Application of Iterative Maximum Weighted Likelihood Estimation in 3-D Target Localization

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Abstract: Time-of-flight (ToF)-based 3-D target localization is a very challenging topic because of the pseudo-targets introduced by ToF measurement errors in traditional ToF-based methods. Although the influence of errors in ToF measurement can be reduced by the probability-based ToF method, the accuracy of localization is not very high. This paper proposes a new 3-D target localization method, Iterative Maximum Weighted Likelihood Estimation (IMWLE), that takes into account the spatial distribution of pseudo-targets. In our method, each pseudo-target is initially assigned an equal weight. At each iteration, Maximum Weighted Likelihood Estimation (MWLE) is adopted to fit a Gaussian distribution to all pseudo-target positions and assign new weight factors to them. The weight factors of the pseudo-targets, which are far from the target, are reduced to minimize their influence on localization. Therefore, IMWLE can reduce the influence of pseudo-targets that are far from the target and improve the accuracy of localization. The experiments were carried out in a water tank to test the performance of the IMWLE method. Results revealed that the estimated target area can be narrowed down to the target using IMWLE and a point estimate of target location can also be obtained, which shows that IMWLE has a higher degree of accuracy than the probability-based ToF method.

Keywords: target localization; Iterative Maximum Weighted Likelihood Estimation

1. Introduction

Structural health monitoring (SHM) is an emerging technology that aims to design systems that are able to continuously monitor structures [1–3]. Structural health monitoring is beneficial to the maintenance of critical structures in many fields [4], such as civil engineering [5] and aerospace [6], where damage detection plays a very important role. The ultrasonic pulse–echo technique is traditionally adopted as a cost-effective SHM strategy. Damage detection of 2-D structures, such as plates and shells, has been widely studied [7–9]. Three-dimensional structures, such as concrete structures, also require periodic inspections and quality control to assess their structural integrity [10,11].

Damage detection can be generalized to target detection in the medium. In target detection, transducer elements act as actuators to emit short ultrasonic pulses, and sensors receive the scattered ultrasound. By recording and analyzing the scattered signals, information related to the target (location, size, type, and orientation) can be obtained. Time-of-flight (ToF)—the time taken by the ultrasonic pulse to travel from an actuator to a sensor—is a feature extracted from the scattered signals that are widely used in target localization [12,13]. Many studies have reported the successful application of ToF-based methods for target localization [1,4,14,15], but the accuracy is affected by ToF measurement errors.
In the traditional ToF-based method, the location of the target is the intersection point of all ellipsoids [4]. This method makes the assumption that ToF measurements are absolutely accurate and error-free. However, ToF measurement errors are inevitable in experiments due to noises and many other factors. Therefore, two problems need to be solved in 3-D target localization. First, ellipsoids will not intersect at only one point if the ToF measurement is inaccurate because multiple intersection points will be obtained instead of one. These intersection points are pseudo-targets and it is impossible to differentiate between pseudo-targets to locate the target using traditional methods. Second, due to ToF measurement errors, the ellipsoids and curves may not intersect, which give rise to disjoint ellipsoids and disjoint curves. In this case, information related to the target location will be lost because scattered signals from the target cannot be employed to construct curves or intersection points.

Probability-based ToF methods [16,17] were introduced so that the points absent in the ellipsoids, which are the loci of ToF measurements, are also considered possible target locations, which can solve the problems caused by ToF measurement errors. The possibility of target occurrence at each point is determined by its distance to the loci. The combination of the possible target location corresponding to all the loci can give the estimated target area. However, the estimation result is less accurate because it involves a series of dispersed points occupying a target area. Therefore, introducing a new method that could narrow down the target area and provide a point estimate of the target location with higher accuracy is crucial in 3-D target localization.

The two problems caused by ToF measurement errors in 3-D target localization result in spatial distribution of pseudo-targets. Some pseudo-targets are located close to the target, while other pseudo-targets are located far from the target and may negatively influence the accuracy of localization. We propose a 3-D localization method, Iterative Maximum Weighted Likelihood Estimation (IMWLE), which takes into account the spatial distribution of pseudo-targets. In this method, each pseudo-target is initially assigned an equal weight. At each iteration, Maximum Weighted Likelihood Estimation (MWLE) is adopted to assign new weight factors to pseudo-targets by fitting a Gaussian distribution to them. This procedure could effectively minimize the influence of the pseudo-targets that are located far from the target. Therefore, the estimated target area could be narrowed down gradually, with the weighted mean of the last MWLE being the point estimate.

In this paper, IMWLE is used to improve the accuracy for 3-D localization. The problems introduced by ToF measurement errors briefly addressed. The resulting spatial distribution of pseudo-targets has analyzed. The IMWLE method used to take into account the spatial distribution of pseudo-targets to narrow down the target area and give a point estimate of target location. The results of our proposed method and probability-based ToF method have been compared. It is shown that the IMWLE method has a higher degree of accuracy than the probability-based ToF method.

2. Experimental Setup and Problem Statement

The problem to be solved was 3-D target localization in an infinitely large media. The experiment was carried out in water, where it is easier to set up targets and make measurements than in solids. Additionally, conducting the experiment in water did not affect the study of localization methods.

Figure 1 is the experimental setup used for 3-D target localization. The experiment was carried out in a water tank with dimensions of 300 mm × 400 mm × 500 mm. A solid sphere (29 mm in diameter) was immersed in water as the target. An actuator and a sensor were used to transmit ultrasonic signals and receive scattered signals, respectively. The sensor was moved $N$ (in our case $N = 12$) times to receive $N$ scattered signals that traveled from the actuator to the sensor along $N$ different paths. A signal generator was used for excitation. The scattered signals were recorded by a Data Acquisition (DAQ) system. A PC was used for signal processing, such as extracting ToFs from the scattered signals and performing localization algorithms (the traditional ToF-based method, the probability-based ToF method, and IMWLE).

The experimental setup consisted of the following:

A solid sphere with a diameter of 29 mm;
An immersion transducer (1 MHz center frequency, 7 mm diameter);
A distributed feedback fiber laser (DFB-FL) ultrasonic sensing system (self-developed);
A water tank with dimensions of 300 mm × 400 mm × 500 mm;
A three-axis mobile platform (self-developed) driven by stepper motor. It was used to move the sensor from one receiving position to another;
A signal generator (Tektronix AFG1022).

The experiment consisted of four steps:
The actuator was excited by a signal generator to transmit ultrasonic signals at a fixed position;
The sensor was used to receive scattered signals from the target. Then, the scattered signals were recorded by a DAQ system;
The sensor was moved to different receiving positions. The scattered waves were also recorded at these positions by the DAQ system;
All recorded signals were processed in PC for target localization.

The localization algorithm was conducted within a virtual calculation domain of 120 mm × 120 mm × 170 mm in the central part of the tank in order to decrease the influence of reflected signals. The coordinate system was established as shown in Figure 1 and the point O was the origin. The coordinates of the actuator, the 12 receiving positions, and the target are listed in Table 1.

This paper focuses on the 3-D target localization method. We make the following statements considering the equipment available in our laboratory.

An immersion transducer (1 MHz center frequency, 7 mm diameter) was chosen to be the actuator. The positions of the target and the actuator were fixed. In the experimental setup, the actuator and the target center were nearly aligned, as shown in Table 1. This was done to increase the energy of the received signals from the target. This knowledge was not utilized in the localization method to ensure that the location of the target remained unknown. In practical applications, an actuator with uniform directivity would be a better choice.

The DFB-FL ultrasonic sensing system we developed [18] was used to receive and record scattered signals due to its high sensitivity [19]. The sensing system includes a sensor, a DAQ, and a PC. The sensor was clamped and moved by a stepping motor. In practical applications, other types of sensors can also be used as long as the scattered signals can be received. It is not necessary for sensors to move from one receiving position to another. A sensor array composed of multiple sensors is also a good choice.

Figure 1. Experimental setup illustration. The target, actuator, and sensor are immersed in water.
In the virtual calculation domain. For any target in the calculation domain with the coordinate \((x, y, z)\), \(T_i\) was the ToF along an actuator-target-sensor path \(A\)–target-\(R_i\)

\[
T_i = \frac{1}{V}(\sqrt{(x^A - x)^2 + (y^A - y)^2 + (z^A - z)^2} + \sqrt{(x^{R_i} - x)^2 + (y^{R_i} - y)^2 + (z^{R_i} - z)^2}),
\]

where \((x^A, y^A, z^A)\) and \((x^{R_i}, y^{R_i}, z^{R_i})\) were coordinates of the actuator position and of the \(i\)-th receiving position, respectively. \(V\) was the speed of ultrasound in water.

There were three unknown target parameters, \((x, y, z)\), in Equation (1). The solution to Equation (1) was a root locus, as shown in Figure 2. For any point on the ellipsoid, the sum of the distance to the receiver and the transmitter was one distance measurement \((T_i; V)\), which was a constant. Therefore, the locus was an ellipsoid whose focuses were actuator position \(A\) and receiving position \(R_i\). Since calculation was conducted in the calculation domain, only part of the ellipsoid in the calculation domain was shown. The ellipsoid implied the possible locations of the target for a certain ToF.

3. Traditional ToF-Based Method

Consider the actuator position as denoted by \(A\) and the 12 receiving positions of the sensor denoted by \(R_i\) \((i = 1, 2, \ldots, 12)\) in the virtual calculation domain. For any target in the calculation domain with the coordinate \((x, y, z)\), \(T_i\) was the ToF along an actuator-target-sensor path \(A\)–target-\(R_i\)

\[
T_i = \frac{1}{V}(\sqrt{(x^A - x)^2 + (y^A - y)^2 + (z^A - z)^2} + \sqrt{(x^{R_i} - x)^2 + (y^{R_i} - y)^2 + (z^{R_i} - z)^2}),
\]

where \((x^A, y^A, z^A)\) and \((x^{R_i}, y^{R_i}, z^{R_i})\) were coordinates of the actuator position and of the \(i\)-th receiving position, respectively. \(V\) was the speed of ultrasound in water.

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![Figure 2. The relation between a single time-of-flight (ToF) measurement and the ellipsoid.](image)

In the traditional ToF-based method, there are two steps involved in target localization. First, the curves—the intersection of any two ellipsoids—are obtained. Then, the target location is given by seeking the intersection of all curves. As shown in Figure 3, in the case of using four receiving positions, there were four ellipsoids. Curve 1 is the intersection of the blue and green ellipsoids. Curve 2 is the...
intersection of the blue and yellow ellipsoids. Then, the point at which all curves (Curve1, Curve2, etc.) passed through was considered the target location.

![Diagram](image-url)

**Figure 3.** Target localization result using the traditional ToF-based method. Four ellipsoids are given by accurate ToF.

However, the measured ToF values were not absolutely accurate due to noises and many other factors. There was no point at which all curves passed through when the ellipsoids were drawn based on real-life ToF measurements. The points at which some, but not all, curves passed through were considered as pseudo-targets. As shown in Figure 4, we marked down Curve1–Curve6. Curve1 is the intersection of the blue and green ellipsoids. Curve2 is the intersection of the blue and pink ellipsoids. Curve3 is the intersection of the green and pink ellipsoids. Curve4 is the intersection of the blue and yellow ellipsoids. Curve5 is the intersection of the pink and yellow ellipsoids. Curve6 is the intersection of the green and yellow ellipsoids. Pseudo-target 1 is the point at which Curve1–Curve3 passed through but Curve4–Curve6 did not. Pseudo-target 2 and pseudo-target 3 were obtained in the same way.
In addition, ToF measurement errors also gave rise to disjoint ellipsoids and disjoint curves. For disjoint ellipsoids, as shown in Figure 5a, there was no curve intersected by the ellipsoids that were drawn based on errors contained in $T_i$ and $T_j$. For disjoint curves, as shown in Figure 5b, zooming in from another angle, the two curves do not cross. The lack of intersections affected the construction of curves and intersection points.

In a word, ToF measurement errors caused two problems in 3-D target localization, pseudo-targets, and disjoints. First, it was impossible to differentiate between pseudo-targets to obtain the target using traditional methods. Some scattered signals from the target are not employed to construct curves or intersection points, so part of the target information is lost. The traditional ToF-based method cannot solve these problems.

4. Probability-Based ToF Method

By assigning a target occurrence probability for each point in the virtual calculation domain, the probability-based ToF method can give an estimated target area, whether these points are located on the ellipsoids or not. Therefore, the two problems caused by ToF measurement errors could be solved.
In the probability-based ToF method, the virtual calculation domain in water was evenly meshed. For a certain measured ToF, $T_i$, the possibility of each mesh node being the target, was estimated using a probability density function [4],

$$f_i(x, y, z) = \left(\frac{1}{\pi \tau^2}\right) \exp\left(-\frac{|T(x, y, z) - T_i|^2}{2\tau^2}\right),$$

where $(x, y, z)$ were the coordinates of each node. $T(x, y, z)$ was the theoretical ToF at the node. $T_i$ was the measured ToF from the actuator to the $i$-th receiving position of the sensor. $\tau$ was a decay factor representing the decay rate of an exponential function. Equation (2) indicates that the possibility of target occurrence at each node is determined by the deviation from its theoretical ToF to the measured ToF. The mesh nodes right located on the established ellipsoid corresponding to $T_i$ were estimated to be the highest degree of probability. For the others, the greater the deviation from the measured ToF, the lower the probability of finding the target at these nodes.

All estimation results for each measured ToF were combined to give the localization result in a matrix form. Each element of the matrix represented the probability of the presence of target for one mesh node,

$$F(x, y, z) = \sum_{i=1}^{12} f_i(x, y, z),$$

where $N$ was the total number of actuator–target–sensor paths, referring to the total number of receiving positions.

In our experiment, ToFs were extracted from the scattered signals, which were recorded at 12 receiving positions of the sensor. The localization result estimated by the probability-based ToF method is shown in Figure 6. The yellow dot represents the real target location where ultrasonic waves interacted with the target. The gray dot represents the node meshed in the virtual calculation domain. The darker the grayscale, the higher the possibility of finding the target in that area.

![Figure 6. Target localization result using the probability-based ToF method.](image)

The probability-based ToF method assigned the probability of target occurrence to the nodes that were not located on the ellipsoids. The method solved the two problems caused by ToF measurement errors to give an area estimate. However, the estimated target area was dispersive, meaning that it was not accurate enough. Moreover, the estimation result was a target area, not a target point. A new localization method should be proposed to narrow down the target area and give an accurate target location.
5. Iterative Maximum Weighted Likelihood Estimation

The IMWLE method is an improvement of the traditional ToF-based method. It is a statistical method that improves the accuracy of 3-D target localization using the distribution of pseudo-targets. The way to get pseudo-targets is the same as that in Section 3. If the problem of disjoint ellipsoids appears, the curve is defined as a region where the distance to the two disjoint ellipsoids is smaller than a given number. If the problem of disjoint curves appears, the pseudo-target is defined as the midpoint of the line segment whose length is the closest distance between the two disjoint curves.

IMWLE is only related to the spatial distribution of pseudo-targets, so it is necessary to understand the rules of distribution. ToF measurement errors affect the spatial distribution of pseudo-targets. Some pseudo-targets are located close to the target, while other pseudo-targets are far from the target and may have a negative influence on the accuracy of localization.

IMWLE takes into account the spatial distribution of pseudo-targets for localization. The flow chart of the method is shown in Figure 7.

The pseudo-targets, whose coordinates are denoted by vector $x^k$, are assigned weight factors $\omega^j_k$. Superscript $k$ refers to the pseudo-target location numbers. Subscript $j$ refers to the iteration number.

Figure 7. The flow chart of the Iterative Maximum Weighted Likelihood Estimation (IMWLE) method.
When $j = 1$, $\alpha_j^k$ is initialized to the same values 1. By assuming that all pseudo-locations satisfied the Gaussian distribution, mean $\mu_1$ and covariance matrix $\Omega_1$ are calculated by the following equations:

$$
\mu_1 = \frac{1}{L} \sum_{k=1}^{L} x^k,
$$

(4)

$$
\Omega_1 = \frac{1}{L} \sum_{k=1}^{L} (x^k - \mu_1)(x^k - \mu_1)^T,
$$

(5)

where $L$ is the number of pseudo-targets.

In the next iteration, the weight factors of the pseudo-targets are defined as the probability density of the Gaussian distribution.

$$
\omega_j^k = \frac{1}{(2\pi)^{3/2} |\Omega_{j-1}|^{1/2}} \exp\left\{-\frac{1}{2} (x^k - \mu_{j-1})^T (\Omega_{j-1})^{-1} (x^k - \mu_{j-1}) \right\}.
$$

(6)

MWLE is used to fit the Gaussian distribution of pseudo-targets.

$$
\mu_j = \frac{\sum_{k=1}^{L} \omega_j^k x^k}{\sum_{k=1}^{L} \omega_j^k},
$$

(7)

$$
\Omega_j = \frac{\sum_{k=1}^{L} \omega_j^k (x^k - \mu_j)(x^k - \mu_j)^T}{\sum_{k=1}^{L} \omega_j^k}.
$$

(8)

The mean $\mu_j$ of the Gaussian distribution can be considered as the estimated target location in the $j$-th iteration.

If $|\mu_j - \mu_{j-1}| < \text{threshold}$ (in our case, we set threshold = 1%), the stop iteration and $\mu_{j-1}$ are the final estimated target location obtained by IMWLE; otherwise, set $j = j + 1$ and repeat from 2.

At each iteration, MWLE assigns new weight factors to pseudo-targets by fitting a Gaussian distribution to their positions, and small weight factors are assigned to the pseudo-targets that are distant from the target so as to minimize their influence on the accuracy of localization. Using the proposed method, the spatial distribution of pseudo-targets were taken into account in order to achieve accurate target localization.

6. Result and Discussion

6.1. Scattered Signals Processing and ToF Extraction

The signals scattered from the target were processed by wavelet threshold denoising to decrease the influence of white Gaussian noise. In order to extract ToF from the scattered signals, a Hilbert transform was applied in order to extract its envelope and the ToF information. The processed scattered signals recorded at receiving position 1 and its Hilbert transform are shown in Figure 8.
6.2. Target Localization Results

All pseudo-targets are shown in Figure 9 (in our case, the number of pseudo-targets was 2145). The black dots represent different pseudo-targets. Some pseudo-targets were close to the target, while others were far from it.

![Distribution of pseudo-targets](image)

**Figure 9.** Distribution of pseudo-targets. The number of pseudo-targets is 2145.

The IMWLE method considers the distribution of pseudo-targets to improve the accuracy of localization. The localization results after multiple iterations and the real target location are shown in Figure 10. The yellow dot represents the real target location, (64.33, 93.93, 43.33), where the ultrasonic waves interacted with the target. The blue dot represents the estimated target location. The grey dots and their grayscale represent different pseudo-targets and their weight factor. The higher the greyscale, the greater the possibility of detecting the target.

![Localization results](image)

**Figure 10.** The localization results after multiple iterations and the real target location are shown.
Figure 10. Target localization results using IMWLE after each iteration. The distribution of pseudo-targets after (a) the first iteration, (b) the second iteration, (c) the third iteration, (d) the fourth iteration, (e) the fifth iteration, and (f) the sixth iteration.
Figure 10a shows the distribution of pseudo-targets after the first iteration. All grey dots had the same grayscale representing same weight factors. Therefore, the estimated target location \( \mu_1 \) was the mean of coordinates of all pseudo-targets. Figure 10b–f show the distribution of pseudo-targets after the second, third, fourth, fifth, and sixth iterations, respectively. The weight factors of the pseudo-targets that were far away from the real target were gradually reduced after iteration so that corresponding greyscale became too low to observe. Therefore, the greyscale region became more converged with each iteration and the estimated target location came closer to the real target location.

The error of localization can be defined as the distance between real target location and estimated target location,

\[
error = \sqrt{(x^{\text{real}} - x^{\text{estimated}})^2 + (y^{\text{real}} - y^{\text{estimated}})^2 + (z^{\text{real}} - z^{\text{estimated}})^2},
\]

where \((x^{\text{real}}, y^{\text{real}}, z^{\text{real}})\) and \((x^{\text{estimated}}, y^{\text{estimated}}, z^{\text{estimated}})\) are the coordinates of the real target and the estimated target, respectively. The errors of localization after each iteration are listed in Table 2 and shown in Figure 11. In early iterations (in our case \( j = 1–6 \)), the influence of pseudo-targets far from the real target were reduced. The error gradually decreased from the first to the sixth iteration. After the sixth iteration, the influence of pseudo-targets that were close to the real target had also reduced. The number of pseudo-targets that took effect in localization algorithm were reduced, which means that the statistical method gradually became inappropriate. Therefore, the fluctuation of error appeared. The stopping condition could stop the iteration at \( j = 7 \) and \( \mu_6 \), which was the final estimated target location.

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Coordinates/mm</th>
<th>Error/mm</th>
<th>Iteration Number</th>
<th>Coordinates/mm</th>
<th>Error/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(57.14, 86.39, 41.07)</td>
<td>10.92</td>
<td>8</td>
<td>(63.67, 93.15, 44.64)</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>(58.12, 89.99, 42.38)</td>
<td>7.61</td>
<td>9</td>
<td>(63.66, 93.15, 44.55)</td>
<td>1.06</td>
</tr>
<tr>
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<td>3.40</td>
<td>10</td>
<td>(63.64, 93.14, 44.44)</td>
<td>1.05</td>
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<td>11</td>
<td>(63.62, 93.15, 44.30)</td>
<td>1.06</td>
</tr>
<tr>
<td>5</td>
<td>(63.64, 93.11, 44.57)</td>
<td>1.10</td>
<td>12</td>
<td>(63.60, 93.14, 44.11)</td>
<td>1.10</td>
</tr>
<tr>
<td>6</td>
<td>(63.71, 93.15, 44.67)</td>
<td>1.05</td>
<td>13</td>
<td>(63.55, 93.13, 44.00)</td>
<td>1.16</td>
</tr>
<tr>
<td>7</td>
<td>(63.69, 93.15, 44.68)</td>
<td>1.07</td>
<td>14</td>
<td>(44.00, 63.50, 93.11)</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Figure 11. The errors of localization results at each iteration.

The probability-based ToF method and the IMWLE method both provided the estimated target area. By comparing the two results, it can be observed that the probability-based ToF method gave
a dispersive estimated target area, as shown in Figure 6. However, the IMWLE method, which was based on the density of pseudo-targets, gradually narrowed down the estimated target area during iteration to increase localization accuracy, as shown in Figure 10. Moreover, IMWLE gave a point estimate of the target, making it possible to evaluate the quality of the localization results, while the probability-based ToF method was not able to provide a point estimate.

7. Conclusions

In this paper, the influence of ToF measurement errors was analyzed in the traditional ToF-based method for 3-D localization. A new localization method based on the density of pseudo-targets, IMWLE, was proposed for 3-D localization to improve accuracy. The spatial distribution of pseudo-targets were taken into account. At each iteration, MWLE adjusted the weight factors assigned to the pseudo-targets by fitting a Gaussian distribution to their positions. By repeating this procedure, the accuracy of estimation had improved because the influence of pseudo-targets distant from the target were minimized. Compared with the probability-based ToF method, the IMWLE method performed better in terms of accuracy. Although multiple target localization still needs further study, the method we proposed is suitable for single 3-D target localization.


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Conflicts of Interest: The authors declare no conflicts of interest.

References

5. Li, H.-N.; Li, D.-S.; Song, G.-B. Recent applications of fiber optic sensors to health monitoring in civil engineering. Eng. Struct. 2004, 26, 1647–1657. [CrossRef]
12. Flynn, E.B.; Todd, M.D.; Wilcox, P.D.; Drinkwater, B.W.; Croxford, A.J. Maximum-likelihood estimation of
467, 2575–2596. [CrossRef]

plate-like structures using an active guided wave structural health monitoring system. Smart Mater. Struct.
2010, 19, 045022. [CrossRef]

14. Hong, M.; Su, Z.; Lu, Y.; Sohn, H.; Qing, X. Locating fatigue damage using temporal signal features of

15. Li, B.; Liu, Y.; Gong, K.; Li, Z. Damage localization in composite laminates based on a quantitative expression
of anisotropic wavefront. Smart Mater. Struct. 2013, 22, 065005. [CrossRef]

Damage Detection Based on Data Fusion. In Proceedings of the 14th Asia-Pacific Vibration Conference,
Hong Kong, China, 5–8 December 2011; The Hong Kong Polytechnic University: Hong Kong, China, 2011.

17. Niri, E.D.; Salamone, S. A probabilistic framework for acoustic emission source localization in plate-like
structures. Smart Mater. Struct. 2012, 21, 035009. [CrossRef]

distributed feedback fiber grating laser Sensors. In Proceedings of the 2017 National Acoustic Conference of


20. Zhao, X.; Gao, H.; Zhang, G.; Ayhan, B.; Yan, F.; Kwan, C.; Rose, J.L. Active health monitoring of an aircraft
wing with embedded piezoelectric sensor/actuator network: I. Defect detection, localization and growth
monitoring. Smart Mater. Struct. 2007, 16, 1208. [CrossRef]