axioms

Article

On Fixed Point Results for Modified JS-Contractions with Applications

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Abstract: In [Fixed Point Theory Appl., 2015 (2015):185], the authors introduced a new concept of modified contractive mappings, generalizing Ćirić, Chatterjea, Kannan, and Reich type contractions. They applied the condition (θ_4) (see page 3, Section 2 of the above paper). Later, in [Fixed Point Theory Appl., 2016 (2016):62], Jiang et al. claimed that the results in [Fixed Point Theory Appl., 2015 (2015):185] are not real generalizations. In this paper, by restricting the conditions of the control functions, we obtain a real generalization of the Banach contraction principle (BCP). At the end, we introduce a weakly JS-contractive condition generalizing the JS-contractive condition.

Keywords: metric space; fixed point; weakly JS-contraction

1. Introduction

The Banach contraction principle (BCP) [1] is one of the famous results in fixed point theory which has attracted many authors. Many extensions and generalizations have been appeared in literature by weakening the topology itself of the space or by considering different contractive conditions (for single and valued mappings). For more details, see ([2–23]).

Definition 1. *Given a mapping* $\Upsilon : X \to X$ *on a metric space* (X, d).

(a) *Such* Υ *is a C-contraction if there is* $\mu \in \left(0, \frac{1}{2}\right)$ *such that for all* $\Omega, \omega \in X$, *[24]*

$$d(\Upsilon\Omega, \Upsilon\omega) \leq \mu \left(d(\Omega, \Upsilon\omega) + d(\omega, \Upsilon\Omega)\right).$$

(b) *Such* Υ *is a K-contraction if there is* $\mu \in \left(0, \frac{1}{2}\right)$ *such that for all* $\Upsilon\Omega \in X$, *[25]*

$$d(\Upsilon\Omega, \Upsilon\omega) \leq \mu \left(d(\Omega, \Upsilon\Omega) + d(\omega, \Upsilon\omega)\right).$$

(c) *Such* Υ *is a Reich contraction if there are* q, r *and* $s \geq 0$ *with* $q + r + s < 1$ *such that for all* $\Omega, \omega \in X$,

$$d(\Upsilon\Omega, \Upsilon\omega) < q \cdot d(\Omega, \omega) + r \cdot d(\Omega, \Upsilon\Omega) + s \cdot d(\omega, \Upsilon\omega).$$

Denote by Θ the set of functions $\theta : (0, \infty) \to (1, \infty)$ satisfying the following assertions:

(θ_1) θ is non-decreasing;

(θ_2) for each $\{h_k\} \subseteq (0,\infty)$, $\lim_{k\to\infty} \theta(h_k) = 1$ if and only if $\lim_{k\to\infty} h_k = 0$;

(θ_3) there are $m \in (0,1)$ and $\tau \in (0,\infty]$ so that

$$\lim_{u\to 0^+} \frac{\theta(u)-1}{u^m} = \tau;$$

(θ_4) $\theta(i+j) \leq \theta(i)\theta(j)$ for all $i,j > 0$.

By Δ we denote the class of functions $\theta \in \Theta$ without condition (θ_4).

Theorem 1. *([26, Corollary 2.1]) Let $\Upsilon : X \to X$ be a self-mapping on a complete metric space (X,d). Suppose there are $\theta \in \Delta$ and $\mu \in (0,1)$ so that*

$$\Omega,\omega \in X, \quad d(\Upsilon\Omega,\Upsilon\omega) \neq 0 \quad \textit{implies} \quad \theta\left(d(\Upsilon\Omega,\Upsilon\omega)\right) \leq \left(\theta\left(d(\Omega,\omega)\right)\right)^{\mu}.$$

Then T has a unique fixed point.

Note that the BCP comes immediately from Theorem 1. Motivated by [26], Hussain et al. [27] gave sufficient conditions for the existence of a fixed point of a class of generalized contractive mappings via a control function $\theta \in \Theta$ in the setting of complete metric spaces and b-complete b-metric spaces. Denote by Λ the set of functions $\theta : (0,\infty) \to (1,\infty)$ verifying (θ_1), (θ_2) and (θ_4). On the other hand, when considering (X,d) as a metric space and $\theta \in \Lambda$ (that is, the condition (θ_3) is omitted from Θ), Jiang et al. [28] proved that $D(x,y) = \ln(\theta(d(x,y)))$ defines itself a metric on X (see Lemma 1 in [28]) and proved that the results in [27] are not generalizations of Ćirić, Chatterjea, Kannan, and Reich results.

In this paper, we more restrict the conditions on the control function θ. For this, denote by Θ' the set of functions $\theta : (0,\infty) \to (1,\infty)$ so that

(θ_1) θ is continuous and strictly increasing;

(θ_2) for each $\{h_k\} \subseteq (0,\infty)$, $\lim_{k\to\infty} \theta(h_k) = 1$ if and only if $\lim_{k\to\infty} h_k = 0$.

Let (X,d) be a metric space. For $\theta \in \Theta'$ (that is, without the condition (θ_4)), note that $D(x,y) = \ln(\theta(d(x,y)))$ does not define a metric on X (we can not ensure the triangular inequality for a metric). Consequently, we are not in same direction as Jiang et al. [28]. Even for such restricted control function θ, we also obtain a real generalization of the Banach contraction principle. In fact, we will complete the work of Hussain et al. [27]. We refer the readers to Theorem 3 of [16].

2. Main Results

Definition 2. *Let $\Upsilon : X \to X$ be a self-mapping on a metric space (X,d). Such Υ is said to be a $\mathcal{P}$-contraction, whenever there are $\theta \in \Theta'$ and $\tau_1,\tau_2,\tau_3,\tau_4 \geq 0$ with $\tau_1+\tau_2+\tau_3+\tau_4 < 1$ such that the following holds:*

$$\theta\left(d(\Upsilon\Omega,\Upsilon\omega)\right) \leq \left(\theta\left(d(\Omega,\omega)\right)\right)^{\tau_1} \left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_2} \left(\theta\left(d(\omega,\Upsilon\omega)\right)\right)^{\tau_3} \left(\theta\left(\tfrac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}\right)\right)^{\tau_4}, \quad (1)$$

for all $\Omega,\omega \in X$.

As a new generalization of the BCP, we have

Theorem 2. *Each $\mathcal{P}$-contraction mapping on a complete metric space has a unique fixed point.*

Proof. Let $\Omega_0 \in X$ be arbitrary. Define $\{\Omega_n\}$ by $\Omega_n = \Upsilon\Omega_{n-1}$, $n \geq 1$. If there is $\Omega_N = \Omega_{N+1}$ for some N, nothing is to prove. We assume that $\Omega_n \neq \Omega_{n+1}$ for each $n \geq 0$.

We claim that

$$\lim_{n\to\infty} d(\Omega_n,\Omega_{n+1}) = 0.$$

In view of (1), we have

$$\begin{aligned}\theta\left(d(\Omega_{n+1},\Omega_n)\right) &= \theta\left(d(\Upsilon\Omega_n,\Upsilon\Omega_{n-1})\right) \\ &\leq \left(\theta\left(d(\Omega_n,\Omega_{n-1})\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_n,\Upsilon\Omega_n)\right)\right)^{\tau_2} \\ &\quad \left(\theta\left(d(\Omega_{n-1},\Upsilon\Omega_{n-1})\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Omega_n,\Upsilon\Omega_{n-1})+d(\Omega_{n-1},\Upsilon\Omega_n)}{2}\right)\right)^{\tau_4} \\ &\leq \left(\theta\left(d(\Omega_n,\Omega_{n-1})\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_n,\Omega_{n+1})\right)\right)^{\tau_2} \\ &\quad \left(\theta\left(d(\Omega_{n-1},\Omega_n)\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Omega_{n-1},\Omega_{n+1})}{2}\right)\right)^{\tau_4} \\ &\leq \left(\theta\left(d(\Omega_n,\Omega_{n-1})\right)\right)^{\tau_1+\tau_3}\left(\theta\left(d(\Omega_n,\Omega_{n+1})\right)\right)^{\tau_2}\left(\theta\left(\max\{d(\Omega_{n-1},\Omega_n),d(\Omega_n,\Omega_{n+1})\}\right)\right)^{\tau_4}.\end{aligned} \tag{2}$$

If for some N, we have

$$d(\Omega_{N-1},\Omega_N) < d(\Omega_N,\Omega_{N+1}),$$

then in view of (θ_1), we get that

$$\theta(d(\Omega_{N-1},\Omega_N)) < \theta(d(\Omega_N,\Omega_{N+1})). \tag{3}$$

Using (2), we have

$$\theta\left(d(\Omega_{N+1},\Omega_N)\right) \leq \left(\theta\left(d(\Omega_N,\Omega_{N-1})\right)\right)^{\tau_1+\tau_3}\left(\theta\left(d(\Omega_N,\Omega_{N+1})\right)\right)^{\tau_2+\tau_4}. \tag{4}$$

Therefore,

$$\theta\left(d(\Omega_{N+1},\Omega_N)\right) \leq \left(\theta\left(d(\Omega_N,\Omega_{N-1})\right)\right)^{\frac{\tau_1+\tau_3}{1-\tau_2-\tau_4}} \leq \theta\left(d(\Omega_N,\Omega_{N-1})\right),$$

which is a contradiction with respect to (3).

Consequently, for all $n \geq 1$,

$$\max\left\{d(\Omega_{n-1},\Omega_n),d(\Omega_n,\Omega_{n+1})\right\} = d(\Omega_{n-1},\Omega_n),$$

which yields that

$$1 < \theta(d(\Omega_{n+1},\Omega_n)) \leq (\theta(d(\Omega_1,\Omega_0)))^{[\frac{\tau_1+\tau_3+\tau_4}{1-\tau_2}]^n}.$$

At the limit, we have

$$\lim_{n\to\infty}\theta(d(\Omega_n,\Omega_{n+1})) = 1.$$

According to (θ_2), we get

$$\lim_{n\to\infty} d(\Omega_n,\Omega_{n+1}) = 0. \tag{5}$$

In order to show that $\{\Omega_n\}$ is a Cauchy sequence, suppose the contrary, i.e., there is $\varepsilon > 0$ for which we can find m_i and n_i so that

$$n_i > m_i > i, \qquad d(\Omega_{m_i},\Omega_{n_i}) \geq \varepsilon. \tag{6}$$

That is,

$$d(\Omega_{m_i},\Omega_{n_i-1}) < \varepsilon. \tag{7}$$

From (6), one writes

$$d(\Omega_{m_i-1},\Omega_{n_i-1}) \leq d(\Omega_{m_i-1},\Omega_{m_i}) + d(\Omega_{m_i},\Omega_{n_i-1}).$$

In view of (5) and (7), we get

$$\limsup_{i\to\infty} d(\Omega_{m_i-1},\Omega_{n_i-1}) \le \varepsilon. \tag{8}$$

Analogously,

$$\limsup_{i\to\infty} d(\Omega_{m_i-1},\Omega_{n_i}) \le \varepsilon. \tag{9}$$

On the other hand, we have

$$\begin{aligned}
\theta\left(d(\Omega_{m_i},\Omega_{n_i})\right) &= \theta\left(d(\Upsilon\Omega_{m_i-1},\Upsilon\Omega_{n_i-1})\right)\\
&\le \left(\theta\left(d(\Omega_{m_i-1},\Omega_{n_i-1})\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_{m_i-1},\Upsilon\Omega_{m_i-1})\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(d(\Omega_{n_i-1},\Upsilon\Omega_{n_i-1})\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Omega_{m_i-1},\Upsilon\Omega_{n_i-1})+d(\Omega_{n_i-1},\Upsilon\Omega_{m_i-1})}{2}\right)\right)^{\tau_4}\\
&\le \left(\theta\left(d(\Omega_{m_i-1},\Omega_{n_i-1})\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_{m_i-1},\Omega_{m_i})\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(d(\Omega_{n_i-1},\Omega_{n_i})\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Omega_{m_i-1},\Omega_{n_i})+d(\Omega_{n_i-1},\Omega_{m_i})}{2}\right)\right)^{\tau_4}.
\end{aligned}$$

Using now (θ_1) and (5)–(8), we have

$$\begin{aligned}
\theta(\varepsilon) &\le \theta\left(\limsup_{i\to\infty} d(\Omega_{m_i},\Omega_{n_i})\right)\\
&\le \left(\theta\left(\limsup_{i\to\infty} d(\Omega_{m_i-1},\Omega_{n_i-1})\right)\right)^{\tau_1}\left(\theta\left(\limsup_{i\to\infty} d(\Omega_{m_i-1},\Omega_{m_i})\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(\limsup_{i\to\infty} d(\Omega_{n_i-1},\Omega_{n_i})\right)\right)^{\tau_3}\left(\theta\left(\limsup_{i\to\infty}\frac{d(\Omega_{m_i-1},\Omega_{n_i})+d(\Omega_{n_i-1},\Omega_{m_i})}{2}\right)\right)^{\tau_4}\\
&\le (\theta(\varepsilon))^{\tau_1}(\theta(\varepsilon))^{\tau_4}.
\end{aligned}$$

This implies that

$$1 < \theta(\varepsilon) \le (\theta(\varepsilon))^{\tau_1+\tau_4},$$

which is a contradiction. Thus, $\{\Omega_n\}$ is a Cauchy sequence. The completeness of X implies that there is $\Omega \in X$ so that $\Omega_n \to \Omega$ as $n \to \infty$. On the other hand,

$$\begin{aligned}
\theta\left(d(\Omega_n,\Upsilon\Omega)\right) &= \theta\left(d(\Upsilon\Omega_{n-1},\Upsilon\Omega)\right)\\
&\le \left(\theta\left(d(\Omega_{n-1},\Omega)\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_{n-1},\Upsilon\Omega_{n-1})\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Upsilon\Omega,\Omega_{n-1})+d(\Omega,\Upsilon\Omega_{n-1})}{2}\right)\right)^{\tau_4}\\
&\le \left(\theta\left(d(\Omega_{n-1},\Omega)\right)\right)^{\tau_1}\left(\theta\left(d(\Omega_{n-1},\Omega_n)\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_3}\left(\theta\left(\frac{d(\Upsilon\Omega,\Omega_{n-1})+d(\Omega,\Omega_n)}{2}\right)\right)^{\tau_4}.
\end{aligned}$$

Taking $n \to \infty$ and using (θ_1) and (5), we have

$$\begin{aligned}
\theta\left(d(\Omega,\Upsilon\Omega)\right) &\le \left(\theta\left(d(\Omega,\Omega)\right)\right)^{\tau_1}\left(\theta\left(d(\Omega,\Omega)\right)\right)^{\tau_2}\\
&\quad \left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_3}\left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_4}\\
&= \left(\theta\left(d(\Omega,\Upsilon\Omega)\right)\right)^{\tau_3+\tau_4}.
\end{aligned}$$

We deduce that $\Omega = \Upsilon\Omega$, so Ω is a fixed point.

Let there are two points Ω, ω which are two different fixed points of Υ. So,

$$\begin{aligned}\theta\left(d(\Upsilon\Omega, \Upsilon\omega)\right) &\leq \left(\theta\left(d(\Omega, \omega)\right)\right)^{\tau_1}\left(\theta\left(d(\Omega, \Upsilon\Omega)\right)\right)^{\tau_2} \\ &\quad \left(\theta\left(d(\omega, \Upsilon\omega)\right)\right)^{\tau_3}\left(\theta\left(d(\Omega, \Upsilon\Omega)\right)\right)^{\tau_4} \\ &= \left(\theta\left(d(\Omega, \Upsilon\Omega)\right)\right)^{\tau_3+\tau_4}.\end{aligned}$$

We deduce that $\Omega = \Upsilon\Omega$, so Ω is a fixed point.
Let Ω, ω be two distinct fixed points of Υ. We have

$$\begin{aligned}\theta\left(d(\Omega, \omega)\right) = \theta\left(d(\Upsilon\Omega, \Upsilon\omega)\right) &\leq \left(\theta\left(d(\Omega, \omega)\right)\right)^{\tau_1}\left(\theta\left(d(\Omega, \Omega)\right)\right)^{\tau_2} \\ &\quad \left(\theta\left(d(\omega, \omega)\right)\right)^{\tau_3}\left(\theta\left(d(\Omega, \omega)\right)\right)^{\tau_4} \\ &= \left(\theta\left(d(\Omega, \omega)\right)\right)^{\tau_1+\tau_4} < \theta\left(d(\Omega, \omega)\right),\end{aligned}$$

which is a contradiction. So, Ω has a unique fixed point. □

Remark 1. *In Theorem 2, we can substitute the continuity of θ by the continuity of Υ.*

By setting $\theta(t) = e^{\sqrt{t}}$, we have

Corollary 1. *Let $\Upsilon : X \to X$ be a mapping on a complete metric space (X, d) such that the following holds:*

$$\sqrt{d\left(\Upsilon\Omega, \Upsilon\omega\right)} \leq \tau_1\sqrt{d\left(\Omega, \omega\right)} + \tau_2\sqrt{d\left(\Omega, \Upsilon\Omega\right)} + \tau_3\sqrt{d\left(\omega, \Upsilon\omega\right)} + \tau_4\sqrt{\frac{d\left(\Omega, \Upsilon\omega\right) + d\left(\omega, \Upsilon\Omega\right)}{2}},$$

for all $\Omega, y \in X$, where $\theta \in \mathcal{P}$ and $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$ so that $\tau_1 + \tau_2 + \tau_3 + \tau_4 < 1$. Then Υ has a unique fixed point.

Remark 2. *Taking $\tau_1 = \tau_4 = 0$ in the Corollary 1, we get Theorem 2.6 of [27].*
Taking $\tau_4 = 0$ in Theorem 1, we get Theorem 2.8 of [27].

Setting $\theta\left(t\right) = e^{\sqrt[n]{t}}$ in Theorem 2, we have

Corollary 2. *Let (Ω, d) be a complete metric space and let $\Upsilon : X \to X$ be such that the following holds:*

$$\sqrt[n]{d\left(\Upsilon\Omega, \Upsilon\omega\right)} \leq \tau_1\sqrt[n]{d\left(\Omega, \omega\right)} + \tau_2\sqrt[n]{d\left(\Omega, \Upsilon\Omega\right)} + \tau_3\sqrt[n]{d\left(\omega, \Upsilon\omega\right)} + \tau_4\sqrt[n]{\frac{d\left(\Omega, \Upsilon\omega\right) + d\left(\omega, \Upsilon\Omega\right)}{2}},$$

for all $\Omega, \omega \in X$, where $\theta \in \mathcal{P}$ and $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$ such that $\tau_1 + \tau_2 + \tau_3 + \tau_3 < 1$. Then Υ has a unique fixed point.

Remark 3 ([12]). *Other examples of functions in the set $\mathcal{P}$ are*

$$\begin{array}{lll} f\left(t\right) = \cosh t, & f\left(t\right) = e^{te^t}, & f\left(t\right) = e^{\sqrt{t}e^{\sqrt{t}}}, \\ f\left(t\right) = \frac{2\cosh t}{1+\cosh t}, & f\left(t\right) = \frac{2e^{te^t}}{1+e^{te^t}}, & f\left(t\right) = \frac{2e^{\sqrt{t}e^{\sqrt{t}}}}{1+e^{\sqrt{t}e^{\sqrt{t}}}}, \\ f\left(t\right) = 1 + \ln\left(1+t\right), & f\left(t\right) = e^{\sqrt{te^t}}, & \\ f\left(t\right) = \frac{2+2\ln(1+t)}{2+\ln(1+t)}, & f\left(t\right) = \frac{2e^{\sqrt{te^t}}}{1+e^{\sqrt{te^t}}}, & \end{array}$$

for all $t > 0$.

By setting $\theta(t) = e^{te^t}$, we have

Corollary 3. *Let $\Upsilon : X \to X$ be a continuous mapping on a complete metric space (X, d). Suppose that there are $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$ with $\tau_1 + \tau_2 + \tau_3 + \tau_4 < 1$ such that the following holds:*

$$d(\Upsilon\Omega, \Upsilon\omega)\, e^{d(\Upsilon\Omega,\Upsilon\omega)} \leq \tau_1 d(\Omega,\omega)\, e^{d(\Omega,\omega)} + \tau_2 d(\Omega,\Upsilon\Omega)\, e^{d(\Omega,\Upsilon\Omega)} + \tau_3 d(\omega,\Upsilon\omega)\, e^{d(\omega,\Upsilon\omega)} + \tau_4 d(\omega,\Upsilon\Omega)\, e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}},$$

for all $\Omega, \omega \in X$. Then there is a unique fixed point of Υ.

Corollary 4. *Let $\Upsilon : X \to X$ be a continuous mapping on a complete metric space (X, d). Suppose that there are $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$ with $\tau_1 + \tau_2 + \tau_3 + \tau_4 < 1$ such that the following holds:*

$$\frac{2e^{d(\Upsilon\Omega,\Upsilon\omega)e^{d(\Upsilon\Omega,\Upsilon\omega)}}}{1+e^{d(\Upsilon\Omega,\Upsilon\omega)e^{d(\Upsilon\Omega,\Upsilon\omega)}}} \leq \left[\frac{2e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}}{1+e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}}\right]^{\tau_1} \left[\frac{2e^{d(\Omega,\Upsilon\Omega)e^{d(\Omega,\Upsilon\Omega)}}}{1+e^{d(\Omega,\Upsilon\Omega)e^{d(\Omega,\Upsilon\Omega)}}}\right]^{\tau_2} \left[\frac{2e^{d(\omega,\Upsilon\omega)e^{d(\omega,\Upsilon\omega)}}}{1+e^{d(\omega,\Upsilon\omega)e^{d(\omega,\Upsilon\omega)}}}\right]^{\tau_3} \left[\frac{2e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}}}}{1+e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}}}}\right]^{\tau_4},$$

for all $\Omega, \omega \in X$. Then there is a unique fixed point of Υ.

Corollary 5. *Let $\Upsilon : X \to X$ be a continuous mapping on a complete metric space (X, d). Suppose that there are $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$ with $\tau_1 + \tau_2 + \tau_3 + \tau_4 < 1$ such that the following holds:*

$$1 + \ln(1 + d(\Upsilon\Omega, \Upsilon\omega)) \leq [1 + \ln(1 + d(\Omega,\omega))]^{\tau_1} [1 + \ln(1 + d(\Omega,\Upsilon\Omega))]^{\tau_2} [1 + \ln(1 + d(\omega,\Upsilon\omega))]^{\tau_3} \left[1 + \ln\left(1 + \frac{d(\Omega,\Upsilon\omega) + d(\omega,\Upsilon\Omega)}{2}\right)\right]^{\tau_4},$$

for all $\Omega, \omega \in X$. Then Υ has a unique fixed point.

Example 1. *Let $X = [0, 5]$ be endowed with the metric $d(\Omega, \omega) = |\Omega - \omega|$ for all $\Omega, \omega \in X$. Define $\Upsilon : X \to X$ and $\theta : (0, \infty) \to (1, \infty)$ by*

$$\Upsilon\Omega = \begin{cases} \frac{2}{3\pi}\Omega \arctan\Omega, & \text{if } \Omega \in [0, \alpha], \\ \frac{1}{3}\sinh^{-1}\Omega & \text{if } \Omega \in [\alpha, +5], \end{cases}$$

where α ($\simeq 2.06$) is the positive solution of the equation

$$\frac{2}{3\pi}\Omega \arctan\Omega = \frac{1}{3}\sinh^{-1}\Omega.$$

Take $\theta(t) = e^{te^t}$. Choose $\tau_1 = \frac{37}{100}$ and $\tau_i = \frac{1}{5}$ for $i = 2, 3, 4$.

Let $\Omega, \omega \in X = [0, 5]$. We have the following cases:

Case 1: $\Omega, \omega \in [0, \alpha]$. According to the mean value Theorem for $t \longmapsto g(t) := \frac{2}{3\pi}t \arctan t$ on the interval $J = (\min(\omega, \Omega), \max(\omega, \Omega)) \subset [0, \alpha]$, there is some $c \in J$ such that

$$d(\Upsilon\Omega, \Upsilon\omega) = \left|\tfrac{2}{3\pi}\Omega \arctan\Omega - \tfrac{2}{3\pi}\omega \arctan\omega\right| \leq g'(c) d(\Omega, \omega),$$

where

$$g'(c) = \frac{2}{3\pi}\arctan c + \frac{2}{3\pi}\frac{c}{1+c^2} \leq \frac{2}{3\pi}\frac{6}{5} + \frac{2}{3\pi}\frac{1}{2} \leq \frac{17}{15\pi} \leq \frac{37}{100},$$

because that $\arctan c \leq \frac{6}{5}$*, for each* $c \in [0, \alpha]$*, and* $\frac{c}{1+c^2} \leq \frac{1}{2}$*, for each* $c \geq 0$*.*

Therefore,

$$\begin{aligned}\theta\left(d(\Upsilon\Omega, \Upsilon\omega)\right) &= e^{d(\Upsilon\Omega,\Upsilon\omega)e^{d(\Upsilon\Omega,\Upsilon\omega)}} \\ &= e^{d(\frac{2}{3\pi}\Omega\arctan\Omega, \frac{2}{3\pi}\omega\arctan\omega)e^{d(\frac{2}{3\pi}\Omega\arctan\Omega, \frac{2}{3\pi}\omega\arctan\omega)}} \\ &\leq \left[e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}\right]^{\frac{37}{100}} \\ &\leq \left[e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}\right]^{\frac{37}{100}} \cdot \left[e^{d(\Omega,\Upsilon\Omega)e^{d(\Omega,\Upsilon\Omega)}}\right]^{\frac{20}{100}} \cdot \\ &\quad \left[e^{d(\omega,\Upsilon\omega)e^{d(\omega,\Upsilon\omega)}}\right]^{\frac{20}{100}} \cdot \left[e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}}}\right]^{\frac{20}{100}}.\end{aligned}$$

Case 2: $\Omega \in [0, \alpha]$ *and* $\omega \in [\alpha, 5]$*. Here,*

$$\frac{2}{3\pi}\omega\arctan\omega \geq \frac{1}{3}\sinh^{-1}\omega$$

for all $\omega \in [\alpha, 5]$*. Using the mean value Theorem on the function* $t \to \frac{2}{3\pi}t\arctan t$ *on the interval* $[\Omega, \omega]$*, we have*

$$\begin{aligned}d\left(\Upsilon\Omega, \Upsilon\omega\right) &= \left|\frac{2}{3\pi}\Omega\arctan\Omega - \frac{1}{3}\sinh^{-1}\omega\right| = \frac{1}{3}\sinh^{-1}\omega - \frac{2}{3\pi}\Omega\arctan\Omega \\ &\leq \frac{2}{3\pi}\omega\arctan\omega - \frac{2}{3\pi}\Omega\arctan\Omega \\ &\leq \frac{37}{100}d(\Omega,\omega),\end{aligned}$$

Therefore, as in case 1,

$$\begin{aligned}\theta\left(d(\Upsilon\Omega, \Upsilon\omega)\right) &= e^{d(\Upsilon\Omega,\Upsilon\omega)e^{d(\Upsilon\Omega,\Upsilon\omega)}} \\ &\leq \left[e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}\right]^{\frac{37}{100}} \cdot \left[e^{d(\Omega,\Upsilon\Omega)e^{d(\Omega,\Upsilon\Omega)}}\right]^{\frac{20}{100}} \cdot \\ &\quad \left[e^{d(\omega,\Upsilon\omega)e^{d(\omega,\Upsilon\omega)}}\right]^{\frac{20}{100}} \cdot \left[e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}}}\right]^{\frac{20}{100}}.\end{aligned}$$

Case 3: $\omega \in [0, \alpha]$ *and* $\Omega \in [\alpha, 5]$*. It is similar to case 2.*

Case 4: $\Omega, \omega \in [\alpha, 5]$*. Here, one writes*

$$d(\Upsilon\Omega, \Upsilon\omega) = \left|\tfrac{1}{3}\sinh^{-1}\Omega - \tfrac{1}{3}\sinh^{-1}\omega\right| \leq \tfrac{37}{100}d(\Omega,\omega).$$

Similarly,

$$\begin{aligned}\theta\left(d(\Upsilon\Omega, \Upsilon\omega)\right) &\leq \left[e^{d(\Omega,\omega)e^{d(\Omega,\omega)}}\right]^{\frac{37}{100}} \cdot \left[e^{d(\Omega,\Upsilon\Omega)e^{d(\Omega,\Upsilon\Omega)}}\right]^{\frac{20}{100}} \cdot \\ &\quad \left[e^{d(\omega,\Upsilon\omega)e^{d(\omega,\Upsilon\omega)}}\right]^{\frac{20}{100}} \cdot \left[e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}e^{\frac{d(\Omega,\Upsilon\omega)+d(\omega,\Upsilon\Omega)}{2}}}\right]^{\frac{20}{100}}.\end{aligned}$$

Hence, Υ *is a* $\mathcal{P}$*-contraction. Thus all the conditions of Theorem 2 hold and* Υ *has a fixed point* ($\Omega = 0$)*.*

3. Weak-JS Contractive Conditions

Let Φ be the class of functions $\phi : [1, \infty) \to [0, \infty)$ satisfying the following properties:

(ϕ_1) ϕ is continuous;
(ϕ_2) $\phi(1) = 0$;
(ϕ_3) or each $\{b_n\} \subseteq (1, \infty)$, $\lim_{n\to\infty} \phi(b_n) = 0$ iff $\lim_{n\to\infty} b_n = 1$.

Remark 4. *It is clear that* $\Upsilon(t) = t - \sqrt[n]{t}$ $(n \geq 1)$ *belongs to* Φ*. Other examples are* $\Upsilon(t) = e^{t-1} - 1$ *and* $\Upsilon(t) = \ln t$*.*

Definition 3. *Let* (X, d) *be a metric space and let* Υ *be a self-mapping on* X*.*
We say that Υ *is a weakly JS-contraction if for all* $\Omega, \omega \in X$ *with* $d(\Upsilon\Omega, \Upsilon\omega) > 0$*, we have*

$$\theta\big(d(\Upsilon\Omega, \Upsilon\omega)\big) \leq \theta\big(d(\Omega, \omega)\big) - \phi\big(\theta(d(\Omega, \omega))\big) \tag{10}$$

where $\phi \in \Phi$ *and* $\theta \in \Theta'$*.*

Theorem 3. *Let* (X, d) *be a complete metric space. Let* Υ *be a self-mapping on* X *so that*

(i) Υ *is a weakly JS-contraction;*
(ii) Υ *is continuous.*

Then Υ *has a unique fixed point.*

Proof. Let $\Omega_0 \in X$ be arbitrary. Define $\{\Omega_n\}$ by $\Omega_n = \Upsilon^n\Omega_0 = \Upsilon\Omega_{n-1}$. Without loss of generality, assume that $\Omega_n \neq \Omega_{n+1}$ for each $n \geq 0$. Since Υ is a weakly JS-contraction, we derive

$$\theta\big(d(\Omega_n, \Omega_{n+1})\big) = \theta\big(d(\Upsilon\Omega_{n-1}, \Upsilon\Omega_n)\big) \leq \theta\big(d(\Omega_{n-1}, \Omega_n)\big) - \phi\big(\theta(d(\Omega_{n-1}, \Omega_n))\big). \tag{11}$$

So, we deduce that $\{\theta\big(d(\Omega_n, \Omega_{n+1})\big)\}$ is decreasing, and so there is $r \geq 1$ so such $\lim_{n\to\infty} \theta\big(d(\Omega_n, \Omega_{n+1})\big) = r$. We will prove that $r = 1$.
Taking $n \to \infty$, we have

$$r - \phi(r) = r. \tag{12}$$

So,

$$\lim_{n\to\infty} \phi\big(\theta(d(\Omega_{n-1}, \Omega_n))\big) = 0. \tag{13}$$

That is,

$$\lim_{n\to\infty} \theta(d(\Omega_{n-1}, \Omega_n)) = 1, \tag{14}$$

i.e.,

$$\lim_{n\to\infty} d(\Omega_{n-1}, \Omega_n) = 0. \tag{15}$$

We claim that $\{\Omega_n\}$ is a Cauchy sequence.
We argue by contradiction, i.e., there is $\varepsilon > 0$ for which there are $\{\Omega_{m_i}\}$ and $\{\Omega_{n_i}\}$ of $\{\Omega_n\}$ so that

$$n_i > m_i > i \text{ and } d(\Omega_{m_i}, \Omega_{n_i}) \geq \varepsilon. \tag{16}$$

From (16) and using the triangular inequality, we get

$$\begin{aligned}\varepsilon &\leq d(\Omega_{m_i}, \Omega_{n_i}) \\ &\leq d(\Omega_{m_i}, \Omega_{m_i+1}) + d(\Omega_{m_i+1}, \Omega_{n_i}) \\ &\leq d(\Omega_{m_i}, \Omega_{m_i+1}) + d(\Omega_{m_i+1}, \Omega_{n_i+1}) + d(\Omega_{n_i+1}, \Omega_{n_i}).\end{aligned}$$

Taking $i \to \infty$, and using (15), we get

$$\varepsilon \leq \limsup_{i\to\infty} d(\Omega_{m_i+1}, \Omega_{n_i+1}). \tag{17}$$

Also,

$$d(\Omega_{n_i}, \Omega_{m_i}) \leq d(\Omega_{n_i}, \Omega_{n_i-1}) + d(\Omega_{n_i-1}, \Omega_{m_i}).$$

Then, from (15),

$$\limsup_{i\to\infty} d(\Omega_{n_i}, \Omega_{m_i}) \leq \varepsilon. \tag{18}$$

As $d(\Upsilon\Omega_{m_i}, \Upsilon\Omega_{n_i}) > 0$, we may apply (10) to get that

$$\begin{aligned}\theta(d(\Omega_{m_i+1}, \Omega_{n_i+1})) &= \theta(d(\Upsilon\Omega_{m_i}, \Upsilon\Omega_{n_i})) \\ &\leq \theta(d(\Omega_{m_i}, \Omega_{n_i})) - \phi(\theta(d(\Omega_{m_i}, \Omega_{n_i}))).\end{aligned}$$

Now, taking $i \to \infty$ and using $(\theta 1)$, (17) and (18), we have

$$\begin{aligned}\theta(\varepsilon) &\leq \theta(\limsup_{i\to\infty} d(\Omega_{m_i+1}, \Omega_{n_i+1})) \\ &\leq \theta(\limsup_{i\to\infty} d(\Omega_{m_i}, \Omega_{n_i})) - \liminf_{i\to\infty} \phi(\theta(d(\Omega_{m_i}, \Omega_{n_i}))) \\ &\leq \theta(\varepsilon) - \liminf_{i\to\infty} \phi(\theta(d(\Omega_{m_i}, \Omega_{n_i}))).\end{aligned}$$

This implies that

$$\liminf_{i\to\infty} d(\Omega_{m_i}, \Omega_{n_i}) = 0,$$

which is a contradiction with respect to (16).

Thus, $\{\Omega_n\}$ is a Cauchy sequence in the complete metric space (Ω, d), so there is some $\Omega \in X$ such that $\lim_{n\to\infty} d(\Omega_n, \Omega) = 0$.

Now, since Υ is continuous, we get that $\Omega_{n+1} = \Upsilon\Omega_n \to \Upsilon\Omega$ as $n \to \infty$. That is, $\Omega = \Upsilon\Omega$. Thus, Υ has a fixed point.

Let $\Omega, \omega \in Fix(T)$ so that $\Omega \neq \omega$. Consider

$$\theta(d(\Omega, \omega)) = \theta(d(\Upsilon\Omega, \Upsilon\omega)) \leq \theta(d(\Omega, y)) - \phi(\theta(d(\Omega, \omega))).$$

Thus,

$$\phi(\theta(d(\Omega, \omega))) = 0.$$

which is a contradiction. Hence, $\Omega = \omega$. $\square$

One can obtain many other contractive conditions by substituting suitable values of θ and ϕ in (10).

Taking $\phi(t) = t - t^{\alpha}$ for all $t \geq 1$ and $\alpha \in [0, 1)$, we obtain the JS-contractive condition.

Without the continuity assumption of Υ, we have

Theorem 4. *Let (X,d) be a complete metric space. Let $\Upsilon : X \to X$ be a mapping. Suppose that*

$$\theta(d(\Upsilon\Omega,\Upsilon\omega)) \leq \theta(d(\Omega,\omega)) - \phi(\theta(d(\Omega,\omega))), \tag{19}$$

for all $\Omega,\omega \in X$, where $\theta \in \Theta'$ and $\phi \in \Phi$. Then Υ has a unique fixed point.

Proof. For $\Omega_0 \in X$, let $\{\Omega_n\}$ be defined by $\Omega_{n+1} = \Upsilon\Omega_n$ for $n \geq 0$. Note that there is $\Omega \in X$ such that

$$\lim_{n\to\infty} d(\Omega_n,\Omega) = 0.$$

We also have

$$d(\Omega,\Upsilon\Omega) \leq d(\Omega,\Upsilon\Omega_n) + d(\Upsilon\Omega_n,\Upsilon\Omega). \tag{20}$$

From (19),

$$1 \leq \theta(d(\Upsilon\Omega_n,\Upsilon\Omega)) \leq \theta(d(\Omega_n,\Omega)) - \phi(\theta(d(\Omega_n,\Omega))), \tag{21}$$

Hence, we get that $\lim_{n\to\infty} \theta(d(\Upsilon\Omega_n,\Upsilon\Omega)) = 1$. Thus, we have $\lim_{n\to\infty} d(\Upsilon\Omega_n,\Upsilon\Omega) = 0$ which by (20), implies that $\Upsilon\Omega = \Omega$. □

Example 2. *Let $\Omega = [2,\infty)$. Take the metric*

$$d(\rho,\varrho) = |\rho - \varrho|$$

for all $\rho,\varrho \in \Omega$. Define $\Upsilon : \Omega \to \Omega$, $\varphi : [1,\infty) \to [0,\infty)$ and $\theta : [0,\infty) \to [1,\infty)$ by

$$\Upsilon\rho = \ln(100+\rho),$$

$$\varphi(\rho) = \ln(\rho),$$

and $\theta(t) = e^t$. Note that for all $x \geq 0$, one has $e^{\frac{x}{100}} \leq e^x - x$. Now, for all $\rho,\varrho \in \Omega$, we have

$$\begin{aligned}
\theta(d(\Upsilon\rho,\Upsilon\varrho)) &= e^{d(\Upsilon\rho,\Upsilon\varrho)} \\
&= e^{(|\ln(100+\rho)-ln(100+\varrho))|} \\
&\leq e^{\frac{|\rho-\varrho|}{100}} \\
&\leq e^{|\rho-\varrho|} - |\rho-\varrho| \\
&= e^{d(\rho,\varrho)} - d(\rho,\varrho) \\
&= \theta(d(\rho,\varrho)) - \varphi(\theta(d(\rho,\varrho))).
\end{aligned}$$

Thus, Υ is a weakly JS-contraction. All hypotheses of Theorem 3 are verified, so Υ has a unique fixed point, which is, $u \simeq \frac{4651}{1000}$.

4. Application to Nonlinear Integral Equations

Consider the following nonlinear integral equation

$$\Omega(t) = \phi(t) + \int_a^b \chi(t,s,\Omega(s))ds, \tag{22}$$

where $a,b \in \mathbb{R}$, $\Omega \in C[a,b]$ (the set of continuous functions from $[a,b]$ to $\mathbb{R}$), $\phi : [a,b] \to \mathbb{R}$ and $\chi : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$ are given functions.

Theorem 5. *Assume that*

(i) $\chi : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$ *is continuous and there is* $\theta \in \theta$ *so that* $\theta(\sup_{t\in[a,b]} f(t)) \leq \sup_{t\in[a,b]} \theta(f(t))$ *for arbitrary function* f *with*

$$\theta(\int_a^b |(\chi(t,s,\Omega(s))ds - \chi(t,s,\omega(s))|ds) \leq \int_a^b \theta(|\chi(t,s,\Omega(s)) - \chi(t,s,\omega(s))|)ds;$$

(ii) *there is* $\tau_i \in (0,1)$ *so that*

$$\begin{aligned}&\theta(|\chi(t,s,\Omega(s)) - \chi(t,s,\omega(s))|)\\ &\leq \frac{[\theta(|\Omega(t) - y(t)|)]^{\tau_1}[\theta(|\Omega(t) - \int_a^b \chi(t,s,\Omega(s)ds)|)]^{\tau_2}[\theta(|\omega(t) - \int_a^b \chi(t,s,\omega(s))ds|)]^{\tau_3}}{b-a}\\ &[\theta(|\omega(t) - \int_a^b \chi(t,s,\Omega(s))ds|)]^{\tau_4}\end{aligned}$$

for all $\Omega, \omega \in C[a,b]$ *and* $t, s \in [a,b]$.

Then (22) has a unique solution.

Proof. Let $X = C[a,b]$. Define the metric d on X by $d(\Omega,\omega) = \sup_{t\in[a,b]} |\Omega(t) - \omega(t)|$. Then (X,d) is a complete metric space. Consider $\Upsilon : X \to X$ by $\Upsilon\Omega(t) = \phi(t) + \int_a^t \chi(t,s,\Omega(s))ds$. Let $\Omega, \omega \in X$ and $t \in [a,b]$. We have

$$\begin{aligned}&\theta(|\Upsilon\Omega(t) - \Upsilon\omega(t)|)\\ &= \theta(|\int_a^t \chi(t,s,\Omega(s))ds - \int_a^t \chi(t,s,\omega(s))ds|)\\ &\leq \int_a^b \theta(|\chi(t,s,\Omega(s)) - \chi(t,s,\omega(s))|)ds\\ &\leq \int_a^b \frac{[\theta(|\Omega(t) - \omega(t)|)]^{\tau_1}[\theta(|\Omega(t) - \int_a^b \chi(t,s,\Omega(s)ds)|)]^{\tau_2}[\theta(|\omega(t) - \int_a^b \chi(t,s,\omega(s))ds|)]^{\tau_3}}{b-a}\\ &[\theta(|\omega(t) - \int_a^b \chi(t,s,\Omega(s))ds|)]^{\tau_4}ds\\ &\leq \frac{1}{b-a}\int_a^b [\theta(d(\Omega,\omega))]^{\tau_1}[\theta(d(\Omega,\Upsilon\Omega))]^{\tau_2}[\theta(d(\omega,\Upsilon\omega))]^{\tau_3}[\theta(d(\omega,\Upsilon\Omega))]^{\tau_4}ds\\ &= [\theta(d(\Omega,\omega))]^{\tau_1}[\theta(d(\Omega,\Upsilon\Omega))]^{\tau_2}[\theta(d(\omega,\Upsilon\omega))]^{\tau_3}[\theta(d(\omega,\Upsilon\Omega))]^{\tau_4}.\end{aligned}$$

Thus Υ is a $\mathcal{P}$-contraction. All the conditions of Theorem 2 hold, and so Υ has a unique fixed point, that is, (22) has a unique solution. □

5. Conclusions

In this paper, we restricted the conditions on the control function θ (with respect to the ones given in [27,28]) and we obtained a real generalization of the Banach contraction principle (BCP). We also initiated a weakly JS-contractive condition that generalizes its corresponding of Jleli and Samet [26], and we provided some related fixed point results.

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