Commentary

Understanding and Addressing the Deficiencies in UK Mathematics Education: Taking an International Perspective

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Abstract: This paper reflects on UK mathematics education following the poor performance in the Programme for International Student Assessment (PISA) metric, which compares reading, science, and mathematics across 27 countries. We compared a range of features within secondary school mathematics in the UK with the countries outperforming the UK. We note disparities in the depth of the curriculum and the use of high-stakes testing which could be disadvantaging UK students. We also reflect on key factors that may underpin teacher effectiveness in the UK, including teacher expectations, in part driven by early use of ability sets, a lack of teacher autonomy, and poor continuous professional development. On this basis, we make several recommendations to strengthen UK mathematics education.

Keywords: PISA; teacher effectiveness; teacher autonomy; professional development; assessment; curriculum

1. Introduction

Mathematics is a core discipline that sits at the heart of primary and secondary education in the UK. Despite this, it has recently been noted by Andreas Schleicher, the Education Director of the Organisation for Economic Co-operation and Development (OECD), that if the UK education system continues to progress at its current rate, “it would take a very long time to catch up with the highest achieving countries” [1]. These remarks were in response to the UK’s performance in the Programme for International Student Assessment (PISA) 2018, an assessment conducted every three years, across 79 different countries, which explores the abilities of 15-year-olds to solve problems set in real-world contexts, in reading, science, and mathematics. In PISA 2018, the UK was ranked 14th for reading and science, but only 18th for mathematics, with students in 12 countries with maths abilities that were at least four months ahead of the comparable UK cohort [2]. Within the top scoring countries, there were gender differences in performance. In some cases, girls outperformed boys (e.g., Finland), but in most countries, including the UK, boys outperformed girls. Moreover, within the UK, the gender gap was greater than for all countries that outperformed the UK. Given this poor performance in mathematics, it is timely to reflect on what can be done to improve this in the UK. One approach is to examine practices in PISA’s top performing countries, such as Estonia, Finland, the Netherlands, and several in East Asia, and compare these to the UK.

There are, of course, likely to be many factors that have led to these countries being more successful, including performance in other areas—research suggests that mathematics ability is, in part, underpinned by reading performance [3]. However, it is not possible to examine all of these and, as such, we have chosen to focus on two core areas: 1. curriculum and assessment and 2. teacher effectiveness, considering several factors within each of these (Figure 1).
Furthermore, we focus on secondary education (ages 11–18 years, Year 7–13) because this period is associated with a significant drop-off in mathematics performance [4], and evidence suggests that students often find secondary school mathematics particularly difficult because the informal approach taken at primary school is replaced with a need to understand increasingly abstract concepts [5]. Finally, it is also apparent that in the UK, this is a period which ends with students very disinterested in mathematics, with a review of post-16 education revealing that when those in upper secondary school are given the choice whether to study this subject, only 13–14% engage [6]. Additionally, this analysis focuses on state schools (as opposed to those run privately or independently), because the majority of students attend this type of institution and the PISA mathematics scores obtained by this sector are significantly lower than those achieved in independent schools [7].

2. Curriculum and Assessment

2.1. Re-Shaping the Curriculum

It has been suggested that a strong curriculum, which is essential for facilitating high-quality learning, should focus on only fundamental concepts and principles [8]. By employing a more focused curriculum, material can be taught in a way that supports deeper learning allowing students to engage with material in a way that creates a deeper understanding [9]. This can facilitate longer term retention of key concepts [10], and the development of critical and analytical skills, which are essential for mathematics and employability [11]. Despite this ideal, over the past two decades, the UK has moved away from a mathematics curriculum focused on key concepts and principles [8,12] to one that has been described by Schleicher as a “mile wide and an inch deep” [13]. The result of this shallow and broad curriculum is that no individual topic or concept can be explored in depth. Thus, the focus of lessons becomes the memorisation of facts, associated with surface learning, rather than teaching students to adopt the thoughts of a mathematician, an outcome that can be achieved by giving them the opportunity to work through and solve mathematical problems by themselves [8]. Furthermore, the UK curriculum typically overcomplicates mathematics, giving students problems where the mathematical skill requirements are low but the complex and confusing context in which the problems are embedded makes them difficult to solve.

In contrast to the UK approach, better performing countries in PISA typically take a narrower but deep approach, in line with research. For example, in Finland, the curriculum is centred on inquiry-based learning, with an inherent focus on transforming students into active, independent learners, who can acquire knowledge for themselves [14,15]. The emphasis placed on achieving this has increased in recent years, such that since 2015, the Finnish education system has required students to take one module a year where they select a real-life issue that is of personal interest and use a multidisciplinary approach to explore and solve the problem by themselves [15]. Similarly, in Singapore, there is a focused, coherent, and challenging curriculum grounded in inquiry-based learning [16,17] and an ethos of teach less, learn more [18]. Looking specifically at mathematics, the strong Japanese curriculum concentrates on a few key topics. This is supported by an analysis...
of English and Japanese mathematics textbooks, which found that whilst the English specification for geometry aims to cover various topics, the Japanese curriculum focuses solely on geometric proof using congruency. This allows for the proof to be concentrated on in depth when students learn geometry, whereas in England, the concept is dispersed throughout topics, such as number and algebra, as well as geometry, and therefore, it is never studied in detail [19].

Based on the success of these other countries, we would recommend that the UK consider reshaping its mathematics curriculum to focus only on key concepts and principles, supporting in-depth study. This study should be grounded in inquiry-based learning, rather than memorisation of facts. Inquiry-based learning is associated with a number of positive outcomes, including increased student engagement [20], improved critical thinking [21,22], a more positive attitude towards problem solving [21], and the development of flexible mathematics knowledge [23]. Furthermore, inquiry-based learning should improve students’ abilities to solve mathematical problems embedded in complex contexts, which can provide a more authentic experience. Within inquiry-based learning, there are a range of approaches that could be adopted, depending on the cohort. For example, problem-based learning, where students are encouraged to collaborate to solve complex, real-world problems [24] or project-based learning, where students master new material through the creation of an original product or presentation, for example, a play or video, which is typically presented to an outside audience [24]. The latter corresponds to Bloom's taxonomy, where the creation of original work is positioned at the top of the hierarchy of learning and therefore can be argued to lead to the deepest and thus most effective learning [25].

2.2. Changing the Stakes of Assessments

The type of assessment used clearly interacts with, and is informed by, the shape of the curriculum. Assessment can be described and classified in many ways, one of which is whether it is high- or low-stakes. High-stakes assessments take the form of tests used to make decisions about the students or educators, for example, supporting student progression, and the purpose is centred on accountability. By contrast, low-stakes assessments focus on measuring achievement and identifying any problems, without significant consequences. In England, students take several compulsory high-stakes mathematics exams, comprising Standard Assessment Tests (SATS) at age 10/11 and the General Certificate of Secondary Education (GCSEs) at age 15/16. Students can also elect to sit mathematics A-level, where they take high-stakes examinations at age 17/18. The high-stakes nature of SATS is demonstrated by these results often being used to set students upon arrival at secondary school, whilst GCSE and A-level mathematics grades can determine which sixth form or college [26], and then which university a student can attend [27]. Several recent reforms have also increased the stakes associated with each mathematics exam. GCSE students must now achieve at least a grade 4 (previously a C) to avoid a resit [28], and all coursework has been removed from the qualification [29]. Moreover, in A-level mathematics, students’ abilities are now assessed after two years of study, rather than at the end of each year.

The emphasis on high-stakes assessment is at odds with research findings which indicate that this approach can be detrimental. For example, there is evidence that high-stakes testing can indirectly reduce the quality of teaching and learning [30], as teachers focus on teaching what they know will be tested, resulting in a narrowing of the curriculum [31]. Given the discussion above, narrowing of the UK curriculum is not necessarily a bad thing, but evidence also shows that high-stakes tests drive teachers to spend increased classroom time coaching for, and practising, tests, meaning the opportunity to enhance students’ conceptual knowledge through deep learning is subsequently disregarded [32]. Other areas may also go undeveloped, such as development of higher-order skills (e.g., critical thinking) [33]. Research suggests that teachers end up ignoring best practice to manage the high-stakes testing environment [33]. Teachers also report teaching to the lowest com-
mon denominator, which can adversely impact more gifted students, effectively creating a ceiling effect for them [34]. The combination of a shallow and broad curriculum and high-stakes testing in the UK, therefore, likely results in a shallow and narrow curriculum, arguably the worst of all options.

As well as changes in what is taught within a specific subject, high-stakes assessments can lead to schools devoting extra instructional time to the examined core subjects, of which mathematics is one, at the cost of other subjects, in the belief that this will improve performance [35]. However, research has found that this is counterproductive. Academic achievement is not enhanced when the time allocated to creative subjects is limited [36], as engagement in Physical Education lessons, for example, can improve students' mathematics grades [37].

There is also evidence of increased negative effects on everyone involved in the process—teachers, students, parents [38–40]. Landry (2005) reports that students are demotivated by the pressure created from high-stakes assessment which can lead to greater drop-out rates. Health effects reported include fear, anxiety, stress, physical illness, and powerlessness in students [41], and studies demonstrating negative effects of high-stakes assessment hugely outnumber studies indicating positive effects [42]. As with the effects derived from changes in teaching, the impacts on stress are not uniform. Students from non-white cultures or those in poverty appear to experience more severe negative effects [43]. Anxiety is particularly pertinent when discussing mathematics because maths anxiety is a recognised phenomenon. In its most severe form, maths anxiety is found in 2–6% of UK secondary school students and is defined as “a feeling of tension and anxiety that interferes with the manipulation of number and the solving of mathematics problems in a wide variety of ordinary life and academic situations” [44] (p. 551). High-stakes assessments may be a potential trigger for maths anxiety [45]. This is particularly concerning because once an individual has maths anxiety, it is difficult to tackle, manifesting as a vicious cycle whereby anxiety leads to poor performance, but also poor performance increases anxiety [46]. Therefore, triggering a student’s maths anxiety with one high-stakes assessment is likely to disadvantage them in all further tests, as the condition that stops them from demonstrating their true abilities would persist [47]. Notably, maths anxiety is more prevalent in girls [48], indicating that, as with other effects, the impact of high-stakes tests may not be uniform.

Given the above, it would be wise for the UK to consider reducing high-stakes assessment in all subjects but particularly mathematics, which is already associated with high levels of anxiety. This approach would be in line with the countries outperforming the UK in PISA. For example, in Finland, students do not take any high-stakes examinations until age 18 [49]. The lack of such assessments is also likely to have contributed to lower levels of stress and higher life satisfaction in this cohort [50]. In Finland, 78% of students are satisfied with their lives, a figure that is significantly higher than the OECD average of 67% and the UK average of 53%, which over the past three years has fallen faster than any other country with comparable PISA results [51–53]. In turn, high levels of life satisfaction not only improve wellbeing but also have a positive impact on exam grades [54]. Furthermore, having only one set of high-stakes examinations to take gives these students a much longer period to prepare. This can reduce self-doubt, anxiety and fear of failure, and subsequently enhance performance and welfare [55–57].

Whilst removing some or all of the current high-stakes assessments would bring the UK in line with Finland and other high-performing countries, such a change is unlikely to be achievable in the short term, given the amount of high-stakes assessment the UK system has, and the educational reform that would be needed to remove it. This was evidenced in the chaos caused by the emergency exam removal in light of the COVID-19 pandemic. Therefore, the options that follow should be considered as either long-term solutions or intermediate measures. One such option would be to reintroduce mathematics coursework. This would reduce the weighting and subsequent pressure associated with exams, which is often detrimental to student performance [58]. Coursework would also allow students to demonstrate they can apply their mathematics knowledge to real-life problems in a
more authentic way than an exam [59]. Furthermore, evidence suggests that the removal of mathematics coursework in 2006 had a greater negative impact on girls compared with boys, a group already disadvantaged by higher levels of maths anxiety [29,60], and so, reintroducing coursework may reduce overall gender differences. It should be noted, however, that analysis of the impact of coursework and examinations on boys and girls suggests that any coursework advantage is relatively small for girls and so will not provide a silver bullet for the gender differences [61]. Another option is to retain high-stakes assessments but to switch to open book examinations. This is because they encourage deeper learning, as students can spend more time engaging with the material, rather than learning facts [62]. A slight amendment to this could be to use modified closed book examinations, with an adaptation that permits students taking in handwritten note cards or cheat sheets, again preventing the process of rote learning [63,64]. Evidence also suggests that open book exams can also be less stressful for students [65] and that performance is comparable with closed book exams, indicating that this would not be a case of lowering academic standards [66].

3. Teacher Effectiveness

The positive impact that an effective teacher has cannot be underestimated. The Sutton Trust [67] found that having a very effective teacher, in comparison to an average one, can cause students’ mathematics scores to increase by 25–45%, an outcome that is greatest amongst disadvantaged students. Teacher effectiveness is dependent on a range of factors, but here, we have chosen to focus on (i) teacher expectations, (ii) teacher autonomy, and (iii) continuous professional development (CPD). Although we discuss these as separate entities, we recognise that they are related—for example, appropriate professional development can increase teacher autonomy [68].

3.1. Teacher Expectations

The impact of teacher expectations on student performance was first demonstrated in 1968 [69] in the now famous study which showed that randomly labelling some students as potential high achievers caused them to make significantly more progress than their peers, despite there being no prior differences in their abilities. These findings have been supported by work focusing specifically on mathematics which showed that high teacher expectations led to grade improvements [70]. Furthermore, these effects are not dependent on individual student expectation; when teachers place high expectations on groups of students, their self-perceptions of their mathematics abilities increase across the course of a school year, whilst the opposite effect occurs when low expectations are assigned [71]. For secondary school teachers, recognising this is particularly important, because low expectations may accentuate the typical decline seen in students’ self-perceptions of their scholastic abilities that occurs during adolescence [72].

One group of particular interest in this review is girls. The PISA data show that the UK has one of the biggest gender gaps in mathematics performance, similar to countries which overall perform much worse. Girls are being outperformed by boys by 12 points (the average across all countries is 5). Part of this could arise from the impact of teacher expectations. Research has shown that immediately prior to secondary school, teachers perceive boys to be more able than girls, and this can result in parents holding the same views [73]. Furthermore, in secondary age students, teacher expectations correlated to students’ maths achievement over a year later and the students on self-concept in maths was impacted by the teacher expectation [74]. Secondary pupils have been shown to perceive STEM subjects, including mathematics, as more masculine, and this in turn is associated with reduced likelihood of continuing with study in this area, for example, at university [75]. Any factors which increase these gendered views within the pupils themselves, including teacher expectations, are likely to result in decreased interest in a topic, which could subsequently result in lowered performance in girls.
As well as teacher expectations potentially reinforcing gender stereotypes, there are also likely to be more general effects. Research suggests that teachers in Western countries, including the UK, are likely to discount a student after a single, weak result because they do not expect to see any further improvement and thus have low expectations of their students [76]. Such teachers will often comfort students following a poor performance, using phrases such as “it’s just not the case that everyone is a math person” [76] (p. 735). This can be problematic as the teacher’s failure to offer strategies for improvement may cause students to assume that they have little belief in their abilities and are disinterested in helping them to progress [76]. Furthermore, teachers in Western countries often overuse praise, which can create issues. For example, if a student receives undue praise for completing an easy task, they may interpret this to mean that the teacher has low expectations of their abilities [77]. Finally, in the UK, almost half of Year 5 and nearly all secondary school students are placed in sets [78], a process which can significantly impact the level of mathematics attainment they can achieve [79]. This setting is highly likely to influence teachers’ perceptions of their students’ abilities, which in turn stipulates the level of work they set; thus, the student can determine whether the teacher has low or high expectations [80].

The research into teacher expectations in Western countries is in stark contrast to the approaches taken by the high-performing countries of China, Singapore, Japan, and South Korea, where teachers hold very high expectations of their students [81,82]. These high expectations may be caused by teachers’ adoption of the incremental theory of intelligence, which states that with the necessary effort, anyone can improve, regardless of their intelligence [83]. In addition to this, Asian teachers are acutely aware of overpraising students, such that it is only offered when both performance and effort levels are very high [83]. Furthermore, in Japanese schools, students are not placed into ability sets until the age of 15 [84]. This instils a belief amongst teachers that all students can attain the desired level of mathematical proficiency.

Given the research emphasising the importance of teacher expectations and approaches taken in high-performing countries, we would suggest some changes to the UK approach. For example, teachers should avoid overly praising students, particularly those who are less able, when they complete an easy mathematics question and ensure that a poor result is followed with advice that enables progression, rather than comfort [76,77]. Similarly, placing students in sets based on their perceived mathematical competency should be delayed. The complete eradication of sets may have the greatest impact on teacher expectations, and it may also narrow the achievement gap by improving the grades of students who would be placed in the lower sets [79,85] but would likely be met with resistance for several reasons. Firstly, high achievers who, because of setting, achieve GCSEs that are half a grade higher than that predicted from their Key Stage 3 scores [79] would likely be in favour of keeping them. Secondly, the middle-class parents who schools want to attract, as they often invest social capital to ensure high standards for their offspring, typically demand sets, as they are confident that their child will be placed in a higher set and hence benefit [86]. Thirdly, a survey showed that only 14% of UK mathematics teachers felt mixed-ability classes were suitable for maths [87]. Finally, the school inspectorate, Ofsted, perceives setting to be an indicator of high standards and in recent years has emphasised gifted and talented programmes [88], meaning setting is very much engrained within the education system. A sensible compromise would therefore be to delay setting until Year 10, which has been successfully implemented in Japan and a small sample of schools in the UK [79]. Finally, to avoid teacher expectations widening or sustaining the gender difference in maths performance, it is important to educate teachers about gender differences and gender stereotypes. This type of education should begin within initial training because research suggests biases develop prior to classroom teaching [89], but projects that extend beyond this to incorporate connections between pupils, parents and teachers and university STEM departments should also be considered [90].
3.2. Teacher Autonomy

Teacher autonomy can be defined as the “freedom to construct a personal pedagogy which entails a balance between personality, training, experience and the requirements of the specific education context” [91] (p. 92), and it is strongly related to teacher effectiveness, with more effective teachers having more autonomy [92]. There is also a positive correlation between student attainment and teachers’ curricula and assessment autonomy [93], meaning that a lack of autonomy could plausibly contribute to poor performance in indicators such as PISA.

In the UK, teacher autonomy has gradually declined with the rise of neoliberalism, which has meant that for maintained schools, the government no longer offers a broad advisory outline of what they should teach but instead creates detailed unit plans, covering all aspects of the curriculum. This is accompanied with targets that teachers should look to address in each lesson [94]. Neoliberalism has also led to the introduction of school league tables collated from the results of standardised tests, which forces schools to compete and prove their worth. Subsequently, the focus of lessons has become helping students to pass examinations, and hence, teaching is rigid, and there is deprivation of opportunity to experiment with new pedagogy [95]. This effect is most pronounced in mathematics, as it is considered the single best way to rank a school [94]. Furthermore, a misalignment between teachers’ current roles and professional perspectives has resulted in a decline in autonomy [96]. UK teachers are experiencing an increasing demand to carry out worthless bureaucratic tasks [97]; thus, they are unable to allocate the desired amount of time to activities they value or find rewarding, such as helping students to progress academically and personally [98].

This lack of autonomy can impact students in several ways. Figures show that in the UK, 32.5% of newly qualified teachers leave the profession in the first five years [99–101], with an increase in central government control, and hence a lack of autonomy, being specified as one of the top ten reasons for doing so [102]. Furthermore, those that do not leave but struggle with the lack of autonomy will often be drawn to the independent school sector, where traditionally they have more autonomy [103]. Consequently, a lack of autonomy for UK teachers can result in a shortage of experienced teachers in the state school sector, which may explain their lower PISA mathematics scores [7]. Another way in which lower autonomy can contribute to poorer performance is through decision making. Evidence indicates that educational decisions are likely to be of a higher standard when made by those responsible for their implementation; indeed, they are often better informed and subsequently more successful [104]. Where teachers lack autonomy and decision-making powers, poor decisions are likely to be made by those with no role in implementing them in the classroom.

The lack of teacher autonomy in the UK is in stark contrast to higher performing nations such as Estonia and the Netherlands, whose respective rankings of 8th and 9th in PISA 2018 may be partly attributable to high rates of teacher autonomy. In Estonia, educators can spend additional time on teaching and learning as they have few administrative tasks to complete [18], whilst headteachers, rather than government, have the power to decide upon the school’s educational priorities and development plans [105]. In the Netherlands, 94% of decisions are made within the school, rather than at state level [106]. Teachers have the freedom to select curriculum and testing methods [107] and headteachers are able to prescribe class size, teacher evaluation methods, and the start and end times of the school day for each year group [108].

Given the stark differences between teacher autonomy in the UK and high performing countries, the UK should return some autonomy to teachers (both classroom teachers and headteachers). This process has already begun with many secondary schools in the UK converting to academies [109] either as part of a Multi-Academy Trust (MAT) where the trust may retain much of the autonomy [110] or as a Single Academy Trust (SAT), meaning that the school runs itself and thus has complete autonomy [111]. In terms of greater autonomy, a SAT may look like the obvious answer, but a MAT can still be advantageous
because there can be some autonomy within the constraints of the trust, which can provide consistency. This corresponds with the aligned autonomy model, which demonstrates that institutions are most effective when high autonomy is coupled with high alignment. This is because giving all staff the opportunity to experiment with no overarching structure creates a chaotic culture [112].

Irrespective of whether schools become academies, the UK education system should be resourced in a way that allows the introduction of more administrative roles, which should mean, as is the case in Estonia, teachers spend less time on administration and thus more time on teaching. This should enhance teachers’ autonomy and hence their commitment to the profession [97,113,114].

3.3. Continuous Professional Development (CPD)

One way to ensure teacher effectiveness is developed and maintained is to provide a wide range of high-quality CPD opportunities for teachers. Professional development opportunities have been shown to have a positive impact on students’ mathematics, problem solving, and reasoning abilities [115], as well as increasing teachers’ knowledge, self-efficacy, and their willingness to experiment with new teaching methods [115,116]. Such experiences also show teachers that there are pathways for improvement and progression to obtain expert or higher status. Despite the clear value of appropriate CPD, there is evidence that CPD is poor in the UK and appears to have declined in recent years. For example, between 1972 and 1990, teachers could enrol on the Secondary Mathematics Individualised Learning Experiment (SMILE), a mathematics programme where one day a week they were released from their teaching responsibilities to meet with other educators to collaborate, develop, and improve upon a student-centred mathematics curriculum [94]. Currently, this type of CPD, both in terms of quantity and approach, is rare. The decline in quality CPD is not driven by a lack of interest; indeed, there is a clear desire to engage in CPD, with participation rates high at 92% [117]. Rather, it seems that the quantity and quality of CPD has declined. On average, teachers in England now spend just four days per year on CPD, which is significantly below the average of 40 days in Shanghai, and the OECD average of ten and a half days [117]. Furthermore, such opportunities often consist of one-off sessions that are not delivered by experts, not research-informed, nor clearly contextualised to classroom practice [118]. Subsequently, only 9% lead to educators incorporating new ideas into their teaching, whilst just 1% facilitate improvements in practice [119].

In contrast to the little and superficial approach to CPD that is commonplace in the UK, higher performing PISA countries take a very different approach. For example, in 2010, Singapore introduced the Academy of Singapore Teachers, which has successfully created “a teacher-led culture of professional excellence for the teaching fraternity” [120] (p. 168). Teachers participate in subject-specific groups, led by either teachers identified as excellent, or specialists from the Ministry of Education and National Institute of Education (NIE), to enhance their knowledge of the curriculum, assessment literacy, and pedagogy [120]. Focusing specifically on CPD within mathematics, Germany’s Enhancing the Efficacy of Teaching in Science and Mathematics (SINUS) programme, which began in 1998, adopts a similar network model based on discipline, without relying on identified experts. SINUS encourages teachers from within a school or across the network to meet to discuss teaching methods that enhance student learning [18]. Whilst country-wide implementation no longer exists, many decentralised programmes continue to run, and the programme has been shown to be effective, with students who attended SINUS schools significantly outperforming those who did not in mathematics (and science), an outcome that was accentuated amongst those of lower ability. Thus, the programme also helped to reduce the attainment gap [121]. Whilst Germany no longer outperforms the UK, this CPD stems from a period when they did. A similar approach is taken in Japan, which still outperforms the UK, where CPD includes a strategy known as Lesson Study to help teachers to improve
upon their practices through collaboratively designing lessons, reflecting on them and revising them [122].

A common feature of CPD from these countries is that they are discipline-specific and teacher-led as part of a community of practitioners. They are also continuous rather than single day events that are not connected. We recommend that the UK consider adopting a similar approach to CPD, supporting the fostering of networks which allow teachers to critically evaluate their own and their peers’ approaches and co-construct lessons. Such an approach should be possible despite increasingly tight school budgets because rather than sending teachers on external costly courses, the CPD can arise from within the school, or network of schools. Furthermore, if additional administrators were brought in, as recommended above, teacher time would be freed up to support greater CPD. An approach similar to SINUS where groups of teachers congregate to critically evaluate their teaching methods can lead to the generation of more innovative ideas and create a sense of community, which supports positive change [123,124] and in many cases can be done in-house, depending on the size of the school. Research has also shown that the Japanese Lesson Study approach can be successfully employed by mathematics teachers in Western countries [122], although there are some caveats. In Japan, the collectivist culture means that teaching is considered a public activity, and hence, educators are accustomed to peer scrutiny, which may differ from the UK [125]. Additionally, Lesson Study is a long-standing tradition in Japan, and thus, the structural flexibility required for teachers to observe lessons and engage in debriefs has always been in place [125]. This contrasts to the UK where flexibility is lacking; the average teacher in England works an additional eight hours a week, in comparison to those in similar OECD countries [126,127], and there is a shortage of mathematics teachers [108], meaning finding the time and flexibility for this could be challenging. Therefore, the CPD may have to start off with a similar programme in mind as a long term rather than immediate goal.

4. Conclusions

The UK is underperforming in mathematics on the international platform, and the data from PISA 2018 force serious consideration of the issue. In this paper, we have reflected on the differences in UK practice in several areas of the educational landscape and identified specific pitfalls associated with mathematics education in UK secondary state schools. Based on this, we have made a series of recommendations summarised in Table 1. Specifically, we believe that narrowing and deepening the curriculum and reducing the emphasis on traditional high-stakes testing, either by reintroducing coursework or opting for open book examinations, would be beneficial. We have also suggested that it may be beneficial to use praise more carefully and delay setting students by ability to better manage teacher expectations and the expectations students have of themselves. Although gender differences are not unique to mathematics, they may also play a role in the poor PISA performance and this could, in part, be addressed by teacher training and collaboration with industry/university to reduce gender stereotyping in teachers and pupils. There is also a clear need to reduce the administrative burden on UK teachers, which would in turn increase the amount of time they have to teach and engage in effective collaborative CPD, as well as increase autonomy. Both will likely increase teacher effectiveness and reduce the high levels of attrition in the profession, keeping effective teachers in state schools for longer. These recommendations range from relatively small-scale, for instance, avoiding praise when a student completes an easy task, to large-scale interventions, such as modifying high-stakes assessments. The scale of these changes corresponds with the level of buy-in needed to adopt them. Some require political support, funding, and government action, but others can be implemented locally. Irrespective of scale, buy-in at the relevant level may not be achieved if the change is perceived as additional work [128]. Buy-in can be encouraged by offering access to research which demonstrates why change is important and the positive impact it will have [129]. Additionally, in the case of schools and teachers, giving ownership or allowing them to help to design the changes increases the likelihood of
successful implementation [129,130]. However, engaging in the process of change requires teachers to leave their comfort zone and overcome bad habits, which is notoriously difficult and takes practise even if the teacher has good intentions [131–133], and therefore, any recommendations for change must be carefully communicated and implemented.

Table 1. Summary of recommendations to improve UK mathematics performance.

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<th>Area</th>
<th>Recommendation</th>
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<tr>
<td>Curriculum</td>
<td>1. Increase focus on key concepts and principles</td>
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<td>2. Increase inquiry-based learning</td>
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<td>Assessment</td>
<td>1. Reduce high-stakes assessment or introduce of open book exams</td>
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<td></td>
<td>2. Reintroduce of coursework</td>
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<td>Teacher expectations</td>
<td>1. Reduce use of praise for easy tasks</td>
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<td>2. Remove or delay ability setting</td>
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<td>3. Increase teacher training around gender stereotyping</td>
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<td>4. Collaborate with universities/industries to challenge gender stereotypes</td>
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<td>Teacher autonomy</td>
<td>1. Increase teacher autonomy in the classroom</td>
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<td>2. Increase resources for administrative roles to allow teachers to teach</td>
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<td>Continuous Professional development</td>
<td>1. Increase opportunities for longitudinal discipline specific, teacher-led CPD</td>
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<td></td>
<td>2. Foster development of peer support networks</td>
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<td>3. Allow greater time for CPD</td>
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It is also important to note that although we have made specific recommendations based on the identified deficiencies in the UK, it would be naïve to assume that every problem can be addressed by following the practices of high performing countries. Certainly, other countries’ accomplishments may not be completely attributable to superior educational practices or systems—cultural values are also likely to be influential—and these clearly vary both between and within countries. For example, research found that despite attending the same school, white Americans believed that mathematics ability is innate, whereas Asian Americans thought it was dependent on effort. Consequently, Asian Americans demonstrated enhanced levels of intrinsic motivation and persevered with challenging work, which led to the achievement of higher mathematics grades [134]. Similarly, East Asian students are typically motivated by failure. One study discovered that when East Asian students failed a test, they spent significantly longer answering the questions in a second similar one. Contrastingly, Western students devoted less time to the second test [135,136]. Furthermore, East Asian students are normally more disciplined than their Western counterparts, meaning teachers spend less time managing behaviour and more time teaching mathematics [137]. Such students are also more likely to study in their spare time [40,137] and receive additional tuition [138]. Therefore, whilst the recommendations outlined here are believed to be feasible for the UK, based on previous practice or research, it is necessary to conduct evaluations of changes and consider pilot programmes where possible, for example, with individual exam boards including coursework or regional CPD networks and funding initially. In this way, the UK can take steps to gradually changing its direction of travel and improving mathematics performance for the next generation.
Author Contributions: Conceptualization, C.S.S. and E.J.D.; Literature Reviewing, C.S.S.; writing—original draft preparation, C.S.S. and E.J.D.; writing—review and editing, E.J.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

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