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Article

Modeling the Distribution of Solar Radiation on a Two-Axis Tracking Plane for Photovoltaic Conversion

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Abstract: The paper presents selected isotropic and anisotropic mathematical models to calculate the distribution of solar radiation on the photovoltaic module plane with any spatial orientation. A comparison of solar radiation models for Poland is based on measured data and received from the Institute of Meteorology and Water Management. Power density of solar radiation for different angular positions, especially for representative days of each month, was calculated. Based on the statistical analysis of the percentage root-mean-square error (RMSE%), mean-bias error (MBE%) and the Pearson correlation coefficient of an individual mathematical model, our own correction factor for diffuse radiation was proposed. A modified Liu-Jordan model was compared with six common mathematical models showing better agreement of measured and calculated values of solar radiation density. The presented analysis explains which mathematical model is the most suitable for central Poland (Poznań, 52°25’ N, 16°56’ E) and shows the validity of applying the modified model to improve the accuracy of determination of the radiation power density for a given elevation and azimuth angle using values for a horizontal plane.

Keywords: solar radiation; mathematical model; tracking system; elevation angle; photovoltaic module

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1. Introduction

Determination of irradiation values on the inclined plane relative to the ground, at different latitudes, requires mathematical models that use relationships between the global intensity of solar radiation on a horizontal plane and the inclined plane at an angle $\beta \neq 0$ [1–5].

Previous studies show many mathematical methods which try to build the relationship between solar radiation on an inclined surface and on the horizontal plane with different accuracies [6–8]. Harrison and Coombes [9] compared simulation results for angular positions of 30°, 60°, 90° for 10 variants of cloudiness degree and showed that Liu-Jordan isotropic model lead to an underestimation of radiation power density, while the anisotropic Klucher and Hay models cause overestimation by a small percentage. Kudish and Ianetz [10], using the same three mathematical models (Liu-Jordan, Klucher, Hay) for the geographical location 31°15’ N, 34°47’ E, demonstrated strong variability of tested models as a function of the season of the year, measurement time and location described primarily by latitude angle. In 2002 Bilbao, comparing the four mathematical models of Perez, Klucher, Temps and Coulson and Liu-Jordan [11], using data from five meteorological stations for the Spanish region of Castille, demonstrated the superiority of the Perez anisotropic model, which was then used to determine the annual insolation map for the Valladolid region. Bilbao showed the necessity to select the appropriate mathematical model every time for the new analyzed locations. Three years later, Kamali *et al.* [12] presented a comparison of the eight mathematical models of Liu-Jordan (1962), Hay (1979), Steven anf Unsworth (1980), Koronakis (1986), Skartveit and Olseth (1986), Reindl (1990), Tian (2001) and Badescu (2002) for the geographical location of Karai (Iran, 35°55’ N, 50°56’ E). It was found that the best fit for the plane directed to the south and west are the Reindl and Koronakis models, while the Steven and Unsworth model has the largest percentage RMSE%, for the south, west and east directions—32.01%, 26.43% and 29.29%, respectively. A comparative analysis carried out in 2007, on the basis of two 25-day periods, for six mathematical models, *i.e.*, Klucher, Hay and Davies, Reindl, Liu-Jordan, Muneer and Perez [6], showed that most of the models are characterized by an underestimation of insolation values for the afternoon hours with simultaneous overestimation for the remaining hours of the day, in particular for the months of March and April. A comprehensive analysis of twelve models, isotropic (Liu-Jordan, Koronakis, Tian, Badescu) and anisotropic (Perez (1990), Reindl, Perez (1986), Skartveit and Olseth, Steven and Unswarth, Hay, Temps and Coulson, Klucher) for latitude 35°55’ N (Iran) was conducted in 2007 [13] declaring mean square error RMSE% in the range of 30.71%–65.53% for Perez and T and C models for west exposure, and for south direction 10.16%–54.89% for Skartveit and Olseth and Temps and Coulson models. Large RMSE and MBE were performed for settings deviating from south direction. In 2011, Benkaciali and Gairaa, based on a comparative analysis of the Liu-Jordan and Brichambaut models [14] with the results of measurements for angles 32°, 60° and 90°, showed small errors for the Liu-Jordan and Brichambaut models: 1.55%, 1.56%, 3.92% and −0.43%, −0.6%, 0.76%, respectively, which was considered satisfactory for most energetic analysis. Jakhrani *et al.* [15], 2013, presented an analysis of Liu-Jordan, HDKR, Klucher and Perez for two sources of data computing (NASA, MMS), which showed that the Klucher model is characterized by the smallest value of the statistical mean error SME and seems to be the most suitable for the analyzed location—Kuching, Malaysia.

Włodarczyk and Nowak [16,17] found that for climatic conditions of southern Poland (Wrocław, Lower Silesia), the Perez anisotropic model is the only one which reaches the required level of accuracy compared with other analyzed models. Calculations based on data from the Wrocław University of Technology for the period 2002–2006 showed the advantage of anisotropic models as well as the isotropic Liu-Jordan over selected models (Skartveit and Olseth, Ma and Iqbal or Temps and Coulson). Models taken under consideration were: Liu-Jordan (isotropic), Jimenez and Castro and Koronakis (pseudoisotropic) and Bugler, Temps and Coulson, Klucher, Hay, Ma and Iqbal, Skartveit and Olseth, Reindl, Gueymard, Muneer, Perez (anisotropic).

2. Methods of Describing Solar Radiation

As source data for the analysis of solar radiation intensity on any inclined surface of a photovoltaic receiver, data provided by the Ministry of Infrastructure and Development was used. A typical meteorological year, generated on the basis of 30-year (1971–2000), one-hour or three-hour (eight times a day) measurement sequences made available by the Institute of Meteorology and Water Management, Poznań Hydrological and Meteorological Station [18]. A typical meteorological year is an hour set of 8760 lines, including solar power radiation for various spatial settings and different solar radiation components in the total radiation. Data for analysis also come from own measurements conducted between 2012 and 2014 at the Poznań University of Technology Renewable Energy Sources laboratory. Figure 2 presents the distribution of monthly total, direct and diffuse solar radiation on a horizontal plane for the typical meteorological year for the city of Poznań.

Inter-hourly values can be determined by spline interpolation of order 3, where large value intervals eliminate some measurement sequences [19,20]. TMY subsequent months are generated on the basis of a statistical comparison of a given month with the 30-year values.

When the calculations are performed for a full month, solar declination characteristic for the month is not calculated for the middle days of the month, but for a so-called recommended day. These are the days with average value of insolation for the analyzed month. In order to obtain the number of the recommended day it is necessary to calculate (on the basis of long-term measurements) daily and monthly insolation and average value for the analyzed month, which is compared with the daily value. A day with similar insolation was used as the recommended day of the month.

Using the typical meteorological year for the city of Poznań, the authors determined the days recommended in particular months of the year, ones that were subsequently employed in developing an optimization program to define a tilt angle of a PV module plane installed in a stationary unit. The system consists of a two-axis Sun tracking unit installed at a close distance to a fixed one on the roof of the Electrical Engineering Faculty of the Poznań University of Technology in order to ensure identical conditions of operation and to enable a comparative analysis [21].

In the analysis isotropic (Liu-Jordan, Badescu, Tian), pseudo-isotropic (Koronakis) and anisotropic (Hay, Steven and Unsworth) models are used to determine the radiation power density distribution on PV module plane inclined at 30°, 45°, 60° for south orientation. The way of describing the contribution of direct and diffuse radiation (isotropic, heliocentric, gleaming horizon) forced to use different mathematical models and correction factors for diffuse radiation. Modification of the Liu-Jordan model by the proposed R_d factor makes it possible to obtain low values of RMSE% and MBE%.

Hottel and Woertz described the first mathematical model of solar radiation for a receiver surface in any location, which assumed the isotropy of scattered radiation without taking into account inclined planes, seeing radiation as falling on a horizontal surface. Such a correction was allowed only for the direct component.

A mathematical model typically used in energy calculations is the Liu-Jordan model. The isotropic Liu-Jordan and the Hay anisotropic models are identical in respect of assessing direct and reflected solar radiation, but differ in the method of describing the diffuse component. Certain publications claim that both models are characterized by almost the same accuracy of assessing the average daily insolation on an inclined plane [22].

In the Liu-Jordan isotropic model, with correction coefficients for all components, the diffuse part (scattered and reflected radiation) is isotropic in character and dispersed from the whole sky hemisphere. For a photovoltaic receiver facing south, a total solar radiation on a plane tilted at the β angle is defined by relation [23]:

$$G_\beta = G_b \cdot \left(\frac{\cos(\varphi-\beta)\cdot\cos\delta(t)\cdot\cos\omega(t)+\sin(\varphi-\beta)\cdot\sin\delta(t)}{\sin\varphi\cdot\sin\delta(t)+\cos\varphi\cdot\cos\delta(t)\cdot\cos\omega(t)}\right) + G_d\cdot\left(\frac{1+\cos\beta}{2}\right)+\left(G_b+G_d\right)\cdot\rho_o\cdot\left(\frac{1-\cos\beta}{2}\right) \tag{1}$$

where: G_b, G_d– direct and diffuse solar radiation for a horizontal surface; G_β—total solar radiation for a plane tilted at β angle; φ—latitude angle; δ(*t*)—declination angle; ω(*t*)—hour angle; ρ_o—reflectance factor.

The Hay anisotropic model allows for a twofold nature of diffuse radiation, seeing it as heliocentric and isotropic radiation dispersed evenly from the remaining part of the horizon. Diffuse radiation on an inclined plane is defined by the relation [24]:

$$G_{d,\beta} = G_d \cdot \left\{\frac{G-G_d}{G_{atm}}\cdot R_b + \left[\frac{1+\cos\beta}{2}\right]\cdot\left[1-\frac{G-G_d}{G_{atm}}\right]\right\} \tag{2}$$

where: $G_{d,\beta}$—diffuse solar radiation for a plane tilted at β angle; G—total solar radiation for a horizontal plane; R_b—correction factor for direct radiation.

The solar radiation intensity on a plane parallel to the surface of the earth outside the atmosphere is in the form of [25]:

$$G_{atm}=1367\cdot\left[1+0.033\cdot\cos\left(\frac{360\cdot n}{365}\right)\right]\left[\sin(|\omega_{ws}|)\cdot\cos\varphi\cdot\cos\delta+|\omega_{ws}|\cdot\sin\varphi\cdot\sin\delta\cdot\frac{\pi}{180}\right] \tag{3}$$

where: n—day number in the year; ω_{ws}, ω_{zs}—hour angle for sunrise and sunset.

In the Badescu isotropic model, the density of solar radiation on an inclined plane is expressed by the relation [26]:

$$G_{d,\beta} = G_d \cdot \left[\frac{3+\cos(2\beta)}{4}\right] \tag{4}$$

The value of a solar radiation intensity of the diffuse component for an inclined surface according to the Tian model is defined by the relation [13]:

$$G_{d,\beta} = G_d \cdot [1 - \frac{\beta}{180}] \quad (5)$$

A diffuse radiation value on an inclined surface, based on the Koronakis mathematical model is expressed by the equation [13]:

$$G_{d,\beta} = G_d \cdot [\frac{(2 + \cos\beta)}{3}] \quad (6)$$

The Steven and Unsworth anisotropic model defines diffuse radiation on a plane tilted at the β angle, its source being the heliocentric radiation of the Sun's disk and the gleaming horizon [27,28]:

$$G_{d,\beta} = G_d \cdot [(0.51 \cdot R_b) + (\frac{1+\cos\beta}{2}) - \frac{1.74}{1.26\pi} \cdot \{\sin\beta - \frac{\beta \cdot \pi}{180} \cdot \cos\beta - \pi \cdot \sin^2 \frac{\beta}{2}\}] \quad (7)$$

The Klucher anisotropic model is based on the Temps and Coulson model and the Liu-Jordan model which allows for the TandC model to be characterized by great accuracy for the clear sky, and by overestimation of values for the remaining conditions, and by underestimation of values and prediction accuracy for the clear sky and the extent of cloudiness respectively (<300 W/m^2) in the case of the Liu-Jordan model. Total solar radiation value on an inclined plane based on the Klucher model is defined by the relation [29]:

$$\begin{aligned} G_\beta = {} & G_b \cdot (\frac{\cos(\varphi - \beta) \cdot \cos\delta(t) \cdot \cos\omega(t) + \sin(\varphi - \beta) \cdot \sin\delta(t)}{\sin\varphi \cdot \sin\delta(t) + \cos\varphi \cdot \cos\delta(t) \cdot \cos\omega(t)}) + \\ & + G_d \cdot (\frac{1+\cos\beta}{2}) \cdot [1 + F \cdot \sin^3 \frac{\beta}{2}][1 + F \cdot \cos^2\theta \cdot \sin^3\theta_z] + \\ & + (G_b + G_d) \cdot \rho_o \cdot (\frac{1-\cos\beta}{2}) \end{aligned} \quad (8)$$

where:

$$F = 1 - (\frac{G_d}{G})^2 \quad (9)$$

The F factor has a value close to zero for heavy sky cloudiness, with the model being reduced to an isotropic one. Thus, the model very accurately assesses solar radiation on any inclined plane in the case of heavy clouds, and generates errors for partial cloudiness.

The Sun zenith distance θ as a complement of the elevation angle may be determined using the relation [7]:

$$\begin{aligned} \cos\theta = {} & \sin\delta \cdot \sin\varphi \cdot \cos\beta - \sin\delta \cdot \cos\varphi \cdot \sin\beta \cdot \cos\gamma + \\ & + \cos\delta \cdot \cos\varphi \cdot \cos\beta \cdot \cos\omega + \cos\delta \cdot \sin\varphi \cdot \sin\beta \cdot \cos\gamma \cdot \cos\omega + \\ & + \cos\delta \cdot \sin\beta \cdot \sin\gamma \cdot \sin\omega \end{aligned} \quad (10)$$

where: γ—azimuth angle.

For considered spatial orientation (PV module is facing south), the relation (10) has the following form:

$$\cos\theta = \sin\delta \cdot \sin(\varphi - \beta) + \cos\delta \cdot \cos(\varphi - \beta) \cdot \cos\omega \tag{11}$$

The Reindl model is a mathematical model defining solar radiation, assuming the isotropic character of diffuse radiation, the influence of the gleaming horizon and the heliocentric radiation of the Sun's disk [6]:

$$\begin{aligned} G_\beta &= (G_b + G_d \cdot A) \cdot \left(\frac{\cos(\varphi-\beta)\cdot\cos\delta(t)\cdot\cos\omega(t) + \sin(\varphi-\beta)\cdot\sin\delta(t)}{\sin\varphi\cdot\sin\delta(t) + \cos\varphi\cdot\cos\delta(t)\cdot\cos\omega(t)}\right) + \\ &+ G_d \cdot (1-A) \cdot \left(\frac{1+\cos\beta}{2}\right) \cdot \left(1 + \sqrt{\frac{G_b}{G}} \cdot \sin^3\left(\frac{\beta}{2}\right)\right) + (G_b + G_d) \cdot \rho_o \cdot \left(\frac{1-\cos\beta}{2}\right) \end{aligned} \tag{12}$$

where: A—index of anisotropy.

An example of pseudo-isotropic mathematical model which is a modification of the Koronakis model, and which assumes a 20% share of diffuse radiation in the total radiation is the Jimenez and Castro model defined in the following form [16]:

$$\begin{aligned} G_\beta &= G_b \cdot \left(\frac{\cos(\varphi-\beta)\cdot\cos\delta(t)\cdot\cos\omega(t) + \sin(\varphi-\beta)\cdot\sin\delta(t)}{\sin\varphi\cdot\sin\delta(t) + \cos\varphi\cdot\cos\delta(t)\cdot\cos\omega(t)}\right) + \\ &+ 0.2 \cdot (G_b + G_d) \cdot \left(\frac{1+\cos\beta}{2}\right) + (G_b + G_d) \cdot \rho_o \cdot \left(\frac{1-\cos\beta}{2}\right) \end{aligned} \tag{13}$$

Diffuse component of solar radiation (scattered and reflected radiation) has isotropic character in the basic assumption (especially correct in the case of cloudiness). In other cases this may be a cause of the underestimation of radiation intensity reaching the receiver. The total solar radiation is therefore a sum of direct radiation, diffuse isotropic radiation and reflected radiation, the value of which depends on the base reflectivity coefficient.

Examples of base reflectivity coefficients for various types of bases are presented in Table 1.

Table 1. Reflectivity coefficients for various types of bases [30].

Base type	Coefficients ρ_0
Water surface	0.7–0.9
Soil not covered	0.2–0.5
Green vegetation	0.15–0.33
Fresh snow	0.87
Old snow	0.46
Dry asphalt	0.07
Water for Sun elevation $\alpha_s > 40$ °	0.05
Water for Sun elevation $\alpha_s < 40$ °	0.05–1.0
Soil after rain	0.16
Dry soil	0.32
Vegetation after rain	0.15
Dry vegetation	0.33
Dry concrete	0.35

Total solar radiation for different spatial orientation described by elevation and azimuth angle was also measured using prepared photovoltaic stand consisting of two-axis tracking system and fixed unit presented in Figure 1. Direct and diffuse components were obtained from Poznań meteorological station.

Figure 1. Two-axis tracking system with photovoltaic module, radiation power density meter and microinverter.

Respective coefficients for direct, diffuse, and reflected radiation are defined by relations [23]:

$$R_b(t) = \frac{\sin\delta(t)\cdot[\sin\varphi\cdot\cos\beta - \cos\varphi\cdot\sin\beta\cdot\cos\gamma]}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \frac{\cos\delta(t)\cdot[\cos\varphi\cdot\cos\beta\cdot\cos\omega(t) + \sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega(t) + \sin\beta\cdot\sin\gamma\cdot\sin\omega(t)]}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} \quad (14)$$

$$R_d = \frac{1+\cos\beta}{2} \quad (15)$$

$$R_o = \frac{1-\cos\beta}{2} \quad (16)$$

For analyzed case, assuming a photovoltaic receiver facing south, the $R_b(t)$ coefficient is [30,31]:

$$R_b(t) = \frac{\cos(\varphi-\beta)\cdot\cos\delta(t)\cdot\cos\omega(t) + \sin(\varphi-\beta)\cdot\sin\delta(t)}{\sin\varphi\cdot\sin\delta(t) + \cos\varphi\cdot\cos\delta(t)\cdot\cos\omega(t)} \quad (17)$$

In order to improve the prediction of the power density of solar radiation in Polish climatic conditions, considering share of the diffuse component in the total solar radiation, as shown in Figure 2, a modified correction coefficient is proposed by the authors for diffuse radiation, reducing the root—mean-square error (RMSE) and mean-bias error (MBE) in relation to the six analyzed mathematical models (Liu-Jordan, Hay, Badescu, Tian, Koronakis, Steven and Unsworth) in the form of:

$$R_d = 0.046p + \frac{2^p(p+q)(1+\cos\beta)+q}{2p+q+2} + 0.006q \quad (18)$$

Empirically determined correction coefficient modifies the general form of the Liu—Jordan model for Polish conditions. The coefficient is a function of the parameters "p" and "q" (value 0 or 1), depending on the period of the year. For the winter months (Polish climatic conditions) the Koronakis model is characterized by good accuracy and low RMSE%. The general form of the proposed R_d factor, using parameters "p" and "q", can be modified to $R_{d,K}$ (Koronakis diffuse radiation correction coefficient), which is increased by 10%–15% for the inclination of 30°–60° (on the basis of the own measurements). According to the analysis, it is noticed that Liu-Jordan model has good accuracy for the spring and summer months, when the contribution of diffuse radiation in total amount is lower. Proposed R_d correction factor is modified to the form similar to the Liu-Jordan correction coefficient.

The objective function that was taken into account in the analysis was a minimum value of the RMSE. Calculations were performed using Matlab software.

In the case of the months January and October-December, when the diffuse component of solar radiation in the typical meteorological year exceeds 75% (Figure 2), the weight parameters "p" and "q" have the values 0 and 1 respectively, and the proposed correction coefficient has the form of:

$$R_d = 0.67 + 0.33\cos\beta \tag{19}$$

In the remaining months of the year, when the direct radiation increases in the total radiation, the parameters "p" and "q" modify the R_d coefficient, having the values 1 and 0, to the following form:

$$R_d = 0.546 + 0.5\cos\beta \tag{20}$$

With the use of the modified Liu-Jordan method, solar radiation power density on a plane of any inclination, based on the correction coefficient proposed by the author, can be expressed by the following relation:

$$\begin{gathered} G_\beta = G_b \cdot \frac{\sin\delta(t)\cdot[\sin\varphi\cdot\cos\beta - \cos\varphi\cdot\sin\beta\cdot\cos\gamma]}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ \frac{\cos\delta(t)\cdot[\cos\varphi\cdot\cos\beta\cdot\cos\omega(t) + \sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega(t) + \sin\beta\cdot\sin\gamma\cdot\sin\omega(t)}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ + G_d\cdot[0.046\cdot p + \frac{2^p\cdot(p+q)\cdot(1+\cos\beta)+q}{2\cdot p+q+2} + 0.006\cdot q] + (G_b + G_d)\cdot\rho_o\cdot(\frac{1-\cos\beta}{2}) \end{gathered} \tag{21}$$

For the period of February-September the relation has the following form:

$$\begin{gathered} G_\beta = G_b \cdot \frac{\sin\delta(t)\cdot[\sin\varphi\cdot\cos\beta - \cos\varphi\cdot\sin\beta\cdot\cos\gamma]}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ \frac{\cos\delta(t)\cdot[\cos\varphi\cdot\cos\beta\cdot\cos\omega(t) + \sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega(t) + \sin\beta\cdot\sin\gamma\cdot\sin\omega(t)}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ + G_d\cdot[0.546 + 0.5\cos\beta] + (G_b + G_d)\cdot\rho_o\cdot(\frac{1-\cos\beta}{2}) \end{gathered} \tag{22}$$

For the months January and October-December the relation is as follows:

$$\begin{gathered} G_\beta = G_b \cdot \frac{\sin\delta(t)\cdot[\sin\varphi\cdot\cos\beta - \cos\varphi\cdot\sin\beta\cdot\cos\gamma]}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ \frac{\cos\delta(t)\cdot[\cos\varphi\cdot\cos\beta\cdot\cos\omega(t) + \sin\varphi\cdot\sin\beta\cdot\cos\gamma\cdot\cos\omega(t) + \sin\beta\cdot\sin\gamma\cdot\sin\omega(t)}{\sin\delta(t)\cdot\sin\varphi + \cos\delta(t)\cdot\cos\varphi\cdot\cos\omega(t)} + \\ + G_d\cdot[0.67 + 0.33\cos\beta] + (G_b + G_d)\cdot\rho_o\cdot(\frac{1-\cos\beta}{2}) \end{gathered} \tag{23}$$

A statistical analysis based on the root-mean-square error (*RMSE*), mean-bias error (*MBE*) and Pearson correlation coefficient was made, according to relations [32]:

$$RMSE = \sqrt{\frac{\sum (C_i - M_i)^2}{n}} \tag{24}$$

$$\%RMSE = \frac{RMSE}{\bar{M}} 100\% \tag{25}$$

$$MBE = \frac{\sum (C_i - M_i)}{n} \tag{26}$$

$$\%MBE = \frac{MBE}{\bar{M}} 100\% \tag{27}$$

where M—average measured value of radiation power density on an inclined plane.

The correlation coefficient of the measured values M_i and values determined by modeling C_i may be defined by the relation [7]:

$$k = \frac{\sum_{i=1}^{N} (C_i - \bar{C})(M_i - \bar{M})}{\sqrt{[\sum_{i=1}^{N} (C_i - \bar{C})^2][\sum_{i=1}^{N} (M_i - \bar{M})^2]}} \tag{28}$$

Normalized within the range [–1,1], the Pearson correlation coefficient defines the power and direction of the relationship between a measured value and one determined on the basis of the analyzed mathematical models. The sign of the coefficient defines the character of the stochastic dependence, which can be positive or negative, and the module determines the correlation degree (strong linear dependence or lack of dependence).

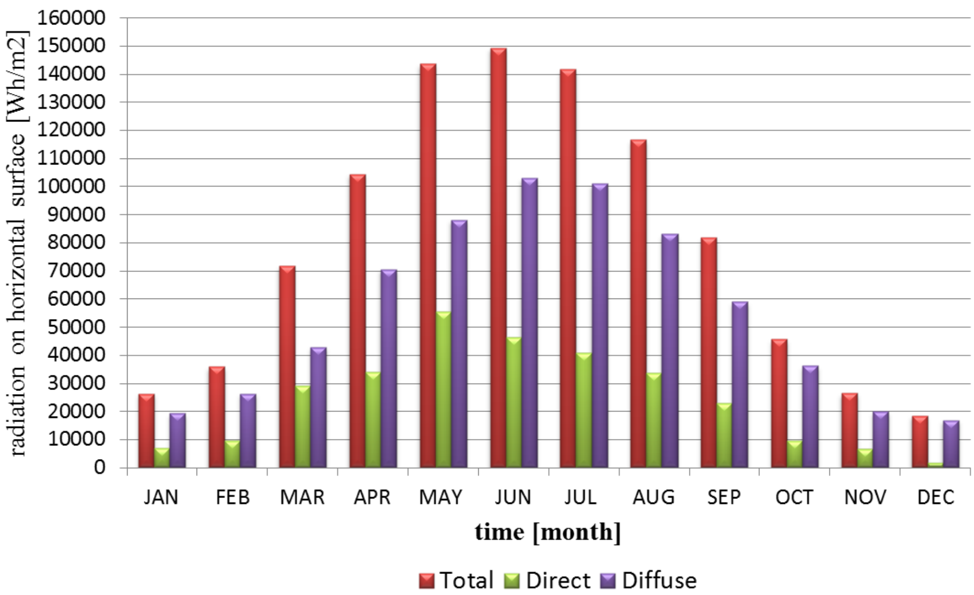

Figure 2. Distribution of solar radiation during the year including total, direct and diffuse components for the city of Poznan, based on [18].

3. Results and Discussion

The global component on tilted and horizontal surfaces was obtained from the Polish Institute of Meteorology and Water Management and from own measurements using a constructed 2-axis Sun tracking system and microprocessor radiation power density sensors. The ground reflectance factor was set to be 0.5.

For tested mathematical models of solar radiation (Liu-Jordan, Badescu, Hay, Tian, Koronakis, Steven and Unsworth) the best results for Polish climatic conditions, considering real mean square error, were achieved by the Hay and Koronakis models. For the analyzed angular settings (30°, 45°, 60°) the largest RMSE% between measured and calculated values were obtained using the Tian and Badescu models. For the analyzed location it is always the underestimation. Increase of elevation angle for constant azimuth causes higher values of RMSE% changing from 6.1% to 13.91% for the Hay model during May, where the total amount of contribution of diffuse radiation reaches the highest value. The Hay anisotropic model is characterized by a comparable level of accuracy with the Liu-Jordan model (Figure 3). It is typical for cloudy days, when the Hay model is reduced to isotropic form (diffuse radiation is considered as isotropic). An over—and underestimation is observed during the year. The MBE% in the case of underestimation exceeds 10% and 1.7% for the overestimation. With regard to the RMSE%, maximum value reaches 29%.

The most common Liu-Jordan isotropic model is characterized by the highest RMSE% during summer months where the anisotropy index reaches higher values. The lowest RMSE% was obtained for February and March, reaching 2.4%. In many cases it is preferred to use anisotropic models for summer months (July, August, September) where circumsolar radiation and radiation coming from brightening of the horizon are significant.

The highest RMSE and MBE, in many cases exceeding 40%, were obtained using the Steven and Unsworth model which is considered inappropriate for Polish latitude. RMSE% and MBE% for elevation angles 30°, 45°, 60° were respectively equal to 44%, 50.49%, 63.2% and 30.79%, 38.60%, 47.88%.

Using data from the Institute of Meteorology and Water Management for conditions prevailing in Poland the lowest mean bias error was achieved by the Koronakis model. For the analyzed elevation angles and time period, except June and September where MBE% equals respectively (1.18%, 1.29%, 0.73%), (0.47%, 0.46%, 0.34%), this model leads to underestimation of radiation power density. The MBE% in the case of underestimation and overestimation generally does not exceed respectively 4% and 1.3% (Figures 4, 6, 8). The RMSE% that shows how accurately global solar radiation can be divided into direct and diffuse components is in the range of up to 12.5%.

Taking into consideration the low complexity of the Liu-Jordan and Koronakis models and sufficient accuracy, mixed models for different months of the year are preferred. Using a formula describing total solar radiation on an inclined surface one diffuse radiation R_b factor was proposed considering parameters of each model.

The modified Liu-Jordan model is characterized by low statistical errors and high coefficients of correlation between measured and theoretical data. The lowest value of RMSE% reaches 0.79% in October for low elevation angles, while for the Liu-Jordan, Hay, Koronakis, Badescu and Tian models it is 4.34%, 2.77%, 1.25%, 12.96%, 19.15%, respectively.

According to Włodarczyk and Nowak [16,17], for 14 models of solar radiation, the best results were obtained by the Perez, Koronakis and isotropic Liu-Jordan models. These models are characterized by low RMSE%, MBE% and a Pearson correlation coefficient exceeding 0.9. For the inclination of 35° low MBE% equal to −1.05% and −2.76% was reached by the Koronakis and Liu-Jordan models. Low MBE% value (−2.67%) was also scored by the Hay anisotropic model. Higher inclination angle (β = 50°) increases MBE% for the Koronakis, Liu-Jordan and Hay models to −2.53%, −5.01% and 3.14%. These are the models with the lowest MBE%.

Results of calculations presented in [16] are similar to the values obtained by the authors, where for the inclination angle of 30° the lowest average MBE% is reached by the Koronakis (−0.64%), Hay (−0.95%) and Liu-Jordan models (−1.96%). For higher inclination of 45° and 60°, MBE% increases to respectively −0.89%, −3.25%, −4.25% and −1.43%, −5.18%, −7.05%.

For west-and east-facing surfaces relatively high RMSE% and MBE% values were found which means that the photovoltaic surface receives less direct radiation than the south-oriented surface. It was calculated that the most accurate analyzed models like Liu-Jordan, Hay, Koronakis were characterized by yearly RMSE% respectively 49.59%, 41.05%, 42.11%. For the geographical localization of Poland it is preferred to set PV modules in the south direction, therefore east and west variants were abandoned.

Figures 3–8 show the differences in RMSE% and MBE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 30°, 45°, 60° located in Poznań, Poland. Table S1 shows the forecast error results of solar radiation power density for the horizontal plane and for the following angular settings: β = 30°, 45°, 60°.

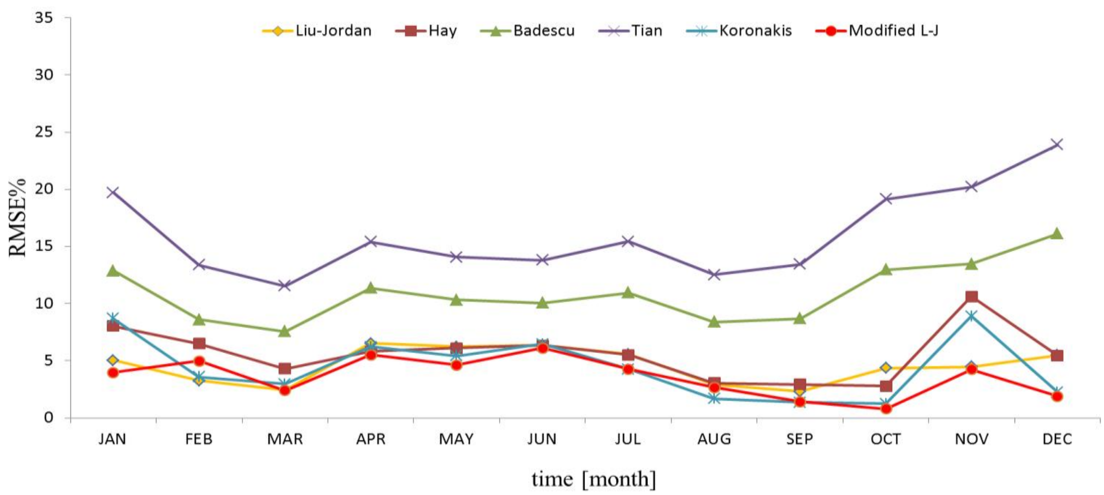

Figure 3. The differences in RMSE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 30° located in Poznań, Poland.

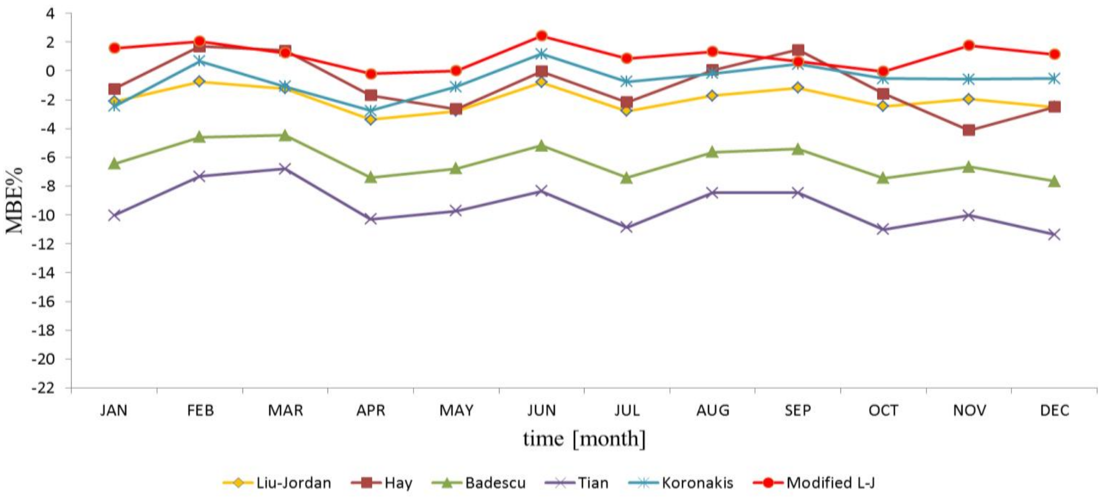

Figure 4. The differences in MBE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 30° located in Poznań, Poland.

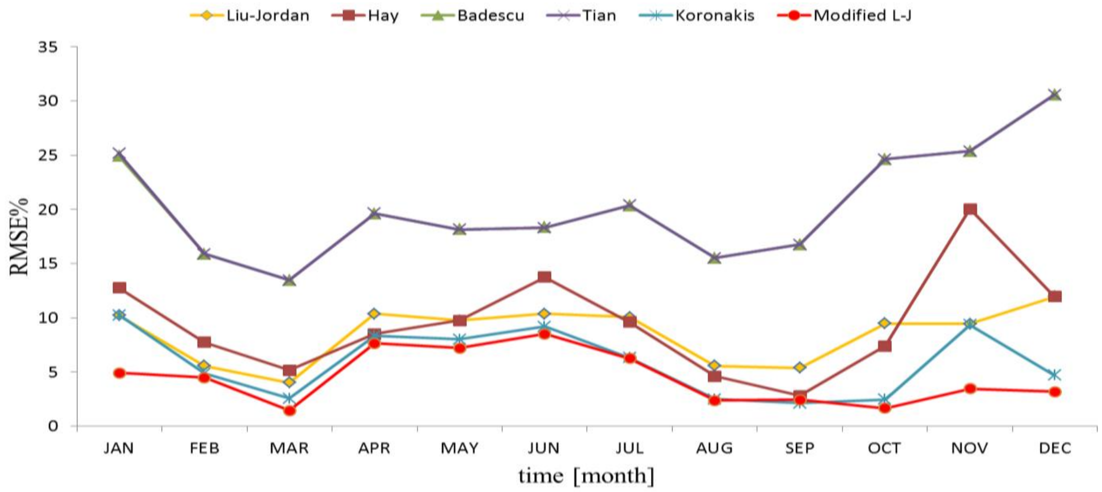

Figure 5. The differences in RMSE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 45° located in Poznań, Poland.

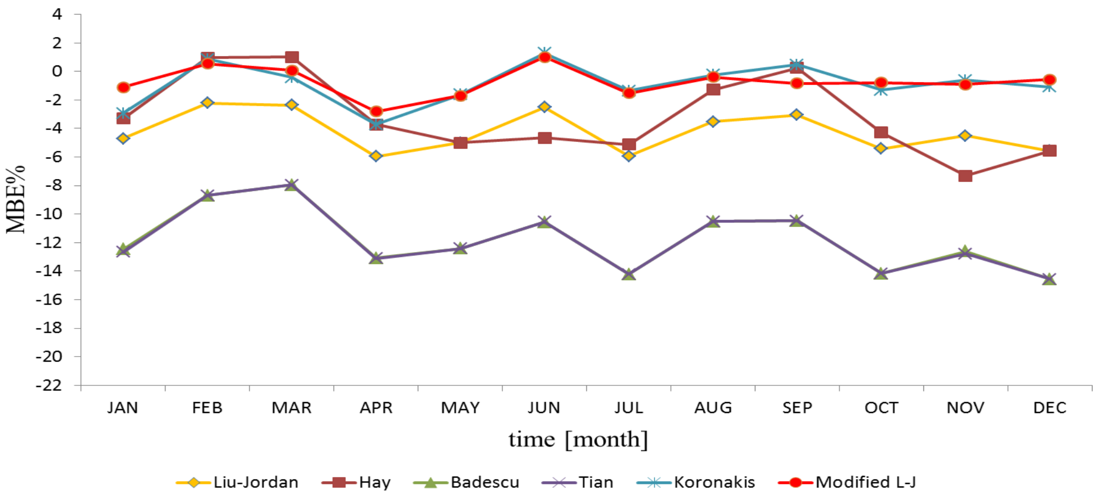

Figure 6. The differences in MBE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 45° located in Poznań, Poland.

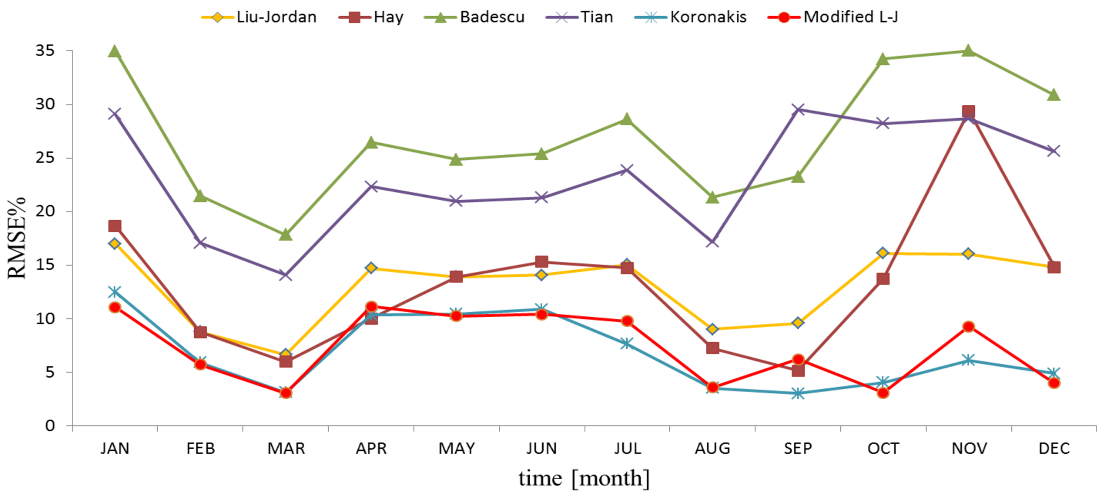

Figure 7. The differences in RMSE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 60° located in Poznań, Poland.

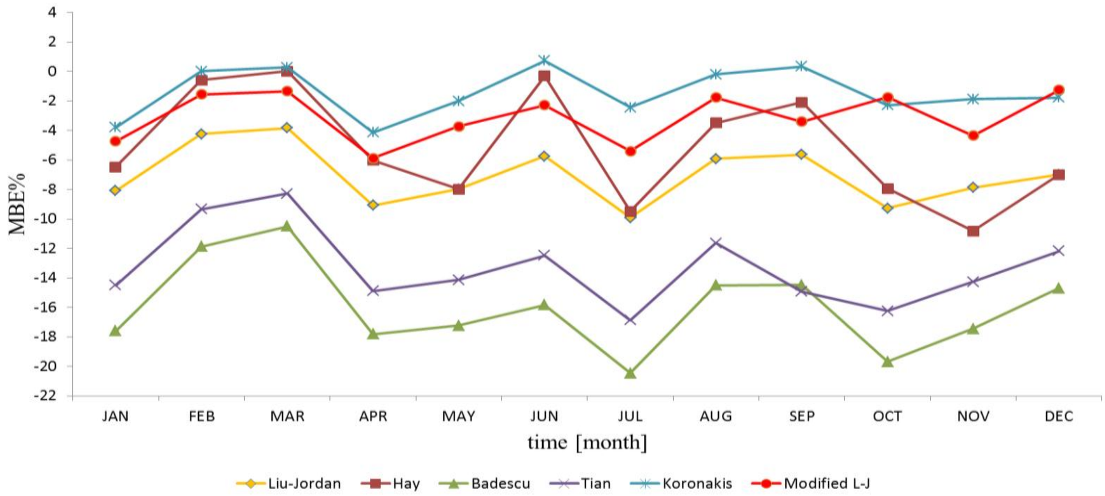

Figure 8. The differences in MBE% of incident solar radiation of multiple models for a plane facing south at an inclination angle of 60° located in Poznań, Poland.

Recommended days for the analyzed months for the city of Poznan are shown in Table 2. Calculated values of hourly insolation for representative days for six month period are presented in Table S2.

Table 2. Recommended days for each month of the year for the city of Poznan (own calculations).

Recommended days					
Month	**Day of the month**	**Day of the year**	**Month**	**Day of the month**	**Day of the year**
January	7	7	**July**	9	190
February	10	41	**August**	29	241
March	16	75	**September**	22	265
April	7	97	**October**	12	285
May	8	128	**November**	15	319
June	9	160	**December**	24	358

4. Conclusions

The analysis was conducted to investigate the effect of choosing the proper mathematical model of solar radiation on the accuracy of determination of radiation power density on any spatially oriented photovoltaic plane for Polish climatic conditions. The following conclusions can be drawn from this study:

- The highest differences between analyzed models relate to surfaces with high inclination angle to the ground. For south orientation and low inclination angle ($\beta = 30°$) the RMSE% for the Hay, Koronakis and Liu-Jordan models does not exceed 10% during the year. For spring and summer months, the models are characterized by similar RMSE% values approximately equal to 6%.
- The Liu-Jordan isotropic model, which shows good accuracy in determining solar radiation on the inclined surface for the city of Poznań, is characterized by higher RMSE% for the summer months, which is associated with a higher index of anisotropy and the influence of heliocentric and the gleaming horizon radiation. The Liu-Jordan model reaches a lower RMSE% during winter months. An underestimation of solar radiation was confirmed in Figures 4, 6, 8, where MBE% reaches negative values in the analyzed period of time. It is important in the case of designing photovoltaic installations.
- For Polish latitude the anisotropic model makes it possible to obtain higher insolation during the year compared to isotropic models. An overestimation of solar radiation increases with inclination angle. A higher impact of positioning of the PV module plane on obtained solar radiation is observed. When the sky is cloudless, the anisotropic component in the Hay model has large values, whereas in the case of heavy clouds this model is reduced to isotropic form. This means a comparable level of accuracy between the Liu–Jordan isotropic model and the Hay anisotropic one under the conditions analyzed, where the total amount of the contribution of diffuse radiation reaches even 75% during winter months.
- The modification of the Liu-Jordan model with proposed R_d diffuse correction coefficient takes into account the different levels of direct and diffuse radiation during the year. The presented correction factor takes two forms depending on the total amount of the contribution of diffuse radiation reducing annual average MBE% for inclination angles 30°, 45°, 60° to 1.1%, −0.6%, −3.1% from −1.97%, −5.58%, −7.05% for the well-known Liu-Jordan model. As a result monthly RMSE% and MBE% were reduced, for example for selected settings of 30°–45° to <0.79%–8.5%> and <−2.8%–2.4%>, respectively.
- The analysis of selected types of isotropic, pseudoisotropic and anisotropic models for central Poland also showed good accuracy of the Liu-Jordan, Koronakis and Hay models. It is stated that the Steven and Unsworth model should not be used due to its significant RMSE% exceeding even 40%.
- The analysis of mathematical models may be useful in assessing the potential of solar radiation in respect of legitimacy of investments incorporating photovoltaic installations for purposes other than optimal ones as well as for the purposes where PV surfaces are exposed to solar radiation for a long time in view of the durability of such installations.
- In order to obtain a high convergence of the results of calculations and measurements, each mathematical model should be modified for local latitude.

Supplementary Materials

Supplementary materials can be accessed at: http://www.mdpi.com/1996-1073/8/2/1025/s1.

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Author Contributions

Grażyna Frydrychowicz-Jastrzębska contributed to the theoretical analysis of solar radiation models and manuscript preparation. Artur Bugała contributed to result analysis, manuscript preparation and experimental design. All authors have approved the submitted manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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