Abstract: The present investigation addresses the flow of hybrid (nickel–zinc ferrite and ethylene glycol) nanoliquid with entropy optimization and nonlinear thermal radiation coatings past a curved stretching surface. Analysis was carried out in the presence of magnetohydrodynamic, heat generation/absorption, and convective heat and mass flux conditions. Solution of the modeled problem was attained numerically using MATLAB built-in function bvp4c. Impacts of prominent parameters on betrothed distributions were depicted through graphs and were well supported by requisite discussions. Numerically calculated values of Sherwood number were established in a tabulated form and were scrutinized critically. An excellent concurrence was achieved when results of the presented model were compared with previously published result; hence, dependable results are being presented. It was observed that concentration field diminished with increasing values of curvature parameter, though the opposite trend was noticed for velocity and temperature distributions. Further, it was detected that Nusselt number decreased with augmented values of radiation and curvature parameters.

Keywords: coatings; curved stretched surface; nanoliquid; nonlinear thermal radiation; entropy generation

1. Introduction

Numerous applications of heat transfer liquids or coolants can be found in a variety of fields, such as automobiles, industry, electronics, and cooling processes. In all such industrial applications, cooling by liquids has been used for years. The process of cooling by fluids may be the single phase (where there is no phase change in the coolant) or two-phase (where coolant liquid will experience a phase change). In the latter, latent heat influences the cooling efficiency [1]. Several coolants, such as water, ethylene glycol, blend of water and glycol, propylene glycol and amalgamation of water, and propylene glycol, are used as coolants in automobiles and industrial cooling processes. In the last two decades, several researchers have devoted their efforts to increasing thermal conductivity of coolants, thereby improving heat transfer capabilities. The pioneering work of Choi et al. [2] introduced nanofluids by insertion of solid nanoparticles into liquids, thus enhancing the thermal properties of these liquids.
This pioneering work has remarkably revolutionized modern engineering and the industrial world. Nanofluids are an amalgamation of suspended solid material particles and customary liquids (ethylene glycol, water). This new type of advanced material possesses amazing capabilities that trigger the process of heat transfer and augments the thermal conductivity of the base fluid. Enhancement in the thermal conductivity and heat transfer is visualized once ferrite nanoparticles are added into the base liquid. Several examples featuring heat transfer can be quoted, including chemicals, cooling and heating system of buildings, and avionics cooling systems. Nanofluids exhibit potentially exceptional features in comparison to macrometer-sized particles. This is because nanoparticles have sufficiently larger surface area compared to micrometer-sized particles; this is the reason nanofluids possess incomparable capabilities of heat transfer [3].

In several electromagnetic applications with high permeability, e.g., electromagnetic wave absorbers and inductors, usage of nickel–zinc ferrite can be noticed. To minimize energy losses related to bulk powders, usage of nickel–zinc nanoparticles has been recommended by a number of researchers [4–6]. In addition, a majority of electronic gadgets require such materials to be compressed into outsized shapes with the required thickness, which is reasonably challenging if the size of these particles is large enough. Several methods have been proposed to get nickel–zinc ferrite, including ball milling, precipitation, and hydrothermal. Ferrofluids are colloidal fluids comprising ferromagnetic or ferrimagnetic nanoparticles suspended in an electrically insulated hauler fluid. In the current examination, ethylene glycol (C\textsubscript{2}H\textsubscript{6}O\textsubscript{2}) was taken as a carrier fluid. The assumed ferrite nanoparticle was nickel–zinc ferrite (NiZnFe\textsubscript{2}O\textsubscript{4}) crystallize in the normal spinal structure. Typically, at room temperature, the inverted spinals are ferromagnetic and normal spinals are paramagnetic. Moreover, zinc ferrites act like antiferromagnetic in nature at low temperature. This feature makes ferromagnetic nanofluids more relevant in different real-world applications [7,8]. The ferrofluid’s flow with the effect of thermal gradients and the magnetic field was discussed by Neuringer [9]. Majeed et al. [10] demonstrated the heat transfer investigation in a ferromagnetic fluid flow.

The subject of fluid flow past stretched surfaces has diverse engineering and industrial applications, including paper production, glass blowing, crystal growing, hot rolling, manufacturing of rubber sheets, annealing of copper wires, etc. The coined work of Crane [11] discussing the flow past a linearly stretching surface urged fellow researchers to discover more avenues in this exciting and interesting subject. This was followed by the remarkable work of Gupta and Gupta [12] who pondered on the flow past a spongy surface. Then, Chakrabarti and Gupta [13] examined the hydromagnetic flow past a stretched surface. Andersson et al. [14] considered the flow of power-law fluid past a surface, which was linearly stretched under the influence of magnetic forces. The flow of an Oldroyd-B fluid with the impact of generation/absorption was deliberated by Hayat et al. [15]. Muhammad et al. [16] discussed the effect of thermal stratification in the ferromagnetic fluid on a stretching sheet. Ramzan and Yousaf [17] demonstrated that the elastic viscous nanofluid finished a bi-directional stretching surface in view of Newtonian heating. Hussain et al. [18] utilized the exponentially stretching sheet to scrutinize the flow of Jeffrey nanofluid with radiation effects. Some recent explorations highlighting various fluid flows past stretched surfaces with coatings can be found in references [19–22].

In today’s cutting-edge engineering technology, curved stretching has a broad relevance because of its different uses in industry, for example, in transportation and electronics. Sanni et al. [23] attained a numerical solution for the viscous fluid flow on a curved stretched channel. Sajid et al. [24] inspected the ferrofluid (Fe\textsubscript{3}O\textsubscript{4}) flow on a curved sheet with effects of Joule heating and magnetic forces. Rosca and Pop [25] studied time-dependent flow along a spongy curved surface. Imtiaz et al. [26] introduced the effect of homogeneous/heterogeneous reactions in ferrofluid embedded in a stretching surface. Naveed et al. [27] calculated heat transfer and used the micropolar fluid to analyze the effects over a curved surface with thermal radiation.

A literature review has specified that copious studies are available relating to nanofluids with linear/nonlinear/exponential stretching surfaces but comparatively less research work is available highlighting curved stretched surfaces. This gets even narrower when we talk about the study of hybrid
nanoliquid with entropy optimization past curved surfaces. Therefore, our task here is to discuss hybrid nanoliquid flow comprising ferromagnetic nanoparticle, i.e., nickel–zinc ferrite (NiZnFe₂O₄), and the base fluid, i.e., ethylene glycol (C₂H₆O₂), over a curved surface with entropy optimization coating. The whole analysis was performed with added impressions of nonlinear thermal radiation with entropy optimization coatings. The analysis was supported by the convective heat and mass flux boundary conditions. Numerical solution of the envisioned model was obtained by utilizing bvp4c from MATLAB. The traits of the sundry parameters on involved distributions were thoroughly discussed keeping their physical justification in mind.

2. Mathematical Formulation

We considered a 2D steady, incompressible nanoliquid flow over a curved stretching channel looped in the form of a circle with a radius \( R \) about the curvilinear directions \( r \) and \( x \), as shown in Figure 1. Here, a higher value of \( R \) corresponds to a marginally curved surface. The stretching velocity is taken as \( u = \dot{u}_w \) along the \( x \)-direction. A magnetic field is applied normal to the fluid flow and along the \( r \)-direction. The electric and induced magnetic fields were overlooked owing to our assumption of small Reynolds number.

![Figure 1. Flow geometry.](image)

The assumed system is governed by the following equations:

\[
\frac{\partial}{\partial r} \{ (R+r)v \} + R \frac{\partial u}{\partial x} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial r} + \frac{Ru}{r+R} \frac{\partial u}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R} \frac{\partial u}{\partial r} \right) + \frac{1}{r+R} \frac{\partial u}{\partial r} - \frac{u}{(r+R)^2} \frac{\partial \tilde{u}}{\partial r} = \frac{\sigma}{\rho_{nf}} B_0^2 u \tag{2}
\]

\[
u \frac{\partial T}{\partial r} + \frac{Ru}{r+R} \frac{\partial T}{\partial x} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R} \frac{\partial T}{\partial r} \right) + \frac{Q_0}{(\rho C_p)_{nf}} (T_\infty - T) + \frac{1}{(\rho C_p)_{nf}} \frac{1}{r+R} \frac{\partial}{\partial r} (r+R) q_r \tag{3}
\]

\[
u \frac{\partial C}{\partial r} + \frac{Ru}{r+R} \frac{\partial C}{\partial x} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r+R} \frac{\partial C}{\partial r} \right) \tag{4}
\]

The system of Equations (1)–(5) is supported by the following boundary conditions:

\[
\begin{align*}
v |_{r=0} &= 0, \\ u |_{r=0} &= \dot{u}_w(x) = sx, \\ k_f \frac{\partial T}{\partial r} |_{r=0} &= h^* (T_f - T), \\ -D_B \frac{\partial C}{\partial r} |_{r=0} &= j_w \\
\end{align*}
\]

\[
\begin{align*}
u |_{r\to\infty} \to 0, \\ \frac{\partial u}{\partial r} |_{r\to\infty} \to 0, \\ T |_{r\to\infty} \to T_\infty, \\ C |_{r\to\infty} \to C_\infty
\end{align*}
\]

The thermophysical traits of the hybrid nanoliquid are appended in Table 1.

\[
\begin{array}{c|c|c|c|c|c}
\hline
\text{Parameter} & \text{Symbol} & \text{Value} \\
\hline
\text{Mass density} & \rho & \text{g/cm}^3 \\
\text{Dynamic viscosity} & \mu & \text{Pa·s} \\
\text{Heat capacity} & C_p & \text{J/(g·K)} \\
\text{Heat conductivity} & k & \text{W/(m·K)} \\
\text{Surface tension} & \sigma & \text{N/m} \\
\text{Magnetic permeability} & \mu_0 & \text{H/m} \\
\text{Magnetic field strength} & B_0 & \text{T} \\
\text{Magnetic susceptibility} & \chi & \\
\hline
\end{array}
\]
The mathematical form of thermophysical properties are given as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)\frac{2}{3\nu}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}$$  \hspace{1cm} (7)

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s$$ \hspace{1cm} (8)

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$ \hspace{1cm} (9)

In Equation (4), the nonlinear radiation heat flux term via Rosseland’s approximation is given as follows:

$$q_r = \frac{4\sigma^* \partial T^4}{3K^*} = \frac{16\sigma^* T^3 \partial T}{3K^*}$$ \hspace{1cm} (10)

3. Solution Procedure

Here, we used the following dimensionless transformations:

$$\zeta = \sqrt{\frac{K}{K_f}} r, \quad p = \rho_f s^2 \chi^2 P(\zeta), \quad T = T_\infty (1 + (\theta_w - 1)\theta),$$

$$C = C_\infty + \frac{\nu}{2\pi} \sqrt{\frac{K}{K_f}} h(\zeta), \quad u = sx f'(\zeta), \quad v = -\frac{R}{\sqrt{\pi}} \sqrt{\frac{K_f}{\nu}} f(\zeta)$$ \hspace{1cm} (11)

Here, prime denotes the derivative \(w, r, T, \zeta\) and \(\theta_w = T_1/T_\infty\). The above transformation Equation (11) satisfies Equation (1) identically and Equations (2)–(6) are given by the following:

$$P' = \left(1 - \phi + \phi \frac{\rho_s}{\rho_f}\right) \frac{f'^2}{\zeta + K_1}$$ \hspace{1cm} (12)

$$\frac{1}{(1 - \phi + \phi \frac{\rho_s}{\rho_f})^{2K_1}} P = \frac{1}{(1 - \phi + \phi \frac{\rho_s}{\rho_f})^{10/3}} \left(f'' - \frac{f'}{(K_1 + \zeta)^2} + \frac{f'''}{(K_1 + \zeta)^3}\right) \frac{K_1}{(K_1 + \zeta)^3} f'^2$$

$$+ \frac{K_1}{(K_1 + \zeta)^2} f'' + M f'$$ \hspace{1cm} (13)

$$\frac{1}{\nu} \left(\frac{k_{nf}}{K_f} + R_d(1 + (\theta_w - 1)\theta)^3\right) \left(\theta'' + \frac{1}{(K_1 + \zeta)^2} \theta\right) + \left(1 - \phi + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}\right) \frac{K_1}{(K_1 + \zeta)^2} f\theta' = 0$$ \hspace{1cm} (14)

$$h'' + \frac{1}{\zeta + K_1} h' + S_c \left(\frac{K_1}{(K_1 + \zeta)}\right) f h' = 0$$ \hspace{1cm} (15)

and

$$f(\zeta) = 0, \quad f'(\zeta) = 1, \quad \theta'(\zeta) = (1 - \theta(\zeta))Bi, \quad h'(\zeta) = -1, \quad \text{as} \quad \zeta = 0$$

$$f'(\zeta) \to 0, \quad f''(\zeta) \to 0, \quad \theta'(\zeta) \to 0, \quad h(\zeta) \to 0, \quad \text{as} \quad \zeta \to \infty$$ \hspace{1cm} (16)

Here, \(K_1 = R \sqrt{\frac{\pi}{\nu} Bi}, \quad S_c = \frac{\nu f'}{\nu f'}, \quad R_d = \frac{16\sigma^* T_\infty^3}{3K^*}, \quad \lambda_1 = \frac{\theta_0}{s(\rho C_p)_f}, \quad \text{and} \quad Pr = \frac{\nu f'}{\alpha f'}.
results and discussion

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5. Results and Discussion

The MATLAB built-in function bvp4c was applied to integrate the numerical solution for the system of Equations (14), (15), and (17) with initial and boundary conditions, Equations (16) and (18), for numerous values of \( K_1, M, R_d, \lambda_1, \) and \( S_c \) graphically. For this technique, we first changed differential equations with the higher order to the equations of order one by utilizing new variables. The function bvp4c needs an initial guess for the solution and with the tolerance of \( 10^{-7} \). The guess we chose needed to satisfy the boundary conditions (Equations (16) and (18)) and the solution. The validation of our presented results is depicted in Table 2. An excellent agreement with Sanni et al. [23] was observed when \( M = 1, \varphi = 0.0, \) and in the absence of temperature and concentration profile.
The values given to other parameters were $Pr = 10$, $Sc = 0.5$, $ϕ = 0.1$, $Rd = 0.5$, $θw = 0.5$, $λ1 = 0.5$, and $M = 0.1$. Further, the momentum and thickness of the thermal boundary layers were boosted with a larger value of $ϕ$. The values of other parameters were fixed as $Pr = 10$, $Sc = 0.5$, $ϕ = 0.1$, $Rd = 0.5$, $θw = 0.5$, $λ1 = 0.5$, and $M = 0.1$.

Figures 2 and 3 show the impression of solid volume fraction $ϕ$ on velocity and temperature profiles. Both velocity fields increased with increasing values of solid volume fraction $ϕ$. Further, the momentum and thickness of the thermal boundary layers were boosted with a larger value of $ϕ$. The values given to other parameters were $Pr = 10$, $Sc = 0.5$, $K_1 = 10$, $Rd = 0.5$, $θw = 0.5$, $λ1 = 0.5$, and $M = 0.1$.

<table>
<thead>
<tr>
<th>$K_1$</th>
<th>Sanni et al. [23]</th>
<th>Present Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.1576</td>
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<td>10</td>
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<td>20</td>
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<td>50</td>
<td>1.0140</td>
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</tr>
<tr>
<td>100</td>
<td>1.0070</td>
<td>1.00690</td>
</tr>
<tr>
<td>1000</td>
<td>1.0008</td>
<td>1.00079</td>
</tr>
</tbody>
</table>

Table 2. Comparison of presented results for skin friction coefficient $\frac{1}{2} C_f (Re)^{\frac{1}{2}}$ when $M = 1$ and $ϕ = 0.0$.

The impact of the curvature parameter $K_1$ on velocity, concentration, and temperature profiles are depicted in Figures 4–6. Increasing values of $K_1$ resulted in an increase in fluid velocity and temperature field, while the concentration profile diminished. This was because of the radius of the surface augment when curvature parameter $K_1$ was increased. As a result, the flow increased but it offered more resistance, therefore the temperature rose. The values of other parameters were fixed as $Pr = 10$, $Sc = 0.5$, $ϕ = 0.1$, $Rd = 0.5$, $θw = 0.5$, $λ1 = 0.5$ and $M = 0.1$. 

![Figure 2](image2.png)

Figure 2. Upshot of $ϕ$ on velocity distribution $f'(ζ)$.

![Figure 3](image3.png)

Figure 3. Upshot of $ϕ$ on temperature field $θ(ζ)$.
Figure 4. Upshot of $K_1$ on velocity profile $f'(\zeta)$.

Figure 5. Upshot of $K_1$ on temperature profile $\theta(\zeta)$.

Figure 6. Upshot of $K_1$ on concentration field $h(\zeta)$.

Figure 7 demonstrates the variation in the velocity field for numerous estimates of magnetic parameter $M$. Here, increments in $M$ led to a decline in the magnitude of fluid’s velocity. This was because of the resistive force (called Lorentz force) triggered by the magnetic field, which lowered the velocity of the fluid’s velocity flow. The values of the other parameters were fixed as $K_1 = 10$, $S_c = 0.5$, $\varphi = 0.1$, $R_d = 0.5$, $\theta_w = 0.5$, $\lambda_1 = 0.5$, and $Bi = 0.1$.
A quick augmentation in the Prandtl number Pr lessened the temperature and thickness of the thermal boundary layer. The temperature profile increased with increment in nonlinear radiation parameter. As Prandtl number is linked in a reciprocal way to the thermal diffusivity, a quick augmentation in the Prandtl number Pr lessened the temperature and thickness of the thermal boundary layer. The temperature profile increased with increment in nonlinear radiation parameter.

The characteristics of Biot number Bi and heat generation/absorption parameter $\lambda_1$ on temperature field are displayed in Figures 8 and 9, respectively. Figure 8 shows that the convective heat transfer coefficient intensified for higher estimates of Bi and the temperature subsequently rose. Figure 9 illustrates the behavior of $\lambda_1$. To increase the estimation values of heat absorption/generation parameter, the temperature profile and thermal boundary layer thickness were increased. The values of other parameters were fixed as $K_1 = 10$, $M = 0.3$, $\varphi = 0.5$, $R_d = 0.1$, $\theta_w = 0.5$, and $Pr = 10$.

Figures 10 and 11 show the impacts of nonlinear radiation parameter $R_d$ and Prandtl number Pr on temperature distribution, respectively. It can be seen that the temperature field fell with increasing Prandtl number Pr. As Prandtl number is linked in a reciprocal way to the thermal diffusivity, a quick augmentation in the Prandtl number Pr lessened the temperature and thickness of the thermal boundary layer. The temperature profile increased with increment in nonlinear radiation parameter $R_d$. Physically, the radiative heat flux increased with increasing values of $R_d$ which ultimately boosted
the temperature of the fluid. The values assigned to other parameters were \( K_1 = 10, M = 0.3, \varphi = 0.5, Bi = 0.1, \theta_w = 0.5, \) and \( S_c = 5. \)

![Figure 10. Upshot of \( R_d \) on temperature field \( \theta(\zeta) \).](image)

![Figure 11. Upshot of \( Pr \) on temperature field \( \theta(\zeta) \).](image)

The impression of Schmidt number \( S_c \) on concentration distribution is portrayed in Figure 12. A decrease in concentration field was detected with increasing values of \( S_c \). As the Schmidt number has a converse proportion with the Brownian diffusion coefficient, an increment in the \( S_c \) yielded a decay in Brownian diffusion coefficient that brought about a diminishment in concentration and its interrelated boundary layer thickness. The values allocated to other parameters were \( K_1 = 10, M = 0.3, \varphi = 0.1, Bi = 0.1, \theta_w = 0.5, \) and \( R_d = 0.1. \)

![Figure 12. Upshot of \( S_c \) on concentration profile \( h(\zeta) \).](image)

The influence of curvature parameter \( K_1 \) and magnetic parameter \( M \) on skin friction coefficient \( -\frac{1}{2}C_f\text{Re}_v^{1/2} \) is depicted in Figure 13. It can be noticed that the surface drag force diminished with increasing value of \( K_1 \). A contradictory trend was demonstrated in case of \( M \). In Figure 14,
the consequence of magnetic parameter $M$ and solid volume fraction $\varphi$ on shear wall stress is demonstrated. The skin friction profile rose with increase in magnetic parameter $M$ and solid volume fraction $\varphi$ for fixed values of parameters $Pr = 10$, $S_c = 0.5$, $R_d = 0.5$, $\theta_w = 0.5$, $\lambda_1 = 0.5$ and $Bi = 0.1$.

Figure 13. Upshot of $K_1$ and $M$ on wall shear stress $-\frac{1}{2} C_f Re_x^{1/2}$.

Figure 14. Upshot of $M$ and $\varphi$ on wall shear stress $-\frac{1}{2} C_f Re_x^{1/2}$.

Figure 15 shows the effect of Biot number $Bi$ and solid volume fraction $\varphi$ on Nusselt number $Nu_x (Re_x)^{-\frac{1}{2}}$. It was detected that for higher value of $Bi$ and $\varphi$, the surface heat transfer rate upsurged when values of parameters were given as $K_1 = 10$, $M = 0.3$, $S_c = 5.0$, $R_d = 0.1$, $\theta_w = 0.5$, $\lambda_1 = 0.5$ and $Pr = 10$.

Figure 15. Upshot of $Bi$ and on $\varphi$ Nusselt number $Nu Re_x^{-1/2}$.

The outcome of curvature parameter $K_1$ and nonlinear radiation parameter $R_d$ on Nusselt number $Nu_x (Re_x)^{-\frac{1}{2}}$ is examined in Figure 16. Here, a reduction in Nusselt number was noted for increasing values of curvature parameter $K_1$ and the opposite trend was seen for nonlinear radiation parameter $R_d$ for fixed values of $\varphi = 0.1$, $M = 0.3$, $S_c = 5.0$, $\theta_w = 0.5$, $\lambda_1 = 0.5$ and $Pr = 10$. 

The skin friction profile rose with increase in magnetic parameter $M$ and solid volume fraction $\varphi$ for fixed values of parameters $Pr = 10$, $S_c = 0.5$, $R_d = 0.5$, $\theta_w = 0.5$, $\lambda_1 = 0.5$ and $Bi = 0.1$. 

The reason behind this was that more heat was generated between the layers of the fluid because of augmented values of (Br). Figure 18 displays the relationship between the magnetic parameter (M) and the entropy generation. Again, the same trait as depicted in case of (Br) was witnessed here. Higher values of (M) meant stronger Lorentz force and ultimate strengthening of the dissipation energy, and this was the main cause of irreversibility.

Table 3 shows the behavior of Sherwood number $Sh_x (Re_x)^{-1/2}$ for varied values of $S_c$ (Schmidt number), $K_1$ (curvature parameter), and $M$ (magnetic parameter). It can be seen that for snowballing values of $S_c$, the Sherwood number $Sh_x (Re_x)^{-1/2}$ increased; however, it diminished for increasing value of $K_1$ and $M$. 

**Figure 16.** Upshot of $K_1$ and $R_d$ on Nusselt number $NuRe_x^{-1/2}$.

**Figure 17.** Upshot of $Br$ on $NG (\eta)$.

**Figure 18.** Upshot of $M$ on $NG (\eta)$. 

**Figure 16.** Upshot of $K_1$ and $R_d$ on Nusselt number $NuRe_x^{-1/2}$.
Table 3. Numerical value of Sherwood number $S_h \left( R_e x \right)^{-1/2}$ for various value of parameter with fixed value of $Pr = 10$, $R_d = 0.1$, $\varphi = 0.1$, $Bi = 0.1$, $\theta_w = 0.5$, $\lambda_1 = 0.5 f'(Q)$.

<table>
<thead>
<tr>
<th>$S_c$</th>
<th>$K_1$</th>
<th>$M$</th>
<th>$S_h \left( R_e x \right)^{-1/2}$</th>
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6. Concluding Remarks

In this paper, the flow of nanoliquid comprising nickel–zinc ferrite–ethylene glycol (NiZnFe$_2$O$_4$–C$_2$H$_6$O$_2$) accompanied by entropy optimization coating past a curved stretching surface with convective heat and mass flux boundary was examined. The solution of the envisioned system of equations was found numerically by applying MATLAB built-in function `bvp4c`. The impact of numerous arising parameters on involved profiles was depicted via graphical illustrations with requisite discussions.

The conclusions of the current study are as follows:

- An increase in curvature parameter accounted for increasing velocity and temperature fields and diminishing concentration distribution.
- Under the considerable influence of magnetic parameter, an increased axial velocity field was attained.
- For the increasing estimates of the solid volume fraction, the temperature and velocity profiles showed increasing behavior.
- The temperature profile improved with increasing values of Biot number and heat generation/absorption parameter.
- The value of friction factor profile augmented for larger values of $M$ and $\varphi$, but it decreased for $K_1$ and $M$.
- The Nusselt number $Nu R_e^{-1/2}$ declined with increasing values of $K_1$ and $R_d$.


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Nomenclature

- $u, v$: velocity components
- $r, x$: coordinate
- $R$: radius of circle
- $P$: pressure
- $B_0$: strength of magnetic field
- $Q_0$: volumetric rate of heat generation/absorption
\( q_r \) nonlinear radiative heat flux
\( T, T_\infty \) temperature
\( C, C_{w}, C_{\infty} \) concentration
\( D_B \) Brownian diffusion coefficient
\( T_f \) convective temperature at the sheet
\( j_w \) mass flux
\( u_w \) stretching velocity along \( x \)-direction
\( k^* \) mean absorption coefficient
\( k_1 \) curvature parameter
\( M \) magnetic parameter
\( S \) positive stretching constant
\( S_c \) Schmidt number
\( R_d \) radiation parameter
\( h^* \) convective heat transfer coefficient
\( C_f \) surface drag force
\( Sh_e \) Sherwood number
\( Re_x \) local Reynolds number
\( E_{\text{gen}}^{\text{vir}} \) volumetric rate of local entropy generation
\( E_0^{\text{vir}} \) characteristic entropy generation rate
\( N_G \) entropy generation
\( Br \) Brinkman number
\( Bi \) Biot number
\( Pr \) Prandtl number
\( Nu_{w} \) Nusselt number
\( C_s \) heat capacity of surface

**Greek Symbols**
\( \rho \) density
\( \nu \) dynamic viscosity
\( \sigma \) electrical conductivity
\( \sigma_{nf} \) modified thermal diffusivity
\( (\rho C_p)_{nf}, (\rho C_p)_{f} \) heat capacity
\( k_f, k_{nf}, k \) thermal conductivity
\( \phi \) solid volume fraction of nanofluid
\( \zeta \) a scaled boundary-layer coordinate
\( \sigma^* \) Stefan–Boltzmann constant
\( \theta_w \) temperature difference
\( \lambda_1 \) heat generation parameter
\( f \) dimensionless stream function,
\( \theta \) dimensionless temperature
\( \tau_{rx} \) wall’s shear stress
\( \Sigma \) a constant parameter
\( \alpha \) dimensionless temperature difference

**Subscripts**
\( w \) for wall surface
\( nf \) for the nanofluid
\( f \) for the base fluid
\( s \) for the solid (nanoparticles)
\( \infty \) use for ambient
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