Abstract: This paper studies the role of the exchange rate regime for trade of new products. It first provides VAR evidence that a rise in external productivity shifts trade away from new products and more so in fixed regimes. Then, it presents a model with firm dynamics in line with this evidence. We argue that exchange rate policy can affect firms’ entry decisions with consequences for the competitiveness of a country’s exports well beyond the short run. In our setup, fixed exchange rates can foster the competitiveness of firms that trade new products, while flexible rates favor firms that produce mature products.

Keywords: trade margins; firm entry; exchange rate policy; international business cycle; panel var; dsge model; comparative advantage

JEL Classification: E31; E32; E52; F10; F44

1. Introduction

This paper belongs to a recent line of research that incorporates exporters’ entry into dynamic macroeconomic models.\(^1\) In departing from the workhorse international business cycle model, which typically considers an exogenous number of traded goods and a constant share of exporting firms, these studies stress the role of foreign market access for the international propagation of shocks. Entry (exit) implies the creation (destruction) of new trade relations, in the form of brand new products and trade of previously non-traded goods. This is known as the extensive margin of trade.

Our focus will be on the role of the exchange rate regime for trade of new products. Previous studies have mainly considered how exchange rate variability affects the decision to access foreign markets in the first place.\(^2\) It is now well-understood that fixed rates stimulate entry because a low exchange rate risk increases the expected revenue of investing in a new trade relationship.\(^3\) Moreover, it can affect the incentive of firms to move production across sectors.\(^4\) An important lesson from this literature is that adjustment at the extensive margin can have relevant consequences for the

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1 Seminal studies in this area include Atkeson and Burstein (2008), Alessandria and Choi (2007), and Ghironi and Mélot (2005). See also, inter alia Auray et al. (2012), Cavallari (2013a), Bergin and Corsetti (2015), Cacciatore et al. (2015), and de Blas and Russ (2015).


3 Exchange rate uncertainty has been extensively studied in connection with trade hysteresis (see Belke et al. (2013) and Belke and Kronen (2019)). According to those contributions, the impact of exchange rate uncertainty and thus the exchange rate regime is not just negative but non-linear due to firm entry and exit decisions.

4 Flexible exchange rates induce production shifts in and out of the export sector that help explain the positive correlation between the relative prices of traded and non-traded goods observed in the data (Naknoi 2008). In a setup where all goods are traded, Bergin and Corsetti (2015) show that monetary stabilization under free floating can induce relocations toward sectors producing differentiated goods.
propagation of shocks. Yet, how firms adjust export margins in response to shocks and whether the extent of these responses depends on the exchange rate regime remains unclear. Our contribution is twofold.

First, we provide VAR evidence about the dynamics of export margins in the wake of external shocks to productivity, demand, and monetary policy, contrasting the propagation mechanism in fixed and floating exchange rate regimes.\(^5\) The analysis is based on a panel VAR model with exogenous factors (VARX for short) in a sample of 22 developed economies over the period 1988–2011. The vector of endogenous includes bilateral extensive and intensive margin of exports together with a measure of relative country size. The vector of exogenous, common to all panels, includes productivity, aggregate demand, and monetary policy shocks in the United States, reflecting unexpected changes in external cyclical conditions. The shocks are identified with a combination of long-run and sign restrictions. The model is estimated separately for country pairs that adopt fixed exchange rates (“peggers”) and for country pairs under free floating (“floaters”).

We find that a rise in external productivity has a negative impact on exports of new products (the extensive margin) and more so in fixed regimes. On average, the extensive margin of exports drops by 1% and 1.5% below the mean in the sample of, respectively, floaters and peggers. Interestingly, the shock has no consequences for the average volume of exports (the intensive margin). In contrast, a rise in external demand increases exports mostly at the intensive margin. The average trade volume rises by almost 9% above the mean among peggers and by 3% among floaters. The responses to a monetary policy easing are either insignificant (for intensive margins) or negligible (for extensive margins the impact effect ranges between 0.25% and 0.6% above the mean in, respectively, fixed and floating regimes). Overall, the evidence suggests that trade of new products responds mainly to productivity shocks, while trade of previously traded goods responds mainly to demand shocks. The extent of these responses is affected by the exchange rate regime.

Second, we develop a model with firm entry that helps explain the role of exchange rates for trade of new products. The model builds on Cavallari (2013a), which we extend to incorporate a firm’s decision to export its products abroad. All domestic producers are potential exporters, yet only a fraction of them will effectively become an exporter because doing so is costly. We consider firm-specific trade costs à la Bergin and Glick (2009), which are paid at the beginning of the period, on a period by period basis, before production and pricing decisions are made. These costs generate a procyclical export threshold.\(^6\)

Simulations show that the exchange rate regime indeed affects the incentive of firms to adjust their exports at the extensive or intensive margin. A rise in domestic productivity increases the number of exporters (the extensive margin) under fixed exchange rates far more than under flexible rates. In addition, the average volume of imports (the intensive margin) drops in fixed regimes while it increases under floating rates. These dynamics reflect a shift toward trade of new products in fixed regimes that is in line with VAR evidence. Remarkably, the cyclical properties of trade margins in our model match those observed in the data: (i) the extensive margin of imports and exports are more volatile than output; (ii) they are negatively correlated between each other; (iii) extensive margins are procyclical relative to output of the exporting country.

An important contribution of our analysis is to clarify that exchange rates, by affecting firms’ dynamics in the export sector, can have an impact on a country’s competitiveness well beyond the short run. Entry of new exporters implies lower export prices and therefore a competitive advantage in sectors that produce new products and previously nontraded goods. Fixed rates, by leading more producers to become exporters, can strengthen the competitiveness of a country’s exports in these

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\(^5\) Early studies have documented a positive relation between a country’s extensive margin of exports and its terms of trade (Cavallari and D’Addona 2015), and a positive relation with external demand shocks (Cavallari and D’Addona 2017).

\(^6\) More general assumptions about the structure of export costs can generate a richer (and more realistic) export dynamics, as in Ruhl and Willis (2017).
sectors: this is exactly what happens in our setup during a cyclical upturn. The argument may appear at odds with Bergin and Corsetti (2015), who show that flexible rates can foster the competitiveness of firms that produce differentiated goods compared to firms in sectors that produce homogeneous goods. The contrast is, however, only apparent. In their model, all goods—both homogeneous and differentiated—are traded and exchange rates have no impact on the decision to export in the first place. We suggest a complement view based on entry into the export sector. In our setup, fixed exchange rates imply more entrants in the export sector when productivity is high and more exits when productivity is low, compared to a floating regime. Therefore, fixed regimes are appealing for high-productivity countries, which can export a large variety of new products at a lower price. In contrast, flexible rates help reduce the (relative) price of mature products in low-productivity countries.

The paper is organized as follows. Section 2 provides the VAR evidence. Section 3 presents the model and Section 4 discusses the simulation results. Section 5 concludes.

2. VAR Evidence

This section provides VAR evidence on the dynamics of export margins in response to external shocks, contrasting the transmission mechanism in fixed and floating regimes. In earlier work (Cavallari and D’Addona 2015; Cavallari and D’Addona 2017), we have focused on the shock absorption properties of flexible exchange rates by considering the dynamics of output and trade margins in the wake of, respectively, shocks to the terms of trade of a country and shocks to global demand factors. In both cases flexible rates are found to stabilize output by reducing the response of the extensive margin. Here, we focus on the dynamics of trade margins in response to a wide range of external shocks, including productivity shocks at the heart of international business cycle models. The scope of the analysis is descriptive.

2.1. Data

Our sample includes 22 developed countries over the period from 1988 to 2011 together with the United States. GDP—measured in domestic currency at constant prices and logged—is from the OECD StatExtracts database (Details about countries included in the dataset and data transformation are provided in Appendix B).7

Export margins are calculated from bilateral trade data made available by the World Bank through the World Integrated Trade Solution website.8 Trade categories are defined according to the four-digit Standard International Trade Classification maintained by the United Nations.9

We follow Hummels and Klenow (2005) in calculating the extensive margin of exports for a country-pair as the weighted sum of exported categories relative to all categories exported in the importer country, using the relevance of each category in world’s export as weights. Namely:

\[ XM_{jm} = \frac{\sum_{i \in I_{jm}} X_{m,i} W_{m}}{X_{m}}, \]

where \( \sum_{i \in I_{jm}} X_{m,i} W_{m} \) is the sum across categories of the values of world exports toward country \( m \), \( I_{jm} \) is the set of categories where a positive value of exports is recorded from country \( j \) to country \( m \), and \( X_{m} \) is the aggregate value of world exports to country \( m \). The extensive margin is, by construction, a number between zero and one, increasing with the variety of categories exported.

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Similarly, the intensive margin of exports is the value of $j$’s exports to country $m$ relative to the weighted categories in which country $j$ exports to country $m$:

$$IM_m^j = \frac{X_m^j}{\sum_{i \in I_{m,j}} X_{m,i}^W},$$ (2)

where $X_m^j$ is the total export value from country $j$ to country $m$. By definition the intensive margin has a lower bound at 0 and is increasing in the volume of exports of previously traded categories.

2.2. VAR Specification

We consider a panel VAR model with a vector of exogenous variables (VARX for short). The model includes three endogenous variables and five exogenous variables. Endogenous variables are measured on a country-pair basis where $j = 1, 2, \ldots, 22$ denotes the exporting country, $m = 1, 2, \ldots, 22$ with $m \neq j$ denotes the destination country (including the United States), and $t$ indicates time. They include relative GDP and bilateral exports, measured at the extensive and the intensive margin. The exogenous vector represents global factors that do not depend on the dynamics of any of the endogenous variables. It is common to all panels and comprises innovations to productivity, real GDP, inflation, energy prices, and monetary policy rates in the United States.

The model is given by:

$$Y_{j \times m,t} = \alpha_{j \times m} + \beta(L)Y_{j \times m,t-1} + \gamma(L)X_t + \epsilon_{j \times m,t},$$ (3)

where $Y_{j \times m,t} = (\log TFP_j, \log GDP_{jt}, \Delta \log CPI_{jt}, \Delta \log Energy_{jt}, \text{FFR}_t)$ is the vector of endogenous variables; $\alpha_{j \times m}$ captures country-pair fixed effects; $\beta(L)$ and $\gamma(L)$ are matrix polynomials in the lag operator; $\epsilon_{j \times m,t}$ is the vector of errors in the system, and $X_t$ is a vector of exogenous shocks. The latters are estimated structural errors obtained from the model:

$$y_t = a + b(L)y_{t-1} + \epsilon_t,$$ (4)

where $y_t$ includes total factor productivity, real output, consumer price inflation, energy prices, and the Federal funds rate, $y_t = (\log TFP, \log GDP_t, \Delta \log CPI_t, \Delta \log Energy_t, \text{FFR}_t)$; $a$ is a vector of intercepts; $b(L)$ is a matrix polynomial in the lag operator; $\epsilon_t$ is the vector of exogenous errors with variance $E(\epsilon_t \epsilon_t') = \Sigma$ for all $t$. Energy prices are considered for their ability to forecast future inflation and avoid a price puzzle, namely a drop in inflation after a monetary easing. Notice that the structural shocks are by construction orthogonal to any of the endogenous variables in the system and can be therefore treated as exogenous in (3), though US variables may in principle be correlated with the export margins of US trading partners.

The structural shocks are identified with a combination of long-run and sign restrictions. Since Blanchard and Quah (1989), many studies use long-run restrictions for identifying shocks that have permanent effects, as technology shocks in Gali (1999). We draw on this idea to identify productivity as the only force in our system that has a permanent effect on output. By contrast, monetary policy is neutral and innovations to the policy rate have no long-run impact on output, either directly or through any other variable in the system. Real demand shocks are identified with sign restrictions, which select a minimum set of common predictions by an ample class of theoretical models, including our own model. The strategy is based on a simple intuition: demand shocks move quantities and nominal prices in the same direction while supply shocks move them in opposite directions. Hence, an increase in aggregate demand leads to a rise in both output and prices while an

10 The exogenous VAR model is estimated over the period 1970–2011 for efficiency reasons.
increase in productivity is associated with a rise in output and a drop in prices. The restrictions are summarized in Table 1.

<table>
<thead>
<tr>
<th>5 year/Long Run Response of Vbl. in Column to A Positive Shock</th>
<th>Productivity</th>
<th>GDP</th>
<th>Inflation</th>
<th>Energy Price</th>
<th>FFR</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>no restr</td>
</tr>
<tr>
<td>AD shock</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>no restr</td>
</tr>
<tr>
<td>FFR shock</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>no restr</td>
<td>no restr</td>
</tr>
</tbody>
</table>

Operationally, the sign restrictions remain in place for five years, reflecting a prior of fairly persistent shocks. The long run restrictions refer to the cumulated effect of the shock over the entire horizon.

The model is estimated separately for countries with fixed and for countries with floating rates. Country pairs are classified using the IMF de facto classification (see Born et al. 2013), updated to match our sample period. For each year, a country pair is classified in the sample of “peggers” if both countries have adopted a regime of “peg within horizontal bands”, or tighter, within the year. In all other cases, the country pair is classified in the sample of “floaters”. The sample of peggers comprises only European country pairs, and reflects intra-EMU trade. The complete list of “peggers” and “floaters” is reported in Appendix B.2.

Finally, we estimate the model (3) using the bootstrap bias-corrected estimator (BSBC) in Pesaran and Zhao (1999) and Everaert and Pozzi (2007). The bootstrap sampling is modified to suit our unbalanced panel as in Fomby et al. (2013). In this way, we address concerns about the consistency of the least-squares dummy variable (LSDV) estimator in dynamic models with a small time dimension (Nickell 1981).

2.3. Results

We consider the mean responses of trade margins in the wake of positive supply, demand, and monetary policy shocks. The supply shock is a one standard deviation increase in US productivity; the demand shock is a one standard deviation increase in US GDP, and the monetary policy shock is a one standard deviation cut in the Federal funds rate.

Figures 1 and 2 report the impulse response functions of, respectively, the extensive and intensive margins together with 90% confidence intervals, generated by Monte Carlo simulations with 1000 replications. The top row of each figure shows the mean responses in the sample of peggers while the bottom row refers to the mean responses in the sample of floaters.

We document a significant effect of productivity on extensive margins. On impact, the extensive margin of exports falls by 1% and 1.5% below the mean in the sample of, respectively, floaters and peggers. The effect is quite persistent and takes up to six years before vanishing. The drop reflects exits from sectors that produce new products and previously non-traded goods. Interestingly, productivity shocks have only negligible effects on the average volume of exports per product. Except for a small increase on impact in flexible regimes, the response of the intensive margin is not different from zero. Therefore, adjustment to external productivity shocks occurs mainly at the extensive margin and more so in fixed regimes.

Aggregate demand shocks, in contrast, have negligible effects on extensive margins. A rise in external demand boosts trade mainly at the intensive margin. The average volume of exports rises

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11 We refer to Cavallari and D’Addona (2017) for an extensive discussion of the reasons for adopting a dichotomous classification with country-pair data.
by almost 9% above the mean among peggers and by 3% among floaters. In the sample of peggers, the demand boost is accommodated also through a moderate increase in the number of products (0.35% on impact). Monetary policy shocks have a moderate impact. The responses of the intensive margins are not significantly different from zero throughout the whole transition in all samples. The extensive margins hike on impact, by 0.25% and 0.6% above the mean in, respectively, fixed and floating regimes and quickly revert to the mean.

To gauge the statistical relevance of differences in the propagation of shocks across exchange rate regimes, we bootstrap the samples for which we compute the difference in the responses of peggers and floaters as is done in Born et al. (2013). Results are shown in Figure 3. Extensive margins are indeed more sensitive to real shocks in fixed regimes compared to floating regimes and these differences are significant at the 90% level. As regard monetary policy shocks, we find no significant differences in the coefficient of the impulse responses between fixed and floating regimes.

![Figure 1. Extensive margins. Mean responses of extensive margins to external shocks in fixed regimes (top row) and in flexible regimes (bottom row).](image-url)
Overall, the evidence above suggests that trade of new products responds mainly to productivity shocks, while trade of previously traded goods responds mainly to demand shocks. The extent of these responses is higher in fixed compared to flexible regimes. In what follows we concentrate on the propagation of productivity shocks under fixed and floating regimes and present a model in line with the evidence.  

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Results provided in this section were further scrutinized with an extensive robustness check on the (i) lag structure, (ii) parameter restrictions (iii) sample selection. Results, available upon request, are qualitatively the same under all the tested specifications.
Figure 3. Differences of responses in the sample of floaters and in the sample of peggers.

3. The Model

The model is based on Cavallari (2013a) and is modified as in Cavallari and D’Addona (2015) to account for the endogenous selection of exporters. In what follows we present linearized equations while Appendix A describes the complete model. To save on space we describe only the home economy with the understanding that similar relations hold in the foreign country. A star superscript denotes a foreign variable.

3.1. Demand Block

Households in each country consume an identical basket of imported and domestic goods, \( C = (C_D)\gamma (C_X)^{1-\gamma} \) and \( C^* = (C_D^*)\gamma (C_X^*)^{1-\gamma} \), whose value is \( P \) in the home country and \( P^* \) in the foreign country. Each basket contains imperfectly substitutable varieties of mass \( N_t \) (\( N_t^* \)) for home (foreign) domestic varieties and \( N_{Xt} \) (\( N_{Xt}^* \)) for home (foreign) imported varieties. Each variety corresponds to a firm, so \( N_t \) is also the number of home firms and \( N_{Xt} \) is the number of home exporters and similarly for \( N_t^* \) and \( N_{Xt}^* \). The elasticity of substitution among all varieties is constant and equal to \( \theta > 1 \).

Households hold home and foreign riskless bonds \( B_t \) and \( B_t^* \), which are denominated in the local currency and yield a gross nominal interest rate \( i_t \) and \( i_t^* \) respectively and shares in a mutual fund of domestic firms. The assumption that shares cannot be traded internationally is inconsequential.

Inter-temporal optimization requires that the marginal rate of substitution between current and one-period ahead consumption equals the real return on bonds and shares. A first set of Euler
equations, one for each country, therefore describes the link between current and expected one-period ahead consumption and relate it to the risk-free return in units of consumption. A second set of Euler equations, again one for each country, relates the dynamics of consumption to the real return on shares. The real value of the firm \( v_t \), equal to the entry cost in equilibrium, is the forward solution to the Euler equations on shares.

The bond Euler equation in the home country is:

\[
E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{\nu}_t - E_t \pi_{t+1}^C \right),
\]

where a hat over a variable denotes the log-deviation from the steady state, \( \pi_{t+1}^C = \ln \frac{P_{t+1}^h}{P_t} - 1 \) is consumer price inflation and \( \rho > 0 \) is the inter-temporal elasticity.

The Euler on shares is:

\[
E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{\nu}_t + E_t \left( i + \delta \tau_d t_{t+1} - i - \delta \hat{\nu}_{t+1} \right) \right),
\]

where \( d \) are real dividends and \( \delta \) is the exogenous probability of firm exit.

International bond trade implies the uncovered interest parity condition \( E_t \Delta \hat{C}_{t+1} = \hat{\nu}_t - \hat{\nu}_t^f \), linking the expected nominal exchange rate to the interest rate differential across countries. Notice that bond trade provides a useful means for sharing consumption risk: movements in the real exchange rate mimic changes in relative consumption \( \hat{\nu}_t = \rho (\hat{C}_t - \hat{C}_t^*) \).

3.2. Supply Block

The supply block comprises the pricing and entry decisions of firms, together with labor supply and equilibrium conditions in goods and financial markets.

Markets are monopolistically competitive. Each firm produces a unique variety \( h \in (0, N_t^*) \) in the home country and \( f \in (0, N_t^f) \) in the foreign country and sets the price of its product in its own currency, \( p(h) \) and \( p^*(f) \). Export prices entail melting transport costs so that for one unit of a good to reach the foreign market \( 1 + \tau \) units must be shipped. They vary with the exchange rate at a constant elasticity \( \eta \). So, the home-currency price of the imported good \( f \) is \( p_i(f) = c_i^f (1 + \tau) p_t^*(f) \).

All nominal prices are staggered à la Calvo and in each period a fraction \( \alpha \) of firms in each country faces pre-determined prices. Define the relative price of variety \( i = (h,f) \) in units of good \( j = (D,X) \) as \( \rho_{D,X}(h) \equiv \frac{\rho_{D,X}(h)}{\rho_{D,X}} \) indicates the price of variety \( h \) relative to the aggregate price of domestic goods. In a symmetric equilibrium where \( p(h) = \rho p^\tau(1 + \tau)^{\alpha} \), real price fluctuations are driven by:

\[
\hat{\rho}_{D,I} = \frac{\alpha}{1 - \alpha} \pi_t^D + \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t^I - \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1}^I
\]

\[
\hat{\rho}_{X,I} = \frac{\alpha}{1 - \alpha} \pi_t^X + \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t^X - \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1}^X,
\]

where \( \pi_t^D = \ln \frac{P_{t+1}^{D_h}}{P_{t+1}^h} - 1 \) is producer price inflation and \( \pi_t^X = \ln \frac{P_{t+1}^{X_h}}{P_{t+1}^h} - 1 \) is imported inflation. With \( \alpha = 0 \), an increase in the range of available varieties leads to a fall in aggregate prices (the so-called variety effect): the higher the fall the lower the elasticity of substitution \( \theta \). The variety effect is dampened when prices are sticky, \( \alpha > 0 \), implying a relative price distortion.

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13 The nominal exchange rate is defined as units of home currency per one unit of foreign currency. The real exchange rate is defined as \( q = \epsilon D^P / P \). An increase in both \( q \) and \( \epsilon \) is therefore a depreciation. The home terms of trade are the price of home exports relative to the price of home imports \( \Delta T = p(h)/p(f) \).

14 With symmetric demand elasticity, the optimal strategy is to set prices in the producers’ currency and let the final price vary with exchange rate at a constant rate (Corsetti and Pesenti 2005).
The Phillips curve is:
\[
\pi^D_t = \frac{(1-\beta(1-\delta))}{\delta} \left( \hat{N}_t - Z_t \right) + \beta (1-\delta) E_t \pi^D_{t+1} \left( \frac{\theta}{1-\beta(1-\delta)} \right) E_t \pi^X_{t+1} - \frac{1}{\theta-1} \hat{N}_t \tag{7}
\]
where \(\beta\) is the discount factor, \(W\) is the nominal wage, and \(Z\) is the aggregate productivity shock\(^{15}\).

Inflation is driven by marginal costs and expected inflation, as in the traditional forward-looking Phillips curve and also by changes in the variety of goods available for consumption. These changes reflect entry of new firms and have the effect of increasing the persistence of inflation.

Inflation for imported goods is given by:
\[
\pi^X_t = \eta \hat{e}_t + \frac{1}{1-\beta} \left( \hat{N}^*_{X,t} - \hat{N}^*_{X,t-1} \right) + \pi^D_t.
\]

It depends on foreign inflation, the nominal exchange rate (recall that exchange rate changes are passed-through into final prices at the rate \(\eta\)), as well as on changes in the variety of imported goods (more variety implies lower prices).

Entry is subject to an exogenous sunk cost: in each period potential entrants \(N_e\) must purchase \(f_x\) units of the consumption basket to set up a new firm. Notice that entry costs are constant in consumption units.\(^{16}\) Start-up investments are tied to output \(Y\) through the aggregate resource constraint. Equilibrium in international financial markets requires that bonds are in zero net supply worldwide, implying that world output must be equal to world spending (consumption plus investments). Borrowing and lending in international markets allows countries to run current account imbalances. The home current account implies:
\[
\hat{N}_{x,t} = \frac{\theta}{\delta} \left( 1 - \frac{1}{\beta} \right) \hat{Y}_t + \left( 1 - \frac{(1-\delta)}{\beta} \right) \hat{C}_t - \hat{v}_t - \frac{(1-\delta)}{\delta} n \hat{f} a_t \tag{8}
\]
where net foreign assets are \(n \hat{f} a_t = \hat{b}_t - \frac{1}{\beta} \hat{b}_{t-1}\) and \(b_t = \frac{\beta (1-\delta)}{\beta} \hat{b}_{t-1}\). Clearly net foreign assets in the foreign country are \(-n \hat{f} a_t\). Notice that the aggregate constraint implies a trade-off between start-up investments and consumption (the coefficient on \(C\) is negative).

The dynamics of entry is based on Ghironi and Mélich (2005): entrants start producing with a one-period lag and all firms entered in each period are subject to an exogenous exit shock \(\delta\). Therefore, the law of motion of firms is:
\[
\hat{N}_t = (1-\delta) \hat{N}_{t-1} + \delta \hat{N}_{x,t-1} \tag{9}
\]

The selection of exporters draws on Cavallari and D’Addona (2015). All incumbent firms are in principle able to export, yet only a fraction of them will do so. Exports entail a fixed cost \(f_x\) for accessing foreign markets, which is independent of the volume of exports and is paid on a period by period basis, before production and pricing decisions are made. Export costs are firm-specific as in Bergin and Glick (2009) and drawn from a Pareto distribution with lower bound \(f_{x,\min}\) and shape parameter \(\gamma > \theta - 1\). A firm decides to export whenever the expected profits from doing so are higher than its specific trade cost. It is worth stressing that the decision is made before uncertainty about the ability to set prices optimally is resolved. The share of exporting firms is given by:
\[
\hat{N}^*_{X,t} - \hat{N}_t = \kappa \left( \hat{\nu}_{X,t} + \gamma (\pi^D_t - \pi^X_t) \right), \tag{10}
\]

\(^{15}\) The derivation of the Phillips curve draws on Cavallari (2013a).

\(^{16}\) In Cavallari (2013a) and Cavallari and D’Addona (2015) the composition of the investment and consumption baskets may differ and entry costs are time-varying in units of consumption. For a quantitative assessment of the business cycle implications of entry costs see Cavallari (2013b) .
where export markups $\hat{\mu}_{X,t}$ are:

$$\hat{\mu}_{X,t} = \alpha \beta (1 - \delta) \left( E_t \hat{\rho}_{Xt+1} - \hat{\rho}_{Xt} + E_t \pi_{Xt+1}^C \right).$$

An increase in markups $\hat{\mu}_{X,t}$ and/or in the price of exports (the second addend in expression (10)) will boost export profits and raise the share of producers who will be able to cover export costs. Note that the share of exporters would be constant in a flexible price environment. With flexible prices, in fact, producers are able to stabilize profits in their own currency and have therefore no incentive to move in and out of the export sector.

Labor supply is derived from the consumers’ optimization condition:

$$\hat{L}_t = -\rho \phi \hat{C}_t + \phi \left( \hat{W}_t - \pi_{Ct} \right), \quad (11)$$

where $\phi > 0$ is the inverse of the Frisch elasticity.

Finally, goods market clearing reads:

$$\hat{Y}_t = \gamma \left[ (1 - \varrho) \hat{C}_t + \varrho \left( \hat{N}_{et} + f_x \hat{N}_{Xt} \right) \right] + (1 - \gamma) \left[ (1 - \varrho) \hat{C}_t + \varrho \left( \hat{N}_{et}^* + f_x \hat{N}_{Xt}^* \right) + \hat{q}_t \right],$$

where $\varrho = \frac{\delta \beta}{\theta (1 - \beta (1 - \delta))}$. The terms in the first square bracket represent domestic demand for consumption and investment purposes, while the terms in the second square bracket reflect foreign demand.

3.3. Exchange Rate Regimes

The model is closed with the interest rate rule. We consider fixed and floating exchange rate regimes. Under flexible exchange rates, monetary policy follows a symmetric Taylor rule with interest rate smoothing, $\hat{i}_t = \phi \hat{i}_{t-1} + \phi_i \pi_{Ct} + \phi_y \hat{y}_t$ in the home country and $\hat{i}_t^* = \phi \hat{i}_{t-1} + \phi_i \pi_{Ct}^* + \phi_y \hat{y}_t^*$ in the foreign economy. The Taylor principle, $\phi_i > 1$, ensures determinacy (Taylor 1993). The fixed regime is a unilateral (hard) peg to the home currency with a fixed exchange rate at all dates. The home monetary policy is the Taylor rule described above while the foreign interest rule is $\hat{i}_t^* = \hat{i}_t - \varsigma \hat{\epsilon}_t$ with $\varsigma > 0$. The exchange rate target (normalized to zero) ensures determinacy.

4. Simulations

We start with an intuitive illustration of the propagation mechanism at work in the model by means of impulse responses and contrast the dynamics of trade in fixed and floating regimes. Then, we assess the quantitative performance at replicating stylized facts of the international business cycle.

4.1. Calibration

The parameterization reflects the usual quarterly frequency. The home country represents the US, while the foreign country is an aggregate of 22 OECD economies in our sample. We set $\gamma = 0.4$ to match the degree of openness in the US economy. For ease of comparison, we calibrate the preference parameters as in Bilbiie et al. (2012): the inter-temporal elasticity of substitution is $\rho = 1$, the Frisch elasticity is $\phi = 4$, the disutility of labour is normalized so that the steady state level of employment is equal to one and the elasticity of substitution across varieties is $\theta = 3.8$. The choice of $\theta$ implies a markup as high as 35% in steady state. Many studies suggest a higher $\theta$ and a lower markup for aggregate data, Rotemberg and Woodford (1999), for instance, document a markup of about 18% in US data. We have checked that using $\theta = 7.8$ so as to reproduce a steady state markup of 18% does not

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17 We refer to Cavallari and D’Addona (2015) for details about the derivation.
affect the qualitative properties of the model. The discount factor is $\beta = 0.99$, in line with an annual interest rate of 4%.

The rate of firm exit is $\delta = 0.025$ to match a 10% rate of job destruction per year in US data. The entry cost $f_e$ and the level parameter of the distribution of export costs $f_{x_{\min}}$ are inconsequential for the dynamics and can be normalized to unity without loss of generality. The shape parameter reproduces the average standard deviation of the extensive margin in our sample, implying $\kappa = 2.6$.\footnote{The shape parameter is such that $\sqrt{\frac{\mu(f_{x_{\min}})^2}{(\kappa-1)^2(\kappa-2)}} = 6.5$, where 6.5 is the average standard deviation of the extensive margin in our sample.}

The iceberg cost is $\tau = 0.49$ in the middle of the range of values suggested by Corsetti et al. (2013).

The degree of nominal rigidity is $\alpha = 0.59$ per year, implying an average duration of nominal contracts of about seven months. This value is the middle point of the range of estimates found by Gali et al. (2001) for major developed countries. The degree of exchange rate pass-through is $\eta = 0.6$ to match the average degree of long-run pass-through documented by Campa and Goldberg (2005) for major developed economies. Under free floating, monetary policy in each country follows a Taylor rule with parameters $\phi_i = 0.8$, $\phi_y = 0$ and $\phi_\pi = 0.3$ as in Bilbiie et al. (2007). These parameters imply a long-run response to inflation equal to 1.5 and no role for output stabilization. Under fixed exchange rates, only the home country follows the Taylor rule.

The parameters of the US productivity process, $Z_t = \rho Z_{t-1} + \epsilon_{Z,t}$, are from King and Rebelo (1999): the degree of persistence is $\rho_z = 0.979$ and the standard deviation $\sigma_z = 0.0072$. When computing the moments, we consider symmetric productivity processes across countries and set the cross-correlation of innovations equal to 0.025 as in Backus et al. (1992).

### 4.2. Impulse Responses

We consider a 1% productivity rise in the home economy and simulate the model in the benchmark calibration with symmetric Taylor rules, flexible exchange rates, and no technology spillovers. Figures 4–6 report the responses of key variables. In all figures, the y-axes report percent deviations from the steady state while the x-axes display the periods (quarters) after the shock. Solid lines refer to home variables and dashed lines to foreign variables.

![Figure 4. IRF to a 1% rise in home productivity. Solid (dashed) lines refer to home (foreign) variables.](image-url)
The productivity rise creates a favorable business environment that stimulates the creation of new firms. Over time, entry translates into a prolonged, U-shaped rise in the number of producers, which reaches a peak after 10 quarters. As long as more varieties become available in the home market, the relative price of domestic goods $P_D / P_X$, the “internal terms of trade”, drops and shifts demand away from imported goods. Since not all firms are able to revise the price of their products in each period, aggregate prices move sluggishly. Lower marginal costs (not shown in the figures) imply a deflationary pressure on producers’ prices in the early part of the transition. Consumer prices, on the contrary, hike for a while because of imported inflation and the depreciation of the home currency.

Notice that absorption (consumption plus investment in new firms) raises above output, implying a deficit in the current account of the balance of payments. Since initial financial wealth is zero, net exports drop on impact and then gradually return toward the steady state, moving countercyclically as in the data (for instance Engel and Wang 2011). The external deficit is financed by borrowing from abroad, i.e., with an increase in net foreign liabilities.

The productivity rise spread its effects abroad through changes in international prices and the composition of trade. A larger share of domestic firms will now be able to export its products abroad, increasing the variety of this country’s exports. The home terms of trade deteriorate, switching world expenditure towards home products. Traditional analysis based on the Mundell–Fleming model suggests that expenditure switching is favoured in flexible regimes, since the depreciation of the domestic currency makes a country’s products more competitive in foreign markets. As it will be clear soon, this may not hold in our setup with endogenous entry. Exchange rate variability, in fact, can deter firms from exporting in the first place, reducing the variety of a country’s exports and increasing average export prices.
To see the point, it is useful to contrast the trade dynamics in fixed and flexible regimes. Figure 7 reports the responses of the extensive margin (number of new products and previously non-traded goods, $N_{X,t}$), the intensive margin (the volume of exports per traded good, variable $\hat{y}_{X,t}$ in the Appendix A), and of export markups. Solid lines now represent the responses in floating regimes while dashed lines refer to fixed exchange rates.

![Figure 7. IRF to a 1% rise in home productivity in flexible regimes (solid lines) and in fixed regimes (dashed lines).](image)

Qualitatively, the impact of the shock is similar in fixed and floating regimes: in the home country, the extensive margin rises above the steady state while the intensive margin falls below the steady state for most of the transition. Therefore, trade shifts toward new products and previously nontraded goods. The relative abundance of new products implies a drop in their price, compared to the price of mature products (the internal terms of trade in Figure 5) and compared to the price of imports (the terms of trade), reflecting a “comparative advantage” in these sectors. In the foreign country, trade moves in the opposite direction: foreign exporters increase production of mature products and previously traded goods (the intensive margin).

The exchange rate regime indeed affects the strength of these responses, in accordance with the evidence presented above. On impact, the rise (fall) in the home (foreign) extensive margin is more than twice as large under fixed than under floating rates, while the home (foreign) intensive margin drops (increases) only in fixed regimes. Export markups, on the contrary, are smoother when exchange rates are fixed. These dynamics reflect an incentive to adjust trade at the extensive margin in fixed regimes, with firms moving in and out of the export sector. By contrast, flexible regimes imply an incentive to adjust trade at the intensive margin, through changes in the scale of production of incumbent firms.

These outcomes shed new light on the debate about exchange rate policy. The conventional argument stresses the competitive gains from currency devaluations, though competitive devaluations may bear risks of retaliation and currency wars. We stress a further motive to stay away from currency devaluations: they may discourage the creation of new export varieties compared to fixed rates. In the model, this is certainly so for high-productivity countries (the home country in the simulation).

The view that fixed rates may strengthen a country’s competitiveness in sectors that produce new products appears in contrast to what was argued by Bergin and Corsetti (2015). They show that fixed rates reduce the share of differentiated products in the overall exports of a country to the US, in line with the predictions of a model where all goods are traded and entry is free in the sector that produces differentiated goods. Their argument is that monetary stabilization, by reducing markup uncertainty, stimulates entry and fosters the competitiveness of firms operating in sectors that produce differentiated goods compared to firms in homogeneous goods sectors. On the contrary,
constraining policy with an exchange rate peg shifts production away from differentiated goods (toward homogeneous goods) and weakens a country’s comparative advantage in these sectors. However, observing a lower share of differentiated goods in a country’s exports per se is far from decisive. It may reflect production shifts away from differentiated goods sectors (their argument), but it may also reflect an increase in the share of products that are traded (our argument). In principle, fixed rates may affect the share of exporters and the variety of exports in any direction. Which sectors are mostly affected by exchange rate variability and what are the consequences for the composition of a country’s exports are challenging questions for empirical research.

4.3. Second Moments

Having established that exchange rates can affect the incentive to adjust trade at the extensive or the intensive margin in our model, we now turn to evaluate the quantitative performance in capturing key properties in the data. We want to be sure that the propagation mechanism we stress does not come at the expense of a plausible macroeconomic dynamics. Table 2 reports the statistics of selected variables, drawn from stochastic simulations of a first-order approximation of the model evaluated in the standard calibration with flexible exchange rates. The reported moments are the medians of 500 simulations, each 2100 periods long, and all variables are detrended with the HP filter and smoothing parameter 1600. The first panel refers to home variables, the second panel reports the correlation between home and foreign variables, and the third panel considers trade variables. All variables that in the model are measured in variety units are multiplied by the relative price of the corresponding variety for consistency with the data (see Bilbiie et al. 2012).

The model matches key business cycle facts. The volatility and persistence of consumption, labor, and investments are close to the data. Notice that investments in our setup represent the creation of new businesses and do not entail adjustment costs. The comovements with output are plausible, although the correlation of consumption is much higher than in the data. An excessively procyclical consumption is a common outcome in dynamics macroeconomic models, reflecting a strong incentive to smooth consumption over time. As regards the international business cycle, output and consumption are positively correlated across countries, though the correlation of consumption is high compared to the data. Once again this is a consequence of consumption smoothing. Notice that international bond trade provides a useful means for smoothing consumption risk across countries: home consumers borrow from abroad to finance a rise in imports well above the rise in exports (imports are more procyclical than exports, not shown in Table 2, implying countercyclical net exports as in the data, see, among others, Engel and Wang 2011). The model is less successful in capturing the comovements of investments. These reflect the incentive to create new businesses where the returns of starting up a new firm are high and behave similarly to investments in the standard real business cycle model.

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19 A natural extension is to consider multi-sector models with entry in both homogeneous and differentiated goods sectors. This is beyond the scope of this paper.

20 Investments and entry variables behave similarly in the data (Chatterjee and Cooper 1993). For a recent assessment of the cyclical properties of business formation, see Cavallari (2015).

21 Backus et al. (1992) find cross-correlations of output and consumption between US and Europe, respectively, 0.66 and 0.51. In a large sample of developed economies, Ambler et al. (2004) document even smaller cross-country comovements.
Interestingly, the model can match stylized facts of trade margins. Using US trade data on more than 10,000 products and 99 trading partners, Naknoi (2015) documents the business cycle properties of the extensive margin of exports to the US (corresponding to the variable $N_X^e$ in the model), the extensive margin of imports from the US (the variable $N_X$ in the model), and the intensive margins of exports and imports (respectively, the variables $y_X$ and $y_X^*$ defined in the Appendix A). She concludes that reasonable models of trade dynamics should yield the following properties: (i) The extensive margin of exports and the extensive margin of imports are more volatile than output; (ii) the extensive margin of exports is negatively correlated with the intensive margin of exports; (iii) The extensive margin of exports is procyclical (relative to output in the exporting country). Facts (i) and (iii) are shown in Table 2 while the correlation between the extensive and the intensive margin of exports (not shown in the Table) is $-0.53$. To the best of our knowledge, we are the first to reproduce all these facts in a stochastic dynamic model.

We finally want to quantify the impact of the exchange rate regime on macroeconomic dynamics, by looking at the second moments of macro and trade variables. Table 3 reports standard deviations and correlations with output under fixed and floating exchange rates. In the first column, the monetary authorities in both countries follow symmetric Taylor rules and exchange rates are flexible; in the second column the home country follows the Taylor rule and the foreign country adopts an exchange rate peg.

Three facts stand out. First, the volatility of output and its components is higher in fixed regimes both in absolute and relative terms (but consumption is smoother). The stabilization properties of flexible rates are by no means new, what is surprising is the magnitude of the effect: constraining monetary policy with a peg more than doubles the standard deviation of output compared to a situation where monetary policy can actively contrast the shocks. Second, trade of new products is more volatile under fixed rates: the standard deviation of the extensive margin of exports to and from the home country increases by, respectively, 41% and 42% compared to floating regimes.22 Third, export markups are smoother when exchange rates are fixed. These facts suggest that exchange rates indeed matter for macroeconomic dynamics. A constrained policy with an exchange rate peg leads to

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22 In a sample of European data, Auray et al. (2012) document a rise in the extensive margin of exports of intra-EMU trade as large as 21% after European monetary unification.
large shifts to and from the export sectors that amplify the propagation of shocks and increase output volatility. Understanding how these production shifts affect the competitiveness of a country’s exports beyond the business cycle is left to future research.

Table 3. Fixed and flexible regimes.

<table>
<thead>
<tr>
<th></th>
<th>Floaters</th>
<th>Peggers</th>
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<tr>
<td><strong>Panel A: Standard deviation</strong></td>
<td></td>
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<tr>
<td>$Y$</td>
<td>0.81</td>
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<tr>
<td>$C$</td>
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<td>1.03</td>
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<td>$L$</td>
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</tr>
<tr>
<td>$y_X$</td>
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<td>3.57</td>
</tr>
<tr>
<td>$y_X^*$</td>
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<tr>
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<td>$\mu_X^*$</td>
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<tr>
<td><strong>Panel B: Standard deviation relative to $Y$</strong></td>
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<td>$C$</td>
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<td>$\mu_X^*$</td>
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<td><strong>Panel C: Correlation with $Y$</strong></td>
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<td>$C$</td>
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<td>$L$</td>
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</tr>
<tr>
<td>$\mu_X^*$</td>
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<td>0.57</td>
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5. Conclusions

This paper investigated the role of exchange rate variability for trade of new products from both a theoretical and an empirical perspective. VAR evidence documents that trade shifts away from new products and previously non-traded goods in response to a rise in external productivity and more so in fixed regimes. Then, we propose a DSGE model with firm dynamics in line with this evidence. The model is characterized by the endogenous determination of the number of products and the endogenous selection of the share of products that will be exported.

We show that a rise in domestic productivity induces the creation of new products in the home market and leads a higher share of domestic firms to export their products abroad. In the partner economy, on the contrary, the variety of domestic products declines and a lower share of firms become exporters. In fixed regimes, strong shifts in and out of the export sector amplify the propagation of shocks within and across countries.

Our analysis has relevant implications for exchange rate policy. Exchange rate variability, by affecting the incentives to shift production in and out of the export sector, may have an impact on a country’s competitiveness well beyond the short run. In high-productivity economies, fixed exchange
rates foster the competitiveness of firms that trade new products and previously non-traded goods. Flexible rates, on the other hand, favor the competitiveness of firms that trade mature products and already traded goods. A testable implication of our model, which we leave to future research, is that exchange rate variability should have a negative impact on trade of new products and a positive effect on trade of mature products.

Author Contributions: While both authors contributed to the final version of the manuscript and finalized the manuscript collaborating closely, Prof. Cavallari developed the theoretical formalism, Prof. D’Addona performed the analytic calculations. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Appendix A.1. The Complete Model

Appendix A.1.1. Households

Lifetime utility of the representative home household is:

\[
\Omega_t = E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{(C_t)^{1-\rho} \varphi Y}{1 - \rho \varphi} (L_t)^{1+\varphi} \right) \right]
\]  (A1)

and the consumption bundle comprises domestic and imported goods

\[
C_t = (C_D)^{\gamma} (C_X)^{1-\gamma},
\]

where

\[
C_D = \left[ \int_0^N C(h)^{\theta-1} \, dh \right]^{\frac{\theta}{1-\theta}}
\]

\[
C_X = \left[ \int_0^{N_e} C(f)^{\theta-1} \, df \right]^{\frac{\theta}{1-\theta}}
\]  (A2)

and \( \theta > 1 \). The corresponding consumer price index is \( P = (P_D)^{\gamma} (P_X)^{1-\gamma} \) while the indexes for producer and imported prices are:

\[
P_D = \left[ \int_0^N p(h)^{1-\theta} \, dh \right]^{\frac{1}{1-\theta}}
\]

\[
P_X = \left[ \int_0^{N_e} p(f)^{1-\theta} \, df \right]^{\frac{1}{1-\theta}}
\]  (A3)

and \( p(h) \) and \( p(f) \) denote the home-currency price of, respectively, domestic and foreign products (similarly, \( p^*(f) \) and \( p^*(h) \) are foreign-currency prices). Exports prices are \( p_t(f) = \epsilon_t^\eta (1 + \tau) p_t^* (f) \) and \( p_t^*(h) = \epsilon_t^{-\eta} (1 + \tau) p_t (h) \).

The household budget constraint is:

\[
\frac{B_t}{P_t} + \frac{\epsilon_t B_t^*}{P_t} + s_t (N_t + N_e,t) v_t = \frac{B_{t-1}}{P_{t-1}} + \frac{\epsilon_{t-1} B_{t-1}^*}{P_{t-1}} + s_{t-1} N_t (v_t + d_t) + \frac{W_t}{P_t} L_t - C_t
\]  (A4)

where \( s_t \) is the share of a mutual fund of domestic firms including incumbent firms, \( N_t \), and entrants, \( N_e,t \). Note that only \((1 - \delta) (N_t + N_e,t)\) of these firms will survive and pay dividend at the end of the period but since households do not know which firm will survive, they finance all of them during period \( t \)
Consumer first order conditions are:

\[
\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{\bar{z}_t}{(1 + \pi_{t+1}^C)} \right] = 1 \tag{A5}
\]

\[
E_t \left[ \frac{C_{t+1}}{C_t} \left( \frac{i_t - \varepsilon_{t+1} \theta_t}{\varepsilon_t} \right)^{-\rho} \right] = 0 \tag{A6}
\]

\[
(C_t)^{-\rho} = \beta (1 - \delta) E_t \left[ \frac{d_{t+1} + v_{t+1}}{v_t} (C_{t+1})^{-\rho} \right] \tag{A7}
\]

\[
\frac{W_t}{P_t} = \chi \left( L_t \right)^\frac{1}{\gamma} (C_t)^\rho. \tag{A8}
\]

Combining the bond Euler equation for home households (A6) with the equivalent condition for foreign households and using the uncovered interest parity \( E_t(\varepsilon_{t+1}/\varepsilon_t) = (i_t) / (i^*_t) \) yields the risk-sharing condition:

\[
\left( \frac{C_t}{C_t^*} \right)^\rho = q_t
\]

Notice that the purchasing power parity, PPP, does not hold in our setup. PPP would require no export costs, \( \tau = 0 \) and \( \eta = 1 \). In these conditions, all firms will export and all goods will be traded, \( N_t = N_{X,t} \) and \( N_t^s = N_{X,t}^s \), and \( q_t = 1 \) in all periods.

Intra-temporal substitution implies the following demands:

\[
C_{D,t}(h) = \rho_{D,t}(h)^{-\gamma} \left( \frac{P_{D,t}}{P_{X,t}} \right)^{\gamma-1} C_t \tag{A9}
\]

\[
C_{X,t}(f) = \rho_{X,t}(f)^{-\gamma} (1 - \gamma) \left( \frac{P_{D,t}}{P_{X,t}} \right)^{\gamma} C_t
\]

where \( \rho_{D,t}(h) = \frac{p(h)}{P_{X,t}} \), \( \rho_{X,t}(f) = \frac{p(f)}{P_{X,t}} \), and the price \( \frac{P_{D,t}}{P_{X,t}} \) represents the “internal terms of trade”.

Appendix A.1.2. Firms

Firms face a linear technology in the labor factor:

\[
y_t(h) = Z_t L_t(h) \tag{A10}
\]

where \( Z \) is a country-specific shock to labor productivity. All firms produce for the domestic market while only a subset of these firms serve foreign markets. We first determine the number of firms in the economy, \( N_t \). Given \( N_t \), we then determine the share of exporters.

Given the exogenous sunk entry cost \( f_e \), entrants start a new firm whenever its real value \( v_t \), equal to the present discounted value of the expected stream of profits \( \{d_s\}_{s=t+1}^\infty \) and expected capital gains, covers entry costs:

\[
v_t = E_t \left[ \sum_{s=t+1}^\infty \beta (1 - \delta) \left( \frac{C_{s+1}}{C_s} \right)^{-\rho} (d_s + v_s) \right] = f_e. \tag{A11}
\]

The timing of entry and the one-period production lag imply the usual law of motion for producers:

\[
N_t = (1 - \delta) (N_{t-1} + N_{e,t-1}) \tag{A12}
\]

We can now determine the subset of firms that export their products abroad, \( N_{X,t} \). Access to foreign markets is subject to a period, firm-specific trade cost \( f_{X,t}(h) \) which is paid at the beginning
of the period before production and pricing decisions are made. This cost is drawn from a Pareto
distribution with cumulative density function \( \Gamma = 1 - \left( \frac{f_{x, \min}}{f_x} \right)^{-\alpha} \). The cut-off exporting firm, i.e., the
last firm with export costs low enough to earn profits, is determined by the zero-profit condition:

\[
\bar{a}_{X,t}(h) = E_{t-1} \left( \frac{\epsilon p^*_t(h)}{p_t} - \frac{W_t (1 + \tau)}{p_t Z_t} \right) y^*_X(h) = f_{x,t}(h)
\]  

(A13)

where \( y^*_X(h) \) is foreign demand for good \( h \). The share of exporters is therefore given by:

\[
\frac{N_{X,t}}{N_t} = \left[ 1 - \left( \frac{\bar{a}_{X,t}}{f_{x, \min}} \right)^{-\alpha} \right].
\]  

(A14)

The share of exporters is an increasing function of the profit threshold \( \bar{a}_{X,t} \): all firms with profits
higher than the threshold will serve foreign markets. For the property of the Pareto distribution,
a small fraction of firms operating in domestic markets will decide to export after a large rise in export
profits (or a large fall in export costs).

Appendix A.1.3. Price Setting

Firms are monopolistic competitors.

A firm \( h \) faces the following demand in the domestic market:

\[
y_D(h) = (\rho_{D,t}(h))^{-\theta} \gamma \left( \frac{P_{D,t}}{P^*_X} \right)^{\gamma-1} (C_t + f_x N_{e,t} + f_{x,t} N_{X,t})
\]  

(A15)

and in the foreign market:

\[
y_X(h) = (\rho^*_X(h))^{-\theta} (1 - \gamma) \left( \frac{P^*_D}{P^*_X} \right)^{\gamma} (C^*_t + f^*_x N^*_e + f^*_x N^*_X).
\]  

(A16)

She will set the price for its product so as to maximize the present discounted value of future
profits, taking into account market demand (A15) and (A16) as well as the probability that she might
not be able to change the price in the future. Optimal pricing gives:

\[
p_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\alpha \beta (1 - \delta))^k W_{t+k} y_{t+k}(h)}{E_t \sum_{k=0}^{\infty} (\alpha \beta (1 - \delta))^k P_{t+k} C_{t+k}^p}.
\]  

(A17)

where \( y_{t+k}(h) = y_{D,t+k}(h) + y_{X,t+k}(h) \).

With \( \alpha = 0 \) optimal pricing implies a constant markup \( \frac{\theta}{\theta - 1} \) on marginal costs, while time-varying
markups emerge when prices are sticky. The producer price index is given by:

\[
(P_{D,t})^{1-\theta} = \alpha \frac{N_t}{N_{t-1}} (P_{D,t-1})^{1-\theta} + (1 - \alpha) N_t (p_t(h))^{1-\theta}.
\]  

(A18)

Notice that an increase in the number of producers reduces aggregate prices because of love for
variety: more varieties imply a higher value of consumption per unit of expenditure and hence lower
producer prices.

Similarly, the price index for imported goods is:

\[
(P_{X,t})^{1-\theta} = \alpha \frac{N^*_X}{N^*_{X,t-1}} (P_{X,t-1})^{1-\theta} + (1 - \alpha) N^*_X (p_t(f))^{1-\theta}.
\]
Appendix A.1.4. Equilibrium and Aggregate Accounting

Assuming symmetry in asset holdings in each economy (so that, \( s_t = s_{t-1} \) and \( s^*_t = s^*_{t-1} \)), and defining home GDP as \( Y_t = \int_0^N \rho_D(h)y(h)dh \) and foreign GDP as \( Y^*_t = \int_0^N \rho^*_D(f)y(f)df \), a competitive equilibrium is defined as a sequence of quantities:

\[
\{Q_t\}_{t=0}^\infty = \{Y_t, Y^*_t, C_t, C^*_t, L_t, L^*_t, N_{t, f}^*, N_t, N^*_t, N_{X, t, f}^*, N^*_X, t, d_t, d_t^*, d_{X, t}^*, d^*_{X, t}, B_t, B^*_t, B_{t, f}^*, B^*_{t, f} \}_{t=0}^\infty
\]

where \( B_{t, f}, B^*_{t, f} \) denote foreign holdings of home and foreign bonds, respectively, and a sequence of prices:

\[
\{P_t\}_{t=0}^\infty = \left\{ \left( \frac{P_{D, t}}{P_t}, \frac{P_{D, t}^*}{P^*_t}, \rho_{D, t}(h), \rho_{D, t}^*(f), \rho_{X, t}(h), \rho_{X, t}^*(f), W_t, W_t^*, P_{D, t}^*, P_{D, t}^*, \gamma, L_t, L_t^* \right) \right\}_{t=0}^\infty
\]

such that, for a given sequence of shocks \( \{Z_t, Z^*_t\}_{t=0}^\infty \), and conditional on given monetary policies in the two economies:

1. for a given \( \{P_t\}_{t=0}^\infty \), the sequence \( \{Q_t\}_{t=0}^\infty \) satisfies first order conditions of domestic and foreign households and maximizes domestic and foreign firms’ dividends;
2. for a given \( \{Q_t\}_{t=0}^\infty \), the sequence \( \{P_t\}_{t=0}^\infty \) guarantees the equilibrium of goods markets:

\[
\begin{align*}
Y_t &= \gamma \left( \frac{P_{D, t}}{P_t} \right)^{\gamma - 1} (C_t + N_{t, f}^* - N_{t, f}^*) + \left( \frac{P_{D, t}}{P_t} \right)^\gamma (1 - \gamma) (C^*_t + f^*_t - f_{t, f}^*) \\
Y^*_t &= \gamma \left( \frac{P_{D, t}^*}{P^*_t} \right)^{\gamma - 1} (C^*_t + N_{t, f}^* - N_{t, f}^*) + \left( \frac{P_{D, t}^*}{P^*_t} \right)^\gamma (1 - \gamma) (C^*_t + N_{t, f}^* - N_{t, f}^*)
\end{align*}
\]

(A19)

the equilibrium of labor markets:

\[
L_t \geq \int_0^{N_t} \frac{y_t(h)}{Z_t} dh
\]

(A20)

and the equilibrium of financial markets:

\[
B_t + B_{t, f} = 0, \\
B^*_t + B^*_{t, f} = 0.
\]

Aggregating the budget constraint across households and assuming zero initial financial wealth in both economies, the accounting equations read:

\[
\begin{align*}
Y_t - C_t - N_{t, f} v_t &= \frac{B_t - \epsilon_t B^*_t}{P_t} \\
Y^*_t - C^*_t - N_{t, f}^* v_t^* &= -\frac{B_t - \epsilon_t B^*_t}{\epsilon_t P^*_t}
\end{align*}
\]

(A21)

where the RHS is the net foreign asset position.

Appendix A.2. Steady State

The model is solved in log-deviation from a symmetric steady state where all shocks are muted and inflation is zero. For reasons of determinacy, we solve the steady state under the assumption of an exogenously given share of exporters equal to \( \psi = 0.2 \). It is immediate to verify that symmetry implies \( q = \epsilon = ToT = 1 \). The steady state number of firms is obtained from:
\[
\frac{(1 - \beta (1 - \delta)) \theta N}{\beta (1 - \delta)} = \left( \frac{\theta}{(\theta - 1)} \right) \frac{\phi t_{-1}}{1 + \tau} \left( \phi t_{-1} \right)^{2\phi (1 - \gamma)} N^{\frac{\theta - \phi}{\theta}} \left( \frac{\theta (1 - \beta (1 - \delta) - \delta \beta}{\beta (1 - \delta)} \right)^{-\phi}. 
\]

Other variables are given by:

\[
i = \frac{1 - \beta}{\rho}, \quad \frac{\nu}{\theta} = \frac{\phi t_{-1}}{1 + \tau}, \quad v = 1, \quad d = \frac{(1 - \beta (1 - \delta))}{\beta (1 - \delta)}, \quad \mu = \frac{\theta}{(\theta - 1)}, \quad \rho_D = N^{\frac{\theta}{\theta - 1}}, \quad \rho_X = \theta N \left[ \frac{1 - \beta (1 - \delta)}{\theta (1 - \delta)} \right], \quad L = \theta d N^{\frac{\theta}{\theta - 1}}, \quad Y = \theta d N, \quad N_c = \left( \frac{\delta}{\theta - 1} \right) N.
\]

Appendix A.3. Loglinear Model

Loglinearized conditions for households are:

\[
E_i \hat{C}_{t+1} = \hat{C}_i + \frac{1}{\rho} \left( \hat{\mu}_t - E_i \pi^C_{t+1} \right)
\]

\[
E_i \hat{C}^*_t = \hat{C}_i^* + \frac{1}{\rho} \left( \hat{\mu}_t^* - E_i \pi^C_{t+1}^* \right)
\]

\[
E_i \hat{C}_{t+1} = \hat{C}_i + \hat{\nu}_t + \frac{1}{\rho} E_i \left( \frac{i + \delta}{1 + i} \hat{\rho}_{d_t+1} + \frac{1 - \delta}{1 + i} \hat{\rho}_{\delta_t+1} \right)
\]

\[
E_i \hat{C}^*_t = \hat{C}_i^* + \hat{\nu}_t^* + \frac{1}{\rho} E_i \left( \frac{i + \delta}{1 + i} \hat{\rho}^*_d_{t+1} + \frac{1 - \delta}{1 + i} \hat{\rho}^*_\delta_{t+1} \right)
\]

\[
\hat{L}_t = -\rho \phi \hat{C}_t + \phi \left( \hat{W}_t - \pi^C_t \right)
\]

\[
\hat{L}_t^* = -\rho \phi \hat{C}_t^* + \phi \left( \hat{W}_t^* - \pi^C_t^* \right).
\]

Loglinearized conditions for firms are:

\[
\hat{N}_t = (1 - \delta) \hat{N}_{t-1} + \delta \hat{N}_{c,t-1}
\]

\[
\hat{N}_t^* = (1 - \delta) \hat{N}_{t-1}^* + \delta \hat{N}_{c,t-1}^*
\]

\[
\hat{N}_{X,t} = \hat{\nu}_t + \gamma \left( \hat{\rho}_{X,t} + \gamma (\pi^D_{t+1} - \pi^X_{t+1}) \right)
\]

\[
\hat{N}_{X,t}^* = \hat{\nu}_t^* + \gamma \left( \hat{\rho}_{X,t}^* + \gamma (\pi^D_{t+1}^* - \pi^X_{t+1}^*) \right)
\]

\[
\hat{\mu}_t = \alpha \beta (1 - \delta) \left( \hat{\rho}_{d,t+1} - \hat{\rho}_{d,t} + E_i \pi^D_{t+1} \right)
\]

\[
\hat{\mu}_t^* = \alpha \beta (1 - \delta) \left( \hat{\rho}^*_d_{t+1} - \hat{\rho}^*_d_{t} + E_i \pi^D_{t+1}^* \right)
\]

\[
\hat{\mu}_{X,t} = \alpha \beta (1 - \delta) \left( \hat{\rho}_{X,t+1} - \hat{\rho}_{X,t} + E_i \pi^X_{t+1} \right)
\]

\[
\hat{\mu}_{X,t}^* = \alpha \beta (1 - \delta) \left( E_i \hat{\rho}^*_X_{t+1} - \hat{\rho}_{X,t}^* + E_i \pi^X_{t+1} \right)
\]

\[
\pi^D_t = \frac{(1 - \alpha \beta (1 - \delta)) (1 - \alpha)}{\alpha} \left( \hat{W}_t - Z_t \right) + \beta (1 - \delta) E_i \pi^D_{t+1} + \frac{\beta (1 - \delta)}{\theta - 1} E_i \hat{N}_{t+1} - \frac{1 + \alpha \beta (1 - \delta)}{\theta - 1} \hat{N}_t + \frac{1}{\theta - 1} \hat{N}_{t-1}
\]

\[
\pi^D_t^* = \frac{(1 - \alpha \beta (1 - \delta)) (1 - \alpha)}{\alpha} \left( \hat{W}_t^* - Z_t^* \right) + \beta (1 - \delta) E_i \pi^D_{t+1} + \frac{\beta (1 - \delta)}{\theta - 1} E_i \hat{N}_{t+1}^* - \frac{1 + \alpha \beta (1 - \delta)}{\theta - 1} \hat{N}_t^* + \frac{1}{\theta - 1} \hat{N}_{t-1}^*.
\]
Other log-linear equilibrium conditions are:

\[
\begin{align*}
\hat{p}_{Dt} &= \frac{s}{1-\pi} \pi^D + \frac{1}{(1-\pi)(1-\pi)} \tilde{N} - \frac{s}{1-\pi} \tilde{N}^* \\
\hat{p}_{Df} &= \frac{s}{1-\pi} \pi^D + \frac{1}{(1-\pi)(1-\pi)} \tilde{N}^* - \frac{s}{1-\pi} \tilde{N}_{t-1}^* \\
\hat{p}_{Xt} &= \frac{s}{1-\pi} \pi^X + \frac{1}{(1-\pi)(1-\pi)} \tilde{N}_{X,t} - \frac{s}{1-\pi} \tilde{N}_{X,t-1}^* \\
\hat{p}_{Xf} &= \frac{s}{1-\pi} \pi^X + \frac{1}{(1-\pi)(1-\pi)} \tilde{N}_{X,t} - \frac{s}{1-\pi} \tilde{N}_{X,t-1}^* \\
\pi^g_t &= \eta \tilde{e}_t + \frac{1}{(1-\pi)} (\tilde{N}^* - \tilde{N}_{X,t-1}) + \pi^D \\
\pi^c_t &= \gamma \pi^D + (1-\gamma) \pi^X \\
\pi^c_t &= \gamma \pi^D + (1-\gamma) \pi^X
\end{align*}
\]

The model is closed with the interest rate rules indicated in the text.

**Appendix B**

**Appendix B.1. Data**

<table>
<thead>
<tr>
<th>Original Series</th>
<th>Source</th>
<th>Data Transformation</th>
</tr>
</thead>
<tbody>
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<td>Peggers and Floaters Nominal GDP</td>
<td>OECD.StatExtracts</td>
<td>log difference after deflating with the GDP Deflator</td>
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<td>Peggers and Floaters GDP Deflator</td>
<td>OECD.StatExtracts</td>
<td>None</td>
</tr>
<tr>
<td>Peggers and Floaters Export Price index</td>
<td>IFS-IMF database</td>
<td>Used to calculate the terms of trade</td>
</tr>
<tr>
<td>Peggers and Floaters Import Price Index</td>
<td>IFS-IMF database</td>
<td>Used to calculate the terms of trade</td>
</tr>
<tr>
<td>Peggers and Floaters Trade Margins</td>
<td>UN Comtrade database</td>
<td>None</td>
</tr>
</tbody>
</table>
Appendix B.2. Peggers and Floaters

Table A2. List of countries and their classification.

<table>
<thead>
<tr>
<th>Peggers</th>
<th>Floaters</th>
</tr>
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<tbody>
<tr>
<td>Belgium</td>
<td>Australia</td>
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<td>Canada</td>
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<tr>
<td>Finland</td>
<td>Czech Republic</td>
</tr>
<tr>
<td>France</td>
<td>Iceland (After 2001)</td>
</tr>
<tr>
<td>Germany</td>
<td>Japan</td>
</tr>
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<td>Mexico</td>
</tr>
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<td>New Zealand</td>
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References


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