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Energy Considerations of Classical Electromagnetic Zero-Point Radiation and a Specific Probability Calculation in Stochastic Electrodynamics

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Abstract: The zero-point (ZP) radiation field in stochastic electrodynamics (SED) is considered to be formally infinite, or perhaps bounded by mechanisms yet to be revealed someday. A similar situation holds in quantum electrodynamics (QED), although there the ZP field is considered to be "virtual". The first part of this article addresses the concern by some about the related disturbing concept of "extracting energy" from this formally, enormous source of energy. The second part of this article introduces a new method for calculating probabilities of fields in SED, which can be extended to linear oscillators in SED.

1. Introduction

This article is focused on certain aspects of the theory called stochastic electrodynamics (SED). Three distinct parts will be discussed. Section 1 outlines this article and then briefly reviews our current understanding of what SED predicts as compared to observed phenomena in nature. Section 2 turns to a specific aspect of SED questioned in the past regarding the exchange of energy between charges and electromagnetic energy, particularly concerning the possible "extraction of energy from the vacuum." This section is relatively qualitative, but has largely not been discussed elsewhere. This section addresses a physical conception regarding SED that may not be well recognized. In contrast, Section 3 turns to a relatively new means of calculating probabilities within SED, to emphasize that there are still possible new techniques, both analytical or computational, for investigating SED's predictions. Thus, these two sections are quite different, but tied together in that Section 2 focuses on a physical conception for SED, while Section 3 deals with a different calculational method for SED, both of which are essentially new.

Without question, SED has quite a ways to go before being an adequate description of atomic physics, despite many fascinating and tantalizing results since its main inception in the 1960s. This early work was largely carried out by Marshall [1–3] and Boyer [4,5]. The original hope by these physicists and others was that SED can provide a deeper understanding and even derivation of quantum mechanical (QM) phenomena, once the interaction of classical electromagnetic charges and radiation are properly taken into account.

If SED does eventually correctly describe atomic physics, then there will be subtle, as well as not so subtle, physical issues to be understood. Section 2, on energy extraction, will address one such point. To clarify, perhaps it is useful to recall a few subtle, but important points that have come up in the past. For example, the work by Boyer in [6] showed that upon comparing N-point correlation electromagnetic zero-point (ZP) fields and zero-point plus Planckian (ZPP) radiation, the correlation

functions agreed perfectly with those in quantum electrodynamics (QED), provided the quantum operators were symmetrized. A similar result was found upon comparing the properties of a linear electric dipole oscillator immersed in these radiation fields. Boyer argued that the symmetrized QED operators were the ones that were physically observable and hence the important quantities to be compared. Thus, one cannot expect a one-to-one correspondence between QM and SED, but of course, the theories still need to agree regarding physical prediction and observation.

There are, and will be, other such points. For example, only a subset of systems considered in standard QM textbooks actually occur in nature, at least in detail. In atomic physics, the binding force of electrons to atoms and molecules is always based on the Coulombic potential. Trying to force SED to yield predictions of, say, a nonlinear oscillator, when such things are only approximations at best in nature, may simply be an invalid requirement. In the mid-1970s to mid-1980s, many such difficult pursuits on nonlinear oscillators were attempted by physicists in SED, resulting in failures when compared with QM predictions. As emphasized by Boyer, these attempts to impose such binding potentials may have caused the problem [7,8]. It is for that reason that the hydrogen atom, the simplest of atomic systems [9–12], has in recent years been emphasized in SED.

However, there are many other physical issues to be addressed. In particular, excited states, multi-electron atoms, spin, and a description of photon behavior, are all concerns.

This discussion leads us to what is here in Section 2, namely, a concern by many folks about the formal infinite energy in the classical electromagnetic ZP radiation, often loosely referred to as the vacuum in SED (the term "vacuum" is also used similarly in QED, although here it refers to the lowest energy state, or the ground state, of the electromagnetic field when the fields are quantized). Section 2 will discuss possible implications regarding violations of thermodynamic concepts, particularly regarding "extracting energy from this vacuum." One aim is to demystify this concept [13], by showing that this process is an extremely common practice, at least from the point of view of SED.

In contrast, Section 3 becomes more detailed and analytical, showing a calculational method to deduce the full probability distribution for radiation fields in SED and indicating how this method can be extended to the linear electric dipole harmonic oscillator in SED. To my knowledge, the latter has not been worked out in detail before, although the result will not be a surprise. Since this result has clear connections between SED and quantum theory (QT) [6], these details should be of interest.

Thus, the combination of Sections 2 and 3 provides, first, in Section 2, a more physical, but subtle, insight involving SED and, second, in Section 3, a clear calculational method in SED for treating linear dynamic systems. Finally, Section 4 ends the article with some concluding comments.

Stochastic electrodynamics (SED) is certainly the best classical physical theory for describing physical phenomena, as very nicely argued in [14]. As will shortly be discussed, there is a large range of phenomena that have always been considered unexplainable within classical physics, but now are described in a coherent, consistent, and logical manner in SED, without needing to draw on any extraneous "physical or phenomenological" concepts.

What forms the key basis of SED? The motion of charged particles is taken to obey the relativistic Lorentz–Dirac classical equation of motion, while the classical electromagnetic radiation behavior is described by the classical electromagnetic Maxwell's equations. Thermodynamic equilibrium systems of classical charged particles and classical electromagnetic radiation have been analyzed in some detail in SED, revealing that for thermodynamic equilibrium to exist at temperature $T = 0$, there must be nonzero radiation present at $T = 0$ to fulfill this equilibrium.

The following sounds trivial, but it turns out to be a cornerstone of SED: electromagnetic radiation acts on charged particles, and charged particles radiate electromagnetic energy when accelerated or deaccelerated, while both aspects must be consistently taken into account. Charges and radiation are in a sense tied together in terms of achieving any sort of equilibrium. This seems like common sense, yet the Rutherford classical hydrogen model was not treated this way; instead, the classical electron was just assumed to radiate all its energy away and spiral into the nucleus. (Incidentally, the time for

decay would be about 1.3×10^{-11} s, starting from the Bohr radius [15]. Whether physicists in 1910 knew the decay was this fast or not, they certainly realized this was a serious problem.) However, taking into account $T = 0$ radiation, or radiation for $T > 0$, makes a huge difference in the electron behavior [9–12], evidently working against the otherwise tendency to decay, on average.

Thus, the "coupling" between charged particles and radiation is essential for analyzing a system as simple as a Rutherford classical hydrogen atom; by doing so, one can in principle calculate, stochastically, the correct probability of stability versus collapse, as originally suggested in [16]. At $T = 0$, the ZP radiation provides a stochastic "mechanism" that constantly perturbs the electron's orbital motion, but in such a way that atomic collapse is resisted on average.

This ZP radiation is also the mechanism by which many otherwise purely quantum effects can be understood in a natural classical manner. For example, when deducing the thermodynamic equilibrium between the fluctuating atoms and molecules that make up the walls of a cavity at a temperature T, the matter walls and the blackbody radiation in the interior must be treated consistently. The correct thermodynamic equilibrium between the cavity walls and the blackbody electromagnetic radiation in the cavity is accounted for in SED via this way, which also requires that as $T \to 0$, ZP radiation must still exist for consistent physical behavior to be realized [17].

There are other fundamental points on which SED is based. For example, the ZP radiation spectrum must be Lorentz invariant [2,4], and the fundamental definition of $T = 0$ must be obeyed by ZP radiation, namely, that no heat flows during isothermal quasistatic changes in physical systems. As with the Lorentz invariance condition, this $T = 0$ requirement can only be fulfilled by the classical electromagnetic ZP radiation spectrum [17–20]. Other interesting and important properties of ZP radiation also exist, many mentioned at the end of [19]; most of these, plus conformal and scaling investigations, were investigated and uncovered by Boyer.

The work in [21] provides perhaps the best history of the development of SED, but key, independent, and early papers that nicely lay out the core ideas are by Marshall [1–3] and Boyer [4,5]. Other reviews of interest are [14,16,22]. The recent reference of [14] does an excellent job explaining, summarizing, and providing new insights into how quantum effects can be understood within SED. What becomes clear is that a wide range of physical phenomena, assumed to be described only by QM theory, have indeed been explained in detail with SED, such as for van der Waals forces, interacting electric dipole harmonic oscillators, Casimir forces, diamagnetism, blackbody radiation [17,23], and acceleration effects on electrodynamic systems through the vacuum.

However, an enormous amount remains to be done to investigate if SED can predict the full wealth of information that QT predicts today. We presently do not even have the correct prediction for the ground state of hydrogen, never mind an understanding for excited states and spectra, nor detailed predictions for any other atom or molecule. However, we do have the original idea by Boyer on how ZP fields can prevent orbital collapse [16], and some attempts at analytical work along these lines, followed much later by simulation work on the classical hydrogen atom in ZP radiation in an attempt to get around the analytical difficulties [9–12]. Atomic collapse does indeed appear to be prevented by the ZP fields in these simulations, which surely is a surprising and remarkable result for SED. However, "ionization" effects, where the classical electron was eventually removed to "infinity" in a random-walk-like behavior, was seen in [10–12]. Whether this observance was a result of the SED physical theory, and thereby an inherent fault of the theory, or due to something else, such as numerical issues [10], perhaps connected with physical and mathematical notions of chaotic theory, is an important issue still to be wrestled within the future.

These results question that SED might be much more than just a clever way to extend classical physics. Some scientists, myself included, and certainly the founders of this theory, have long speculated that SED might provide a fundamental foundation for QT itself, since it now seems clear that consistently treating classical charges and electromagnetic fields in equilibrium greatly extends classical physics in a very natural and logical way that may provide a solid physical basis to QT. Still, what remains to be shown is daunting, including excited states, photon behavior, spectra,

particle diffraction, etc. What is odd is how difficult it is to make progress, but these are nonlinear stochastic differential equations in SED, so highly nontrivial. Nevertheless, as will be discussed more shortly, SED may well deduce QM and QED, but the reverse is certainly not true.

2. Extracting Energy from the Vacuum

This section will address in qualitative terms a subtle, but important point regarding SED. The intent of [13], published in 1993, was to help examine whether having an infinite or near infinite energy (meaning a cutoff frequency as discussed on p. 245 in [24] or p. 1202ff in [25]) in the so-called "vacuum" at $T = 0$ could result in violations of thermodynamics, or even just common sense. For some reason, this seems to have been more of a concern for SED than QED, perhaps because the QED "vacuum" is considered to consist of "virtual" photons, whereas the SED "vacuum" is considered to consist of physically "real" electromagnetic fields, just as real as "heat" or infrared radiation, X-ray radiation, sunlight, or laser light.

Part of the difficulty here is that, formally (without suggested cutoffs), the energy in ZP fields is infinite. The additional thermal energy by making the temperature $T > 0$ is not infinite. This ZP field energy has both excited some researchers, for the possibility of energy extraction mechanisms, as well as concerned other researchers that this approach might be nonsense, since to have an infinite source of energy all around us seems very contrary to common sense.

The work in [13] mentioned various published proposals such as in [26,27], then described a fairly simple situation that all might agree upon, namely, two neutral, conducting plates held apart. If the plates are close enough, typically much less than a micron or so, the force predicted by Casimir in 1948 [28] will push the plates together, due to the change in the ZP fields via the plates altering the boundary conditions on the fields. There are two very different processes to consider that make this happen: first, the plates might be controlled to move together in a slow, reversible process, such as by "stopper holders" or springs that are slowly squeezed together; second, the plates might be allowed to move together, uninhibited in their motion. The first scenario can be viewed as a reversible thermodynamic process, while the second is an irreversible one. In the former, work is done by the plates on the "stoppers" or springs (i.e., configuration work, as in deforming a material), while in the second, no work is done (free motion of the plates), and heat is released due to acceleration of fluctuating charges in the plates.

In both cases, energy is certainly conserved. The electromagnetic fields obey Maxwell's equations and adhere to an energy conservation law, where changes in electromagnetic energy in a volume are equated to radiation moving in or out of the volume, plus radiation generated by accelerated motion of charges, and changes in kinetic and potential energy of the charges, as discussed in some detail in [22]. Even though the net electromagnetic energy is formally infinite, or huge, but finite due to physical cutoff processes yet to be determined (either in SED or QED), energy is conserved. For the two plate movements just discussed, changes in the net energy of the ZP fields between and outside the plates are finite [29], as are other changes due to (1) the work done upon the stoppers or springs in the reversible process of the first scenario, and (2) an increase in kinetic energy of the plates, as well as (3) energy radiated off in the irreversible process of the second scenario, particularly when the plates come together and kinetic energy is converted into radiated energy.

In any case, [13] analyzed this process and found nothing mysterious about what to expect. Reversible versus irreversible processes are indeed important to distinguish, and of course in practice, all processes are really irreversible ones. The reversible processes are more of idealistic situations, used to calculate entropy changes between equilibrium states. A number of other articles that pertain to energy changes in ZP-related phenomena in QED and SED are: [17–20,23,29]. A related, unusual, yet intriguing idea that cosmic rays may be aided by ZP accelerations in vast empty space was proposed by Rueda [30] and analyzed in a more thermodynamic sense in [31].

Casimir and van der Waals forces are typically weak forces, with the usual explanation in their origin as arising from ZP correlation fluctuations in electromagnetic fields and charged particle motion.

This last statement is typically true, whether in SED or QED, although in QED, an alternative approach was advanced by Schwinger with what he called "source theory," as an attempt to move away from the concept of zero point fluctuations of field quanta [32].

Used as an example of energy extraction in [13], of "colliding" conducting plates, it seems hardly impressive in terms of energy quantity, although, yes, impressive or puzzling on first reflection in the sense that energy is indeed "extracted". At first blush, it is hard to not walk away with the impression that some energy has been obtained from nothing. However, of course that is not the case. When the plates collide, the physical configuration has completely changed; work would need to be done by external forces to pull the plates apart and return them to their original state. Energy is, yes, in a sense extracted from the vacuum, resulting in work done on stoppers or springs for the reversible situation, or radiated energy in the case of the irreversible process. However, this cannot be a continuous process. Indeed, the situation seems quite analogous to lightly kicking the leg out on a heavy table, or lightly pushing a boulder teetering on the edge of a cliff. Both of these are irreversible thermodynamic processes, due to taking the original systems out of equilibrium. In both cases, considerable energy change will occur, with matter crashing into other matter and making physical changes. Returning to the original state of the erect table or the boulder on the cliff would likely require much work in both the pure physics "work" sense, as well as the minute details of truly returning all other objects to their original state that were impacted during the process. Clearly, entropy, in the sense of disorder, increases during these processes and is directly analogous to the irreversible process of letting the Casimir plates collide, with radiation given off.

All of this was discussed in [13], in more detail than here. However, in the present article, we want to go beyond a specific point discussed in [13]. Quoting from [13], "Here we do not comment further on devising practical methods for enabling the vacuum to become a viable, economical alternative to more conventional sources of energy, except to say that, without a doubt, considerable technological effort might need to be expended to adequately harness such energy. Instead, here we will concentrate on the issue of whether fundamental thermodynamic laws are being violated in even considering this possible source of energy."

We will now attempt to go beyond this restriction. Suppose SED is indeed a future fundamental foundation of QT, which as we have said, remains to be established, despite interesting and tantalizing results to date. Suppose the basic premise is true that classical electromagnetic ZP radiation is a key component to include with classical charged particle motion, to deduce part of, if not much, of QT. If so, then SED would describe much of the atomic physics realm, meaning nuclei as positive charged particles and electrons as negative charged point particles. Electrons would follow the classical point charge relativistic Lorentz Dirac equation of motion, with applied electromagnetic fields acting along with ZP and thermal fields. This theory would not describe nucleus changes of splitting and combining, nor other complicated nuclei behavior. (Aside: some SED physicists have speculated that one needs to take into account the ZP fields of weak and strong interactions to proceed further here. However, that is far beyond our present SED scope, which still has only modest atomic success.)

What is different from QED than a different formulation and physical perspective? Actually, quite a bit. SED can deduce the results of the QED, but not vice versa. To put this in context, the Schrödinger equation becomes analogous to the heat conduction equation, the diffusion equation, or even Navier–Stokes equations, which all give "the correct average answers", but do not supply the detailed underlying particle motion and fluctuations. In contrast, the SED formulation, describes the detailed stochastic electron motion while under the interaction of ZP, thermal field, and applied fields. Continuing the analogy, the Schrödinger equation appears analogous to thermodynamics as SED is to statistical mechanics. One can deduce thermodynamics from statistical mechanics, but not vice versa.

How does this affect our energy extraction considerations? Consider the huge realm of chemical reactions, which includes a large part of atomic physics (electrons and nuclei), the very area of physics that SED addresses. Chemical reactions involve electron configuration changes. In terms of typical chemistry descriptions, bonds are formed and broken. Atoms combine with other atoms to form

molecules; molecules combine with other molecules or atoms; and of course reverse processes occur. Common changes we encounter daily are: (1) our stomach acids breaking down proteins, sugars, and carbohydrates, (2) oxidation of iron, copper, aluminum, silicon, etc., (3) fuel combustion in cars and planes, (4) wood and candles burning, (5) photosynthesis, (6) the action of soap on grime, and (7) of course many cooking processes for food consumption.

Yet if SED is valid, then not only would the classical hydrogen atom (the electron) be supported, stochastically, from collapse by the classical electromagnetic ZP fields, but so would the electron configuration in all other atoms in the periodic table, and so would the same apply for "bonds" of atoms to form molecules, whether ionic, covalent, etc. In other words, the classical electromagnetic ZP fields would be the key entity that interacts in concert with atomic matter to result in stable equilibrium states, averaged over the electron constant motion. Yes, there would be a difference of course if temperature $T > 0$, but the ZP fields exert a much stronger effect than the much weaker thermal fields, so these are our key concern.

What this means is that changes in equilibrium states of atoms when they combine or change their original state are due to a close coupling with the classical electromagnetic ZP field. All the things we have learned about chemistry reactions, whether of the exothermic or endothermic type or, more generally, of the exergonic or endergonic type, still come down to changes in electron configurations as atoms interact. From the SED viewpoint, the fluctuations of the ZP field primarily, but also the thermal and applied fields, result in fluctuating particle motions and provide the statistical equilibrium behavior of both fields and particles.

All of these changes involve exchanges in energy between the fields and the kinetic energy of the fluctuating electron motion, plus the binding potential energy between electrons and nuclei. The changes in energy are not necessarily huge, but they are real, and provide a range, for example, from the large chemical reaction energy when TNT explodes, to the smaller, but real example of say, hydration, when cement is formed and heat is released.

In other words, if SED in the distant future is shown to provide stability for electron configurations in atoms and molecules, where stability largely arises between the electrons' motions, the binding forces to nuclei, applied electromagnetic fields, and in particular the ZP and additional thermal fields, then as chemical reactions occur and electron configurations are altered, energy is altered and transferred in various ways. The "vacuum" is altered, as it is when Casimir plates are moved together or apart, or when electron probability densities of position and motion are altered. Energy can be transferred from one of these entities to the other. Chemical reactions with much larger notice, like TNT exploding, or more subtle, like cement forming, or even battery operations where chemical changes occur and electricity is produced, would then all be examples of these energy exchange sources, but, in particular, the ZP electromagnetic fields.

These examples just mentioned were considered when writing [13]. However, since the simplest atom, hydrogen, had not yet been shown to provide the stability discussed here with the ZP fields, adding in chemical reactions as examples where one can "extract energy from the vacuum," seemed too speculative.

Where does this leave us? Extracting energy from the vacuum may be a commonplace operation we see on a daily basis. It may be less mysterious than even the simple examples discussed in [13]. Moreover, it should be acknowledged that having some deeper knowledge about electromagnetic ZP fields and matter (electron/nuclei) interactions can help one to conceive of additional interactions with the vacuum that go beyond "common" chemical reactions and "simple" Casimir plate operations or, say, atomic probe microscope interactions with surfaces. Examples of such possible proposals have been contemplated in [27,33].

Intuitively, "extraction of energy from the vacuum," does sound mysterious and seems like it must be violating some fundamental physical principles. However, matter as composed of charged particles constantly interacting with electromagnetic fields, should result in a dynamic equilibrium of sorts when the combination settles down to a stationary state of fluctuating entities. Upon changes in

barriers and applied forces, like allowing Casimir plates to move together, or chemicals previously separated to come together, this will result in changes in the final equilibrium state, such as an eventual "dead battery" after chemical equilibrium is reached. During the change, energy can be released.

How does this compare with QT? Of course, QM is used to calculate aspects of energy changes occurring in chemical reactions, as physical chemistry calculations, and the notions are based on QM. However, most of this is based on Schrödinger's equation plus impressive computational extensions to this basis for multi-electron scenarios, including Hartree–Fock theory and generalizations, to more recent emphasis in past decades on density functional theory. However, none of these approaches use the ZP field to provide the actual "support" for the electron configuration. The vacuum in QED perturbs the atomic states of QM, to give rise to the Lamb shift for example. The vacuum fields in QED do not create the atomic states of QM. That is a major difference between the underpinnings of SED and QM/QED. In SED, the intimate relationship between classical radiation and classical charges, in stochastic equilibrium, is key to SED's prediction of atomic state equilibrium and results in QM phenomena (again, still much to be shown, but some promising results for hydrogen). In QM and QED, the atomic states arise from Schrödinger's or Dirac's wave equations, extended to multi-electron situations; interactions with QED electromagnetic vacuum change the behavior, but do not explain the origin of the QM states.

Finally, we leave this section with an observation often said by researchers in SED. When someone questions how can these large fluctuating fields exist and not be obvious to us, is that their existence is revealed by the QM phenomena we observe all around us, from holding our bodies together through atoms and molecules existing and not flying apart, to the very existence of the incredible host of chemical reactions everywhere, to semiconductor behaviors, superfluid behavior, "lasing" of light, and so many other QM effects uncovered in physics since about 1900.

3. A Calculational Method within SED

This article began with a broad introduction to SED, then turned to a problem that has bothered many, as to how the infinite, or near infinite, energy of the ZP fields in SED (and secondarily in QED) reduces to the physics we see about us. Now, we change directions to a specific calculation for the probability distribution of the ZP and ZPP fields.

I am not aware of the following calculational method being used before in SED, so perhaps it can help illustrate some new directions. Usually in SED, aspects about the probability distributions of the fields, as well of the electric dipole simple harmonic oscillators (SHO), are examined by looking at the mean and the variance of these quantities. The present method, although long, does provide the full probability distribution analytically in closed form, at least for the fields, and will also work for linear nonrelativistic electric dipole SHOs.

If we consider a rectilinear-piped region in space, with dimensions L_x, L_y, and L_z, along the x, y, and z axes, then any free fields in this region of space can be expressed as plane waves, with periodic boundary conditions imposed. This last condition is assumed here for mathematical simplicity, since we are not attempting to describe the fields outside this volume of space. One can show that the way to write the electric and magnetic free fields (no charges present) in this region of space, with periodic boundary conditions, is [21]:

$$\vec{E}\left(\vec{x},t\right) = \frac{1}{\left(L_x L_y L_z\right)^{1/2}} \sum_{n_x,n_y,n_z=-\infty}^{\infty} \sum_{\lambda=1,2} \hat{\varepsilon}_{\vec{k}_{\vec{n}},\lambda} \left[A_{\vec{k}_{\vec{n}},\lambda} \cos\left(\vec{k}_{\vec{n}}\cdot\vec{x}-\omega_{\vec{n}}t\right) + B_{\vec{k}_{\vec{n}},\lambda} \sin\left(\vec{k}_{\vec{n}}\cdot\vec{x}-\omega_{\vec{n}}t\right) \right], \quad (1)$$

$$\vec{B}\left(\vec{x},t\right) = \frac{1}{\left(L_x L_y L_z\right)^{1/2}} \sum_{n_x,n_y,n_z=-\infty}^{\infty} \sum_{\lambda=1,2} \left(\hat{k}_{\vec{n}}\times\hat{\varepsilon}_{\vec{k}_{\vec{n}},\lambda}\right) \left[A_{\vec{k}_{\vec{n}},\lambda} \cos\left(\vec{k}_{\vec{n}}\cdot\vec{x}-\omega_{\vec{n}}t\right) + B_{\vec{k}_{\vec{n}},\lambda} \sin\left(\vec{k}_{\vec{n}}\cdot\vec{x}-\omega_{\vec{n}}t\right) \right], \quad (2)$$

where:

$$\vec{k}_{\vec{n}} = \frac{2\pi n_x}{L_x}\hat{x} + \frac{2\pi n_y}{L_y}\hat{y} + \frac{2\pi n_z}{L_z}\hat{z}, \quad (3)$$

and n_x, n_y, and n_z are integers, and $\omega_{\vec{\mathbf{n}}} = c \left| \vec{\mathbf{k}}_{\vec{\mathbf{n}}} \right|$, $\vec{\mathbf{k}}_{\vec{\mathbf{n}}} \cdot \hat{\varepsilon}_{\vec{\mathbf{k}}_{\vec{\mathbf{n}}},\lambda} = \vec{\mathbf{k}}_{\vec{\mathbf{n}}} \cdot \hat{\varepsilon}_{\vec{\mathbf{k}}_{\vec{\mathbf{n}}},\lambda'} = 0$, and $\hat{\varepsilon}_{\vec{\mathbf{k}}_{\vec{\mathbf{n}}},\lambda} \cdot \hat{\varepsilon}_{\vec{\mathbf{k}}_{\vec{\mathbf{n}}},\lambda'} = 0$ for $\lambda \neq \lambda'$, where λ and λ' indicate the linear polarization direction. Specifically, λ might be represented by the values one or two, and the same for λ'. Furthermore, $\hat{\mathbf{k}}_{\vec{\mathbf{n}}} = \vec{\mathbf{k}}_{\vec{\mathbf{n}}} / \left| \vec{\mathbf{k}}_{\vec{\mathbf{n}}} \right|$.

Let us consider the thermal radiation field $T \geq 0$, represented here by ZPP fields, while $T = 0$ fields will be indicated by ZP. Specifically, we will assume that the coefficients of the radiation field are randomly distributed in the following way initially, but once fixed, they stay fixed during, say, an analytic study of the interaction of charged oscillators and radiation, or in the simulation runs of [9–12]. Each time a similar "experiment" of radiation and charges is considered, the experiment is treated as another member of the ensemble of similar experiments. These considerations are very much along the lines of the pioneering physicists who considered radiation and charged particle behavior, in particular in the first half of Planck's major treatise [34], Einstein's and Hopf's work [35,36], and as much later picked up by Boyer, except then with the important caveat that equilibrium radiation must exist at $T = 0$ (ZP) [4,5]. Thus, we will assume the following, where $\langle\ \rangle$ means ensemble averages of these similar "experiments."

$$\left\langle A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} \right\rangle = \left\langle B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} \right\rangle = 0, \tag{4}$$

$$\left\langle A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}'},\lambda'} \right\rangle = 0, \tag{5}$$

$$\left\langle A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}'},\lambda'} \right\rangle = \left\langle B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}'},\lambda'} \right\rangle = 0, \text{ if } \tilde{\mathbf{n}} \neq \tilde{\mathbf{n}}' \text{ or } \lambda \neq \lambda', \tag{6}$$

but:

$$\left\langle A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} A_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} \right\rangle = \left\langle B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}},\lambda} \right\rangle = \left[\sigma\left(\omega_{\tilde{\mathbf{n}}}, T \right) \right]^2, \tag{7}$$

and we assume that the $A's$ and $B's$ coefficients are independent random variables, with zero means as in Equation (4), variance $\left[\sigma\left(\omega_{\tilde{\mathbf{n}}}, T \right) \right]^2$, and that they follow Gaussian distributions.

Researchers in SED have derived and written much about $\left[\sigma\left(\omega_{\vec{\mathbf{n}}}, T \right) \right]^2$ [21]. The point here is not to repeat that work, but to assume the value is correct and to show how to carry out an interesting calculation with it for the probability distribution of $\vec{\mathbf{E}}\left(\vec{\mathbf{x}}, t \right)$ in Equation (1), as well as indicate how to obtain more complicated aspects of joint probabilities of these $\vec{\mathbf{E}}\left(\vec{\mathbf{x}}, t \right)$ and $\vec{\mathbf{B}}\left(\vec{\mathbf{x}}, t \right)$ quantities and to how this method can be extended to electric dipole oscillators. Therefore, assuming:

$$\left[\sigma\left(\omega_{\vec{\mathbf{n}}}, T \right) \right]^2 = 2\pi\hbar\omega + \frac{4\pi\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_{\mathrm{B}}T} \right) - 1} = 2\pi\hbar\omega \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T} \right). \tag{8}$$

Note that $\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T} \right) \to 1$ as $T \to 0$. The term $2\pi\hbar\omega$ is considered the ZP radiation contribution, as it exists as $T \to 0$, while the $\frac{4\pi\hbar\omega}{\exp\left(\frac{\hbar\omega}{k_{\mathrm{B}}T} \right) - 1}$ term is the "Planck spectrum" contribution, and the full expression $2\pi\hbar\omega \coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T} \right)$ we will refer to as the ZP plus Planckian spectrum, notated by ZPP.

Thus, we will start by calculating the probability density function of realizing a specific value of the electric field, for the ZPP situation, so involving our expression of $\vec{\mathbf{E}}_{\mathrm{ZPP}}\left(\mathbf{x}, t \right)$ over the ensemble (ensemble of $A's$ and $B's$), at position $\vec{\mathbf{x}}$ and time t in Equation (1):

$$P\left(\vec{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t \right)$$

$$= \int dA_1 \cdots \int dA_N ... \int dB_1 \cdots \int dB_N ... P\left(A_{1,} \cdots, A_N, \cdots, B_N, ... \right) \delta^3 \left[\vec{\mathbf{E}} - \vec{\mathbf{E}}_{\mathrm{ZPP}}\left(\vec{\mathbf{x}}, t \right) \right], \tag{9}$$

where the $A's$ and $B's$ here are symbolically meant to represent the coefficients in Equation (1). $P(A_1, \cdots, A_N, \cdots, B_N, ...)$ represents the probability density function of all these coefficients. In the end, we would let $N \to \infty$. By $\vec{\mathbf{E}}_{\text{ZPP}}\left(\vec{\mathbf{x}}, t\right)$ in the Dirac delta function, we mean Equation (1), but where the ZPP conditions of Equations (4)–(7) hold.

Noting the assumed statistical independence of the coefficients, then inserting each Gaussian probability density function for the coefficients, and re-expressing the Dirac delta function in terms of Fourier integrals, yield:

$$
\begin{aligned}
P\left(\vec{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t\right) = \int dA_1 \cdots \int dA_N ... \frac{1}{\sqrt{2\pi\sigma_{\tilde{\mathbf{n}}_1}^2}} \exp\left[-\frac{(A_1)^2}{2\sigma_{\tilde{\mathbf{n}}_1}^2}\right] ... \frac{1}{\sqrt{2\pi\sigma_{\tilde{\mathbf{n}}_N}^2}} \exp\left[-\frac{(A_N)^2}{2\sigma_{\tilde{\mathbf{n}}_N}^2}\right] ... \\
\times \int dB_1 \cdots \int dB_N ... \frac{1}{\sqrt{2\pi\sigma_{\tilde{\mathbf{n}}_1}^2}} \exp\left[-\frac{(B_1)^2}{2\sigma_{\tilde{\mathbf{n}}_1}^2}\right] ... \frac{1}{\sqrt{2\pi\sigma_{\tilde{\mathbf{n}}_N}^2}} \exp\left[-\frac{(B_N)^2}{2\sigma_{\tilde{\mathbf{n}}_N}^2}\right] ... \\
\times \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_1 e^{is_1(E_x - E_{x,\text{ZPP}})} \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_2 e^{is_2(E_y - E_{y,\text{ZPP}})} \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_3 e^{is_3(E_z - E_{z,\text{ZPP}})},
\end{aligned}
\tag{10}
$$

where:

$$
\left[\sigma\left(\omega_{\vec{\mathbf{n}}}, T\right)\right]^2 \equiv \sigma_{\tilde{\mathbf{n}}}^2,
\tag{11}
$$

to simplify notation. From these independent Gaussian distributions, one can of course deduce the relations Equations (4)–(7).

To evaluate Equation (10), Equation (1) needs to be substituted in three places on the last line. To simplify notation yet again, let us replace Equation (1) via:

$$
\begin{aligned}
\vec{\mathbf{E}}_{\text{ZPP}}\left(\vec{\mathbf{x}}, t\right) &= \frac{1}{(L_x L_y L_z)^{1/2}} \sum_{n_x, n_y, n_z = -\infty}^{\infty} \sum_{\lambda=1,2} \hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}}, \lambda} \left[A_{\mathbf{k}_{\tilde{\mathbf{n}}}, \lambda} \cos\left(\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}} \cdot \vec{\mathbf{x}} - \omega_{\tilde{\mathbf{n}}} t\right) + B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}}, \lambda} \sin\left(\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}} \cdot \vec{\mathbf{x}} - \omega_{\tilde{\mathbf{n}}} t\right)\right] \\
&= \sum_q A_q \tilde{\mathbf{E}}_{cq} + \sum_q B_q \tilde{\mathbf{E}}_{sq},
\end{aligned}
\tag{12}
$$

where q represents all the indices of n_x, n_y, n_z, λ, with their appropriate ranges, A_q still represents $A_{\mathbf{k}_{\tilde{\mathbf{n}}}, \lambda}$, and likewise for B_q and $B_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}}, \lambda}$. Furthermore,

$$
\tilde{\mathbf{E}}_{cq} \equiv \frac{1}{(L_x L_y L_z)^{1/2}} \hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}}, \lambda} \cos\left(\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}} \cdot \vec{\mathbf{x}} - \omega_{\tilde{\mathbf{n}}} t\right),
\tag{13}
$$

and:

$$
\tilde{\mathbf{E}}_{sq} \equiv \frac{1}{(L_x L_y L_z)^{1/2}} \hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}}, \lambda} \sin\left(\tilde{\mathbf{k}}_{\tilde{\mathbf{n}}} \cdot \vec{\mathbf{x}} - \omega_{\tilde{\mathbf{n}}} t\right).
\tag{14}
$$

Putting these expressions into Equation (10) and then integrating over all the $A's$ and $B's$, can be done by completing the squares before integrating the Gaussian expressions. The result is:

$$
\begin{aligned}
&P\left(\tilde{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t\right) = \\
&\frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_1 e^{is_1 E_x} \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_2 e^{is_2 E_y} \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} ds_3 e^{is_3 E_z} \times \\
&\times \exp\left[-\sum_q \left(s_1 E_{cq,x} + s_2 E_{cq,y} + s_3 E_{cq,z}\right)^2 \frac{\sigma_q^2}{2}\right] \exp\left[-\sum_q \left(s_1 E_{sq,x} + s_2 E_{sq,y} + s_3 E_{sq,z}\right)^2 \frac{\sigma_q^2}{2}\right].
\end{aligned}
\tag{15}
$$

Now, only three remaining integrals need to evaluated. Once again, $\sigma_{\tilde{n}}^2$ has been simplified to σ_q^2, for notation purposes. The factor $\exp\left[-\sum_q \left(s_1 E_{cq,x} + s_2 E_{cq,y} + s_3 E_{cq,z}\right)^2 \frac{\sigma_q^2}{2}\right]$ arises from integrating over the $A's$, while $\exp\left[-\sum_q \left(s_1 E_{sq,x} + s_2 E_{sq,y} + s_3 E_{sq,z}\right)^2 \frac{\sigma_q^2}{2}\right]$ arises from integrating over all the $B's$.

The last two exponential terms containing the sums over indices q consist of the electric field sine and cosine terms in Equations (13) and (14). These simplify to:

$$
\begin{aligned}
&\left(s_1 E_{cq,x} + s_2 E_{cq,y} + s_3 E_{cq,z}\right)^2 + \left(s_1 E_{sq,x} + s_2 E_{sq,y} + s_3 E_{sq,z}\right)^2 \\
&= \frac{1}{\left(L_x L_y L_z\right)} \left(s_1^2 \varepsilon_{q,x}^2 + s_2^2 \varepsilon_{q,y}^2 + s_3^2 \varepsilon_{q,z}^2 + 2 s_1 s_2 \varepsilon_{q,x} \varepsilon_{q,y} + 2 s_1 s_3 \varepsilon_{q,x} \varepsilon_{q,z} + 2 s_2 s_3 \varepsilon_{q,y} \varepsilon_{q,z}\right).
\end{aligned}
\tag{16}
$$

The last three "cross terms" here in Equation (16), involving $s_1 s_2$, $s_1 s_3$, and $s_2 s_3$, still make the subsequent evaluation difficult. However, by first summing over the polarization index λ in the q set of indices and by making use of the following identities for the three perpendicular vectors of $\hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{n}},1}$, $\hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{n}},2}$, and $\tilde{\mathbf{k}}_{\tilde{n}}$, this greatly simplifies the subsequent calculations:

$$
\sum_{\lambda=1,2} \left[\left(\hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{n}},\lambda}\right)_i\right]^2 = 1 - \left[\frac{(\tilde{\mathbf{k}}_{\tilde{n}})_i}{k_n}\right]^2,
\tag{17}
$$

$$
\sum_{\lambda=1,2} \left(\hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{n}},\lambda}\right)_i \left(\hat{\varepsilon}_{\tilde{\mathbf{k}}_{\tilde{n}},\lambda}\right)_j = \delta_{ij} - \frac{k_i k_j}{k^2}.
\tag{18}
$$

Summing over λ in the q set of indices and making use of Equations (17) and (18) results in:

$$
\begin{aligned}
&\sum_q \left(s_1^2 \varepsilon_{q,x}^2 + s_2^2 \varepsilon_{q,y}^2 + s_3^2 \varepsilon_{q,z}^2 + 2 s_1 s_2 \varepsilon_{q,x} \varepsilon_{q,y} + 2 s_1 s_3 \varepsilon_{q,x} \varepsilon_{q,z} + 2 s_2 s_3 \varepsilon_{q,y} \varepsilon_{q,z}\right) \sigma_q^2 \\
&= \sum_{\tilde{n}} \left\{s_1^2 \left[1 - \left(\frac{k_{\tilde{n},x}}{k_{\tilde{n}}}\right)^2\right] + s_2^2 \left[1 - \left(\frac{k_{\tilde{n},y}}{k_{\tilde{n}}}\right)^2\right] + s_3^2 \left[1 - \left(\frac{k_{\tilde{n},z}}{k_{\tilde{n}}}\right)^2\right]\right\} \sigma_{\tilde{n}}^2 \\
&\quad - 2 \sum_{\tilde{n}} \left[s_1 s_2 \frac{k_{\tilde{n},x} k_{\tilde{n},y}}{k_{\tilde{n}}^2} + s_1 s_3 \frac{k_{\tilde{n},x} k_{\tilde{n},z}}{k_{\tilde{n}}^2} + s_2 s_3 \frac{k_{\tilde{n},y} k_{\tilde{n},z}}{k_{\tilde{n}}^2}\right] \sigma_{\tilde{n}}^2.
\end{aligned}
\tag{19}
$$

The "cross terms" involving $s_1 s_2$, $s_1 s_3$, $s_2 s_3$ still remain, but upon summing over $\vec{n}$ in the last three terms,

$$
\sum_{\tilde{n}} \frac{k_{\mathbf{n},i} k_{\mathbf{n},j}}{k_{\tilde{n}}^2} \sigma_{\tilde{n}}^2 \approx 0 \text{ for } i \neq j,
\tag{20}
$$

which is valid in the continuum limit of $\tilde{\mathbf{k}}_{\tilde{n}}$, meaning the sum over $\tilde{n}$ is converted to an integral, usually an excellent approximation for large L_x, L_y, and L_z. We still assume that $\sigma_{\tilde{n}}^2$ only depends on the frequency $\omega_{\tilde{n}} = c \left|\tilde{\mathbf{k}}_{\tilde{n}}\right|$ as in Equation (7).

We now have from the steps due to Equation (15):

$$
P\left(\tilde{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t\right) = I_1 I_2 I_3 ,
\tag{21}
$$

where:

$$
I_i \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} ds_i \exp\left(is_i E_i - \frac{1}{2} \frac{s_i^2}{\left(L_x L_y L_z\right)} \sum_{\tilde{n}} \left[1 - \left(\frac{k_{\tilde{n},i}}{k_{\tilde{n}}}\right)^2\right] \sigma_{\tilde{n}}^2\right).
\tag{22}
$$

Defining:

$$
\alpha_i \equiv \frac{1}{2} \frac{1}{\left(L_x L_y L_z\right)} \sum_{\tilde{n}} \left\{1 - \left[\frac{(\mathbf{k}_{\tilde{n}})_i}{k_{\tilde{n}}}\right]^2\right\} \sigma_{\tilde{n}}^2,
\tag{23}
$$

and completing the square in Equation (22) results in:

$$I_i = \frac{1}{2\pi} \exp\left(-\frac{E_i^2}{4\alpha_i}\right)\left(\frac{\pi}{\alpha_i}\right)^{1/2}. \tag{24}$$

Hence:

$$P\left(\tilde{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t\right) = I_1 I_2 I_3 = \frac{1}{\left(2\pi^{1/2}\right)^3} \frac{1}{(\alpha_1\alpha_2\alpha_3)^{1/2}} \exp\left(-\frac{E_x^2}{4\alpha_1} - \frac{E_y^2}{4\alpha_2} - \frac{E_z^2}{4\alpha_3}\right) \tag{25}$$

To put this in perspective, using similar procedures, although not as involved as what led to Equation (25), one can show that:

$$\left\langle E_{\text{ZPP},i}^2 \right\rangle = \frac{1}{\left(L_x L_y L_z\right)} \sum_{\tilde{\mathbf{n}}} \left\{ 1 - \left[\frac{(\mathbf{k}_{\tilde{\mathbf{n}}})_i}{k_{\tilde{\mathbf{n}}}}\right]^2 \right\} \sigma_{\tilde{\mathbf{n}}}^2. \tag{26}$$

Consequently, Equation (25) becomes the more recognizable form of the product of three Gaussians:

$$P\left(\tilde{\mathbf{E}} \text{ at } \mathbf{x}, t\right) = \frac{\exp\left[-\frac{E_x^2}{2\left\langle E_{\text{ZPP},x}^2 \right\rangle}\right]}{\sqrt{2\pi \left\langle E_{\text{ZPP},x}^2 \right\rangle}} \frac{\exp\left[-\frac{E_y^2}{2\left\langle E_{\text{ZPP},y}^2 \right\rangle}\right]}{\sqrt{2\pi \left\langle E_{\text{ZPP},y}^2 \right\rangle}} \frac{\exp\left[-\frac{E_z^2}{2\left\langle E_{\text{ZPP},z}^2 \right\rangle}\right]}{\sqrt{2\pi \left\langle E_{\text{ZPP},z}^2 \right\rangle}}. \tag{27}$$

Thus, this technique enables the entire probability density function to be deduced for the electric field values in the ZPP radiation field. With the above approximations originally considered by Planck in [34], of a large cavity with smooth varying walls and structure sizes large compared to the main wavelength range, we do indeed obtain a Gaussian functional dependence. Moreover the basic "recipe" of the present calculation, but not necessarily taking continuum limits, could be applied to examine smaller structures, as in quantum cavity electrodynamic experiments [37].

As mentioned earlier, this same technique can be applied to much more complicated probability density functions involving these fields, such as $P\left(\tilde{\mathbf{E}}_1 \text{ at } \vec{\mathbf{x}}_1, t_1 \bigcap \tilde{\mathbf{E}}_2 \text{ at } \vec{\mathbf{x}}_2, t_2\right)$, as well as for probability densities $P(x_1, t_1)$, $P(x_1, t_1 \bigcap x_2, t_2)$, etc., for nonrelativistic linear electric dipole oscillators in SED. The latter calculations are considerably lengthier than the example here on radiation calculations, but they can be carried out analytically to the same extent as above. For each extra element in a joint probability function, one would add another Dirac delta function to the integral expression in Equation (9). For example:

$$P\left(\tilde{\mathbf{E}}_1 \text{ at } \vec{\mathbf{x}}_1, t_1 \bigcap \tilde{\mathbf{E}}_2 \text{ at } \vec{\mathbf{x}}_2, t_2\right) =$$
$$\int dA_1 \cdots \int dA_N \ldots \int dB_1 \cdots \int dB_N \ldots P\left(A_1, \cdots, A_N, \cdots, B_N, \ldots\right)$$
$$\times \delta^3\left[\tilde{\mathbf{E}}_1 - \tilde{\mathbf{E}}_{\text{ZPP}}\left(\mathbf{x}_1, t_1\right)\right] \delta^3\left[\tilde{\mathbf{E}}_2 - \tilde{\mathbf{E}}_{\text{ZPP}}\left(\vec{\mathbf{x}}_2, t_2\right)\right]. \tag{28}$$

This is related to the "two-point" correlation functions in [2,6] and in many other articles in SED [38], including for uniform acceleration conditions [39]. In Equation (28), however, instead of only looking at a correlation function, one can obtain the whole probability joint density function. The above calculation then results in the final result being dependent on differences in time and space, which does not show up in Equation (27).

Calculations with the linear SHOs work similarly, where now one works with, for example (and considering a 1D oscillator):

$$P\left(x_1 \text{ at } t_1 \bigcap x_2 \text{ at } t_1\right) =$$
$$\int dA_1 \cdots \int dA_N ... \int dB_1 \cdots \int dB_N ... P\left(A_1, \cdots, A_N, \cdots, B_N, ...\right) \qquad (29)$$
$$\times \, \delta^3\left[x_1 - x\left(t_1\right)\right] \delta^3\left[x_2 - x\left(t_2\right)\right],$$

where $x(t)$ is the solution to the SHO motion in either ZP or ZPP fields. It would be a linear function of all the $A's$ and $B's$, which is what enables the calculation to go through.

4. Concluding Remarks

This article began with an overview on SED, then tackled a physical concept in Section 2 that has both bothered, as well as intrigued many researchers when encountering SED. This problem has to do with the infinite, or extremely large, energy in the ZP fields, which is taken to be real within this classical physical theory. The fluctuations of the ZP field, interacting with the fluctuating charged particles that would make up atomic physics, has been shown in SED to reveal quantum effects, but as explained purely by classical physical concepts. The infinity, or large, energy in the ZP field was discussed in some detail here, offering the explanation that "energy extraction from the vacuum," may actually not be so mysterious an effect, but rather a daily, common occurrence for us all, particularly in the form of chemical reactions. The ZP field is the key element that keeps electrons in stochastically stable situations to form atoms, molecules, bonds, etc., and changes in element proximity to each other result in changes in electron configurations, with the resulting changes in "bonds" and emission or absorption of energy. In turn, the radiation fields can be altered, similar to what happens as Casimir plates are moved closer or farther apart.

Section 3 of this article changed considerably from the more descriptive piece in Section 2. A technique for detailed calculations of probabilities was introduced for the ZPP field. $P\left(\tilde{\mathbf{E}} \text{ at } \vec{\mathbf{x}}, t\right)$ was calculated, and indications were given on how to extend this to even more complicated situations, such as the joint probability of fields at several points in space and time, as well as probabilities of linear electric dipole oscillators in SED.

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