

Editorial

# Deductive Systems in Traditional and Modern Logic

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Since its inception, logic has studied the acceptable rules of reasoning, the rules that allow us to pass from certain statements, serving as premises or assumptions, to a statement taken as a conclusion. The first kinds of such rules were distilled by Aristotle and are known as moduses. Stoics ramified Aristotle's system, and for centuries, the syllogistic remained the main tool for logical deduction. With the birth of formal logic, new types of deduction emerged, and to support this new kind of inference, the deductive systems were used. Since then, the deductive systems have been at the heart of logical investigations. In one form or the other, they are used in all branches of logic.

Contemporary understanding of science, as a theory of a high degree of exactness, requires treating it as a deductive theory (a deductive system). Generally speaking, such a theory (system) is a set of its sentential expressions which are derivable (deducible) from some expressions of the set selected as axioms, by means of deduction (inference) rules. The expressions obtained as a result of derivation from a given set of expressions are consequences; that is, they have a proof. The principal feature of deductive systems (theories) is the deducibility or provability of their theorems. From a very general point of view, there are two methods of deduction: (a) the axiomatic method (Hilbert style method) and (b) the natural deduction method (Jaśkowski–Słupecki–Borkowski, Gentzen or semantic tableaux). Method (b) leads to natural deduction systems, while the most often used method (a) leads to presentation (or characterization) of logical and mathematical theories as axiomatic deductive systems. Methods (a) and (b) are used in different scientific disciplines, such as physics, chemistry, sociology, philosophical and psychological sciences, information sciences, discursive sciences, computer science, and some technical sciences.

Deductive sciences have not always been built explicitly as axiomatic systems. Depending on the degree of methodological precision, three of their forms have been distinguished: pre-axiomatic, non-formalized axiomatic, and formalized axiomatic. As we know, a pre-axiomatic form was commonly used in arithmetic and geometry, and later in set theory and probability theory. Their axiomatization was carried out only at the end of the 19th century and the beginning of the 20th century. In contrast, such mathematical theories as the Boolean system and theories of groups, rings, and fields were built as formalized axiomatic systems since inception. The deductive method (*calculi*) is most often used for formalizations of theories, but these theories also admit formalization as natural deductive systems.

Formalized axiomatic systems are rooted in a tradition originated by G. Frege (1891, 1903), but the first axiomatic system (non-formalized) in the history of science—as it was disclosed by Jan Łukasiewicz in his seminal monograph on Aristotle's syllogistic (1951)—was Aristotle's syllogistic system. J. Łukasiewicz initiated the construction of the first systems of syllogistic satisfying the contemporary requirements, and thus, the requirements of formalized axiomatic systems. He constructed a formalization of syllogistic logic on two levels using (in addition to the commonly used axiomatic method by means of proof) a new axiomatic method by means of rejection—the so-called axiomatic rejection, or refutation, method. He and his disciples (mainly J. Słupecki and his collaborators) applied this method to the bi-level formalization of

some classical and non-classical logical deductive systems of sentences or names. This approach allows one to define two disjoint sets of language expressions of a given system: the set of all its theses (theorems), which are asserted, accepted, intuitively true expressions (called the assertion system), and the set of all the other expressions—non-accepted, or intuitively false, refuted, rejected expressions of the system (called the rejection or the refutation system). In such a way, the bi-level formalization of deductive systems provides some new inspiration to build different sciences.

This book is a collection of articles included in the special issue “Deductive Systems” of Axioms regarding mainly the logical deductive system. They are ordered in accordance with the well-known division of logic into term logic (logic of names) and propositional logic (propositional calculus), which correspond to two historical stages of the development of logic, namely, Aristotelian logic and the logic of stoics, with the latter being a contemporary counterpart of propositional logic. Deductive systems for classical propositional logic are broadly known, and one of them is most often assumed for the term logics. Systems for non-classical propositional logics, which are inspired by philosophy, are introduced in the book later than systems related to term logics. Term logic can be interpreted in predicate logic, that is, the second part of contemporary logic. Predicate logic is the basis of mathematical deductive systems (theories).

The volume is opened with paper [1] by P. Kulicki in which he looks back to the roots of Western logic and compares what we have achieved today with the legacy of Aristotle. Somehow surprisingly, we can find many features of today’s mature deductive systems in Aristotle’s system of syllogism. The paper discusses some of these features, focusing on Aristotle’s approach to the issue of completeness reconstructed by J. Łukasiewicz.

In [2], P. Simons considers term logic (logic of names) which is a successor of Aristotle’s syllogistic along with 19th century algebraic logic. This is a very natural medium for representing many inferences of ordinary discourse. The axiomatic term logic proposed by P. Simons is intuitive and easy to understand without deeper knowledge of predicate logic.

The paper [3] by J.-M. Castro-Manzano introduces an idea of a distribution model for Sommers’ and Englebretsen’s term logic. It provides some alternative formal semantics to aforementioned logic.

In his paper [4], E. Wojciechowski makes a reference to the differentiation between Zahl and Anzahl, which is present in the works of Frege and formulates Peano’s axiomatic for arithmetic of natural numbers, following Leśniewski on the grounds of the names calculus. This differentiation corresponds syntactically to the name (of natural number)-functor (category  $n/n$ ). This functor (equivalent of Anzahl) is a primitive term of the proposed axiomatic system.

In [5], V. Goranko introduces hybrid deduction–refutation systems, which are deductive systems intended to derive both valid and non-valid, i.e., semantically refutable, formulae of a given logical system, by employing together separate derivability operators for each of these and combining “hybrid derivation rules” that involve both deduction and refutation. The concept is illustrated with a hybrid deduction–refutation system of natural deduction for classical propositional logic, for which soundness and completeness for both deductions and refutations are proved.

In [6], K. Mruczek-Nasieniewska and M. Nasieniewski analyze the so called discussive logic introduced by Stanisław Jaśkowski, and this is probably the first fully formally formulated system of paraconsistent logic. In 1974 Jerzy Kotas gave an axiomatization of discussive logic. In the paper, Kotas’ style axiomatization of the minimal discussive logic is presented.

In [7], J. Ciuciura presents an alternative axiomatization for the hierarchy of paraconsistent systems. The main idea behind it is to focus explicitly on the (in)validity of the principle of *ex contradictione sequitur quodlibet*. This makes the hierarchy less complex and more transparent, especially from the paraconsistency standpoint.

In [8], A. Citkin studies the deductive systems with multiple conclusion rules which admit the introduction of meta-disjunction. Using the defined notion of the inference with multiple-conclusion rules, it is shown that in the logics enjoying the disjunction property, any derivable rule can be inferred from the single-conclusion rules and a single multiple-conclusion rule, which represents the disjunction

property. Additionally, the conversion algorithm of single- and multiple-conclusion deductive systems into each other is studied.

In his paper [9], D. Surowik constructs and studies properties of the minimal temporal logic systems built on the basis of classical logic and intuitionistic logics.

In [10], J. Golińska-Pilarek and M. Welle study deductive systems defining the weakest, extensional two-valued, non-Fregean propositional logic, the language of which is obtained by endowing the language of classical propositional logic with a new binary connective that expresses the identity of two statements.

In [11], S. Pkhakadze and H. Tompits present axiomatizations in terms of the well-known sequent method for two variants of default logic, which is a nonmonotonic formalism relevant for artificial intelligence. The distinguishing feature of the calculi is the usage of rejection systems which axiomatize non-theorems.

In [12], H. Antunes, W. Carnielli, A. Kapsner, and A. Rodrigues construct Kripke-style semantics for the natural deduction systems of the logics of evidence and truth  $LET_J$  and  $LET_F$  introduced earlier by W. Carnielli and A. Rodrigues. Such logics were conceived to express the deductive behavior of positive and negative evidence, which can be conclusive or non-conclusive. Here, the logics are interpreted in terms of positive and negative information, which can be either reliable or unreliable.

The paper [13] by A. Malec studies the classical first-order predicate logic. This logic is a sufficient and desirable basis for deontic theories which are free-from paradoxes inherent in propositional deontic logics that are adequate to the domain of law. The specific axioms of these theories proposed in the paper refer to Bogusław Wolniewicz's "Ontology of Situations" and reflect: (i) relations between sets of legal events, (ii) properties of simple acts, and (iii) properties of compound acts.

In [14], D. Leszczyńska-Jasion and S. Chlebowski develop a proof method (synthetic tableaux method) for a class of the first-order theories axiomatized by universal axioms. Completeness of the system is demonstrated, and some similarities between the method of synthetic tableaux and the axiomatic method are discussed.

The paper [15] by U. Wybraniec-Skardowska presents two equivalent axiomatic systems of arithmetic of natural numbers: Peano's (P) and Wilkosz's (W), and two intuitive axiomatic extensions of integer arithmetic modeled on them. All these systems of arithmetic are based on second-order predicate calculus, and the systems P and W differ mainly in that while in both categorical systems P and W, the primitive concept is a set of natural numbers, in the former, the primitive concepts are also zero and a successor of the natural number; in the latter, the primitive concept is the inequality relation.

In [16], J-P. Desclés and A. Pascu study mathematical models of the logic of the determination of objects (LDO) and the logic of typical and atypical instances of concept (LTA). The novelty of the model presented in this book is that it describes the structural level of LDO in the framework of preordered sets and lattices. A mathematical model of LTA is constructed as an extension of LDO model. In the case of LTA, a set of objects related to a concept gets equipped with a quasi-topological structure.

A review [17] of the book "The Significance of the New Logic" by Willard Van Orman Quine, contributed by R. Freire, completes the volume.

**Conflicts of Interest:** The authors declare no conflict of interest.

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