T.M.; Liska, M.T.P. Simulation of the hydrogen ground state in stochastic electrodynamics-2: Inclusion of relativistic corrections. *Phys. Scr.* **2015**, *165*, 014006. [CrossRef]
16. Nieuwenhuizen, T.M.; Liska, M.T.P. Simulation of the hydrogen ground state in stochastic electrodynamics. *Found. Phys.* **2015**, *45*, 1190–1202. [CrossRef]
17. Claverie, P.; Soto, F. Nonrecurrence of the stochastic process for the hydrogen atom problem in stochastic electrodynamics. *J. Math. Phys.* **1982**, *23*, 753–759. [CrossRef]
18. Boyer, T.H. Unfamiliar trajectories for a relativistic particle in a Kepler or Coulomb potential. *Am. J. Phys.* **2004**, *75*, 992–997. [CrossRef]

19. Boyer, T.H. Blackbody radiation in classical physics: A historical perspective. *Am. J. Phys.* **2018**, *86*, 495–509. [CrossRef]

20. Boyer, T.H. Equilibrium for classical zero-point radiation: Detailed balance under scattering by a classical charged harmonic oscillator. *J. Phys. Commun.* **2018**, *2*, 105014. [CrossRef]

21. Boyer, T.H. The blackbody radiation spectrum follows from zero-point radiation and the structure of relativistic spacetime in classical physics. *Found. Phys.* **2012**, *42*, 595–614. [CrossRef]

22. Huang, W.C.; Batelaan, H. Discrete Excitation Spectrum of a Classical Harmonic Oscillator in Zero-Point Radiation. *Found. Phys.* **2015**, *45*, 333–353. [CrossRef]

23. Cole, D.C.; Zou, Y. Analysis of Orbital Decay Time for the Classical Hydrogen Atom Interacting with Circularly Polarized Electromagnetic Radiation. *Phys. Rev. E* **2004**, *69*, 016601. [CrossRef] [PubMed]

24. Cole, D.C.; Zou, Y. Subharmonic resonance behavior for the classical hydrogen atomic system. *J. Sci. Comput.* **2009**, *39*, 1–27. [CrossRef]

25. Cole, D.C. Subharmonic resonance and critical eccentricity for the classical hyrogen atomic system. *Eur. Phys. J. D* **2018**, *72*, 200. [CrossRef]

26. Boyer, T.H. Interference between source-free radiation and radiation from sources: Particle-like behavior for classical radiation. *Am. J. Phys.* **2017**, *85*, 670–675. [CrossRef]

27. Boyer, T.H. Brownian motion due to random classical radiation: Superfluid behavior in the presence of classical zero-point radiation. (Submitted for publication).