A Study on Input Power Factor Compensation Capability of Matrix Converters

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Abstract: In practice, the input filter is an important component in matrix converter (MC) systems for removing high harmonic components from input currents. Due to the input filter, the input power factor (IPF) at the main power supply does not always achieve unity. To investigate the behavior of the IPF, this paper analyzes the IPF compensation capacity of MCs with an LC input filter based on space vector theory and the conservation of energy law. The study shows that the range of voltage transfer ratio (VTR) to achieve unity IPF depends strongly on the quality factor, which is determined by the system parameters. If the quality factor is greater than 0.375, the MC can never achieve unity IPF for the whole range of VTR. If the quality factor is lower than 0.375, the MC can only achieve unity IPF for a certain range of VTR, except at a very low or very high VTR. Experimental results are provided to confirm the correctness of the study.

Keywords: Matrix converter; input power factor; input filter; space vector theory; conservation of energy law

1. Introduction

A matrix converter (MC) is a type of direct ac–ac converter that can transfer input power to the output side directly without intermediate energy storage elements [1–4]. In recent years, MCs have received considerable research interest because they provide many attractive features compared with back-to-back converters, such as sinusoidal input/output waveforms, bidirectional energy-flow capability, controllable input-power factor (IPF), compact realization, high-temperature operating capability, and longevity [5–8]. With these potential advantages, MCs can be employed in specific industrial applications such as electric vehicles, military aircraft, and marine systems, where the converters are required to meet the demands of high-power density, reliability, and high-temperature operation [9,10]. To date, some commercial MC products have been launched by Yaskawa, Eupec, and Fuji companies [11–13].

An MC consists of nine bidirectional switches that connect any output phase directly to any input phase. In addition, an input filter is necessary to remove high harmonic components from the input currents and to meet electromagnetic interference (EMI) requirements at the MC’s input side [14–16]. Among many different topologies, the traditional second-order LC filter is most widely used [17]. To mitigate the oscillations around the LC resonance frequency, it is necessary to include a
damping resistor in parallel with the inductor [18,19]. Figure 1 shows a common MC configuration, including the input filter.

![Figure 1. A common matrix converter configuration.](image)

However, the input filter can result in a displacement angle between the source voltage and source current, which causes an IPF degradation at the main power supply. Several IPF compensation algorithms have been proposed to improve the IPF for MCs [20-23]. Some papers have analyzed and compared many different control strategies regarding aspects including the IPF [24,25]. However, systematic investigation of the IPF compensation capability of MCs has not to date been presented in the literature.

To fulfil IPF control, this paper presents a study on the IPF compensation capacity of MCs. The IPF is analyzed with a traditional LC input filter (due to its widespread use). The analysis for other input filters is similar. The study is conducted using space vector theory and the conservation of energy law. The study of IPF compensation capability shows that MCs can only operate with unity IPF within a certain range of voltage transfer ratio (VTR). The IPF cannot reach unity at very low or very high VTR for every load value. In this case, the maximum achievable IPF versus VTR is presented. These results are useful for designing input filters and optimal IPF compensation algorithms for MCs. Experimental results carried out on an MC prototype with the traditional space vector modulation (SVM) method are presented to confirm the study.

The remainder of this paper is organized as follows. Section 2 briefly presents the traditional SVM method for MCs. Section 3 analyzes the input filter in MCs. Section 4 presents the principal study on the IPF compensation capability of MCs. Experimental results are provided in Section 5 to validate the study. Finally, Section 6 summarizes the contributions of this paper.

2. SVM Method for MCs

In a three-phase ac system, the source voltage is defined as follows:

$$
\mathbf{v}_s = \begin{bmatrix} v_{sa} \\ v_{sb} \\ v_{sc} \end{bmatrix} = V_s \begin{bmatrix} \cos(\omega t + \phi) \\ \cos(\omega t + \phi - 2\pi/3) \\ \cos(\omega t + \phi + 2\pi/3) \end{bmatrix}
$$

(1)

The three instantaneous source voltages can be represented by the space vector $\vec{v}_s$:

$$
\vec{v}_s = 2(v_{sa} + v_{sb}e^{j2\pi/3} + v_{sc}e^{j4\pi/3})/3 = V_s e^{j(\omega t + \phi)}
$$

(2)

Similarly, the space vectors of input voltage, output voltage, input current, and output current are defined, respectively, as follows:

$$
\vec{v}_i = 2(v_{ia} + v_{ib}e^{j2\pi/3} + v_{ic}e^{j4\pi/3})/3 = V_i e^{j\omega t}
$$

(3)
\[ \bar{v}_v = 2(v_a + v_b e^{j2\pi/3} + v_c e^{j4\pi/3})/3 = V_v e^{j\alpha_v} \]  
(4)

\[ i_v = 2(i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3})/3 = I_v e^{j\beta_v} \]  
(5)

\[ i_v = 2(i_a + i_b e^{j2\pi/3} + i_c e^{j4\pi/3})/3 = I_v e^{j\beta_v} \]  
(6)

Basically, the switching states of 9 bidirectional switches must satisfy the two main rules to provide safe operation of the MC: i) no short circuit at the input side, and ii) no open circuit at the output side.

As a result of these constraints, the three-phase MC has 27 possible switching states categorized into three groups as shown in Table 1:

1) Group I includes 18 active-vector states,
2) Group II includes three zero-vector states,
3) Group III includes six rotating-vector states.

<table>
<thead>
<tr>
<th>Switching Configuration</th>
<th>Output Voltage</th>
<th>Input Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>+1</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>-1</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>+2</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>-2</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>+3</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>-3</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>+4</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>-4</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>+5</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>-5</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>+6</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>-6</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>+7</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>-7</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>+8</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>-8</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>+9</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>-9</td>
<td>c</td>
<td>c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group II</th>
<th>No</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>V_v</th>
<th>(\alpha_v)</th>
<th>I_v</th>
<th>(\beta_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 a a a a</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 b b b b</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 c c c c</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Group III</th>
<th>No</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>V_v</th>
<th>(\alpha_v)</th>
<th>I_v</th>
<th>(\beta_v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>r 1 a b c</td>
<td>V_v</td>
<td>(\alpha_v)</td>
<td>I_v</td>
<td>(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 2 a c b</td>
<td>V_v</td>
<td>-(\alpha_v)</td>
<td>I_v</td>
<td>-(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 3 c a b</td>
<td>V_v</td>
<td>2\pi/3+(\alpha_v)</td>
<td>I_v</td>
<td>-2\pi/3+(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 4 b a c</td>
<td>V_v</td>
<td>2\pi/3-(\alpha_v)</td>
<td>I_v</td>
<td>2\pi/3-(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 5 b c a</td>
<td>V_v</td>
<td>-2\pi/3+(\alpha_v)</td>
<td>I_v</td>
<td>2\pi/3+(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>r 6 c a b</td>
<td>V_v</td>
<td>-2\pi/3-(\alpha_v)</td>
<td>I_v</td>
<td>-2\pi/3-(\beta_v)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conventionally, the rotating-vector states in Group III are not used to drive the MC, since their angular positions always change together with the input voltage making it difficult to create a repetitive pattern [6]. Therefore, in the traditional SVM method, active-vector states in Group I are selected to generate the desired output voltage vector [18], then zero-vector states are used to fulfil one sampling period.

For instance, when \(\bar{v}_v\) and \(\bar{i}_v\) are both in sector 1 as shown in Figure 2a and b, active-vector states among \(\pm 1, \pm 2, \pm 3, \text{ and } \pm 7, \pm 8, \pm 9\) are selected to generate the reference output voltage vector \(\bar{v}_v\). Meanwhile, from Figure 2b, to generate the input current vector \(\bar{i}_v\) in sector 1, active-vector states should be among \(\pm 1, \pm 4, \pm 7 \text{ and } \pm 3, \pm 6, \pm 9\). Common states are chosen to generate both \(\bar{v}_v\) and \(\bar{i}_v\) in sector 1. Moreover, the states with higher magnitudes should be considered to
maximize the capacity of the VTR. As can be seen from Figure 3, when the input current vector \( \vec{i} \) is in sector 1, two higher input line-to-line voltages are \( v_{ab} \) and \( v_{ac} \). By looking up Table 1, it is, therefore, possible to conclude that four active-vector states (+1, −3, −7, and +9) are selected to generate \( v_o \) and \( \vec{i} \) in this case.

**Figure 2.** The location of (a) the output voltage vector and (b) the input current vector.

**Figure 3.** Input phase and line-to-line voltages at main power supply.

Using the same procedure, the selected active-vector states for any combinations of output voltage and input current sectors are summarized in Table 2.

**Table 2.** Selected active-vector states for each combination of output voltage and input current sectors in the traditional space vector modulation method.

<table>
<thead>
<tr>
<th>Input Current Vector Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Voltage Vector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1   2   3   4   5   6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duty cycles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| \( d_1 \) \( d_2 \) \( d_3 \) \( d_4 \) \( d_1 \) \( d_2 \) \( d_3 \) \( d_4 \) \( d_1 \) \( d_2 \) \( d_3 \) \( d_4 \) \( d_1 \) \( d_2 \) \( d_3 \) \( d_4 \)
The duty cycles in Table 2 can be calculated by the following equations:

\[ d_i = \frac{2q \sin \tilde{\alpha}_o \sin \left( \pi / 6 - \tilde{\beta} \right)}{\cos \delta} \]  
\[ d_2 = \frac{2q \sin \tilde{\alpha}_o \sin \left( \pi / 6 + \tilde{\beta} \right)}{\cos \delta} \]  
\[ d_3 = \frac{2q \sin \left( \pi / 3 - \tilde{\alpha}_o \right) \sin \left( \pi / 6 - \tilde{\beta} \right)}{\cos \delta} \]  
\[ d_4 = \frac{2q \sin \left( \pi / 3 - \tilde{\alpha}_o \right) \sin \left( \pi / 6 + \tilde{\beta} \right)}{\cos \delta} \]  

where \( \tilde{\alpha}_o \) and \( \tilde{\beta}_i \) are defined as

\[ \tilde{\alpha}_o = \alpha_o - (k_e - 1) \times \pi / 3 \]  
\[ \tilde{\beta}_i = \beta_i - (k_e - 1) \times \pi / 3 \]  

Finally, zero-vector states are applied to fulfil the sampling period with the duty cycle:

\[ d_6 = 1 - \frac{2q \cos(\tilde{\alpha}_o - \pi / 6) \cos \tilde{\beta}_i}{\cos \delta} \]  

All duty cycles must be non-negative and lower than or equal to unity:

\[ 0 \leq d_i \leq 1; \quad n = 0, \ldots, 4 \]  

Expressions (14) lead to the limit of the VTR in an MC:

\[ q \leq \frac{\sqrt{3}}{2} \cos \delta \]  

From (15), the compensated angle is limited as follows:

\[ \cos \delta \geq \frac{2q}{\sqrt{3}}, \quad \text{or} \quad \delta \leq \cos^{-1} \left( \frac{2q}{\sqrt{3}} \right) \]  

3. Input Filter Analysis

In practice, the input filter is an essential component to smooth the input currents and to satisfy the EMI requirements at the MC’s input side [26]. Due to the input filter, the source current leads the source voltage at a high displacement angle, especially at light-load conditions, resulting in poor IPF. Therefore, to utilize maximum IPF at the main power supply, we need to investigate the displacement angle caused by the input filter.

Figure 1 shows a typical input filter configuration for MCs, namely, a second-order LC filter with a damping resistor and an inductor in parallel [16]. Figure 4 shows a simplified model of this input filter, from which the relationship between \( \bar{v}_i, \bar{i}_i \) and \( \bar{v}_i, \bar{i}_i \) is shown as follows:

\[ \bar{i}_i = \frac{j \omega L_i + R_i - \omega^2 R_d L_i C_i}{j \omega L_i + R_d} \bar{i}_i - j \omega C_i \bar{v}_i \]  
\[ \bar{v}_i = \bar{v}_i - \frac{j \omega L_i R_d}{j \omega L_i + R_d} \bar{i}_i \]
To ensure sufficient damping of the input current and voltage oscillations, \( R_f \gg \omega L_f \), so Equations (17) and (18) become (19) and (20), respectively, as follows:

\[
\begin{align*}
\tilde{i}_s &= \tilde{i}_i - j \omega C_f \tilde{v}_s \\
\tilde{v}_s &= \tilde{v}_i - j \omega L_f \tilde{i}_i
\end{align*}
\]

(19)  
(20)

Because the voltage drop across the input filter is very small compared with the source voltage, the input voltage of the MC and the source voltage are the same:

\[
\tilde{v}_i = \tilde{v}_s
\]

(21)

From Equations (19) and (21), the vector diagram of input voltages and currents is illustrated in Figure 5.

![Vector diagram of input voltages and currents.](image)

Figure 5. Vector diagram of input voltages and currents.

To accomplish maximum achievable IPF at the main power source, we need to investigate the displacement angle between the MC input-current vector \( \tilde{i}_i \) and the input-voltage vector \( \tilde{v}_i \), namely, \( \delta_i \).

4. Study on IPF Compensation

The conservation of energy law for MCs is expressed as follows:

\[
V_s I_s \cos \delta = V_i I_o \cos \delta_o
\]

(22)

where \( V_o = qV_i = qV_s \); \( I_o = \frac{V_o}{Z} = \frac{qV_s}{Z} \); \( \cos \delta_o = \frac{R}{Z} \).

Therefore, from Equation (22), the source current amplitude is obtained as follows:

\[
I_s = \frac{qV_s R}{Z \cos \delta_s}
\]

(23)

It should be noted that Equation (23) is true for any modulation case.
4.1. No Compensation

Figure 6 shows the vector diagram of input voltages and currents with no compensated angle, namely, $\delta_i = 0$. From Figure 6, it is possible to obtain the following relationship:

$$\sin \delta_s = \frac{\omega C_f V_s}{I_s}$$  \hspace{1cm} (24)

![Figure 6. Vector diagram of input voltages and currents with no compensated angle ($\delta_i = 0$).](image)

From Equations (23) and (24), we can obtain the expression of displacement angle at the main power supply under the non-compensation condition as follows:

$$\tan \delta_{i,\text{non-com}} = \frac{\omega C_f Z^2}{q^2 R}$$  \hspace{1cm} (25)

For each fixed value of $VTR_q$, $\frac{\omega C_f Z^2}{q^2 R}$ is a constant value.

4.2. General Compensation

Figure 7 shows the vector diagram of input voltages and currents with the compensated angle $\delta_i = 0$. We need to determine the displacement angle at the main power supply $\delta_i$ after applying a compensated angle $\delta_i$ between the MC input-current vector $i_i$ and input-voltage vector $v_i$.

![Figure 7. Vector diagram of input voltages and currents with compensated angle $\delta_i$.](image)

From Equation (23), the power factor at the main power supply is always as follows:

$$\cos \delta_i = \frac{q^2 V R}{Z I_s}$$  \hspace{1cm} (26)

Therefore, from Figure 7, the value of $OH$ is:

$$OH = OA \cos \delta_i = I_s \cos \delta_i = \frac{q^2 V R}{Z^2}$$  \hspace{1cm} (27)
From Figure 7, we can determine the relationship between $\delta_s$ and $\delta_i$ as follows:

$$\tan \delta_s = \frac{AH}{OH} = \frac{AB - HB}{OH} = \frac{AB}{OH} - \tan \delta_i$$

$$= \frac{\omega C_i V_s}{q^2 V R / Z^2} - \tan \delta_i$$

$$\Rightarrow \tan \delta_s + \tan \delta_i = \frac{\omega C_i Z^2}{q^2 R}.$$  \hspace{1cm} (28)

When comparing Equations (25) and (28), the relationship between $\delta_s$ and $\delta_i$ becomes:

$$\tan \delta_s + \tan \delta_i = \tan \delta_{i, \text{non-com}}.$$  \hspace{1cm} (29)

4.3. Unity IPF

From Equation (28), to achieve unity IPF at the main power source, namely, $\delta_s = 0$, the compensated angle $\delta_i$ must be as follows:

$$\tan \delta_i = \frac{\omega C_i Z^2}{q^2 R}.$$  \hspace{1cm} (30)

Let us define the quality factor for the IPF as follows:

$$Q = \frac{\omega C_i Z^2}{R}.$$  \hspace{1cm} (31)

From Equations (16), (30), and (31), the condition for achieving unity IPF is expressed as follows:

$$\begin{cases} 
\tan \delta_i = \frac{Q}{q^2} \\
\cos \delta_i \geq 2q \sqrt{3} 
\end{cases}.$$  \hspace{1cm} (32)

where $Q = \frac{\omega C_i Z^2}{R}$.

Thus, from Equation (32), we can derive the following condition:

$$\tan^2 \delta_i + 1 = \frac{1}{\cos^2 \delta_i}$$

$$\Rightarrow \left( \frac{Q}{q^2} \right)^2 + 1 = \frac{1}{\cos^2 \delta_i} \leq \frac{3}{4q^2}$$

$$\Rightarrow q^4 - \frac{3}{4} q^2 + Q^2 \leq 0.$$  \hspace{1cm} (33)

From Equation (33), it is possible to determine the VTR range to achieve unity IPF as follows:

$$\sqrt{\frac{3}{8}} - \sqrt{\left( \frac{3}{8} \right)^2 - Q^2} \leq q \leq \sqrt{\frac{3}{8} + \left( \frac{3}{8} \right)^2 - Q^2}.$$  \hspace{1cm} (34)

under the following condition:

$$Q = \frac{\omega C_i Z^2}{R} \leq \frac{3}{8} = 0.375.$$  \hspace{1cm} (35)

Figures 8, 9, and 10 present illustrations of the IPF at the main power supply when $Q < 0.375$, $Q = 0.375$, and $Q > 0.375$, respectively. In Figure 8, when the quality factor $Q$ is set to 0.1, the IPF
at the main power supply can achieve unity in the range of $0.116 \leq q \leq 0.858$. This result is in good agreement with the theoretical result in Equation (34). In Figure 9, when the quality factor $Q$ is set to 0.375, the IPF at the main power supply can only achieve unity at $Q = \sqrt{3}/8 = 0.612$. In Figure 10, when the quality factor $Q$ is set to 0.54, the IPF at the main power supply cannot achieve unity within the entire range of VTR.

![Figure 8](image1.png)

**Figure 8.** Maximum allowable compensated angle, required compensation angle, displacement angle, and input power factor at the main power supply at $Q = 0.1$.

![Figure 9](image2.png)

**Figure 9.** Maximum allowable compensated angle, required compensation angle, displacement angle, and IPF at the main power supply at $Q = 0.375$. 
5. Experimental Results

To verify the effectiveness of the study, a prototype three-phase MC was used to supply a three-phase symmetrical passive $RL$ load. The parameters used in the experiments are shown in Table 3. The MC was realized with 18 discrete IGBTs (IRG4PF50WD). The digital control system was implemented with fixed-point digital signal processors (TMS320F2812). A complex programmable logic device (EPM7128SLC84-15) was implemented for a four-step commutation. The input line-to-line voltages were sampled by voltage sensors (AD202) and a 12-bit A/D converter (AD7864).

As can be seen from Equation (35), the quality factor $Q$ is a function of the output angular frequency $\omega_o$, thus one easy way to change $Q$ is to adjust $\omega_o$. To verify the study, the quality factor $Q$ is investigated at three values, i.e., lower than 0.375, equal to 0.375, and greater than 0.375. With reference to the values of the system parameters shown in Table 3, the MC is performed at $f_o = 50$ Hz, 200 Hz, and 250 Hz, corresponding to the quality factor $Q$ of 0.1, 0.375, and 0.54, respectively.

Table 3. System parameters.

<table>
<thead>
<tr>
<th>Power supply</th>
<th>Input filter</th>
<th>Output load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_o = 100V$</td>
<td>$L = 1.4 \text{ mH}$</td>
<td>$R = 10 \Omega$</td>
</tr>
<tr>
<td>$f_i = 60 \text{ Hz}$</td>
<td>$C = 22 \mu\text{F}$</td>
<td>$L = 15 \text{ mH}$</td>
</tr>
<tr>
<td>$R = 20 \Omega$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures 11, 12, and 13 show the input/output waveforms with no compensation at $Q = 0.1$, $Q = 0.375$, and $Q = 0.54$, respectively. As can be seen, with no compensation, the source current inherently leads the source voltage with a certain angle due to the input filter. Figures. 14, 15, and 16 show the input/output waveforms with maximum achievable IPF at $Q = 0.1$, $Q = 0.375$, and $Q = 0.54$, respectively.
Figure 11. Source phase voltage/source current, output line-to-neutral voltage, and output line current at $f_c = 50\ \text{Hz}$ ($Q = 0.1 < 0.375$), $q = 0.5$ with no compensation.

Figure 12. Source phase voltage/source current, output line-to-neutral voltage, and output line current at $f_c = 200\ \text{Hz}$ ($Q = 0.375$), $q = 0.6$ with no compensation.
Figure 13. Source phase voltage/source current, output line-to-neutral voltage, and output line current at $f_c = 250$ Hz ($Q = 0.54 > 0.375$), $q = 0.7$ with no compensation.

Figure 14. Source phase voltage/source current, output line-to-neutral voltage, and output line current at $f_c = 50$ Hz ($Q = 0.1 < 0.375$), $q = 0.5$ with unity IPF compensation.
At $Q = 0.1 < 0.375$, from Equations (25) and (31), we can see that the displacement angle at low value of $Q$ is also small and easy to be compensated, therefore, the IPF at the main power supply can be easily achieved unity. Figure 17 shows a comparison between IPF under no compensation and maximum compensated angle conditions at $Q = 0.1$. With maximum compensated angle condition, the MC can achieve unity IPF for a wide range of the VTR. This result is in good agreement with the theoretical study shown in Figure 8.
Figure 17. Comparison between IPF under no compensation and maximum compensated angle conditions at $Q = 0.1$.

At $Q = 0.375$, the IPF at the main power supply can be achieved unity only at $q = 0.6$ as shown in Figure 18. At $Q = 0.54 > 0.375$, from Equations (25) and (31), the displacement angle at high value of $Q$ is very large and difficult to be compensated, therefore, the IPF at the main power supply can never be achieved unity for entire range of the VTR as shown in Figure 19. These results are in good agreement with the theoretical results shown in Figures 9 and 10.

Figure 18. Comparison between IPF under no compensation and maximum compensated angle conditions at $Q = 0.375$. 
Figure 19. Comparison between IPF under no compensation and maximum compensated angle conditions at $Q = 0.54$.

The study shows that it difficult to achieve the unity IPF for MCs at high values of $Q$. From Equation (35), we can see that the value of $Q$ is dependent on the input frequency, the input filter parameters, the output load parameters, and the output frequency. In practice, the input frequency and the input filter parameters are usually fixed, therefore, the value of $Q$ is greatly dependent on the output load parameters and the output frequency. From this point of view, it is difficult to achieve the unity IPF when the MC operates at a high output frequency.

6. Conclusions

This paper has presented a study on the IPF compensation capacity of MCs with a traditional LC input filter. The study was defined by space-vector theory and the conservation of energy law. The study shows that the MC cannot always achieve unity IPF within the entire range of VTR, regardless of system parameters. The range of VTR for achieving unity IPF depends on the quality factor. The MC can achieve unity IPF for a wide range of VTR at a low-quality factor. The greater the quality factor, the narrower the range of VTR within which to achieve unity IPF. If the quality factor is greater than 0.375, the MC can never achieve unity IPF within the entire range of VTR. Experimental results verified that IPF behavior is in good agreement with the theoretical study.

Nomenclature

$v_s \quad$ Three-phase source voltage vector, $[v_{sa} \ v_{sb} \ v_{sc}]^T$
$v_{sa}, \ v_{sb}, \ v_{sc} \quad$ Instantaneous source phase voltages
$V_s \quad$ Source voltage amplitude
$\omega_s \quad$ Angular frequency of source voltage
$\phi \quad$ Initial phase angle of source voltage
$
\bar{v}_s \quad$ Space vector of three-phase source voltage
$
\bar{i}_s \quad$ Space vector of three-phase source current
$I_s \quad$ Amplitude of source current, $I_s = |\bar{i}_s|$
$v_{ia}, \ v_{ib}, \ v_{ic} \quad$ Instantaneous input phase voltages of MC
$v_{id}, \ v_{id}, \ v_{ic} \quad$ Instantaneous output phase voltages of MC
$V_i, V_o$  
Amplitudes of MC input and output voltages

$\alpha_i, \alpha_o$  
Phase angles of MC input and output voltages

$i_i, i_o$  
Space vectors of MC input and output voltages

$i_a, i_b, i_c$  
Instantaneous input line currents of MC

$I_i, I_o$  
Amplitudes of MC input and output currents

$\beta_i, \beta_o$  
Phase angles of MC input and output currents

$i_i, i_o$  
Space vectors of MC input and output currents

$T_s$  
Sampling period

$d_n(n=0,...,4)$  
Duty cycles of zero and active vectors

$q$  
Voltage transfer ratio of MC, $q = V_o / V_i$

$k_v$  
Output voltage sector

$k_i$  
Input current sector

$L_i, C_i$  
Inductance and capacitance of input filter

$R_d$  
Damping resistance of input filter

$\delta_f$  
Displacement angle due to input filter, $\delta_f = \angle i_i - \angle i_f$

$\delta_i$  
Compensated angle, $\delta_i = \angle i_i - \angle i_i$

$\delta_s$  
Displacement angle at main power supply, $\delta_s = \angle i_i - \angle i_i$

$\delta_{s,non-com}$  
Displacement angle at main power supply with no compensated angle

$\delta_o$  
Load displacement angle at output frequency, $\delta_o = \angle i_o - \angle i_o$

$R, L, Z$  
Load resistance, inductance and impedance

$f_o$  
Output frequency

$\omega_o$  
Output angular frequency

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**References**


