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Coupled Least Squares Identification Algorithms for Multivariate Output-Error Systems

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Abstract: This paper focuses on the recursive identification problems for a multivariate output-error system. By decomposing the system into several subsystems and by forming a coupled relationship between the parameter estimation vectors of the subsystems, two coupled auxiliary model based recursive least squares (RLS) algorithms are presented. Moreover, in contrast to the auxiliary model based recursive least squares algorithm, the proposed algorithms provide a reference to improve the identification accuracy of the multivariate output-error system. The simulation results confirm the effectiveness of the proposed algorithms.

Keywords: coupling identification concept; parameter estimation; auxiliary model; least squares; multivariate system

1. Introduction

Multivariable systems are popular in industrial processes [1–3] and a number of successful methods have been developed to solve the identification and control problems of multivariable systems [4–7]. For example, Zhang and Hoagg used a candidate-pool approach to identify the feedback and feedforward transfer function matrices and presented a frequency-domain technique for identifying multivariable feedback and feedforward systems [8]; Salhi and Kamoun proposed a recursive algorithm to estimate the parameters of the dynamic linear part and the static nonlinear part of multivariable Hammerstein systems [9].

The idea of the auxiliary model is to use the measurable information to construct a dynamical model and to replace the unknown variables in the information vector with the output of the auxiliary model [10,11]. There are two typical identification methods for multivariate output-error systems: stochastic gradient (SG) algorithms [12,13] and the recursive least squares (RLS) algorithms [14,15]. The SG algorithm requires lower computational cost, but the RLS algorithm has a faster convergence rate than the SG algorithm [16]. The RLS algorithm has been applied to the identification of various systems [17,18]. For example, on the basis of the work in [19], Jin et al. proposed an auxiliary model based recursive least squares algorithm for autoregressive output-error autoregressive systems [20]; and Wang and Tang presented an auxiliary model based recursive least squares algorithm for a class of linear-in-parameter output-error moving average systems [21].

Although the RLS algorithm can be applied to identify the parameter of the multivariate output-error systems, it requires computing the matrix inversion (see Remark 1 in the following section), resulting in a large computational burden [22]. This motivates us to study a new coupled least squares algorithm without involving matrix inversion. The coupling identification concept is useful for simplifying the parameter estimation of the coupled parameter multivariable systems [23]. It is based on the coupled relationship of the parameter estimates between the subsystems of a multivariable system [24–26]. The purpose of the coupling identification is to reduce the redundant estimation of the subsystem parameter vectors and to avoid computing the matrix inversion of the RLS algorithm. Recently, a coupled least squares algorithm has been proposed for multiple linear regression systems [22].

This paper focuses on the parameter estimation of multivariate output-error systems, and the main contributions of this paper are the following:

- for multivariate output-error systems, this paper derives two coupled least squares parameter estimation algorithms by using the auxiliary model identification idea and the coupling identification concept;
- the proposed algorithms can generate more accurate parameter estimates, and avoid computing the matrix inversion in the multivariable RLS algorithm, for the purpose of reducing computational load.

The rest of this paper is organized as follows: Section 2 gives some definitions and the identification model of multivariate output-error systems. Section 3 presents two new coupled auxiliary model identification algorithms. Section 4 gives two simulation examples to validate the effectiveness of the proposed methods. Finally, some concluding remarks are offered in Section 5.

2. System Description and Identification Model

Let us introduce some notation. The symbol $\boldsymbol{I}_m$ is an $m \times m$ identity matrix; $\mathbf{1}_n$ is an n-dimensional column vector whose elements are 1; the superscript T denotes the matrix transpose; the norm of the matrix $\boldsymbol{X}$ is defined as $\|\boldsymbol{X}\|^2 := \mathrm{tr}[\boldsymbol{X}\boldsymbol{X}^{\mathrm{T}}]$; the symbol $\otimes$ denotes the Kronecker product or the direct product: if $\boldsymbol{A} = [a_{ij}] \in \mathbb{R}^{m\times n}$, $\boldsymbol{B} = [b_{ij}] \in \mathbb{R}^{p\times q}$, then $\boldsymbol{A} \otimes \boldsymbol{B} = [a_{ij}\boldsymbol{B}] \in \mathbb{R}^{mp\times nq}$; $\mathrm{col}[\boldsymbol{X}]$ denotes the vector formed by the column of the matrix $\boldsymbol{X}$, that is, if $\boldsymbol{X} := [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n] \in \mathbb{R}^{m\times n}$, then $\mathrm{col}[\boldsymbol{X}] = [\boldsymbol{x}_1^{\mathrm{T}}, \boldsymbol{x}_2^{\mathrm{T}}, \cdots, \boldsymbol{x}_n^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{mn}$. $\hat{\boldsymbol{X}}(t)$ denotes the estimate of $\boldsymbol{X}$ at time t and $\tilde{\boldsymbol{X}}(t) := \hat{\boldsymbol{X}}(t) - \boldsymbol{X}$ denotes the estimation error.

Consider the following multivariate output-error system:

$$\boldsymbol{y}(t) = \boldsymbol{x}(t) + \boldsymbol{v}(t), \tag{1}$$

$$\boldsymbol{x}(t) := \frac{\boldsymbol{\Phi}_s(t)\boldsymbol{\theta}}{A(z)} \in \mathbb{R}^m, \tag{2}$$

$$A(z) := 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \in \mathbb{R}, \tag{3}$$

where $\boldsymbol{y}(t) := [y_1(t), y_2(t), \cdots, y_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ is the system output vector and the noisy measurement of $\boldsymbol{x}(t)$, $\boldsymbol{\Phi}_s(t) \in \mathbb{R}^{m\times n}$ is the information matrix consisting of the input–output data, $\boldsymbol{\theta} \in \mathbb{R}^n$ is the parameter vector, and $\boldsymbol{v}(t) := [v_1(t), v_2(t), \cdots, v_m(t)]^{\mathrm{T}} \in \mathbb{R}^m$ is the observation noise vector with zero mean, and z^{-1} is a unit backward shift operator with $[z^{-1}\boldsymbol{y}(t) = \boldsymbol{y}(t-1)]$.

Assume that the degrees m, n, n_a are known and when $t \leq 0$, $\boldsymbol{y}(t) = 0$, $\boldsymbol{\Phi}_s(t) = 0$ and $\boldsymbol{v}(t) = 0$. $\{\boldsymbol{\Phi}_s(t), \boldsymbol{y}(t)\}$ is the available measurement data.

Equations (1) and (2) can be expressed as

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}_x(t)\boldsymbol{a} + \boldsymbol{\Phi}_s(t)\boldsymbol{\theta}, \tag{4}$$

$$\boldsymbol{y}(t) = \boldsymbol{\Phi}(t)\boldsymbol{\vartheta} + \boldsymbol{v}(t), \tag{5}$$

$$\begin{aligned}
\boldsymbol{\Phi}_x(t) &:= [-\boldsymbol{x}(t-1), -\boldsymbol{x}(t-2), \cdots, -\boldsymbol{x}(t-n_a)] \in \mathbb{R}^{m\times n_a},\\
\boldsymbol{\Phi}(t) &:= [\boldsymbol{\Phi}_x(t), \boldsymbol{\Phi}_s(t)] \in \mathbb{R}^{m\times n_0},\\
\boldsymbol{a} &:= [a_1, a_2, \cdots, a_{n_a}]^{\mathrm{T}} \in \mathbb{R}^{n_a},\\
\boldsymbol{\vartheta} &:= \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{\theta} \end{bmatrix} \in \mathbb{R}^{n_0}, \quad n_0 := n_a + n.
\end{aligned}$$

Let $\hat{\boldsymbol{\vartheta}}(t) := [\hat{\boldsymbol{a}}^{\mathrm{T}}(t), \hat{\boldsymbol{\theta}}^{\mathrm{T}}(t)] \in \mathbb{R}^{n_0}$ be the estimate of $\boldsymbol{\vartheta}$ at time t.

For the identification model in (5), $\mathbf{\Phi}_x(t)$ is the information matrix that consists of the unknown inner variables $x(t-j)$'s, so we construct an auxiliary model $\boldsymbol{x}_{\mathrm{a}}(t)$, and define the estimate of $\mathbf{\Phi}_x(t)$ as

$$\hat{\mathbf{\Phi}}_x(t) := [-\boldsymbol{x}_{\mathrm{a}}(t-1), -\boldsymbol{x}_{\mathrm{a}}(t-2), \cdots, -\boldsymbol{x}_{\mathrm{a}}(t-n_a)] \in \mathbb{R}^{m \times n_a}.$$

Then, we use $\hat{\mathbf{\Phi}}_x(t)$ and $\mathbf{\Phi}_s(t)$ to construct the estimate of $\mathbf{\Phi}(t)$ as

$$\hat{\mathbf{\Phi}}(t) := [\hat{\mathbf{\Phi}}_x(t), \mathbf{\Phi}_s(t)] \in \mathbb{R}^{m \times n_0}.$$

Thus, according to (4), we can obtain the auxiliary model,

$$\begin{aligned}\boldsymbol{x}_{\mathrm{a}}(t) &= \hat{\mathbf{\Phi}}_x(t)\hat{\boldsymbol{a}}(t) + \mathbf{\Phi}_s(t)\hat{\boldsymbol{\theta}}(t)\\ &= \hat{\mathbf{\Phi}}(t)\hat{\boldsymbol{\vartheta}}(t).\end{aligned}$$

The objective of this paper is to use the auxiliary model identification idea and the coupling identification concept to derive new methods for estimating the system parameters $\boldsymbol{\theta}, a_1, a_2, \cdots, a_{n_a}$ from the observation data $\{\boldsymbol{y}(t), \mathbf{\Phi}_s(t)\}$ and to confirm the theoretical results with simulation examples.

3. The Multivariate Auxiliary Model Coupled Identification Algorithm

3.1. The Auxiliary Model Based Recursive Least Squares Algorithm

According to the identification model in (5), define a cost function:

$$J_1(\boldsymbol{\vartheta}) := \sum_{j=1}^{t} \|\boldsymbol{y}(j) - \mathbf{\Phi}(j)\boldsymbol{\vartheta}\|^2.$$

Based on the auxiliary model identification idea and on the derivation of the RLS algorithm [27,28], we use the output $\boldsymbol{x}_{\mathrm{a}}(t)$ as the unknown inner vector $\boldsymbol{x}(t)$ and replace the unknown information matrix $\mathbf{\Phi}(t)$ with its estimate $\hat{\mathbf{\Phi}}(t)$, and obtain the following auxiliary model based recursive least squares (AM-RLS) algorithm:

$$\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}(t)[\boldsymbol{y}(t) - \hat{\mathbf{\Phi}}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{6}$$

$$\boldsymbol{L}(t) = \boldsymbol{P}(t-1)\hat{\mathbf{\Phi}}^{\mathrm{T}}(t)[\boldsymbol{I}_m + \hat{\mathbf{\Phi}}(t)\boldsymbol{P}(t-1)\hat{\mathbf{\Phi}}^{\mathrm{T}}(t)]^{-1}, \tag{7}$$

$$\boldsymbol{P}(t) = \boldsymbol{P}(t-1) - \boldsymbol{L}(t)\hat{\mathbf{\Phi}}(t)\boldsymbol{P}(t-1), \tag{8}$$

$$\hat{\mathbf{\Phi}}(t) = [\hat{\mathbf{\Phi}}_x(t), \mathbf{\Phi}_s(t)], \tag{9}$$

$$\hat{\mathbf{\Phi}}_x(t) = [-\boldsymbol{x}_{\mathrm{a}}(t-1), -\boldsymbol{x}_{\mathrm{a}}(t-2), \cdots, -\boldsymbol{x}_{\mathrm{a}}(t-n_a)], \tag{10}$$

$$\boldsymbol{x}_{\mathrm{a}}(t) = \hat{\mathbf{\Phi}}(t)\hat{\boldsymbol{\vartheta}}(t). \tag{11}$$

The steps of computing the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ by the AM-RLS algorithm are listed in the following:

1. Set the initial values: $t = 1$, $\hat{\boldsymbol{\vartheta}}(0) = \mathbf{1}_{n_0}/p_0$, $\boldsymbol{P}(0) = p_0\boldsymbol{I}_{n_0}$, $\boldsymbol{x}_{\mathrm{a}}(t-j) = \mathbf{1}_m/p_0$, $j = 1, 2, \cdots, n_a$, $p_0 = 10^6$. Set the data length L.
2. Collect the observation data $\{\mathbf{\Phi}_s(t), \boldsymbol{y}(t)\}$ and form the information matrix $\hat{\mathbf{\Phi}}_x(t)$ by (10).
3. Form $\hat{\mathbf{\Phi}}(t)$ by (9), and compute $\boldsymbol{L}(t)$ by (7) and $\boldsymbol{P}(t)$ by (8).
4. Update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ by (6).
5. Compute the output $\boldsymbol{x}_{\mathrm{a}}(t)$ of the auxiliary model using (11).
6. If $t = L$, stop the recursive computation and obtain the parameter estimates; otherwise, increase t by 1 and go to Step 2.

Remark 1. *For the multivariable RLS algorithm in (6)–(10), we can see from (7) that it requires computing the matrix inversion* $[\boldsymbol{I}_m + \hat{\boldsymbol{\Phi}}(t)\boldsymbol{P}(t-1)\hat{\boldsymbol{\Phi}}^{\mathrm{T}}(t)]^{-1} \in \mathbb{R}^{m\times m}$ *at each step, resulting in heavy computational load, especially for large m (the number of outputs). This is the drawback of the multivariable RLS algorithm in (6)–(10). This motivates us to study new coupled parameter identification methods.*

3.2. The Coupled Subsystem Auxiliary Model Based Recursive Least Squares Algorithm

The coupling identification is usually used to reduce the redundant estimates of the system parameter vectors, based on the coupled relationship of the parameter estimates between subsystems [22].

Let $\boldsymbol{\phi}_i^{\mathrm{T}}(t) \in \mathbb{R}^{1\times n_0}$ be the ith row of the information matrix $\boldsymbol{\Phi}(t)$, i.e.,

$$\boldsymbol{\Phi}(t) := \begin{bmatrix} \boldsymbol{\phi}_1^{\mathrm{T}}(t) \\ \boldsymbol{\phi}_2^{\mathrm{T}}(t) \\ \vdots \\ \boldsymbol{\phi}_m^{\mathrm{T}}(t) \end{bmatrix} \in \mathbb{R}^{m\times n_0}. \tag{12}$$

From (5), we obtain m identification models (subsystems)

$$y_i(t) = \boldsymbol{\phi}_i^{\mathrm{T}}(t)\boldsymbol{\vartheta} + v_i(t), \quad i = 1, 2, \cdots, m. \tag{13}$$

From here, all subsystems contain a common parameter vector $\boldsymbol{\vartheta} \in \mathbb{R}^{n_0}$. In general, one of the subsystems can be used to identify the parameter vector $\boldsymbol{\vartheta}$; however, in order to improve the parameter estimation precision, we should make full use of the information in all subsystems for identifying $\boldsymbol{\vartheta}$.

Based on the RLS algorithm in (6)–(11), and applying the auxiliary model idea, we replace the unknown variables $\boldsymbol{\phi}_i(t)$ in the identification algorithm with their estimates $\hat{\boldsymbol{\phi}}_i(t)$, and obtain m RLS algorithms from (13), namely, the subsystem recursive least squares (S-RLS) algorithm,

$$\hat{\boldsymbol{\vartheta}}_i(t) = \hat{\boldsymbol{\vartheta}}_i(t-1) + \boldsymbol{L}_i(t)[y_i(t) - \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}_i(t-1)], \quad \hat{\boldsymbol{\vartheta}}_i(0) = \mathbf{1}_{n_0}/p_0, \tag{14}$$

$$\boldsymbol{L}_i(t) = \boldsymbol{P}_i(t)\hat{\boldsymbol{\phi}}_i(t) = \boldsymbol{P}_i(t-1)\hat{\boldsymbol{\phi}}_i(t)[1 + \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\phi}}_i(t)]^{-1}, \tag{15}$$

$$\boldsymbol{P}_i(t) = [\boldsymbol{I}_{n_0} - \boldsymbol{L}_i(t)\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_i(t-1), \quad \boldsymbol{P}_i(0) = p_0\boldsymbol{I}_{n_0}, \quad i = 1, 2, \cdots, m. \tag{16}$$

From here, we can see that there is no coupled relationship between the subsystem parameter estimation vector $\hat{\boldsymbol{\vartheta}}_i(t)$.

Remark 2. *For* $i = 1, 2, \cdots, m$*, we can obtain m estimation vectors* $\hat{\boldsymbol{\vartheta}}_i(t)$ *from (14)–(16), and they are all the estimates of the common parameter vector* $\boldsymbol{\vartheta}$ *in all subsystems, resulting in a large amount of redundant parameter estimates. One way is to use their average as the estimate of* $\boldsymbol{\vartheta}$*, that is*

$$\hat{\boldsymbol{\vartheta}}(t) := \frac{\hat{\boldsymbol{\vartheta}}_1(t) + \hat{\boldsymbol{\vartheta}}_2(t) + \cdots + \hat{\boldsymbol{\vartheta}}_m(t)}{m} \in \mathbb{R}^{n_0}. \tag{17}$$

If we regard the parameter estimate $\hat{\boldsymbol{\vartheta}}(t)$ in (17) as the output parameter vector, then each S-RLS identification algorithm is still independent. According to the coupling identification concept, we use $\hat{\boldsymbol{\vartheta}}(t-1)$ to replace $\hat{\boldsymbol{\vartheta}}_i(t-1)$ in the S-RLS algorithm, and get the coupled subsystem AM-RLS (C-S-AM-RLS) algorithm:

$$\hat{\boldsymbol{\vartheta}}_i(t) = \hat{\boldsymbol{\vartheta}}(t-1) + \boldsymbol{L}_i(t)[y_i(t) - \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}(t-1)], \tag{18}$$

$$\boldsymbol{L}_i(t) = \boldsymbol{P}_i(t)\hat{\boldsymbol{\phi}}_i(t) = \boldsymbol{P}_i(t-1)\hat{\boldsymbol{\phi}}_i(t)[1 + \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_i(t-1)\hat{\boldsymbol{\phi}}_i(t)]^{-1}, \tag{19}$$

$$\boldsymbol{P}_i(t) = [\boldsymbol{I}_{n_0} - \boldsymbol{L}_i(t)\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_i(t-1), \quad i = 1, 2, \cdots, m, \tag{20}$$

$$\hat{\boldsymbol{\vartheta}}(t) = \frac{\hat{\boldsymbol{\vartheta}}_1(t) + \hat{\boldsymbol{\vartheta}}_2(t) + \cdots + \hat{\boldsymbol{\vartheta}}_m(t)}{m}, \tag{21}$$

$$\hat{\boldsymbol{\Phi}}_x(t) = [-\boldsymbol{x}_{\mathrm{a}}(t-1), -\boldsymbol{x}_{\mathrm{a}}(t-2), \cdots, -\boldsymbol{x}_{\mathrm{a}}(t-n_a)], \tag{22}$$

$$\hat{\boldsymbol{\Phi}}(t) = [\hat{\boldsymbol{\Phi}}_x(t), \boldsymbol{\Phi}_s(t)] \tag{23}$$

$$= [\hat{\boldsymbol{\phi}}_1(t), \hat{\boldsymbol{\phi}}_2(t), \cdots, \hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \tag{24}$$

$$\boldsymbol{x}_{\mathrm{a}}(t) = \hat{\boldsymbol{\Phi}}(t)\hat{\boldsymbol{\vartheta}}(t). \tag{25}$$

The steps of computing the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t)$ by the C-S-AM-RLS algorithm in (18)–(25) are listed in the following:

1. Set the initial values: $t = 1$, $\hat{\boldsymbol{\vartheta}}(0) = \mathbf{1}_{n_0}/p_0$, $\boldsymbol{P}_i(0) = p_0\boldsymbol{I}_{n_0}$, $\boldsymbol{x}_{\mathrm{a}}(t-j) = \mathbf{1}_m/p_0$, $j = 1, 2, \cdots, n_a$, $p_0 = 10^6$. Set the data length L.
2. Collect the observation data $\{\boldsymbol{\Phi}_s(t), \boldsymbol{y}(t)\}$ and form the information matrix $\hat{\boldsymbol{\Phi}}_x(t)$ by (22).
3. Form $\hat{\boldsymbol{\Phi}}(t)$ by (23) and read $\hat{\boldsymbol{\phi}}_i(t)$ from $\hat{\boldsymbol{\Phi}}(t)$ in (24).
4. For each i, $i = 1, 2, \cdots, m$, compute $\boldsymbol{L}_i(t)$ by (19), and $\boldsymbol{P}_i(t)$ by (20), and update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}_i(t)$ by (18).
5. Compute $\hat{\boldsymbol{\vartheta}}(t)$ by (21) and $\boldsymbol{x}_{\mathrm{a}}(t)$ by (25).
6. If $t = L$, stop the recursive computation and obtain the parameter estimates; otherwise, increase t by 1 and go to Step 2.

Remark 3. *The C-S-AM-RLS algorithm in (18)–(25) uses the estimate $\hat{\boldsymbol{\vartheta}}(t-1)$ on the right-hand side of (18) instead of $\hat{\boldsymbol{\vartheta}}_i(t-1)$ on the right-hand side of (14) for $i = 1, 2, \cdots, m$. Thus, the C-S-AM-RLS algorithm is different from the S-RLS algorithm.*

3.3. The Coupled Auxiliary Model Based Recursive Least Squares Algorithm

In order to avoid the redundant parameter estimates, we use the coupling identification concept to derive a coupled AM-RLS algorithm based on the C-S-AM-RLS algorithm.

Referring to the partially coupled SG identification method [24], and with the help of the Jacobi or Gauss–Seidel iterative algorithm, replacing $\hat{\boldsymbol{\vartheta}}_m(t-1)$ with $\hat{\boldsymbol{\vartheta}}_1(t-1)$ for $i = 1$, replacing $\hat{\boldsymbol{\vartheta}}_i(t-1)$ with $\hat{\boldsymbol{\vartheta}}_{i-1}(t)$ for $i = 2, 3, \cdots, m$, we can obtain the following coupled auxiliary model based recursive least squares (C-AM-RLS) identification algorithm:

$$\hat{\boldsymbol{\vartheta}}_1(t) = \hat{\boldsymbol{\vartheta}}_m(t-1) + \boldsymbol{L}_1(t)[y_1(t) - \hat{\boldsymbol{\phi}}_1^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}_m(t-1)], \tag{26}$$

$$\boldsymbol{L}_1(t) = \boldsymbol{P}_m(t-1)\hat{\boldsymbol{\phi}}_1(t)[1 + \hat{\boldsymbol{\phi}}_1^{\mathrm{T}}(t)\boldsymbol{P}_m(t-1)\hat{\boldsymbol{\phi}}_1(t)]^{-1}, \tag{27}$$

$$\boldsymbol{P}_1(t) = [\boldsymbol{I}_{n_0} - \boldsymbol{L}_1(t)\hat{\boldsymbol{\phi}}_1^{\mathrm{T}}(t)]\boldsymbol{P}_m(t-1), \tag{28}$$

$$\hat{\boldsymbol{\vartheta}}_i(t) = \hat{\boldsymbol{\vartheta}}_{i-1}(t) + \boldsymbol{L}_i(t)[y_i(t) - \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\hat{\boldsymbol{\vartheta}}_{i-1}(t)], \tag{29}$$

$$\boldsymbol{L}_i(t) = \boldsymbol{P}_{i-1}(t)\hat{\boldsymbol{\phi}}_i(t)[1 + \hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)\boldsymbol{P}_{i-1}(t)\hat{\boldsymbol{\phi}}_i(t)]^{-1}, \tag{30}$$

$$\boldsymbol{P}_i(t) = [\boldsymbol{I}_{n_0} - \boldsymbol{L}_i(t)\hat{\boldsymbol{\phi}}_i^{\mathrm{T}}(t)]\boldsymbol{P}_{i-1}(t), \quad i = 2, 3, \cdots, m, \tag{31}$$

$$\hat{\boldsymbol{\Phi}}_x(t) = [-\boldsymbol{x}_{\mathrm{a}}(t-1), -\boldsymbol{x}_{\mathrm{a}}(t-2), \cdots, -\boldsymbol{x}_{\mathrm{a}}(t-n_a)], \tag{32}$$

$$\hat{\boldsymbol{\Phi}}(t) = [\hat{\boldsymbol{\Phi}}_x(t), \boldsymbol{\Phi}_s(t)] \tag{33}$$

$$= [\hat{\boldsymbol{\phi}}_1(t), \hat{\boldsymbol{\phi}}_2(t), \cdots, \hat{\boldsymbol{\phi}}_m(t)]^{\mathrm{T}}, \tag{34}$$

$$\boldsymbol{x}_{\mathrm{a}}(t) = \hat{\boldsymbol{\Phi}}(t)\hat{\boldsymbol{\vartheta}}(t). \tag{35}$$

In the above algorithm in (26)–(35), $\hat{\boldsymbol{\vartheta}}_i(t) \in \mathbb{R}^{n_0}$ is the parameter estimation vector of the ith subsystem at time t, $\boldsymbol{L}_i(t) \in \mathbb{R}^{n_0}$ is the gain vector of the ith subsystem at time t, $\boldsymbol{P}_i(t) \in \mathbb{R}^{n_0 \times n_0}$ is the covariance matrix of the ith subsystem at time t. $\hat{\boldsymbol{\vartheta}}_{i-1}(t)$ and $\boldsymbol{P}_{i-1}(t)$ are the parameter estimation vector and the covariance matrix of the $(i-1)$th subsystem at time t, respectively; $\hat{\boldsymbol{\vartheta}}_m(t-1)$ and $\boldsymbol{P}_m(t-1)$ are the parameter estimation vector and the covariance matrix of the mth subsystem at time $t-1$, respectively, the system parameter estimation vector is defined by the parameter estimation vector of the mth subsystem at time t: $\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}_m(t)$.

The procedure of computing the parameter estimation vector $\hat{\boldsymbol{\vartheta}}_m(t)$ in (26)–(35) is as follows.

1. Set the initial values: $t = 1$, $\hat{\boldsymbol{\theta}}_m(0) = \mathbf{1}_{n_0}/p_0$, $\boldsymbol{P}_m(0) = p_0 \boldsymbol{I}_{n_0}$, $\boldsymbol{x}_{\rm a}(t-j) = \mathbf{1}_m/p_0$, $j = 1, 2, \cdots, n_a$, $p_0 = 10^6$. Set the data length L.
2. Collect the observation data $\boldsymbol{y}(t)$ and $\boldsymbol{\Phi}_s(t)$, and construct $\hat{\boldsymbol{\Phi}}_x(t)$ and $\hat{\boldsymbol{\Phi}}(t)$ by (32) and (33).
3. Read $\hat{\boldsymbol{\phi}}_i(t)$ from $\hat{\boldsymbol{\Phi}}(t)$ in (34), compute $\boldsymbol{L}_1(t)$ and $\boldsymbol{P}_1(t)$ by (27) and (28), and update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}_1(t)$ by (26).
4. For $i = 2, 3, \cdots, m$, compute $\boldsymbol{L}_i(t)$ and $\boldsymbol{P}_i(t)$ by (30) and (31), and update the parameter estimation vector $\hat{\boldsymbol{\vartheta}}_i(t)$ by (29).
5. Obtain the parameter estimation vector $\hat{\boldsymbol{\vartheta}}(t) = \hat{\boldsymbol{\vartheta}}_m(t)$ and compute $\boldsymbol{x}_{\rm a}(t)$ by (35).
6. If $t = L$, stop the recursive computation and obtain the parameter estimates; otherwise, increase t by 1 and go to Step 2.

Remark 4. *The C-AM-RLS algorithm in (26)–(35) uses the estimate $\hat{\boldsymbol{\vartheta}}_{i-1}(t)$ on the right-hand side of (29) instead of $\hat{\boldsymbol{\vartheta}}_i(t-1)$ on the right-hand side of (14) for $i = 2, 3, \cdots, m$. When computing $\hat{\boldsymbol{\vartheta}}_1(t)$, the C-AM-RLS algorithm uses the estimate $\hat{\boldsymbol{\vartheta}}_m(t-1)$ on the right-hand side of (26) instead of $\hat{\boldsymbol{\vartheta}}_i(t-1)$ on the right-hand side of (14) with $i = 1$. Thus, the C-AM-RLS algorithm is different from the S-RLS algorithm.*

4. Examples

Example 1. *Consider the following multivariate output-error system:*

$$
\begin{aligned}
\boldsymbol{y}(t) &= \frac{\boldsymbol{\Phi}_s(t)\boldsymbol{\theta}}{A(z)} + \boldsymbol{v}(t), \\
\boldsymbol{\Phi}_s(t) &= \begin{bmatrix} y_1(t-2)u_2(t-2) & y_1(t-2)\sin(t/\pi) & u_1(t-1)+u_2(t-2) & u_2(t-1)\sin(u_2(t-2)) \\ y_1(t-2)\sin(y_2(t-2)) & y_2(t-2)u_1(t-2) & u_1(t-2)u_2(t-2) & u_2(t-1)\cos(t/\pi) \end{bmatrix}, \\
\boldsymbol{\theta} &= [\theta_1, \theta_2, \theta_3, \theta_4]^{\mathrm{T}} \\
&= [-0.25, 0.47, -0.50, 0.57]^{\mathrm{T}} \in \mathbb{R}^4, \\
A(z) &= 1 + a_1 z^{-1} + a_2 z^{-2} = 1 + 0.30 z^{-1} + 0.64 z^{-2}, \\
\boldsymbol{\vartheta} &= [a_1, a_2, \theta_1, \theta_2, \theta_3, \theta_4]^{\mathrm{T}} \\
&= [0.30, 0.64, -0.25, 0.47, -0.50, 0.57]^{\mathrm{T}} \in \mathbb{R}^6.
\end{aligned}
$$

In simulation, we generate two persistent excitation signal sequences with zero mean and unit variances as the inputs $\{u_1(t)\}$ and $\{u_2(t)\}$, and take $v_1(t)$ and $v_2(t)$ to be two white noise sequences with zero mean and variances σ_1^2 for $v_1(t)$, and σ_2^2 for $v_2(t)$. Taking $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 0.50^2$, the data length L = 3000, and applying the AM-RLS, C-S-AM-RLS and C-AM-RLS algorithms to estimate the parameters of this system, respectively, the parameter estimates are shown in Tables 1–3, and the estimation errors $\delta := \|\hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}\|/\|\boldsymbol{\theta}\|$ versus t are shown in Figures 1 and 2.

From Tables 1–3 and Figures 1 and 2, we can draw the following conclusions.

- The parameter estimation errors by the presented algorithms become smaller and smaller and go to zero with the increasing of time t.
- In contrast to the AM-RLS algorithm, the proposed C-S-AM-RLS and C-AM-RLS algorithms have faster convergence rates and more accurate parameter estimates with the same simulation conditions.

Table 1. The AM-RLS estimates and their errors for Example 1.

t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	δ (%)
100	0.19169	0.46405	−0.29921	0.51505	−0.69003	0.56147	24.77137
200	0.24129	0.55426	−0.26362	0.52514	−0.61030	0.56009	13.91359
500	0.24361	0.55706	−0.24361	0.47651	−0.56945	0.53294	10.96959
1000	0.29857	0.61286	−0.24654	0.45360	−0.55928	0.57197	5.78088
2000	0.31147	0.64219	−0.23371	0.47639	−0.51822	0.57977	2.53112
3000	0.31743	0.65208	−0.23405	0.47050	−0.48951	0.55819	2.65000
True values	0.30000	0.64000	−0.25000	0.47000	−0.50000	0.57000	

Table 2. The C-S-AM-RLS estimates and their errors for Example 1.

t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	δ (%)
100	0.29097	0.63357	−0.27254	0.45781	−0.49700	0.52648	4.44497
200	0.29760	0.62737	−0.26354	0.45916	−0.50241	0.54735	2.69357
500	0.29689	0.63747	−0.25217	0.46685	−0.50539	0.55960	1.11229
1000	0.29887	0.63910	−0.25196	0.46698	−0.50283	0.55826	1.08848
2000	0.30151	0.63810	−0.24935	0.46850	−0.50098	0.55961	0.92996
3000	0.30313	0.63900	−0.24647	0.46872	−0.50302	0.56643	0.58689
True values	0.30000	0.64000	−0.25000	0.47000	−0.50000	0.57000	

Table 3. The C-AM-RLS estimates and their errors for Example 1.

t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	δ (%)
100	0.29005	0.63637	−0.25615	0.45607	−0.49231	0.58483	2.14162
200	0.29623	0.64525	−0.25140	0.46821	−0.50367	0.57151	0.67931
500	0.29686	0.63477	−0.25512	0.47214	−0.50108	0.56153	1.01842
1000	0.30266	0.64036	−0.24782	0.46668	−0.50268	0.57115	0.48179
2000	0.30091	0.64021	−0.24740	0.46931	−0.50116	0.57056	0.26815
3000	0.30202	0.64160	−0.24831	0.46919	−0.49823	0.56821	0.34814
True values	0.30000	0.64000	−0.25000	0.47000	−0.50000	0.57000	

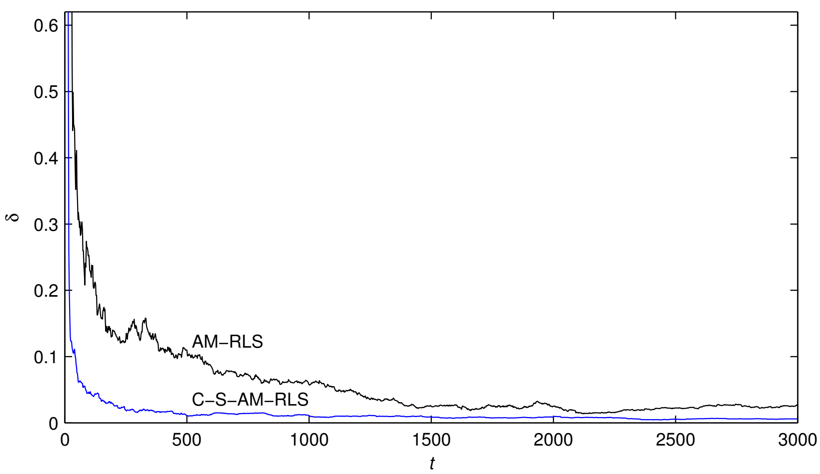

Figure 1. The AM-RLS and the C-S-AM-RLS estimation errors δ versus t for Example 1.

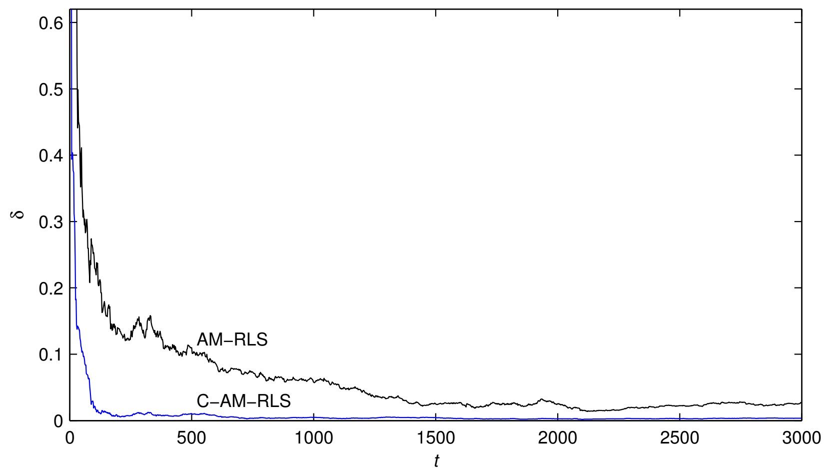

Figure 2. The AM-RLS and the C-AM-RLS estimation errors δ versus t for Example 1.

Example 2. *Consider the following 2-input 2-output system:*

$$\begin{aligned}
\boldsymbol{y}(t) &= \frac{\boldsymbol{Q}(z)}{A(z)}\boldsymbol{u}(t) + \boldsymbol{v}(t),\\
A(z) &= 1 + a_1 z^{-1} + a_2 z^{-2} = 1 - 0.19z^{-1} - 0.15z^{-2},\\
\boldsymbol{Q}(z) &= \boldsymbol{Q}_1 z^{-1} + \boldsymbol{Q}_2 z^{-2}\\
&= \begin{bmatrix} -0.31 & 0.25 \\ 0.28 & -0.23 \end{bmatrix} z^{-1} + \begin{bmatrix} 0.65 & -0.38 \\ 0.41 & 0.62 \end{bmatrix} z^{-2},\\
\boldsymbol{a} &= [a_1, a_2]^{\mathrm{T}} = [-0.19, -0.15]^{\mathrm{T}} \in \mathbb{R}^2,\\
\boldsymbol{\theta}^{\mathrm{T}} &= [\boldsymbol{Q}_1, \boldsymbol{Q}_2] = \begin{bmatrix} -0.31 & 0.25 & 0.65 & -0.38 \\ 0.28 & -0.23 & 0.41 & 0.62 \end{bmatrix} \in \mathbb{R}^{2\times 4}.
\end{aligned}$$

This example system can be transformed into the multivariate output-error system:

$$\begin{aligned}
\boldsymbol{y}(t) &= \frac{\boldsymbol{\Phi}_s(t)\boldsymbol{\theta}}{A(z)} + \boldsymbol{v}(t),\\
\boldsymbol{\varphi}(t) &= [\boldsymbol{u}^{\mathrm{T}}(t-1), \boldsymbol{u}^{\mathrm{T}}(t-2)]^{\mathrm{T}} \in \mathbb{R}^4,\\
\boldsymbol{\Phi}_s(t) &= \boldsymbol{I}_2 \otimes \boldsymbol{\varphi}^{\mathrm{T}}(t)\\
&= \begin{bmatrix} u_1(t-1) & u_2(t-1) & u_1(t-2) & u_2(t-2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_1(t-1) & u_2(t-1) & u_1(t-2) & u_2(t-2) \end{bmatrix} \in \mathbb{R}^{2\times 8},\\
\hat{\boldsymbol{\Phi}}(t) &= \left[\begin{matrix} x_1(t-1) & x_1(t-2) & u_1(t-1) & u_2(t-1) & u_1(t-2) \\ x_2(t-1) & x_2(t-2) & 0 & 0 & 0 \end{matrix} \right.\\
&\qquad \left. \begin{matrix} u_2(t-2) & 0 & 0 & 0 & 0 \\ 0 & u_1(t-1) & u_2(t-1) & u_1(t-2) & u_2(t-2) \end{matrix} \right] \in \mathbb{R}^{2\times 10},\\
\boldsymbol{\vartheta} &= [\boldsymbol{a}^{\mathrm{T}}, \mathrm{col}[\boldsymbol{\theta}]^{\mathrm{T}}]^{\mathrm{T}}\\
&= [a_1, a_2, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8]^{\mathrm{T}}\\
&= [-0.19, -0.15, -0.31, 0.25, 0.65, -0.38, 0.28, -0.23, 0.41, 0.62]^{\mathrm{T}} \in \mathbb{R}^{10}.
\end{aligned}$$

The simulation conditions are similar to those of Example 1. Applying the AM-RLS algorithm, the C-S-AM-RLS algorithm and the C-AM-RLS algorithm with $\sigma^2 = 0.50^2$ and $\sigma^2 = 0.20^2$ to estimate *the parameters of this system, respectively, the parameter estimates are shown in Tables 4–6, and the estimation errors δ versus t are shown in Figures 3–7.*

From Tables 4–6 and Figures 3–7, we can draw the following conclusions:

- In contrast to the AM-RLS algorithm, the proposed C-S-AM-RLS and C-AM-RLS algorithms have faster convergence rates and more accurate parameter estimates with the same simulation conditions, and the C-AM-RLS algorithm can obtain the most accurate estimates for the system parameters.
- The parameter estimation errors given by the proposed algorithms are smaller under a lower noise level—see Tables 4–6 and Figures 3–7.

Table 4. The AM-RLS estimates and errors with different noise variances for Example 2.

σ	t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	δ (%)
0.50	100	−0.07767	−0.05531	−0.36662	0.24956	0.48032	−0.31206	0.31519	−0.06257	0.41747	0.62394	24.42046
	200	−0.12975	−0.19484	−0.39270	0.19552	0.54739	−0.39801	0.25127	−0.15292	0.40869	0.59869	15.11250
	500	−0.19310	−0.17229	−0.36609	0.20923	0.61180	−0.37565	0.30089	−0.22450	0.47133	0.60444	8.75946
	1000	−0.19356	−0.18464	−0.35077	0.26929	0.61180	−0.40097	0.29274	−0.24058	0.41768	0.58457	6.77306
	2000	−0.16577	−0.16642	−0.35510	0.26588	0.63592	−0.38138	0.27095	−0.24643	0.43211	0.58239	6.17573
	3000	−0.17437	−0.15862	−0.34955	0.26143	0.63990	−0.38157	0.26516	−0.24155	0.41820	0.59485	4.64800
0.20	100	−0.14504	−0.09948	−0.34108	0.24926	0.56565	−0.34426	0.29779	−0.13584	0.40947	0.62501	12.55557
	200	−0.16080	−0.17384	−0.35653	0.21984	0.59626	−0.38955	0.26341	−0.18663	0.40729	0.60783	8.16496
	500	−0.19361	−0.16177	−0.34130	0.22732	0.62940	−0.37772	0.29134	−0.22686	0.44372	0.61191	4.82443
	1000	−0.19260	−0.16915	−0.33276	0.26075	0.62898	−0.39165	0.28698	−0.23581	0.41417	0.60050	3.75051
	2000	−0.17637	−0.15883	−0.33513	0.25880	0.64247	−0.38068	0.27492	−0.23911	0.42227	0.59904	3.43136
	3000	−0.18131	−0.15460	−0.33203	0.25635	0.64460	−0.38087	0.27172	−0.23640	0.41457	0.60604	2.58030
True values		−0.19000	−0.15000	−0.31000	0.25000	0.65000	−0.38000	0.28000	−0.23000	0.41000	0.62000	

Table 5. The C-S-AM-RLS estimates and errors with different noise variances for Example 2.

σ	t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	δ (%)
0.50	100	−0.21759	−0.11924	−0.38525	0.15721	0.63656	−0.34223	0.29058	−0.20577	0.37492	0.48917	15.79715
	200	−0.20637	−0.14535	−0.37445	0.17481	0.63617	−0.35041	0.28941	−0.21028	0.38068	0.52643	12.03360
	500	−0.20820	−0.15449	−0.35370	0.20271	0.64497	−0.35998	0.29075	−0.22235	0.40008	0.56361	7.55407
	1000	−0.20219	−0.15807	−0.34231	0.22279	0.64575	−0.36723	0.28829	−0.22609	0.39938	0.57785	5.31882
	2000	−0.19351	−0.15523	−0.33660	0.23115	0.65084	−0.36868	0.28424	−0.22807	0.40408	0.58666	4.04352
	3000	−0.19451	−0.15372	−0.33290	0.23487	0.65196	−0.37059	0.28273	−0.22881	0.40384	0.59231	3.39624
0.20	100	−0.22599	−0.13897	−0.32193	0.22250	0.58332	−0.31192	0.25836	−0.21204	0.39836	0.55805	10.49246
	200	−0.22086	−0.15125	−0.32183	0.22627	0.60226	−0.33263	0.26598	−0.21525	0.40181	0.57614	7.64726
	500	−0.21073	−0.15254	−0.31739	0.23480	0.62239	−0.35282	0.27247	−0.22272	0.40840	0.59331	4.55688
	1000	−0.20374	−0.15441	−0.31579	0.24128	0.63135	−0.36182	0.27507	−0.22497	0.40834	0.60063	3.11396
	2000	−0.19848	−0.15281	−0.31510	0.24388	0.63828	−0.36705	0.27614	−0.22659	0.40919	0.60523	2.17345
	3000	−0.19699	−0.15245	−0.31444	0.24521	0.64095	−0.36950	0.27665	−0.22739	0.40911	0.60772	1.76930
True values		−0.19000	−0.15000	−0.31000	0.25000	0.65000	−0.38000	0.28000	−0.23000	0.41000	0.62000	

Table 6. The C-AM-RLS estimates and errors with different noise variances for Example 2.

σ	t	a_1	a_2	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	δ (%)
0.50	100	−0.14313	−0.08944	−0.29923	0.14967	0.60004	−0.48000	0.27048	−0.10880	0.39393	0.64199	17.32565
	200	−0.15590	−0.16031	−0.30620	0.18954	0.60690	−0.43088	0.26046	−0.17047	0.40328	0.61953	9.53865
	500	−0.18908	−0.15426	−0.31365	0.22262	0.63038	−0.39293	0.27930	−0.21346	0.42501	0.61348	3.57545
	1000	−0.18864	−0.15914	−0.31439	0.24558	0.63487	−0.38941	0.28005	−0.22501	0.41240	0.60972	1.98417
	2000	−0.18100	−0.15438	−0.31763	0.24933	0.64474	−0.38309	0.27657	−0.23004	0.41637	0.60902	1.58563
	3000	−0.18721	−0.15257	−0.31714	0.24959	0.64668	−0.38257	0.27582	−0.23014	0.41296	0.61247	1.06366
0.20	100	−0.18575	−0.14611	−0.32571	0.23906	0.65606	−0.34876	0.27370	−0.22277	0.41431	0.62734	3.27710
	200	−0.18663	−0.15518	−0.32364	0.24072	0.65224	−0.36429	0.27312	−0.22690	0.41301	0.62321	2.08590
	500	−0.19190	−0.15066	−0.31665	0.24569	0.65227	−0.37418	0.27914	−0.23001	0.41543	0.62035	0.96352
	1000	−0.19025	−0.15184	−0.31398	0.25006	0.65002	−0.37822	0.27973	−0.23074	0.41154	0.61862	0.43133
	2000	−0.18790	−0.15067	−0.31339	0.25029	0.65091	−0.37870	0.27896	−0.23105	0.41213	0.61798	0.45028
	3000	−0.18935	−0.15030	−0.31286	0.25024	0.65072	−0.37924	0.27882	−0.23074	0.41111	0.61867	0.31747
True values		−0.19000	−0.15000	−0.31000	0.25000	0.65000	−0.38000	0.28000	−0.23000	0.41000	0.62000	

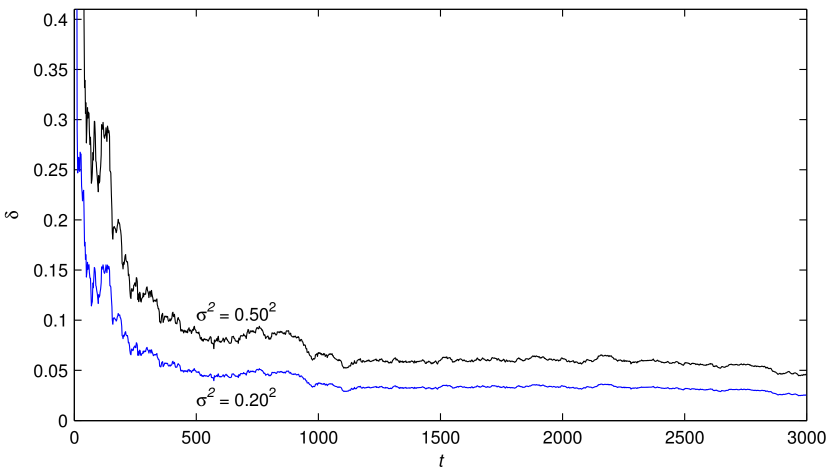

Figure 3. The AM-RLS estimation errors δ versus t with different σ^2 for Example 2.

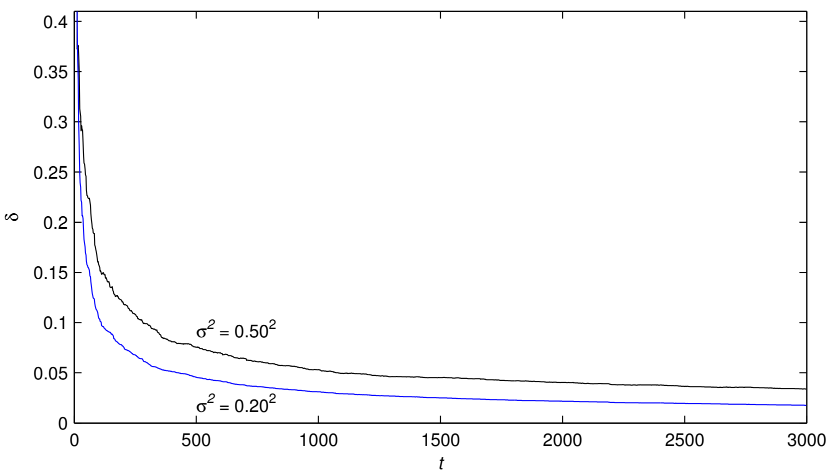

Figure 4. The C-S-AM-RLS estimation errors δ versus t with different σ^2 for Example 2.

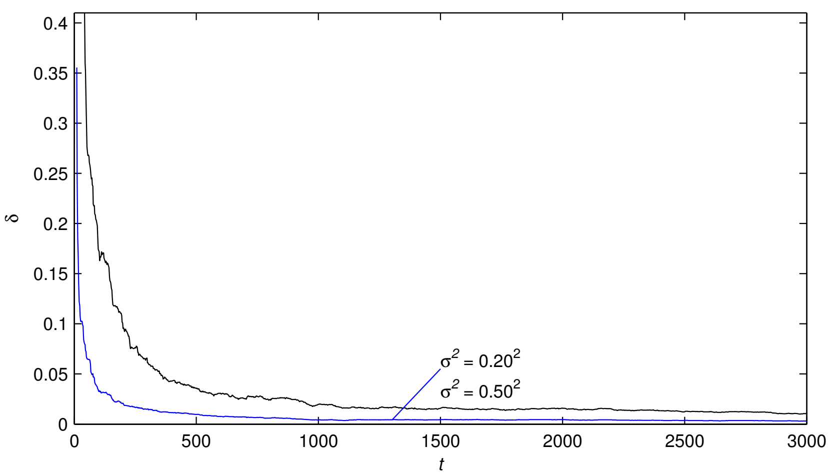

Figure 5. The C-AM-RLS estimation errors δ versus t with different σ^2 for Example 2.

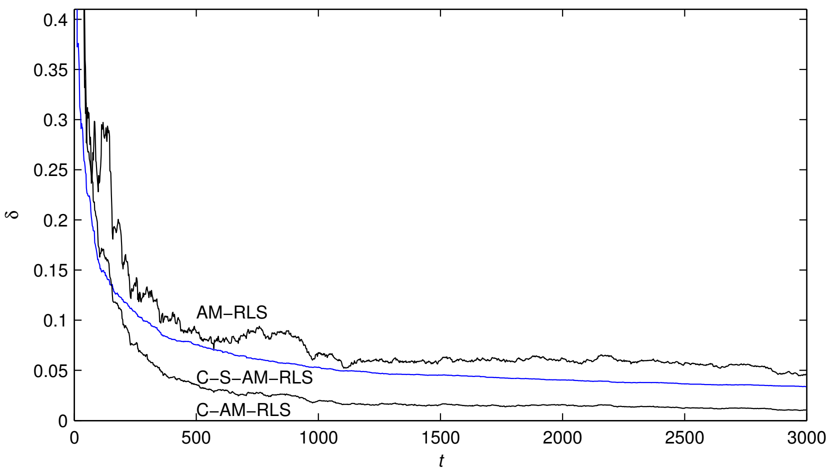

Figure 6. The AM-RLS, C-S-AM-RLS and C-AM-RLS estimation errors δ versus t for Example 2 ($\sigma^2 = 0.50^2$).

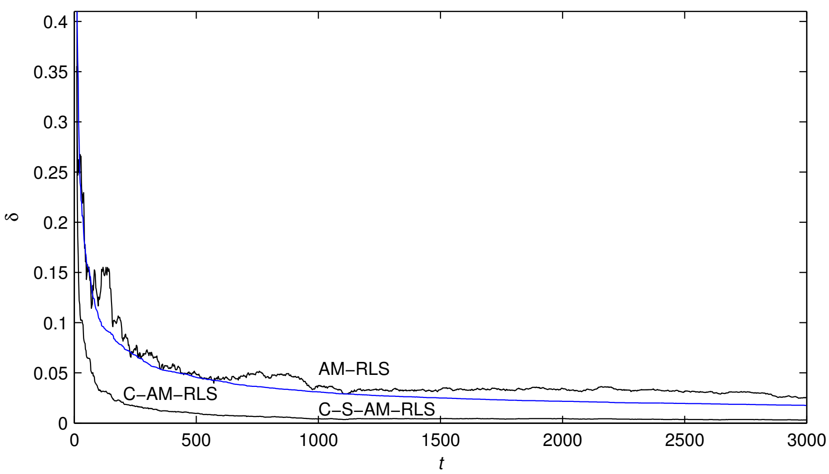

Figure 7. The AM-RLS, C-S-AM-RLS and C-AM-RLS estimation errors δ versus t for Example 2 ($\sigma^2 = 0.20^2$).

5. Conclusions

By means of the auxiliary model identification idea, this paper employs the coupling identification concept to propose a novel recursive identification method for multivariate output-error systems. The proposed methods have the following properties:

- The C-S-AM-RLS algorithm and the C-AM-RLS algorithm are presented by forming a coupled relationship between the parameter estimation vectors of the subsystems, and they avoid computing the matrix inversion in the multivariable AM-RLS algorithm so they require lower computational load and achieve highly accurate parameter estimates.
- With the noise-to-signal ratios decreasing, the parameter estimation errors given by the proposed algorithms become smaller.

The basic idea of the proposed algorithms in this paper can be extended and applied to other fields [29–31].

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Author Contributions: Feng Ding conceived the whole paper and supervised his student Wu Huang to write the paper and Wu Huang designed and performed the simulation experiments. Both authors have read and approved the final manuscript.

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