Significance of Arrhenius Activation Energy and Binary Chemical Reaction in Mixed Convection Flow of Nanofluid Due to a Rotating Disk

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Abstract: This article addresses mixed convective 3D nanoliquid flow by a rotating disk with activation energy and magnetic field. Flow was created by a rotating disk. Velocity, concentration and temperature slips at the surface of a rotating disk were considered. Impacts of Brownian diffusion and thermophoretic were additionally accounted for. The non-linear frameworks are simplified by suitable variables. The shooting method is utilized to develop the numerical solution of resulting problem. Plots were prepared just to explore that how concentration and temperature are impacted by different pertinent flow parameters. Sherwood and Nusselt numbers were additionally plotted and explored. Furthermore, the concentration and temperature were enhanced for larger values of Hartman number. However, the heat transfer rate (Nusselt number) diminishes when the thermophoresis parameter enlarges.

Keywords: rotating disk; mixed convective flow; MHD; binary chemical reaction; nanoparticles; arrhenius activation energy

1. Introduction

A nanoparticle of size under 100 nm deferred into a standard fluid is then named a nanofluid. The essentialness of a nanofluid is expected from its distinctive thermophysical qualities. Nanofluids show enormous capacity to lead power and heat, so they have a critical impact in industry. Nanoliquids have attracted extraordinary enthusiasm for their wide applications; for example, electronic chip cooling, hybrid powered machines, progressed atomic frameworks, solar liquid heating, microchips, excessively proficient magnets and optoelectronics. Thus, Choi [1] exhibited the term nanoparticle inundated into a standard fluid. Buongiorno [2] presented a mathematical model for heat transport in nanoliquid by considering the impacts of Brownian diffusion and thermophoretic dispersion. Further examinations on nanofluids can be seen through the attempts [3–28].

The flow due to a rotating disk plays vital roles in numerous mechanical processes, encompassing psychologist fits, rotors and flywheels. Recently rotating disks became very significant in thermal power creating frameworks, electric-control generation, stopping mechanisms, rotating sawing machines, etc. Fluid flow by a rotating disk is initiated by the Von Karman effect [29]. Turkyilmazoglu and Senel [30] explored the impacts of mass and heat transport because of the porous disk subject to rotating frame. Entropy generation in MHD flow by the rotation of porous disk subject to slip and variable properties is examined by Rashidi et al. [31]. Nanofluid flow because of revolution of disk is discussed by Turkyilmazoglu [32]. Hatami et al. [33] investigated the impacts of contracting rotating disk on nanofluids. They utilized least square technique for solution development. Mustafa et al. [34] analyzed three dimensional nanofluid flow over a stationary disk. Sheikholeslami et al. [35] constructed numerical solutions of nanofluid by a rotating surface. Micropolar liquid flow by a turning disk...
is explored by Doh and Muthamilvelan [36]. Aziz et al. [37] provided a numerical report to nanofluid flow by rotation of disk subject to slip impacts and thermal absorption/generation. Third-grade nanofluid flow over a stretchable rotating surface with heat generation is examined by Hayat et al. [38]. Radiative flow in the presence of nanoparticles and gyrotactic microorganism by the variable-in-thickness surface of a pivoting disk is explained by Qayyum et al. [39]. Hayat et al. [40] provided a numerical solution for radiative flow of carbon nanotubes by the revolution of disk subject to partial slip.

The aim of the present paper is to generalize the analysis of study [11] into four directions. Firstly, to examine magnetohydrodynamic flow of viscous nanofluid due to the rotation of disk. Attention is mainly given to Brownian diffusion and thermophoresis. Secondly, to utilize thermal, concentration and velocity slips at the surface of rotating disk. Thirdly, to consider the effect of mixed convection. Fourth, to analyze the Arrhenius activation energy and binary chemical reaction. The resulting scientific framework is solved numerically via the shooting method. Concentration, temperature and Sherwood numbers are also explored via graphs.

2. Problem Description

Let us examine a mixed convective 3D nanoliquid flow by a pivoting disk with slip features. Arrhenius activation energy, magnetic field and binary chemical reaction are also accounted for. A disk at $z = 0$ rotates with constant angular velocity $\Omega$ (see Figure 1). Brownian dispersion and thermophoretic impacts are additionally present. The velocities are $(u, v, w)$ in the headings of expanding $(r, \varphi, z)$ respectively. The associated boundary-layer equations are [11,37]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0,$$  \hspace{1cm} (1)

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} + w \frac{\partial u}{\partial z} = v \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \frac{\sigma B_0^2}{\rho_f} u + g^* (\beta_T (T - T_\infty) + \beta_C (C - C_\infty)),$$  \hspace{1cm} (2)

$$u \frac{\partial v}{\partial r} + \frac{u v}{r} + w \frac{\partial v}{\partial z} = v \left( \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) - \frac{\sigma B_0^2}{\rho_f} v,$$  \hspace{1cm} (3)

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = v \left( \frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right),$$  \hspace{1cm} (4)

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \kappa_m \left( \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \frac{(\rho C)_p}{(\rho c)_f} \left( D_B \left( \frac{\partial T}{\partial r} \frac{\partial C}{\partial r} + \frac{\partial T}{\partial z} \frac{\partial C}{\partial z} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial z} \right)^2 + \left( \frac{\partial T}{\partial r} \right)^2 \right),$$  \hspace{1cm} (5)

$$u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{\partial^2 C}{\partial z^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) - k^2_\tau (C - C_\infty) \left( \frac{T}{T_\infty} \right)^n \exp \left( -\frac{E_a}{kT} \right),$$  \hspace{1cm} (6)

$$u = L_1 \frac{\partial u}{\partial z}, \quad v = r \Omega + L_1 \frac{\partial v}{\partial z}, \quad w = 0, \quad T = T_w + L_2 \frac{\partial T}{\partial z}, \quad C = C_w + L_3 \frac{\partial C}{\partial z} \text{ at } z = 0,$$  \hspace{1cm} (7)

$$u \rightarrow 0, \quad v \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } z \rightarrow \infty.$$  \hspace{1cm} (8)
Here \( u, v \) and \( w \) stand for velocity components in directions of \( r, \varphi \) and \( z \); \( \rho_f, \mu \) and \( v = \mu / \rho_f \) are for fluid density, dynamic and kinematic viscosities, respectively. \( L_1 \) stands for velocity slip factor; \( C_w \) for ambient concentration; \( g^* \) for acceleration due to gravity; \( T_\infty \) for ambient temperature; \( \beta_T \) for thermal expansion factor; \( (\rho c)_p \) for effective heat capacity of nanoparticles; \( \sigma \) for electrical conductivity; \( E_a \) for activation energy; \( L_3 \) for concentration slip factor; \( (\rho c)_f \) for heat capacity of liquid; \( \beta_C \) for concentration expansion factor, \( \zeta = \left( \frac{2 \Omega \nu}{\Omega^*} \right)^{1/2} z \); \( \alpha_m = \frac{k}{(\rho c)_f} \) and \( k \) for thermal diffusivity and thermal conductivity, respectively; \( k_r \) for reaction rate; \( T \) for fluid temperature; \( D_B \) for Brownian factor; and \( \kappa \) for Boltzmann constant. Selecting \([37]\):}

\[
\begin{align*}
  u &= r \Omega f' (\zeta), \quad w = -(2 \Omega \nu)^{1/2} f (\zeta), \quad \theta (\zeta) = \frac{T - T_\infty}{T_\infty}, \quad \phi (\zeta) = C - C_\infty C_w - C_\infty, \\
  f (0) &= 0, \quad f'(0) = \alpha f''(0), \quad g(0) = 1 + \alpha g'(0), \quad \theta(0) = 1 + \beta \theta'(0), \quad \phi(0) = 1 + \gamma \phi'(0), \\
  f'(\infty) &\to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. 
\end{align*}
\] (9)

Equation (1) is now verified while Equations (2)–(8) yield \([11,37]\):

\[
\begin{align*}
  2 f''' + 2 f f'' - f'^2 + g^2 - (Ha)^2 f' + \lambda_T (\theta + \lambda_C \phi) &= 0, \\
  2 g'' + 2 f g' - 2 f' g - (Ha)^2 g &= 0, \\
  \frac{1}{Pr} \theta'' + f \theta' + N_b \theta' \phi' + N_\theta \theta'^2 &= 0, \\
  \frac{1}{Sc} \phi'' + f \phi' + \frac{1}{Sc} \frac{N_l}{N_b} \theta'' - \sigma (1 + \delta \theta)^n \phi \exp \left( - \frac{E}{1 + \delta \theta} \right) &= 0, \\
  f(0) &= 0, \quad f'(0) = \alpha f''(0), \quad g(0) = 1 + \alpha g'(0), \quad \theta(0) = 1 + \beta \theta'(0), \quad \phi(0) = 1 + \gamma \phi'(0), \\
  f'(\infty) &\to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. 
\end{align*}
\] (10–13, 14, 15)

Here \( \lambda_T \) stands for thermal buoyancy number, \( N_l \) for thermophoresis parameter, \( \alpha \) for velocity slip parameter, \( Pr \) for Prandtl number, \( \lambda_C \) for concentration buoyancy number, \( Sc \) for Schmidt parameter, \( Ha \) for Hartman number, \( \beta \) for thermal slip parameter, \( \sigma \) for chemical reaction number, \( N_b \) for Brownian parameter, \( \delta \) for temperature difference parameter, \( \gamma \) for concentration slip parameter and \( E \) for non-dimensional activation energy. These parameters are defined by
(Ha)² = \frac{\sigma b^2}{\nu r^4}, \quad \Pr = \frac{\nu}{\alpha}, \quad \alpha = L_1 \sqrt{\frac{20}{r}}, \quad \gamma = L_3 \sqrt{\frac{20}{r}},
N_t = \frac{\nu (\rho c_p D_s (T_w - T_m))}{\nu (\rho c_p) \nu}, \quad Sc = \frac{\nu}{\lambda},
N_b = \frac{(\rho c_p) D_s (T_w - T_m)}{(\rho c_p) \nu}, \quad \lambda_T = \frac{\rho^2 \gamma \beta^2 (T_w - T_m)}{\nu},
\lambda_C = \frac{\rho^2 (C_w - C_m)}{\beta^2 (T_w - T_m)}, \quad \beta = L_2 \sqrt{\frac{20}{r}}, \quad \sigma = \frac{\beta^2}{\nu}, \quad \delta = \frac{T_w - T_m}{T_w}, \quad E = \frac{E_s}{\pi r^2}.

(16)

The coefficients of skin friction and Sherwood and Nusselt numbers are

\text{Re}_r^{1/2} C_f = f''(0), \quad \text{Re}_r^{1/2} C_s = g'(0), \quad \text{Re}_r^{-1/2} Sh = -\phi'(0), \quad \text{Re}_r^{-1/2} Nu = -\theta'(0),

(17)

where \text{Re}_r = 2(\Omega r) r / \nu depicts local rotational Reynolds number.

3. Solution Methodology

By employing suitable boundary conditions on the system of equations, a numerical solution was constructed considering NDSolve in Mathematica. The shooting method was employed via NDSolve. This method is very helpful in the situation of a smaller step-size featuring negligible error. As a consequence, both the z and r varied uniformly by a step-size of 0.01 [20].

4. Graphical Results and Discussion

This segment displays variations of various physical flow parameters, such as the thermophoresis parameter \( N_t \), Hartman number \( Ha \), thermal slip parameter \( \beta \), chemical reaction parameter \( \sigma \), Brownian motion parameter \( N_b \), concentration slip parameter \( \gamma \) and activation energy \( E \), on concentration \( \phi(\zeta) \) and temperature \( \theta(\zeta) \) distributions. Figure 2a displays the effect of Hartman number \( Ha \) on temperature \( \theta(\zeta) \). Temperature \( \theta(\zeta) \) is enhanced for higher estimations of \( Ha \). The effect of thermal slip \( \beta \) on temperature \( \theta(\zeta) \) is shown in Figure 2b. Greater \( \beta \) shows diminishing trend of \( \theta(\zeta) \) and associated warmth layer. The impact of \( N_t \) on temperature \( \theta(\zeta) \) is explored in Figure 2c. An increment in \( N_t \) leads to stronger temperature field \( \theta(\zeta) \). Figure 2d depicts change in temperature \( \theta(\zeta) \) for varying Brownian motion number \( N_b \). Physically, the Brownian motion of nanoparticles is enhanced by increasing Brownian motion number \( N_b \). Therefore dynamic vitality is altered into thermal vitality, which depicts an increment in temperature \( \theta(\zeta) \) and the respective warmth layer. Figure 3a shows that how the Hartman number \( Ha \) influences concentration \( \phi(\zeta) \). For a greater Hartman number \( Ha \), both concentration \( \phi(\zeta) \) and the concentration layer are upgraded. Figure 3b displays that concentration \( \phi(\zeta) \) is weaker for a greater concentration slip. Figure 3c demonstrates how thermophoresis \( N_t \) influences concentration \( \phi(\zeta) \). By improving the thermophoresis parameter \( N_t \), the concentration \( \phi(\zeta) \) and associated layer are upgraded. Figure 3d depicts effect of Brownian motion \( N_b \) on concentration \( \phi(\zeta) \). It is noted that higher concentration \( \phi(\zeta) \) is developed by utilizing greater Brownian parameter \( N_b \). Figure 3e explains effect of non-dimensional activation energy \( E \) on concentration \( \phi(\zeta) \). An increment in \( E \) rots change Arrhenius work \( \left( \frac{T}{T_w} \right)^n \exp \left( -\frac{E}{RT_w} \right) \), which inevitably builds up a generative synthetic reaction due to which concentration \( \phi(\zeta) \) increases. Figure 3f introduces the fact that an increment in chemical response number \( \sigma \) causes a rot in concentration \( \phi(\zeta) \). Figure 4a,b displays the effects of \( N_t \) and \( N_b \) on \( \text{Re}_r^{-1/2} Nu \). It is noted that \( \text{Re}_r^{-1/2} Nu \) decreases for greater \( N_t \) and \( N_b \). Contributions of \( N_t \) and \( N_b \) on \( \text{Re}_r^{-1/2} Sh \) are explored in Figure 5a,b. Here \( \text{Re}_r^{-1/2} Sh \) is increasing the factor of \( N_b \) while it is decreasing the factor of \( N_t \).
Figure 2. (a) Variations of temperature distribution $\theta(\zeta)$ for Hartman number $H_a$; (b) variations of temperature distribution $\theta(\zeta)$ for thermal slip parameter $\beta$; (c) variations of temperature distribution $\theta(\zeta)$ for thermophoresis parameter $N_t$; (d) variations of temperature distribution $\theta(\zeta)$ for Brownian motion parameter $N_b$.

Figure 3. Cont.
Figure 3. (a) Variations of concentration distribution $\phi(\zeta)$ for Hartman number $Ha$; (b) variations of concentration distribution $\phi(\zeta)$ for concentration slip parameter $\gamma$; (c) variations of concentration distribution $\phi(\zeta)$ for thermophoresis parameter $N_t$; (d) variations of concentration distribution $\phi(\zeta)$ for Brownian motion parameter $N_b$; (e) variations of concentration distribution $\phi(\zeta)$ for activation energy $E$; (f) variations of concentration distribution $\phi(\zeta)$ for chemical reaction parameter $\sigma$.

Figure 4. (a) Variations of Nusselt number $Re^{-1/2}Nu$ for thermophoresis parameter $N_t$; (b) variations of Nusselt number $Re^{-1/2}Nu$ for Brownian motion parameter $N_b$. 
Figure 5. (a) Variations of Sherwood number $Re_{r}^{-1/2} Sh$ for thermophoresis parameter $N_{t}$; (b) variations of Sherwood number $Re_{r}^{-1/2} Sh$ for Brownian motion parameter $N_{b}$.

5. Conclusions

Mixed convective 3D nanoliquid flow by a rotating disk subject to activation energy, magnetohydrodynamics and a binary chemical reaction was studied. Here, the flow field was considered to contain the chemically reacting species. Moreover, the mass transport mechanism was developed via modified Arrhenius function for the activation energy. Activation energy is the minimum quantity of energy needed by reactants to examine a chemical reaction. The source of the activation energy needed to initiate a chemical reaction is typically heat energy from the surroundings. Furthermore, the scientific system obtained was solved numerically via shooting method. A stronger temperature distribution was seen for $N_{b}$ and $N_{t}$. Both the concentration and temperature display increasing behavior for greater $Ha$. Higher $\gamma$ exhibits a decreasing trend for concentration field. Concentration $\phi(\zeta)$ depicts decreasing behavior for larger $\sigma$. Higher activation energy $E$ shows stronger concentration $\phi(\zeta)$. Concentration $\phi(\zeta)$ displays reverse behavior for $N_{b}$ and $N_{t}$.

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Nomenclature

- $u$, $v$, $w$: velocity components
- $\sigma$: electrical conductivity
- $\mu$: dynamic viscosity
- $\beta_{T}$: thermal expansion coefficient
- $\nu$: kinematic viscosity
- $L_{1}$: velocity slip coefficient
- $L_{3}$: concentration slip coefficient
- $T$: temperature
- $T_{w}$: wall temperature
- $T_{\infty}$: ambient fluid temperature
- $a_{m}$: thermal diffusivity
- $D_{B}$: Brownian diffusion coefficient
- $E_{a}$: activation energy
- $r$, $\varphi$, $z$: coordinate axes
- $B_{0}$: magnetic field strength
- $\rho_{f}$: density of base fluid
- $\beta_{C}$: concentration expansion coefficient
- $g^{*}$: acceleration due to gravity
- $L_{2}$: temperature slip coefficient
- $\Delta$: constant angular velocity
- $C$: concentration
- $C_{w}$: wall concentration
- $C_{\infty}$: ambient fluid concentration
- $k$: thermal conductivity
- $(pc)_{p}$: effective heat capacity of nanoparticles
- $(pc)_{f}$: heat capacity of fluid
- $D_{T}$: thermophoretic diffusion coefficient
- $n$: fitted rate constant
$k_r$ reaction rate  \hspace{1cm} \kappa \text{ Boltzmann constant}

$\zeta$ similarity variable  \hspace{1cm} \eta' , \eta \text{ dimensionless velocities}

$\theta$ dimensionless temperature  \hspace{1cm} \phi \text{ dimensionless concentration}

$S_c$ Schmidt number  \hspace{1cm} \text{Ha} \text{ Hartman number}

$\lambda_T$ thermal buoyancy number  \hspace{1cm} \text{Pr} \text{ Prandtl number}

$N_{b}$ Brownian motion parameter  \hspace{1cm} N_t \text{ thermophoresis parameter}

$\lambda_C$ concentration buoyancy number  \hspace{1cm} \alpha \text{ velocity slip parameter}

$\beta$ thermal slip parameter  \hspace{1cm} \gamma \text{ concentration slip parameter}

$E$ dimensionless activation energy  \hspace{1cm} \delta \text{ temperature difference parameter}

$C_f, C_g$ skin friction coefficients  \hspace{1cm} \text{Re}_r \text{ local rotational Reynolds number}

$Nu$ Nusselt number  \hspace{1cm} \text{Sh} \text{ Sherwood number}

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