

Supplementary Materials

The Matlab codes used for modeling are as:

```
clear all
close all
clc
%%
syms Mh Jh          % Mass and Moment Inertia of Trunk
syms Ms_int Js_int  % Mass and Moment of inertia of torque sensor (Internal Ring)
syms Ms_ext Js_ext  % Mass and Moment of inertia of torque sensor (External Ring)
syms M_act J_act    % Mass and Moment Inertia of Actuator
syms g              % Gravity acceleration
syms rh            % Raidal Displacement of Central mass of trunk with respect rotational axis
syms Kh dh dhs     % Trunk Stiffness, Trunk Damping, Trk_trq Damping
syms Ks ds         % Stiffness of torque sensor, Damping of torque sensor
syms Kg dg dgs     % Gearbox Stiffness, Gearbox Damping, Gea_Trq Damping
syms tetah tetadh tetaddh % Trunk Position, Trunk Velocity, Trunk Acceleration with respect to vertical axis
syms tetah_ext tetadh_ext tetadd_ext % torque sensor Position, Velocity, Acceleration (external ring)
syms tetah_int tetadh_int tetadd_int % torque sensor Position, Velocity, Acceleration (internal ring)
syms tetah_act tetadh_act tetadd_act % BDC motor shaft Position, BDC motor shaft Velocity, BDC motor shaft Acceleration, data from encoder
syms tauh tau_m    % Torque by Trunk, Torque by BDC motor
syms rg            % Gearbox ratio
syms bm           % Friction between motor and gearbox
syms Tm           % Motor Torque

%% Defining auxiliary variables
syms TETA_H TETAD_H TETADD_H
syms TETA_EXT TETAD_EXT TETADD_EXT
syms TETA_INT TETAD_INT TETADD_INT
syms TETA_ACT TETAD_ACT TETADD_ACT

%% Equations of Motion
% eqns = [ Tm Bf*tetadm Jtot*TETADDM (Kg/rg)*(tetam/rg) tetas ) (Dg/rg)*(tetadm/rg) tetads ] == 0 ,
%         Kg*(tetam/rg) tetas ) + Dg*(tetadm/rg) tetads ) Js*TETADDS Ks*(tetas tetal) Ds*(tetads tetadl) == 0 ,
%         Ks*( tetas tetal ) + Ds*( tetads tetadl ) J1*TETADDL K1*(tetal tetah) D1*(tetadl tetadh) == 0 ];

eqns = [ Tm bm*tetad_act J_act*TETADD_ACT (Kg/rg)*(tetah_act/rg) tetah_int ) (dgs/rg)*(tetad_act/rg) tetad_int ) (dg/rg)*(tetad_act/rg) == 0 ,
         Kg*(tetah_act/rg) tetah_int ) + dgs*(tetad_act/rg) tetad_int ) Js_int*TETADD_INT Ks*(tetah_int tetah_ext) ds*(tetad_int tetad_ext) == 0 ,
         Ks*(tetah_int tetah_ext) + ds*(tetad_int tetad_ext) Js_ext*TETADD_EXT Kh*(tetah_ext tetah) dhs*(tetad_ext tetadh) == 0 ,
         Kh*(tetah_ext tetah) + dhs*(tetad_ext tetadh) TETADD_H*Jh tetah*dh == 0 ];

vars = [ TETADD_ACT TETADD_INT TETADD_EXT TETADD_H ];
%%
eqns_mat = [ eqns ,
             TETAD_ACT == tetad_act ,
             TETAD_INT == tetad_int ,
             TETAD_EXT == tetad_ext ,
             TETAD_H == tetad_h ];
vars_mat = [ TETAD_ACT TETADD_ACT TETAD_INT TETADD_INT TETAD_EXT TETADD_EXT TETAD_H TETADD_H ];
[TETAD_ACT TETADD_ACT TETAD_INT TETADD_INT TETAD_EXT TETADD_EXT TETAD_H TETADD_H] = solve(eqns_mat, vars_mat)

%%
%STATE SPACE MODEL
%states = [ Motor position, Motor velocity, Internal torque sensor position, Internal torque sensor velocity, External torque sensor position, ...
           External torque sensor velocity, Trunk position ]
%inputs = [ Motor torque, Trunk velocity ]

eqns_mat_form = [ TETAD_ACT TETADD_ACT TETAD_INT TETADD_INT TETAD_EXT TETADD_EXT TETAD_H ];
vars_mat_form = [ tetah_act tetad_act tetah_int tetad_int tetah_ext tetad_ext tetah ];
ns = length(vars_mat_form); % Number of States
[A,nB] = equationsToMatrix( eqns_mat_form , vars_mat_form );
A_eq = A;
Bt = nB;
[B_eq,-] = equationsToMatrix( Bt , [ Tm , tetad_h ] );
C_eq = [ 0 , 0 , Ks , ds , Ks , ds , 0 ] ;
D_eq = [ 0 , 0 ];

%% Transfer Function
syms s
% H(s) = Y(s) / U(s) = C*(sI A)^(-1)*B + D
disp('C')
disp('The_symbolic_transfer_function_is:')
TF = simplify( C_eq*[ (s*eye(ns,ns) A_eq )^(-1) ]*B_eq + D_eq , 'Steps',100 );
pretty(TF)
disp('C')
```