Energy Efficient Joint Power Control and User Association Optimization in Massive MIMO Enabled HetNets

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Abstract: Massive MIMO enabled heterogeneous cellular networks (HetNets) have a wide application prospect in improving end-to-end performance. To increase the performance of energy efficiency (EE), we formulate the joint optimization problem of power control and user association in this paper. Unfortunately, the problem is a fractional and mixed integer nonlinear programming problem (FMINLP). Nevertheless, an energy efficient joint power control and user association optimization algorithm is proposed to solve the problem. Firstly, based on the Dinkelbach’s theorem, we transform the objective function of the problem into an integral expression and propose the optimal EE iterative algorithm. Then, with the help of the approximately iterative method and Lagrange’s decomposition dual method, the problem can be transformed into a convex optimization problem and the joint optimization algorithm is proposed correspondingly. In the simulation, the performance of EE and its influence factors are analyzed and some interesting points are discussed.

Keywords: Massive MIMO; HetNets; Power control; user association; energy efficiency

1. Introduction

1.1. Motivation

Massive multiple-input-multiple-output (MIMO) and heterogeneous cellular networks (HetNets) are both considered as critical technologies in future 5G wireless system due to the potential to achieve dramatic improvements in network coverage and throughput within the limited system resources [1]. On the one hand, massive MIMO can achieve extra spatial multiplexing gain due to higher degrees of freedom and the beamforming technology [2]. On the other hand, low-cost small cells in HetNets, referred to as pico-cells or femto-cells, are flexibly deployed to provide dense coverage and ubiquitous high throughput. Both of these promising technologies can be effectively combined by employing many antennas at the macro-cell base station (BS) in HetNets [3]. The architecture of massive MIMO in coordination with HetNets can utilize the large-scale antennas installed on the macro-cell BS to manage the interference while greatly enhance the performance of the cell edge users due to the short propagation distance between users and small cell [4].

Although this architecture shows great promises to obtain higher spectral efficiency by universal frequency reuse, it also introduces new challenges and problems. Specifically, traditional user association scheme called max-SINR criterion is no longer applicable. It will cause heavy load imbalance and further damage the system energy efficiency (EE) because different types of BS have large disparities in transmit power. In addition, the use of large-scale antennas and dense deployment of small cells will undoubtedly greatly increase energy consumption of overall network.
Meanwhile, with steadily rising energy costs and environment concerns, green communication has caught substantial attentions and increasing EE has become an essential issue in 5G networks. We note that it has been proven that flexible user association and power control strategies effectively improve the EE of the network [5]. Moreover, there is no practical solution to optimize the user association and power consumption in the meantime. Therefore, how to jointly optimize user association and power control in massive MIMO enabled HetNets under various channel conditions to obtain optimal EE of the network is considered to be an urgent and challenging problem to be solved.

1.2. Related Works

The purpose of this paper is to solve the joint user association and power control optimization problem with EE maximization. The following are some related works about the research. In [6,7], the centralized and distributed power control methods were proposed to improve the spectral efficiency (SE) of HetNets while simultaneously considering the overall network energy consumption. Hirata et al. [8,9] presented a centralized and distributed algorithm for user association in HetNets, respectively, aiming at SE maximization with user’s demand satisfaction. Sun et al. [10] formulated a joint optimization problem of power control and user association to improve the throughput of HetNets, while considering the fairness of users and the minimum QoS requirements of them. Yang et al. [11,12] considered the minimum QoS demand and fairness of users in line-of-sight and non-line-of-sight fading channel environments, respectively, and proposed the power control method in multi-cell network environment. However, all of the above studies focused on single antenna HetNets, which are no longer applicable when considering the impact of massive MIMO on HetNets, i.e., the channel hardening impact will cause the large-scale change of the ergodic rate of users and further influence the user-BS association. To fully profit from the benefits promised by massive MIMO and small cells, it is critical to investigate the user association problem, i.e., how to assign active users to the BS such that the system-wide performance can be maximized or enhanced [13]. Related studies about massive MIMO were also mostly done in isomorphic multi-cell network environment (not HetNets). Therefore, we can conclude that it is necessary and critical to solve the joint user association and power control optimization problem in massive MIMO enabled HetNets.

To improve the EE, some related studies have also been done in massive MIMO enabled HetNets. With the help of stochastic geometry, Dhillon et al. proposed the approach of modeling multi-tier HetNets [14]. Then, the cell density allocation algorithm was proposed to achieve a higher coverage region with saving power consumption [15], which provide the optimal simulation parameter setting of the cell density for this paper. After that, a joint optimization problem of power control and user association was proposed in HetNets [16]. However, the effect of massive MIMO was neglected. Ye et al. [17] investigated the joint optimization problem of user association and interference management to maximize the network throughput. However, the EE of the network was not considered, which is the critical performance metric in 5G network scenarios. Liu et al. [13,18] proposed the centralized and distributed energy efficient user association algorithm in massive MIMO enabled HetNets while considering the fairness among users, respectively. However, the transmit power of each BS was not optimized, which is the main source of the system energy consumption. Kwon et al. [19] proposed a distributed power control method by using non-cooperative game theory, without considering the impact of user association on EE. The above works mainly focused on user association or power control with the EE maximization in HetNets or massive MIMO system. However, an energy efficient joint power control and user association optimization in massive MIMO enabled HetNets is scarce. Recently, Prasad et al. proposed different deployment modes for massive MIMO enabled HetNets [20] and then a joint user association and backhaul routing algorithm for EE maximization in [21]. We further solve the joint user association and power control optimization problem for massive MIMO enabled HetNets in this paper.

In addition, there are also some related works about optimization models and algorithms for wireless network design. For example, Amaldi et al. gave an overview of some of the most significant
optimization problems arising in planning cellular networks [22] and provided an up-to-date view of challenges in modern design of wireless networks [23]. Moreover, the topic of joint optimization of power control and user association has been a very relevant topic of research in the mathematical optimization community working on wireless network design. D’Andreagiovanni et al. [24] presented a mathematical formulation for wireless network design purely based on binary user-base station assignment variables. Then D’Andreagiovanni et al. [25] presented a tight mathematical formulation and cutting plane algorithm for wireless network design based on the discretization of the power emission of transmitters that eliminates all numerical instabilities. Also they [26] proposed a genetic algorithm for better exploring the set of power configurations associated with power-indexed formulations introduced in the previous suggested reference. Moreover, many works have been devoted to develop effective models and efficient algorithms including power control and user association. Some more relevant works are introduced as follows. Chiaraviglio et al. [27] formulated the problem of maximizing different performance on RFB-based network architecture. Marotta et al. [28] proposed novel optimization models that allow the minimization of the energy of the computing and network infrastructure which is hosting a set of service chains that implement the VNFs. Salsano et al. [29] provided an assessment on what can be achieved with current technologies and gives a first confirmation of the validity of the proposed approach. Shojafar et al. [30] designed an efficient algorithm, called P5G, which is based on Particle Swarm Optimization (PSO). Dely et al. [31] introduced a fast decomposition heuristic for designing wireless mesh networks, including signal-to-interference constraints and user-hotspot association. Gendron et al. [32] proposed an effective branch-and-benders-cut algorithm for power minimization in wireless networks. However, these works do not properly address the energy efficient joint optimization problem as a fractional and mixed integer nonlinear programming (FMINLP) problem. We in contrast do this and, using the Dinkelbach’s theorem, an approximately iterative method, and Lagrange’s dual method, we propose a new algorithm with low complexity to solve the problem.

1.3. Main Contribution

Based on the above works, this paper aims at solving an energy efficient joint power control and user association optimization problem while considering the load constraint of each BS in massive MIMO enabled HetNets. The main contributions of this paper are summarized as follows:

- In this paper, we consider the characteristic of channel hardening in the massive MIMO system and calculate ergodic rate of each user with perfect channel state information (CSI). In addition, we establish the energy consumption model in massive MIMO enabled HetNets and then define the network EE in consideration of user fairness. Finally, a joint optimization problem of power control and user association is formulated to maximize the network EE with the constraint of each BS’s maximum load.

- Giving priority to the EE, an energy efficient joint optimization algorithm of power control and user association is proposed in massive MIMO enabled HetNets. We note that the joint optimization problem is a fractional and mixed integer nonlinear programming (FMINLP) problem. Firstly, the objective function of the problem is transformed into an integral expression with weight coefficient based on the Dinkelbach’s theorem and an optimal EE iterative algorithm is proposed correspondingly. Then, the joint optimization problem is decomposed into power control and user association optimization subproblem. Both subproblems can be transformed into convex optimization problems with the help of the approximately iterative method and Lagrange’s dual method, respectively. Finally, the joint optimization algorithm is proposed through the interior point method and the sub-gradient method. In addition, the convergence of the proposed algorithm is analyzed and we discuss the gap between the solution obtained from the proposed algorithm and the optimal one.

- This paper analyzes the performance and its influence factors of the proposed algorithm. First, this paper proves the validity of all the proposed algorithms through convergence results. In addition,
the paper analyzes the EE of the network and its functional relationship with other factors, such as spectrum efficiency, weight coefficient, the number of antennas, and so on. Finally, comparing with other algorithms, this paper analyzes superiority of the proposed algorithm in terms of the EE.

The paper is organized as follows. Section 2 introduces the system model and the joint optimization problem formulation. Section 3 presents the joint optimization problem reformulation and then joint optimization algorithms for power control and user association are proposed. Section 4 simulates and analyzes the performance of EE and its influence factors. Finally, the conclusion and future work of the paper are obtained.

2. System Model

As shown in Figure 1, a multi-user two-tier heterogeneous network scenario is considered in the paper, which includes a single massive MIMO enabled macro-base station (MBS), multiple single-antenna small base stations (SBSs) and single-antenna users. The MBS and SBSs differ in transmit power, size, density and the number of antennas. Let \( J = \{0, 1, 2, \ldots, J\} \) be the set of BSs, where index 0 denotes the MBS and the others are SBSs. \( K = \{1, 2, \ldots, K\} \) is the set of users. It is assumed that all users distribute uniformly within the coverage area of the MBS and each user is associated with only one BS at a time.

![Figure 1. System model. MBS: macro-base station; SBS: small base station; MUE: macro-user; SUE: small cell user.](image)

2.1. Ergodic Rate and Power Consumption

It is assumed the MBS employs large scale antenna array, and the number of active antennas is represented by \( M \gg 1 \). Due to the channel hardening effect introduced by massive MIMO, the channel fluctuations will be averaged. In addition, the user association is always assumed to be carried out in a very large time scale relative to the change of channel. Thus, we can use the ergodic rate as the performance metric for the network.

It is assumed the network is operating in time-division duplexing (TDD) mode and all the BSs share the perfect channel state information (CSI). Moreover, we assumed each BS share the same frequency band and all users associated with the same BS share the orthogonal frequency band. The MBS can serve multiple users on a given resource block, and \( N (N \ll M) \) denotes the maximum number of downlink transmission streams. Then, with linear zero-forcing (ZF) precoding employed for eliminating the intra-cell interference, the downlink ergodic rate from BS \( j \) to user \( k \) can be lower bounded by Equation (1) \([33,34]\).
Here, $r_{jk}$ represents the ergodic rate of user $k$ when associated with BS $j$. $p_j$ denotes the transmit power from BS $j$. $\tau_{jk}$ denotes the channel fading coefficient from BS $j$ to user $k$. We note that the assumption of the perfect CSI can be relaxed by the equation $\tau_{jk} = \hat{\tau}_{jk} + \Delta \tau_{jk}$. $\hat{\tau}_{jk}$ and $\Delta \tau_{jk}$ denote the estimated channel variable and the estimated error variable. The estimated error reduces the network energy efficiency. However, we only analyze the relationship between the network energy efficiency and other influence factors, such as the spectrum efficiency, the number of antennas installed on the MBS and so on. Thus, the assumption of the perfect CSI has little effect on the analysis in the paper. $\sigma^2$ is the variance of the additive white Gaussian noise (AWGN) with zero mean.

In the wireless downlink transmission system, the power consumed when the BS provides the service to the user is mainly considered, and the power consumption generated by the user receiving the signal can always be negligible. The power consumption at the BS mainly includes wireless transmission power consumption and static circuit power. Besides, it is necessary to consider circuit power of each antenna due to the installation of large-scale antenna matrix at the MBS. The transmit power of the MBS and SBSs can be calculated as Equation (2) [35].

$$P_j = \begin{cases} \frac{p_j}{\varepsilon_j} + M\rho + \xi_j, & j = 1 \\ \frac{p_j}{\varepsilon_j} + \xi_j, & j > 1 \end{cases}$$

Here, $\rho$ denotes the circuit power per antenna, $\varepsilon_j$ denotes the power amplifier efficiency of the BS $j$, and $\xi_j$ denotes the static circuit power term independent of the antenna number. The total system power consumption can be expressed as Equation (3).

$$P_{\text{total}} = \sum_j P_j = M\rho + \sum_j (\frac{p_j}{\varepsilon_j} + \xi_j)$$

2.2. Problem Formulation

In this paper, we consider the network EE as the objective function, and study the joint optimization problem of power control and user association in the scenario of massive MIMO enabled HetNets. The network EE is defined as shown in Equation (4).

$$\eta (\mathbf{x}, \mathbf{p}) = \frac{\sum_j \sum_k x_{jk} R_{jk}}{P_{\text{tot}}} = \frac{R_{\text{tot}}}{P_{\text{tot}}}$$

Here, $x_{jk}$ denotes whether the user $k$ is associated with the BS $j$. If the user $k$ is associated with the BS $j$, $x_{jk} = 1$, otherwise $x_{jk} = 0$. $\mathbf{x} = \{x_{jk}\}$, $\mathbf{p} = \{p_j\}$. We consider the proportional fairness and define $R_{jk} = \ln \left( \frac{R_{jk}}{P_{\text{tot}}} \right)$ as the proportional fair rate [36]. Therefore, the EE defined in this paper can be understood as the network proportional fair throughput in the case of energy consumption per unit
spectrum. Then, the energy efficient joint power control and user association optimization problem can be formulated as shown by Problem P0.

\[
P_0 : \max_{\{x, p\}} \eta(x, p)
\]

\[
s.t. \quad C_1 : \quad 0 \leq p_j \leq p_{j,max}, \forall j
\]

\[
C_2 : \quad \sum_j x_{jk} = 1, \forall k
\]

\[
C_3 : \quad x_{jk} \in \{0, 1\}, \forall j, k
\]

Here, \(C_1\) represents the maximum transmit power constraint of each BS. \(C_2\) shows the principle of user association in the network, that is, each user can be associated with only a single base station. \(C_3\), which constrains the variable of each user’s association, is a binary variable.


Problem P0 is a fractional and mixed integer nonlinear programming problem (FMINLP). Firstly, the objective function can be transformed into an integral expression with weight coefficient based on the Dinkelbach’s theorem and an iterative algorithm giving priority to the EE is proposed correspondingly. Then, the joint optimization problem is decomposed into power control and user association optimization subproblems. Both subproblems can be transformed into convex optimization problems based on the approximately iterative method and Lagrange’s decomposition dual method, respectively. Finally, the joint optimization algorithm is proposed through the interior point method and the sub-gradient method.

3.1. Optimal EE Iterative Algorithm Based on Dinkelbach’s Theorem

Considering load optimization of each BS and the simplification of the problem, Problem P0 can be firstly transformed into Problem P1 as follows.

\[
P_1 : \max_{\{x, p, k\}} \eta(x, p, k)
\]

\[
s.t. \quad C_1, C_2, C_3
\]

\[
C_4 : \quad \sum_k x_{jk} = k_j, \forall j
\]

\[
C_5 : \quad \sum_j k_j = K
\]

Here, \(k_j\) represents the load of the BS \(j\) in the network, that is, the number of users associated with the BS \(j\). The vector \(k = \{k_1, k_2, ..., k_J\}\) denotes the load of the network. According to Lemma 1, it is obvious that Problem P1 can be transformed into Problem P2, with the objective function transforming from a fractional expression into integral one.

\[
P_2 : \max_{\{x, p, k\}} T(\omega) = \sum_j \sum_k x_{jk} \ln(r_{jk}) - \omega P_{tot}
\]

\[
s.t. \quad C_1 - C_5
\]

Here, \(\omega\) denotes the weight coefficient and the range of the weight coefficient is \(0 \leq \omega \leq \sum_k \max_j \ln(r_{jk})/P_{tot}(p_j = p_{j,max})\). The weight coefficient can denote the degree of preference for energy consumption in the objective function of Problem P2.
**Lemma 1.** If and only if there is an optimal parameter \( \omega^* \) for Problem P2, such that \( T(\omega^*) = 0 \) holds and Problem P1 obtains the optimal EE value \( \omega^* \).

Based on the Dinkelbach’s theorem [37], Lemma 1 is proved in Appendix A. In fact, Lemma 1 can be easily considered as the specific example of Dinkelbach’s theorem in this paper. Problem P1 is the fractional optimization problem where the functional is given by the ratio of two integrals. With Dinkelbach’s approach, the fractional optimization problem is transformed into an equivalent parametric family \((x, p)\) of the optimization problem, where the ratio disappears and the functional is given by the weighted difference of the numerator and the denominator of the ratio. Finally, the optimization iteration algorithm can be designed as follows to solve Problem P1.

**Algorithm 1** Optimal EE iterative algorithm

**Input:** the maximum iteration number \( \text{iter}_{\text{max}} \), the maximum tolerance error \( \sigma > 0 \);

**Output:** the optimal solution \( x^*, p^*, k^*, \omega^* \);

**Initialize:** the weight coefficient of the first iteration \( \omega(0) = \omega_0 \), iteration number \( i = 0 \), the variable vector \( x_0, p_0, k_0 \)

\[ \text{while } T(\omega(i)) = \sum_{j} x_{jk} \ln(a_{jk}) - \omega(i) P_{all} \geq \sigma \text{ and } i \leq \text{iter}_{\text{max}} \text{ do} \]

- **Update** \( i = i + 1 \);
- **Calculate** \((x(i), p(i), k(i))\), by solving Problem P2 (see Algorithm 2);
- **Update** \( \omega(i) = \frac{R_{all}(x(i), p(i), k(i))}{P_{all}(x(i), p(i), k(i))} \);

**end while**

**Return** \( \omega^* = \omega(i) \) and \((x^*, p^*, k^*) = (x(i), p(i), k(i))\);

**3.2. Joint Optimization Algorithm**

To solve Problem P2, we reformulate the problem by separating the individual variables \((x, k)\) and \(p\) to decompose the joint optimization problem into two dependent Sub-Problems P3 and P4: the Lower level problem (Power control) and Master problem (User association optimization).

\[
P3 : \quad \max_{p} f(p) = \sum_{j} x_{jk} \ln(a_{jk}) - \omega P_{all} \quad \text{s.t.} \quad C1 \tag{15}
\]

\[
P4 : \quad \max_{s,k} g(x,k) = \sum_{j} x_{jk} \ln(a_{jk}) - \sum_{j} k_{j} \ln(k_{j}) \quad \text{s.t.} \quad C2 - C5 \tag{17}
\]

Here, \( a_{jk} = k_{j} r_{jk} \). We can obtain the joint power control and user association optimization algorithm as follows. In addition, the convergence of the algorithm can be seen in the simulation.

**Algorithm 2** Joint optimization algorithm

**Initialize:** the weight coefficient \( \omega \), iteration number \( t = 0 \), the maximum tolerance error \( \sigma_{t} \), the actual error \( \Delta x^t \), each BS’s transmit power \( p_{0}^{t} = p_{j, \text{max}}, \forall j \)

\[ \text{while } \Delta x^t > \sigma_{t} \text{ do} \]

- **Update** \( t = t + 1 \);
- According to \( p^{t-1} \), **calculate** the optimal user association vector \( x^t \) by solving Problem P4;
- According to \( x^{t-1} \), **calculate** the optimal user association vector \( p^t \) by solving the problem P3;
- **Calculate** \( \Delta x^t = \max_{j,k} \left| x_{jk}^t - x_{jk}^{t-1} \right| \);

**end while**
3.2.1. The Lower Level Problem: Power Control

Problem P3 is a non-convex optimization problem. In this paper, we approximate the objective function into a convex function in each iterative step. Then, Problem P3 can be transformed into a convex optimization problem. The transformation process is shown as follows.

Lemma 2. There exists $\log_2 (1 + \gamma) \geq a \log_2 (\gamma) + \beta$, where $a = \alpha \gamma / (1 + \gamma), \beta = \log_2 (1 + \gamma^*) - (\gamma^*/1 + \gamma^*) \log_2 (\gamma^*)$. If only if $\gamma = \gamma^*$, then the inequality can take the equal sign. The proof can be seen in Reference [38].

According to Lemma 2, the equation $f(p) \geq \sum_i \sum_j x_{ik} \ln (y_{ik}) - \omega P_{tot} = f(p)$ is obviously available, where $\gamma_{ik} (q) = a_{jk} \log_2 (\gamma_{jk} (q)) + \beta_{jk}$.

Since the log-sum-exp function in the objective function of Problem P5 is proven to be a concave function [39], the non-convex optimization Problem P3 can be transformed into Problem P5. Note that Problem P5 is a convex optimization and can be solved by the interior point method.

$$\begin{align*}
P5 & : \max_q \mathcal{f}(q) \\
& \text{s.t.} \quad 2^{q_j} \leq p_{j,\max}
\end{align*}$$

Here, $q_j = \log_2 p_j$. To sum up, the power control algorithm based on approximately iterative method (Algorithm 3) can be designed as follows in this paper. In Algorithm 3, the objective function increases monotonously with the iteration number increasing and finally converges to a fixed value, which can be seen in the simulation later. The gap between the solution of the proposed algorithm and the optimal solution are described in [40,41].

Algorithm 3 Power control algorithm based on approximately iterative method

| Initialize: iteration number $n = 0$, the maximum tolerance error $\sigma_2$, the flag of convergence $\xi$, each BS’s transmit power $p^0_j = p_{j,\max}, \forall j$ |
| while $\xi < \sigma_2$ do |
| Update $n = n + 1$; |
| According to $\gamma_{jk}^{n-1}$, calculate $q_{jk}^{n-1}$ and $p_{jk}^{n-1}$; |
| Calculate the optimal solution $q^n$ of Problem P5 by the interior point method; |
| According to $q_j = \log_2 p_j$, calculate $p^n$ and $\gamma_{jk}^n$; |
| Calculate $\Delta \gamma_{jk} = \left| \gamma_{jk}^n - \gamma_{jk}^{n-1} \right| / \gamma_{jk}^{n-1}, \forall j, k$; |
| Update $\xi^n = \max_j \Delta \gamma_{jk}$; |
| end while |

3.2.2. The Master Problem: User Association Optimization Problem

Problem P4 is a mixed-integer nonlinear programming (MINLP) problem, which can be converted into a convex optimization problem with the help of the Lagrange’s dual decomposition method. The Lagrange function of Problem P4 can be expressed as Equation (21).

$$\begin{align*}
\mathcal{L}(x, k, \lambda, \mu) = \sum_j \sum_k x_{jk} \ln (a_{jk}) - \sum_j k_j \ln (k_j) + \sum_j \lambda_j \left( k_j - \sum_k x_{jk} \right) + \mu \left( \sum_j k_j - K \right)
\end{align*}$$

(21)
Correspondingly, the dual Problem P6 can be expressed as Equation (22).

\[ \text{P}_6 : h(\lambda, \mu) = \max_{x, k} \mathcal{L}(x, k, \lambda, \mu) \]

subject to \(C_2, C_5\)

Then, Problem P6 can be decomposed into Equations (23) and (24).

\[ h_1(\lambda) = \max_x \sum_k \left( \sum_j x_{jk} \left( \ln(a_{jk}) - \lambda_j \right) \right) \]

subject to \(C_2, C_5\)

\[ h_2(\lambda, \mu) = \max_k \sum_j k_j \left( \lambda_j + \mu - \ln(k_j) \right) \]

In this paper, the binary variable can be replaced by a continuous variable in the interval \([0,1]\) \cite{42}, which is the main source of the gap between the proposed algorithm and the optimal algorithm. We analyze the gap later in the paper. Note that Problem P6 is a convex optimization problem and can be solved by the sub-gradient method. According to the KKT conditions, the optimal solution of user association and each BS’s load can be obtained by deriving the partial derivative \(\partial h_1(\lambda)/\partial x\) and \(\partial h_2(\lambda, \mu)/\partial k\) as shown in Equations (25) and (26).

\[ x_{jk}^* = \begin{cases} 1, & j = \arg \max_i \ln(a_{jk}) - \lambda_i \\ 0, & \text{otherwise} \end{cases} \]

\[ k_j^* = e^{\lambda_j + \mu - 1} \]

In addition, parameters \(\lambda_j\) and \(\mu\) can be calculated as shown in Equations (27) and (28).

\[ \lambda_j^{n+1} = \left[ \lambda_j^n - z_1^n \times \left( k_j - \sum x_{jk} \right) \right]^+ \]

\[ \mu^{n+1} = \left[ \mu^n - z_2^n \times \left( \sum k_j - K \right) \right]^+ \]

Here, \([y]^+\) denotes \(\max(y, 0)\). \(z_1^n\) and \(z_2^n\) represent iterative step size of \(\lambda_j\) and \(\mu\), respectively.

**Algorithm 4** User association algorithm based on Lagrange’s dual method

- **Initialize**: iteration number \(n = 0\), the maximum tolerance error \(\sigma_3\), the flag of convergence \(\Delta x^n\), each BS’s transmit power \(p_j^0 = p_{j,\text{max}}, \forall j\)

- **while** \(\Delta x^n > \sigma_3\) **do**
  - **Update** \(n = n + 1\);
  - According to Equations (25) and (26), **update** \(x_{jk}^n\) and \(k_j^n\);
  - According to Equations (27) and (28), **update** \(\lambda_j^n\) and \(\mu^n\);
  - **Calculate** \(\Delta x^n = \max_{j,k} \left| x_{jk}^n - x_{jk}^{n-1} \right| \);

- **end while**

As described above, there may be a gap between the solution of the proposed algorithm and the optimal one, due to the conversion process of the variable \(x_{jk}\) from 0–1 integer to the interval \([0,1]\). Next, the gap analysis of the proposed algorithm is shown as follows.

Suppose \((x, k)\) and \((x^*, k^*)\) are the solution of the proposed algorithm and the global optimal solution, respectively. \(g(x, k)\) and \(g(x^*, k^*)\) are the values of corresponding
objective functions, respectively. It can be proven that there will be
\[ g(x^*, k^*) - g(x, k) \leq \Omega, \]
where \( \Omega = \sum_j k_j \ln \left( k_j / \epsilon_k^j + 1 \right) \). The proof can be seen in Appendix B.

3.3. Complexity Analysis

The asymptotic complexity of the proposed algorithm is analyzed in this section. In Algorithm 1, the calculation of each user’s association variable and each BS’s transmit power entails \( KJ \) operations. Correspondingly, the computation in Algorithms 2–4 also calls \( O(KJ) \) operations. Suppose Algorithms 1–4 need \( \Delta_1, \Delta_2, \Delta_3 \) and \( \Delta_4 \) iterations to converge. The total complexity of the proposed algorithm is thus \( O\left(KJ^4\Delta_1\Delta_2\Delta_3\Delta_4\right) \). Compared with the exhaustive search for the joint optimization problem, which has a worst case complexity of \( O\left(2^{KJ}\right) \), the proposed algorithm has a much lower complexity. Moreover, \( \Delta_1, \Delta_2, \Delta_3 \) and \( \Delta_4 \) are small enough, as can be seen in the simulation.

4. Simulation and Analysis

In this paper, the scenario of massive MIMO enabled HetNets is considered, where the coverage radius of MBSs and SBSs are 500 m and 50 m, respectively. The network simulation parameters are shown in Table 1 [13]. Assumed that all the SBSs and users distribute evenly within the coverage area of the center MBS and each SBS is at least 40 m away from the center MBS. The number of antennas installed on the macro-BS and the maximum data streams supported by the macro-BS are supposed to be 100 and 20, respectively. In addition, the maximum transmit power of the MBS and each SBS are 43 dBm and 23 dBm, respectively. All the simulation results are obtained by 5000 Monte Carlo runs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
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<td>( J )</td>
<td>10</td>
<td>( B )</td>
<td>10 MHz</td>
</tr>
<tr>
<td>( R_{MBS} )</td>
<td>500 m</td>
<td>( R_{SBS} )</td>
<td>50 m</td>
</tr>
<tr>
<td>( M )</td>
<td>100</td>
<td>( N )</td>
<td>20</td>
</tr>
<tr>
<td>( p_{\max} )</td>
<td>43 dBm</td>
<td>( p_{\max, j} &gt; 1 )</td>
<td>23 dBm</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.02 W</td>
<td>( \xi_j )</td>
<td>0.38</td>
</tr>
<tr>
<td>( \xi_1 )</td>
<td>10 W</td>
<td>( \xi_j, j &gt; 1 )</td>
<td>0.2 W</td>
</tr>
</tbody>
</table>

Due to the characteristic of channel hardening in the massive MIMO system, the effects of fast fading can be eliminated. Then, we can suppose the channel fading only includes path loss, shadow fading and frequency selective Rayleigh fading. The path loss model can be found in Model A.2.1.1.2-3 for outdoor RRH or hotspot area model 1 [43], as shown in Table 2, where the unit of \( d \) is km. The Rayleigh fading channel gains are modeled as i.i.d. unit-mean exponentially distributed random variables. The variance of the lognormal shadowing from the associated BS to each user is considered to be 10 dB.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Path Loss Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>From MBS to users</td>
<td>( PL = 128.1 + 37.6\log_{10}(d) )</td>
</tr>
<tr>
<td>From SBS to users</td>
<td>( PL = 140.7 + 36.7\log_{10}(d) )</td>
</tr>
</tbody>
</table>

Figure 2 shows the convergence in term of different initial values of each BS’s transmit power for Algorithm 3 versus the number of iterations, where \( p_j = p_{\max, j}, \forall j \) and \( p_j = p_{\max, j}/2, \forall j \), respectively. The simulation setting of the number of users and the number of antennas installed on the MBS are 30 and 100, respectively. The ordinate in Figure 2 represents the change number of the convergence flag \( \xi_n = \max_{j,k} \Delta \gamma_{jk}^n \) in Algorithm 3. Obviously, we can find Algorithm 3 converges after a limited number
of iterations irrespective of different initial values of each BS’s transmit power. This result, together with the previous analysis, ensures that the proposed Algorithm 3 is applicable in the massive MIMO enabled HetNets.

Figures 3 and 4 show the convergence in terms of different initial values of users for Algorithms 1 and 2, respectively. The simulation setting of \( p_j \), \( \forall j \) and the number of antennas installed on the MBS are \( p_{j,\text{max}} \) and 100, respectively. The ordinate in Figures 3 and 4 represent the change number of the convergence flag \( \Delta x^t = \max_{j,k} |x_{jk}^t - x_{jk}^{t-1}| \) in Algorithm 2 and the weight coefficient \( \omega(i) \) in Algorithm 1, respectively. Obviously, we can find Algorithms 1 and 2 converge after a limited number of iterations irrespective of different initial values of users. This result, together with the previous analysis, ensures that the proposed Algorithms 1 and 2 are applicable in the massive MIMO enabled HetNets.
Figure 4. The convergence in terms of different initial values of users for Algorithm 1.

Figure 5 shows the relationship between the network EE and the weight coefficient $\omega$ in Algorithm 2, where the numbers of users $K$ has different values. The simulation setting of the number of antennas $M$ and the initial value of each BS’s transmit power $p_{j}, \forall j$ are 100 and $p_{j,\text{max}}$, respectively. The vertical axis of Figure 5 is the value of the network EE calculated by Algorithm 2 with different values of given weight coefficient $\omega$. We can find two interesting points in Figure 5. The first point is that the network EE is a concave function on the weight coefficient $\omega$. In addition, it can be concluded that the network EE has no change irrespective of the different numbers of users. Then, we analyze and discuss the two points as follows. Firstly, the weight coefficient can denote the degree of preference for energy consumption. With the weight coefficient increasing, the energy consumption increases and the network capacity reduces, correspondingly. Then, the network EE is a concave function on $\omega$. Secondly, the energy consumption of users can be almost neglected in the wireless downlink transmission system. In the OFDMA system, the distribution and the number of users have little effects on the spectrum efficiency after enough number of Monte Carlo run. Based on the above discussion, it can be concluded that the number of users has a negligible impact on the network EE.

Figure 5. The energy efficiency via the weight coefficient.
Figure 6 shows the relationship between the network EE and spectrum efficiency where the number of antennas installed on the MBS is different. The simulation setting of the number of users $K$ and the initial value of each BS’s transmit power $p_j, \forall j$ are 30 and $p_{j,max}$, respectively. The horizontal and vertical axes in Figure 6 represent the spectral efficiency and EE values of the network calculated by Algorithm 2 with different values of given weight coefficient $\omega$, respectively. It can be obviously seen in Figure 6 that the network EE is a concave function on the spectrum efficiency. In addition, the maximum value of the network EE and spectral efficiency increase and reduce with the number of antennas installed on the MBS increasing from 50 to 200. This point is interesting and we have some discussion about it. On the one hand, the number of available spatial channels increases with the more antennas existing in the network. The network obtains greater throughput and spectrum efficiency. On the other hand, the energy consumption of the network also increases as the number of antennas increases. To some extent, the network EE reduces, correspondingly.

Figure 6. The energy efficiency via the spectrum efficiency.

Figure 7 shows the network EE calculated by different algorithms when the number of antennas installed on the MBS increases from 10 to 170. The simulation setting of the number of users $K$ and the initial value of each BS’s transmit power $p_j, \forall j$ are 30 and $p_{j,max}$, respectively. We can find two interesting points in Figure 7. Firstly, it can be obviously found that the network EE is a concave function on the number of antennas installed on the MBS. This point can be proven as described above. The second point is that the performance of the network EE obtained by the proposed Algorithm 1 is superior to other single optimization algorithms, which also demonstrates the available effectiveness of the proposed Algorithm 1 to some extent.
5. Conclusions and Future Work

In the scenario of massive MIMO enabled HetNets, we propose an optimal iterative EE algorithm and joint optimization algorithm. From the simulation results and relevant discussion about the performance of proposed algorithms, we conclude the network EE is a concave function on the weight coefficient, the spectrum efficiency and the number of antennas installed on the MBS. That is, we sacrifice some part of the performance of spectrum efficiency with maximizing the network EE. Furthermore, we also propose the joint user association and power control algorithm for the development of new energy reduction policies in massive MIMO enabled HetNets and optimize the number of antennas installed on the MBS to increase the network EE. In addition, we analyze the validity and effectiveness of all the proposed algorithms. In future works, the joint number of antennas, user association, and power control and channel allocation optimization will be considered in the massive MIMO enabled HetNets. That is, to further increase the network EE, we will also optimize the number of antennas and channel allocation in future work, which is the main source of increasing the network EE.

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Author Contributions: Liangrui Tang and Hailin Hu initiated and discussed the research problem; Hailin Hu conceived and developed the methods; Yanhua He and Hailin Hu performed the simulation and made the figures; Hailin Hu and Liangrui Tang analyzed the data; and Hailin Hu prepared and wrote the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Lemma A1. If and only if there is an optimal parameter $\omega^*$ for Problem P2, such that $T(\omega^*) = 0$ holds and Problem P1 obtains the optimal EE value $\omega^*$.

Proof. Obviously, $T(\omega)$ is a monotonically decreasing function on the variable $\omega$. That is, if $\omega_1 > \omega_2$, then $T(\omega_1) < T(\omega_2)$. Suppose $\omega^*$ is the optimal solution to Problem P1. Then, there is the expression $\frac{R_{\text{tot}}(x^*,p^*,k^*)}{P_{\text{tot}}(x^*,p^*,k^*)} = \omega^* \Rightarrow R_{\text{tot}}(x^*,p^*,k^*) - \omega^* P_{\text{tot}}(x^*,p^*,k^*) = 0$, where the vector $(x^*, p^*, k^*)$ is the optimal solution of P2. Let $(x, p, k) \neq (x^*, p^*, k^*)$ be the arbitrary solution vector of the problem. It is obvious that there is $\eta(x, p, k) \leq \omega^* \Leftrightarrow R_{\text{tot}}(x, p, k) - \omega^* P_{\text{tot}}(x, p, k) \leq 0 \Leftrightarrow \max \{R_{\text{tot}}(x, p, k) - \omega^* R_{\text{tot}}(x, p, k)\} = 0$. □
Appendix B

As described in Section 3.2.1, it can be proven that there is $g(x^*, k^*) - g(x, k) \leq \Omega$, where $\Omega = \sum_j k_j \ln \left( \frac{k_j}{e^{k_j} + \mu - 1} \right)$. The proof is shown as follows.

Proof.

\[
g(x, k) = \sum_j \sum_k x_{jk} \ln \left( a_{jk} \right) + \sum_j k_j \ln \left( e^{k_j} + \mu - 1 \right) - \Omega
\]

\[
\leq \sum_k \max_j \left( \ln \left( a_{jk} \right) - \lambda_j \right) + (1 - \mu) K - \Omega
\]

(a) $\Rightarrow \sum_k \max_j \left( \ln \left( a_{jk} \right) - \lambda_j \right) + (1 - \mu) K - \Omega$

(b) $\Rightarrow h(\lambda, \mu) - \Omega$

(c) $\geq g(x^*, k^*) - \Omega$

Here, (a) shows the result after the solution calculated by Equation (26); (b) is due to the optimization condition of parameter $\mu$; and (c) is due to the weak duality of $h(\lambda, \mu)$.

References


