Article

Metaheuristic Regression Equations for Split-Ring Resonator Using Time-Varying Particle Swarm Optimization Algorithm

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Abstract: This article presents a new technique for determining accurate values of resonant frequency and quality factor pertaining to the split-ring resonator. Different conducting shield materials have been used around a copper split-ring. The split-ring has been designed to operate at about 2.1 GHz. Various equations were worked out earlier to determine the values of resonant frequency and quality factor. However, these equations yielded different solutions. Therefore, simulations were used to obtain the values of the resonant frequency and quality factor of the split-ring resonator with different five-shield materials, using High-Frequency Structure Simulator (HFSS) software. In this work, a novel method has been introduced for obtaining values of resonant frequency which provides results with negligible error. An optimal technique, namely time-varying particle swarm optimization (TVPSO), was then performed to obtain two sets of equations for resonant frequency and quality factor. The two sets of equations, optimized using TVPSO, were compared for their effectiveness in matching the actual frequency and quality factor for each of the five materials. It was found that the TVPSO was significant in achieving the frequency and quality factor regression equation to accurately resemble the actual values portrayed by the low mean absolute error.

Keywords: split-ring resonator; regression equations; time-varying particle swarm optimization (PSO)

1. Introduction

The split-ring resonator (SRR) is a structure that has utility in various microwave applications [1–4]. This structure (in different configurations) has been used in both planar and non-planar forms. These applications include material characterization [5], compositional analysis [6], food grading [4,7], bio-medical applications [2,3], meta-materials [8], quality control etc., in the 1–5 GHz range. The SRR is characterized by ease of fabrication, low cost, moderate quality (Q) factor, low noise interference [9] etc. It comprises a metallic ring with one or more longitudinal gap(s) and generally enclosed in a metallic shield [10–12] for better performance [13]. This work is particularly focused on non-planar SRR. Besides SRR, there are also other resonator topologies which have been presented in the literature [14,15]. A single-gap SRR is shown in Figure 1 [16].

The resonant frequency and Q-factor are important parameters of the SRR which are worked out to determine its applicability [17] in different circumstances. A longitudinal gap of SRR ‘t’ represents capacitance, while the single turn represents inductance of the structure. The resonant frequency is generally obtained on the basis of these quantities, besides the dimensions of the split-ring and shield. This quantity has been worked out by numerous researchers [11–13,18]. One such equation worked

out for obtaining the value of resonant frequency while considering the fringing field and shield effects is given below [19]:

$$f_0 = \frac{c}{2\pi r_0} \sqrt{\frac{t}{\pi W}} \sqrt{1 + \frac{r_0^2}{R_0^2 - (r_0 + W)^2}} \sqrt{\frac{1 + \frac{\Delta Z}{Z}}{1 + \frac{\Delta W}{W}}} \quad (1)$$

where t , W , Z , r_0 , R_0 are different dimensions of SRR and shown in Figure 1. ΔZ is the equivalent length extension due to the magnetic fringe fields at two ends of the resonator, whereas ΔW is the equivalent length extension due to the electric fringe fields at two ends of the resonator [16]. These are calculated as $\Delta Z = 0.18R_0$, $\Delta W = 3.0t$. The Q-factor of SRR is an important figure of merit and is worked out on the basis of material property and dimensions of split-ring and shield. One such equation for the Q-factor while considering ohmic losses on walls of resonator and shield, effects of fringing field and shield effects is given below [19]:

$$Q_s = \frac{r_0}{\delta} \frac{\left(1 + \frac{r_0^2}{R_0^2 - (r_0 + W)^2}\right) \left(1 + \frac{\Delta Z}{Z}\right)}{1 + \left(1 + \frac{W}{r_0} + \frac{R_0}{r_0} + \frac{R_0^2}{3.8Zr_0}\right) \left(\frac{r_0^2}{R_0^2 - (r_0 + W)^2}\right)^2} \quad (2)$$

where δ is skin depth of resonator material. However, the above requires correction due to conductor loss of the capacitor gap and is given as [16]:

$$Q_c = \frac{1.7 \times 10^5 t}{f_0^{3/2} \epsilon_0 W^2 \left(1 + \frac{\Delta W}{W}\right)} \quad (3)$$

Overall Q-factor is, therefore, worked out as:

$$\frac{1}{Q} = \frac{1}{Q_s} + \frac{1}{Q_c} \quad (4)$$

Utilization of the resonance technique for obtaining resonant frequency and Q-factor is a well-established method and has been in use for performing compositional analysis, permittivity sensing, biomedical electron paramagnetic resonance (EPR), etc. [1,3,6,20,21]. Hence, obtaining the exact/accurate value of parameters is essential for optimum utilization of components/devices. For obtaining resonant frequency and Q-factor of the split-ring resonator, numerous models have been formulated [13,18,19]. A comparison of SRR models [13] shows that at times large errors were obtained. So, there exists a need to determine these values using a numerical approach. Basing on these observations, regression equations were obtained [9]. These equations yielded values that are relatively accurate. Since Equations (1)–(4) mentioned above and other similar models only provide approximate values [13], they may provide inaccurate information. Inaccurate values obtained through these models may lead to incorrect design and a wastage of time and effort. Simulation results are also not accurate as mentioned in earlier work [22]. The difference in the two values is mainly due to changes in material properties during machining work, small inaccuracies in fabrication, besides other reasons. However, HFSS simulation yields results better than the mentioned models. So, simulation values can be utilized with a certain degree of confidence. Due to this reason, HFSS [23] simulation has been utilized for this work. The default settings of HFSS have mostly been utilized so that these have equal effects on the obtained values. Regression equations were obtained, with the help of simulated results obtained using MINITAB software [24] to determine values of resonant frequency and Q-factor with high accuracy for different shield material around a copper ring [9]. Optimization is required to use a requisite object in its optimal way. Recently, metaheuristic optimization techniques have found tremendous applications in various fields of engineering and science [25,26]. Particle swarm optimization (PSO) being one of the most used metaheuristic algorithms, has played a substantial role in various problems. Particular to communication and electronics fields, PSO has many applications [27–29]. This optimization algorithm

yields solutions using the iterative method and tries to improve the successive result with regard to a given measure of quality (objective function). It uses predefined data, which are dubbed particles, and these particles are moved around in the search-space over the particle's position and velocity to yield solutions.

Figure 1. Split-ring resonator enclosed in cavity, front (L) and top (R) view.

In this work, a novel method has been introduced for obtaining the values of resonant frequency and Q-factor, which provides results with negligible errors. Two sets of equations for resonant frequency and Q-factor have been obtained for different shield materials around a copper ring, as discussed above. Coefficients of the two sets of equations have been optimized by a variant of standard PSO, namely time-varying particle swarm optimization (TVPSO) which has been effective in finding global solutions to many engineering problems [30,31], by formulating an optimization problem using a minimizing objective function. Comparison of the results is presented for validation and usability of the presented technique.

2. Problem Formulation and Methodology

The resonant frequency and Q-factor of SRR are dependent upon critical dimensions besides material and other properties. Important material properties which affect the values of these parameters include bulk conductivity, relative permeability and mass density of both the split-ring and shield material [9]. Important properties of the shield materials are given in Table 1 [9]. As per guidance available in earlier research [1,10,11], high conductivity is a necessity for a split-ring, so copper has been utilized in this work to design the split-ring. The shield has been designed using aluminum (AL), brass (BR) [21], stainless steel (SS), cast iron (CI), and tin (SN). These have been previously utilized in an earlier work [9].

Table 1. Properties of shield material.

Shield Material	Relative Permeability μ_r	Bulk Conductivity σ , Siemens	Mass Density ρ , kg/m ³
AL	1.000021	38,000,000	2689
BR	1	15,000,000	8600
SS	1	1,100,000	8055
CI	60	1,500,000	7200
SN	1	8,670,000	7304

Equations of the resonant frequency and Q-factor of SRR can be obtained using data. Data can be obtained with the help of simulations. For this purpose, five base-design SRR models were formulated using HFSS, each with a shield of different material. Design parameters of the base design SRR are produced in Table 2. A square cross-section of the ring was developed so that magnetic and electric fringing effects remain uniform in vertical and horizontal directions. A HFSS simulation model

comprising of SRR enclosed in a shield, designed for this work, is shown in Figure 2. Five sets of values pertaining to resonant frequency and Q-factor for the base design SRR were obtained. For studying the effects of variations in shield dimensions on the resonant frequency and Q-factor pertaining to each of these, dimensional parameters had to be varied within allowable limits. These ranges were obtained as per the guidelines worked out by earlier researchers. The range over which parameters were to be varied are tabulated in Table 3 [9]. It can be observed that each of the parameters, namely height of shield ' H ', the inner radius of shield ' R_0 ' and thickness of shield ' T ', had 5 values. So for each shield material, 125 ($5 \times 5 \times 5$) solutions were required. The grand total of solutions for all SRRs with shield, of five defined materials, was 625 solutions.

Figure 2. HFSS software simulation of a split-ring resonator (SRR).

Table 2. Parameters of base design SRR.

Design Parameters	Dimensions (mm)
Inner radius of shield " R_0 "	25
Inner radius of resonator " r_0 "	6
Width of resonator " W "	6
Length of resonator " Z "	6
Gap of resonator " t "	2
Inner height of shield " H "	28
Thickness of shield " T " (in all directions)	4

Table 3. Range of parametric variations.

Parameters	Range of Variation		
	Minimum (mm)	Maximum (mm)	Increment (mm)
Inner height of shield " H "	28	32	1
Inner radius of shield " R_0 "	25	29	1
Thickness of shield " T " (in all directions)	4	8	1

The effects of variations in geometrical dimensions of the shield on the resonant frequency and Q-factor using full factorial method could be studied with the help of a full factorial experiment. In the full factorial experiment, all combinations of variable levels are included to find solutions [32]. In this case, 625 experiments had to be designed because three variables used five levels of values for each shield material, as discussed above. In order to reduce the number of experiments without losing accuracy in the results, the Design of experiment Taguchi approach [33] was adopted. The Taguchi method defines two types of factors, namely control factors and noise factors. An inner design constructed over the control factors finds optimal settings. An outer design over the noise factors looks at how the response behaves for a wide range of noise conditions. The experiment is

performed on all combinations of the inner and outer design runs. A performance statistic is calculated across the outer runs for each inner run. The total experiments required in the Taguchi approach is obtained by formulating a table comprising of orthogonal arrays e.g., L4, L8, L9, L12, L16, L18, L25, L27 etc. Statistical software like Minitab, SPSS etc., are designed for formulating orthogonal arrays and hence a reduced number of experiments. For this work, the Taguchi method was utilized with the help of Minitab software [24]. Taguchi's L-25 orthogonal array required only 25 solutions for the aforementioned problem. This array was formulated using the software. HFSS simulations were designed to obtain solutions of the resonant frequency and Q-factor for each design of the array. The obtained values of resonant frequency and Q-factor for each row is given in Table 4.

Table 4. L-25 array with values of resonant frequency and Q-factor.

Height of Shield (mm)	Radius of Shield (mm)	Thickness of Shield (mm)	Shield Material	Resonant Frequency (GHz)	Q-Factor
28	25	4	AL	2.178	3147.49
28	26	5	BR	2.082	2750.86
28	27	6	SS	2.085	2399.01
28	28	7	CI	2.114	1217.51
28	29	8	SN	2.127	2785.92
29	25	5	SS	2.100	2465.01
29	26	6	CI	2.061	1234.29
29	27	7	SN	2.025	2596.40
29	28	8	AL	2.060	2895.21
29	29	4	BR	2.150	2607.00
30	25	6	SN	2.028	2796.62
30	26	7	AL	1.976	2573.34
30	27	8	BR	2.040	2619.77
30	28	4	SS	2.133	2542.19
30	29	5	CI	2.081	1352.58
31	25	7	BR	2.069	2728.93
31	26	8	SS	2.023	2419.74
31	27	4	CI	2.136	1271.32
31	28	5	SN	2.043	2775.66
31	29	6	AL	2.084	2962.62
32	25	8	CI	1.953	1251.17
32	26	4	SN	2.121	2826.00
32	27	5	AL	2.067	2763.36
32	28	6	BR	1.964	2328.57
32	29	7	SS	2.061	2321.32

Following two regression models, non-interactive and interactive respectively, have been considered for each of the two quantities i.e., resonant frequency (f) and Q-factor (Q) to accommodate the data given in Table 3. Equation (5) presents a general form of the non-interactive regression model for resonant frequency while (6) is the interactive model for the same parameter. Higher order interactive equations can be defined, but this work has been limited to two-factor interaction. (5) is termed as Model-1 and (6) is termed as Model-2 for resonant frequency.

$$f = a_0 + a_1 \times H + a_2 \times R + a_3 \times T \quad (5)$$

$$f = a_0 + a_1 \times H + a_2 \times R + a_3 \times T + a_4 \times H \times R + a_5 \times H \times T + a_6 \times R \times T \quad (6)$$

General forms of regression equations, defined in a similar fashion, for Q-factor calculations are shown in (7) and (8); (7) is termed as Model-1 (non-interactive) and (8) is termed as Model-2 (interactive) for Q-factor.

$$Q = b_0 + b_1 \times H + b_2 \times R + b_3 \times T \quad (7)$$

$$Q = b_0 + b_1 \times H + b_2 \times R + b_3 \times T + b_4 \times H \times R + b_5 \times H \times T + b_6 \times R \times T \quad (8)$$

The two models differ in complexity such as number of coefficients; Model-1 needs four coefficients to be updated whereas Model-2 needs 7 coefficients to be optimized. Furthermore, Model-1 considers the effect of the shield dimensions individually whereas Model-2 considers the effect of the shield dimensions individually as well as their combined effect. The regression problem is converted into an optimization problem by formulating an objective function. For our work, we have used mean absolute error (MAE) as the minimizing objective function as expressed in (9).

$$\min J = \frac{1}{N} \sum_{i=1}^N |e_i| \quad (9)$$

where J represents the objective function to be minimized. N is the number of data points; here $N = 5$ for a particular material. The error (e) is defined as the difference between the actual and the estimated resonant frequency and Q-factor, respectively, as expressed by (10) and (11).

$$e_i = f_{act.} - f_{calc.} \quad (10)$$

$$e_i = Q_{act.} - Q_{calc.} \quad (11)$$

The TVPSO is used to optimize (minimize) the objective value in each iteration and to settle to a global minimum objective value. The optimization problem formulated is four-dimensional for Model-1 and seven-dimensional for Model-2 of both resonant frequency and Q-factor.

3. Time-Varying Particle Swarm Optimization (TVPSO) Algorithm and Parameter Settings

Particle swarm optimization (PSO) is a well-established metaheuristic optimization algorithm; PSO imitates the food search behavior of birds or fishes. The candidate solution is termed as a particle; a swarm of particles is used to explore the search space in which our potential global solution exists. Two variables are updated in each search iteration, namely the velocity and position of the particles as expressed in the following.

$$v_i(t+1) = \omega v_i(t) + c_1 r_1 (pBest_i(t) - x_i(t)) + c_2 r_2 (gBest(t) - x_i(t)) \quad (12)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (13)$$

In (12) and (13) v_i represents the velocity of the i th particle, t is the current iteration, ω is the inertia, c_1 is the personal acceleration coefficient, c_2 is the global acceleration coefficient, $pBest_i$ is the personal best position of the i th particle in current iteration, $gBest(t)$ is the global best position of the entire swarm, r_1 and r_2 are the random numbers between 0 and 1 and x_i is the current particle position. In standard PSO (SPSO), the values of ω , c_1 and c_2 are kept constant.

In our work, a variant of PSO called time-varying PSO (TVPSO) [34] has been used. The TVPSO differs from SPSO [35] in terms of parameters as in TVPSO the parameters are time-dependent rather than fixed in SPSO. The parameters are made variable to better explore and exploit capabilities to avoid premature solutions or trapping in a local solution [36]. The ω is varied between 0.4 and 0.9 whereas c_1 and c_2 are varied between 0.5 and 2.5 in samples defined by current iteration count and maximum number of iterations as expressed by (14)–(16).

$$\omega(t) = (\omega_{max} - \omega_{min}) \times \frac{t_{max} - t}{t_{max}} + \omega_{min} \quad (14)$$

$$c_1 = (c_{1max} - c_{1min}) \times \frac{t_{max} - t}{t_{max}} + c_{1min} \quad (15)$$

$$c_2 = (c_{2min} - c_{2max}) \times \frac{t_{max} - t}{t_{max}} + c_{2max} \quad (16)$$

In (14)–(16), ω_{max} and ω_{min} represent the maximum and minimum values of ω , respectively. c_{1max} and c_{1min} represent maximum and minimum values of c_1 , respectively. c_{2max} and c_{2min} represent maximum and minimum values of c_2 . t is the current iteration count and t_{max} is the maximum number of iterations.

For obtaining coefficients for the two models, separate iterative routines were developed using the TVPSO optimization algorithm in MATLAB. The optimization algorithm requires a number of initial parameter settings. These settings are carefully selected so as to obtain convergence with the desired accuracy. Table 5 shows the parameter settings used in TVPSO, whereas Figure 3 shows the flow diagram of the TVPSO.

Table 5. TVPSO parameters.

Parameters	Setting
Swarm size	5000
Number of iterations	100
ω	(0.4, 0.9)
c_1	(0.5, 2.5)
c_2	(0.5, 2.5)

Figure 3. Flow diagram of the TVPSO.

4. Formation of Regression Equations Using TVPSO Algorithm

Coefficients were obtained for both models using routines described in the previous section. These were subsequently used for the formulation of regression equations for resonant frequency and Q-factor. The algorithm utilized the values of the parameters mentioned in Table 3. However, data were utilized material wise for which the table was sorted accordingly. Regression equations were obtained which are presented model wise for different shield materials

4.1. Model-1

Non-interactive regression equations for resonant frequency and Q-factor were formulated using the coefficients, which were generated by the iterative routine for Model-1. These equations describe dependencies of these parameters over geometrical dimensions of the shield. Regression equations

for resonant frequency and Q-factor pertaining to SRRs of different shield materials were obtained. The generalized forms of equations for resonant frequency and Q-factor are given in Equations (5) and (7), respectively. Coefficients for the two equations pertaining to Model-1 are given in Tables 6 and 7, respectively.

4.2. Model-2

Interactive regression equations for resonant frequency and Q-factor were formulated in a similar manner. Coefficients were generated by the iterative routine for Model-2. These equations describe not only the dependencies of these parameters over geometrical dimensions of the shield but also the effect due to their interactions. Regression equations for resonant frequency and Q-factor pertaining to SRRs of different shield materials were obtained. Generalized forms of equations for resonant frequency and Q-factor are given in Equations (6) and (8), respectively. Coefficients for the two equations pertaining to Model-1 are given in Tables 8 and 9, respectively.

5. Results and Discussion

Results for resonant frequency and Q-factor were obtained using both models. These were compared with the values obtained through HFSS simulations. Material wise coefficients and MAE pertaining to these parameters for Model-1 are tabulated in Tables 6 and 7, whereas Tables 8 and 9 present information for Model-2.

Table 6. Optimized coefficients for resonant frequency using Model-1.

Material	Coefficients				MAE
	a_0	a_1	a_2	a_3	
AL	2.6004	−0.0395	0.0352	−0.0489	1.3×10^{-2}
BR	2.0461	0.0023	0.0033	−0.0205	3.46×10^{-2}
SS	2.1658	−0.0023	0.0049	−0.0245	1×10^{-3}
CI	2.4399	−0.0134	0.0094	−0.0367	2.16×10^{-2}
SN	−1.2008	0.0819	0.0121	0.0780	3.6×10^{-2}

Table 7. Optimized coefficients for Q-factor using Model-1.

Material	Coefficients				MAE
	b_0	b_1	b_2	b_3	
AL	3630.23	−140.1566	161.9139	−151.5583	24.522
BR	4844.7288	−23.4483	−51.8445	−15.2135	51.3
SS	2773.1	14.9930	−20.5774	−39.7109	35.12
CI	569.0005	14.9670	9.9874	−5.8076	19.608
SN	2093.007	21.2604	−0.0816	11.3066	47.972

Table 8. Optimized coefficients for resonant frequency using Model-2.

Material	Coefficients							MAE
	a_0	a_1	a_2	a_3	a_4	a_5	a_6	
AL	−0.6931	−0.0275	−0.0085	1.4727	0.0054	−0.0348	−0.0193	7.1468×10^{-12}
BR	0.0881	0.0503	0.4575	−1.6877	−0.0149	0.0565	3.5958×10^{-4}	2.2723×10^{-12}
SS	4.0439	−0.0364	1.0166	−4.9398	−0.0346	0.1515	0.0119	2.4821×10^{-11}
CI	1.1301	−0.1801	−0.4872	3.2691	0.0237	−0.0733	−0.0380	4.9702×10^{-11}
SN	1.2012	0.0224	1.9063	1.0571	−0.0518	−0.0336	−0.0599	7.0194×10^{-3}

Table 9. Optimized coefficients for resonant Q-factor using Model-2.

Material	Coefficients						MAE
	b_0	b_1	b_2	b_3	b_4	b_5	
AL	4303.33	−85.98	−36.41	452.04	3.73	−27.38	8.6953×10^{-9}
BR	2873.80	−145.95	134.25	718.73	0.33	9.15	1.57×10^{-9}
SS	493.70	−30.25	78.23	529.30	1.24	1.73	1.5623×10^{-9}
CI	1608.40	−189.14	−33.95	914.69	7.77	−3.84	6.1021×10^{-9}
SN	1902.70	35.53	582.76	342.75	−16.20	−10.80	37.5346

Table 10 tabulates the actual and estimated resonant frequency values along with the absolute error values. In Table 10, $f_{act.}$ represents the actual frequency values obtained through simulation whereas f_1 and f_2 represent the estimated frequency values by Model-1 and Model-2, respectively. These are also shown graphically in Figure 4. The difference between $f_{act.}$ and f_1 is represented by e_1 , and the difference between $f_{act.}$ and f_2 is represented by e_2 . Absolute error for frequency is shown graphically in Figure 5. On the whole, it is obvious from this table that the absolute values of e_2 are much less or nearly zero compared to the absolute values of e_1 . However, in some cells the absolute value of e_1 is better than the absolute value of e_2 . In overall comparison, Model-2 outperformed Model-1 in terms of achieving resonant frequency values overlapping actual values.

In a similar manner, the actual and estimated Q-factor values along with the absolute error values have been tabulated in Table 11. In this table, $Q_{act.}$ represents the actual Q-factor values obtained through simulation whereas Q_1 and Q_2 represent the estimated values by Model-1 and Model-2, respectively. These are also shown graphically in Figure 6. The difference between $Q_{act.}$ and Q_1 is represented by e_3 , and the difference between $Q_{act.}$ and Q_2 is represented by e_4 . The absolute error for frequency is shown graphically in Figure 7. On the whole, it is obvious from this table that the absolute values of e_4 are much less or nearly zero compared to absolute values of e_3 . In one instance however, absolute value of e_3 is better than the absolute value of e_4 . In this comparison also, Model-2 outperformed Model-1 in terms of achieving desired values.

Table 10. Comparison of actual and estimated resonant frequency using Model-1 and Model-2.

Material	$f_{act.}$ (GHz)	Model-1		Model-2	
		f_1 (GHz)	$abs(e_1)$	f_2 (GHz)	$abs(e_2)$
AL	2.178	2.178	0	2.1778	0.0002
	2.06	2.0486	0.0114	2.0597	0.0003
	1.976	1.9875	0.0115	1.9777	0.0017
	2.084	2.1025	0.0185	2.0837	0.0003
	2.067	2.0414	0.0256	2.0668	0.0002
BR	2.082	2.0935	0.0115	2.0818	0.0002
	2.15	2.1261	0.0239	2.1498	0.0002
	2.04	2.04	0	2.0397	0.0003
	2.069	2.0563	0.0127	2.0687	0.0003
	1.964	2.0888	0.1248	1.9638	0.0002
SS	2.085	2.085	0	2.0852	0.0002
	2.1	2.0974	0.0026	2.1002	0.0002
	2.133	2.1342	0.0012	2.1332	0.0002
	2.023	2.0242	0.0012	2.0232	0.0002
	2.061	2.0609	0.0001	2.0613	0.0003
CI	2.114	2.0714	0.0426	2.1140	0
	2.061	2.076	0.015	2.0610	0
	2.081	2.1275	0.0465	2.0809	0.0001
	2.136	2.132	0.004	2.1359	0.0001
	1.953	1.953	0	1.953	0
SN	2.127	2.0688	0.0582	2.1272	0.0002
	2.025	2.048	0.023	2.0603	0.0353
	2.028	2.028	0	2.0282	0.0002
	2.043	2.0683	0.0253	2.0432	0.0002
	2.121	2.0479	0.0731	2.1211	0.0001

Figure 4. Estimated frequency values of Model-1 and Model-2.

Figure 5. Absolute errors for frequency pertaining to shields of different materials.

Table 11. Comparison of actual and estimated Q-factor using Model-1 and Model-2.

Material	$Q_{act.}$	Model-1		Model-2	
		Q_1	$abs(e_3)$	Q_2	$abs(e_4)$
AL	3147.49	3147.23	0.26	3147.4899	0.0001
	2895.21	2866.5599	28.6501	2895.2100	0
	2573.34	2574.1399	0.7999	2573.3400	0
	2962.62	3071.27	108.65	2962.6200	0
	2763.36	2758.8499	4.5101	2763.3599	0.0001
BR	2750.86	2764.121	13.261	2750.8747	0.0147
	2607	2600.315	6.685	2607.0169	0.0169
	2619.77	2619.741	0.029	2619.7829	0.0129
	2728.93	2715.196	13.734	2728.9453	0.0153
	2328.57	2551.426	222.856	2328.5886	0.0186

Table 11. Cont.

Material	$Q_{act.}$	Model-1		Model-2	
		Q_1	$abs(e_3)$	Q_2	$abs(e_4)$
SS	2399.01	2399.059	0.049	2399.0311	0.0211
	2465.01	2494.917	29.907	2465.031	0.021
	2542.19	2487.89	54.3	2542.214	0.024
	2419.74	2385.193	34.547	2419.765	0.025
	2321.32	2378.166	56.846	2321.348	0.028
CI	1217.51	1227.07	9.56	1217.57	0.06
	1234.29	1227.8698	6.4202	1234.33	0.04
	1352.58	1278.6066	73.9734	1352.62	0.04
	1271.32	1279.4064	8.0864	1271.347	0.027
	1251.17	1251.1682	0.0018	1251.226	0.056
SN	2785.92	2776.3859	9.5341	2785.92	0
	2596.4	2786.5029	190.103	2784.0729	187.6729
	2796.62	2796.62	0	2796.62	0
	2775.66	2806.3289	30.6689	2775.66	0
	2826	2816.446	9.554	2826	0

Figure 6. Estimated Q-factor values of Model-1 and Model-2.

Figure 7. Absolute errors for Q-factor pertaining to shields of different materials.

It is clear from the above comparisons that Model-2 generated highly precise values both for resonant frequency and Q-factor compared to Model-1. This is primarily because Model-2 has been obtained keeping in mind the interactions between the dimensional parameters of SRR shields, whereas Model-1 does not cater for this aspect. Keeping in view this observation, better results can be expected while considering higher order interactions.

6. Conclusions

Values of the resonant frequency and Q-factor pertaining to SRR with different shield materials were obtained using two multi-coefficient models. A variant of the PSO algorithm with time-varying inertia and acceleration coefficients was used to optimize the models' coefficients. Having the algorithm parameters as time-varying contributes towards good exploration and exploitation capabilities in order to avoid becoming trapped in a local solution. Results obtained through these models were compared with each other. Overall, Model-2 contributed well towards achieving very low objective (MAE) values when compared with Model-1. Average absolute errors generated by Model-1 for resonant frequency and Q factor were less than 4 and 3 percent, respectively; while Model-2 yielded results with average absolute errors even less than 1 percent for both parameters. It can thus be concluded that the interactive models were able to generate the expected results with higher accuracy and show a good match between the actual and calculated resonant frequency and Q-factor.

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