Article

Bucket Trajectory Optimization under the Automatic Scooping of LHD

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Abstract: We propose an optimal planning scheme of the bucket trajectory in the LHD (Load-Haul-Dump) automatic shoveling system to improve the effectiveness of the scooping operation. The research involves simulation of four typical shoveling methods, optimization of the scooping trajectory, establishment of a reaction force model in the scooping process and determination of optimal trajectory. Firstly, we compared the one-step, step-by-step, excavation and coordinated shoveling method by the Engineering Discrete Element Method (EDEM) simulation. The coordinated shoveling method becomes the best choice on account of its best comprehensive performance among the four methods. Based on the coordinated shoveling method, the shape of the optimized trajectory can be roughly determined. Then, we established a model of bucket force during the shoveling process by applying Coulomb’s passive earth pressure theory for the purpose of calculating energy consumption. The trajectory is finally determined through optimizing the minimum energy consumption in theory. The theoretical value is verified by the EDEM simulation.

Keywords: LHD; EDEM simulation; minimum energy consumption; optimal shoveling trajectory

1. Introduction

With the exhaustion of open-pit mineral resources, the use of wheel loaders in the mining industry has gradually moved underground. In the complex underground working environment, the unmanned Load-Haul-Dump (LHD) has an indelible effect on reducing the heavy manual labor, ensuring work safety and improving labor productivity [1,2]. The leading machine properties during operation are productivity, energy efficiency and operability [3]. There are three key technical challenges [4] about automatizing the LHD: (1) obtaining bulk material pile models, (2) planning optimal shoveling trajectory, and (3) developing a controller that responds quickly to rapidly changing external forces [5]. We focus on the second challenge.

In the early work, Hemami minimizes energy consumption without considering resistance force [6]. Hisashi minimizes energy consumption by obtaining the force numerical value through sensors [7]. However, none of these authors directly linked the energy consumption to the resistance received by the bucket theoretically. We derived energy consumption based on the resistance force received by the bucket. The main contributions of this paper are as follows: (1) we screened out the best method in the actual working condition by comparing four typical shoveling methods of LHD [8], (2) we derived the resistance received by bucket by establishing a reaction force model for the purpose of calculating the energy consumption during scooping, and (3) we planned the optimal trajectory through optimizing the minimum energy consumption and verified the theoretical value by EDEM simulation.

In Section 2, we introduced the resistance force received by bucket, which is widely used in the process of scooping. In Section 3, we performed and compared four typical shoveling method simulations. In Section 4, we formulated the relation between energy consumption and insertion depth and then optimized the trajectory with minimum energy consumption.

2. Force of Shoveling Process

2.1. Bulk Material Pile

Granular media is a collection of a large number of similarly sized particles such as sand, loose ore, grain, cement and crushed coal [9]. The particles scooped by the LHD can generally be divided into two categories. One is a single disc or a sphere particle and the other is a single polygon block.

The materials studied in this paper are granular media formed by the minerals generated after mine blasting. As shown in Figure 1, the granular particles are simplified by the complexation of multi-spherical particles [10]. The bulk material pile model is naturally accumulated with a certain accumulation angle, as shown in Figure 2.

![Figure 1. Model of typical granular particle generated after blasting.](image1)

The main characteristics of the bulk materials we studied are varied for different research purposes. Generally speaking, bulk materials have several basic characteristics [11], such as Poisson’s ratio, particle size, density, cohesion, water content, natural accumulation angle, internal and external friction angle, compressive strength, shear strength, and deformation, etc. Because of the focus of this paper, rock density, internal friction angle, and natural accumulation angle are the main considerations of this paper.

2.2. Load-Haul-Dump (LHD) Working Mechanism

LHD is a versatile equipment for the shoveling-transport-unloading operation (Figure 3). There is no storage device and the final execution component of the operation is only the bucket. The working
mechanism of LHD is mainly composed of two parts, the lifting mechanism of the boom and the flipping mechanism of the bucket, as shown in Figure 4.

Figure 3. Reverse six-bar mechanism, Load-Haul-Dump (LHD).


The resistance of LHD during the loading operation varies with the way of shoveling, the bucket geometry, the trajectory and the characteristics of the material. We can achieve the desired trajectory by controlling the path of the front end of the working mechanism, which is the trajectory of the bucket blade [12,13].

2.3. Material Movement Characteristics

During the operation of LHD, the change of the bulk material pile shape is closely related to the resistance received by the bucket. The main theories are dense nuclear theory and slip surface theory. The dense nuclear theory [14] is used to study the motion characteristics of the material when the bucket is inserting into the pile. During the horizontal inserting phase, the bulk materials directly in front of the blade edge are squeezed to form a compacted area called the dense area. The formation of the dense area is related to the geometry of the bucket, the trajectory of the shoveling, the depth of insertion, the characteristics of the material and the height of the pile. When the dense area is generated, the resistance received by the bucket is sharply increasing and the energy consumption of LHD is rising as well.

The slip surface theory [15] is used to study the motion characteristics of materials during the scooping process. There are two slip surfaces generated after inserting into the bulk materials pile. One slip surface is between the bucket bottom plane and the bulk materials pile and the other extends diagonally upward from the blade of the bucket, as shown in Figure 5. The two slip surfaces cut the entire pile into three sections, each of which has different kinematic characteristics.
The particle of the bulk material pile was iron ore. The main physical properties of the materials are shown in Table 1, and the mutual contact properties of the materials are shown in Table 2. Reduce the bucket by a certain ratio according to the actual size.

2.4. External Force Received by the Bucket

The resistance of the shoveling process can be analyzed by equivalently converting each resistance to the force and moment acting on the edge of the bucket. The magnitude of the driving force required by the bucket is determined by analyzing the change in resistance received by the bucket during the scooping phase. The working mechanism-material force system can be established as in Figure 6, according to the change of the bulk materials' shape. The force received by the bucket is divided into five components [16], each of which varies with the different shoveling phase.

![Figure 5](image1.png)

**Figure 5.** Two slip surfaces of the bulk material pile during shoveling.

![Figure 6](image2.png)

**Figure 6.** The force acting on the bucket during the shoveling process.

Where, $F_1$ is the gravity of the material entering the bucket, $F_2$ is the force of the material on the bottom plane of the blade, $F_3$ is the insertion resistance, $F_4$ is the friction force of the material on the blade and $F_5$ is the force of the bulk materials in the bucket acting on the bucket bottom.

3. Four Shoveling Methods Simulation

In this section, we simulated and compared the one-step shoveling method, step-by-step shoveling method, excavation method and coordinated shoveling method with the full bucket rate, the full bucket rate per unit time and resistance peak value as indicators.

We made the following assumptions for the discrete element model in this paper:

1. The focus of this paper is the macroscopic force of the bucket in the scooping process. According to the actual background, it is the stacking of ore materials formed after the mine blasting.
2. The types of particles are ore particles, which belong to the inviscid unit. It is assumed that there is no crushing or compaction bonding of particles during scooping.
3. Before the LHD works, the bulk material pile has already formed a certain accumulation angle and stabilized in overall condition.
4. In this simulation, the velocities of forward, bucket-flipping and bucket lifting are controlled to be the same.

Two kinds of materials were used in the simulation. The material of the bucket was steel, and the particle of the bulk material pile was iron ore. The main physical properties of the materials are shown...
in Table 1, and the mutual contact properties of the materials are shown in Table 2. Reduce the bucket by a certain ratio according to the actual size.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m³)</th>
<th>Shear Modulus (pa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7850</td>
<td>$1 \times 10^{10}$</td>
<td>0.30</td>
</tr>
<tr>
<td>Iron ore</td>
<td>3940</td>
<td>$5 \times 10^{7}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient of Restitution</th>
<th>Coefficient of Static Friction</th>
<th>Coefficient of Rolling Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel-Iron ore</td>
<td>0.5</td>
<td>0.4</td>
<td>0.05</td>
</tr>
<tr>
<td>Iron ore-Iron ore</td>
<td>0.5</td>
<td>0.9</td>
<td>0.15</td>
</tr>
</tbody>
</table>

We simulated four typical methods of scooping commonly used in LHD, which are one-step shoveling, step-by-step shoveling, excavation and coordinated shoveling [17], as shown in Figure 7.

![Figure 7](image-url)

**Figure 7.** Four typical shoveling methods: (a) one-step shoveling, (b) step-by-step shoveling, (c) excavation, (d) coordinated shoveling.

The one-step shoveling method, as shown in Figure 7a, shows that when the bottom of the LHD bucket is parallel to the plane, the bucket edge is inserted along the bottom of the pile. When the inserting depth is about the length of the bottom of the bucket, the LHD stops moving forward and then flips the bucket to complete the shovel. The advantage of this method is that the operation steps are simple and the most widely used in actual production. The disadvantage of this method is that the LHD needs to overcome the large resistance and has high requirements on the dynamic performance of the LHD.

The step-by-step shoveling method, as shown in Figure 7b, is used for step-by-step inserting and lifting when the bucket edge cannot be directly inserted into the bottom of the bucket. After the bucket is inserted into a certain depth, then raised by a certain angle and next rotated back to horizontal direction, finally repeat the steps above. The method is complicated in operation, requires a high level of operators and has a large loss on the LHD.

The excavation method, as shown in Figure 7c, inserts the LHD bucket edge into the bottom of the pile about 1/3 of the length of the bottom of the bucket then flips the boom to fill the bucket.
In the coordinated shoveling method, as shown in Figure 7d, the LHD is advanced while the bucket is turned over to fill the material. The goal of the shoveling method is to keep the trajectory of the bucket edge parallel to the slope of the pile as much as possible so that the bucket has the least resistance to shovel.

3.1. One-Step Shoveling Simulation

The one-step shoveling method of the LHD is carried out by the set parameters. The simulation process is shown in Figure 8.

![Figure 8. Snapshots of the one-step shoveling method.](image)

The post-processor of the EDEM can directly observe changes in the shape of the bulk materials pile during scooping. The center of the bucket is used as a truncation plane to observe the interaction of the bulk materials and the bucket. The parameter velocity of material particles is selected to reflect the shape of the pile during the shoveling process.

At the beginning of the horizontal shoveling phase, both the deformation of the pile and the resistance force received by the bucket are small. As the depth of inserting increases, the cutting edge begins to squeeze the material ahead and the pile deformation becomes larger. In this phase, a certain range of materials in front of the cutting edge begins to be compacted and the resistance received by the bucket gradually increases.

As the depth continues to increase, the bulk materials pressed in front of the cutting edge gradually form a dense core and the resistance begins to rise sharply. Meanwhile, the pressed bulk materials tend to move toward the loose area and the deformation range expands.

When the bucket is flipping, the bucket needs to overcome the inertial force of bulk materials above the bucket. At the same time, destroy the binding force and shear force between the bulk materials, so the resistance received by the bucket is also at a high level. The bucket begins to detach from the pile and the force on the bottom of the bucket gradually decreases to zero.

As the boom is lifted, the bucket is filled with bulk materials leaving the pile and the bulk materials in the bucket tend to stabilize. At this point, the bucket is mainly subjected to weight from the bulk materials in the bucket.

The peak resistance of the entire one-step scooping process is shown in Figure 9, and the volume of bulk materials in the bucket is shown in Figure 10.
Figure 9. Peak resistance of the entire one-step scooping process.

Figure 10. The volume of bulk material in the bucket of the entire one-step scooping process.

3.2. Step-by-Step Shoveling Simulation

The step-by-step shoveling simulation process is shown in Figure 11. The first four steps are similar to the one-step shoveling method so they would be skipped over.

Figure 11. Snapshots of the step-by-step shoveling method.

When the bucket reverses to horizontal, the shape of the pile has changed greatly and gradually reaches a new stable state and there is already a certain amount of bulk material in the bucket when the bucket inserting begins again. The bottom of the bucket pushes the material inside the bucket to
continue forward, squeezing the bulk materials in front of the cutting edge. However, the shape of the pile is not completely stable at this time and the snap-in force and shear stress between the material particles have not yet fully formed.

3.3. Excavation Simulation

The excavation simulation process is shown in Figure 12. The excavation method is often used in a loose working environment in the pile. The bucket is inserted to about 1/3 of the bottom of the bucket, then the arm is swung and the bucket is flipped.

![Figure 12. Snapshots of the excavation method.](image)

When the bucket is flipping and lifting, the bucket is subjected to the upward resistance of the bulk materials before the angle of the bucket is turned over to the stacking angle of the pile. When the bucket is flipping at an angle equal to the stacking angle of the material, the resistance of the bucket is almost at its peak.

When the bucket flip angle is greater than the angle of repose, the bottom of the bucket gradually leaves the pile. At this time, the main shoveling resistance becomes the gravity of the material in the bucket. Shoveling resistance eventually tends to change to a relatively stable value.

3.4. Coordinated Shoveling Simulation

The coordinated shoveling simulation process is shown in Figure 13, which is avoiding the dense core formed by extrusion during the shoveling process and selecting a path with less resistance to work.

![Figure 13. Snapshots of the coordinated shoveling method.](image)

3.5. Calculation of Full Bucket Rate and Peak Resistance

The full bucket rate $\zeta$, which is the ratio of shoveled mass (volume) of bulk materials to the capacity of bucket [18], can be calculated by using Equations (1):

$$\zeta = \frac{M_w}{M_0} = \frac{V_w}{V_0}$$

(1)
\[ \bar{\zeta} = \frac{\zeta}{t_w} \]  

where, \( M_w (V_w) \) is the mass (volume) of bulk materials finally shoveled in the bucket, \( M_0 (V_0) \) is the mass (volume) of bulk materials in the full bucket condition, \( t_w \) is the working time of shoveling, and \( \bar{\zeta} \) is the full bucket rate per unit time which can be expressed by Equations (2).

The parameters of four typical shoveling methods are shown in Table 3.

<table>
<thead>
<tr>
<th>Method</th>
<th>One-Step Shoveling</th>
<th>Step-by-Step Shoveling</th>
<th>Excavation</th>
<th>Coordinated Shoveling</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_w (m^3) )</td>
<td>1.93 \times 10^{-4}</td>
<td>2.88 \times 10^{-4}</td>
<td>9.40 \times 10^{-5}</td>
<td>2.82 \times 10^{-4}</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>35.93%</td>
<td>53.53%</td>
<td>17.47%</td>
<td>52.42%</td>
</tr>
<tr>
<td>( t_w (s) )</td>
<td>2.5</td>
<td>5.6</td>
<td>2.5</td>
<td>3.1</td>
</tr>
<tr>
<td>( f_{\text{max}} (N) )</td>
<td>18.82</td>
<td>795.9</td>
<td>12.72</td>
<td>31.5</td>
</tr>
<tr>
<td>( \bar{\zeta} )</td>
<td>14.37%</td>
<td>9.56%</td>
<td>6.99%</td>
<td>17.23%</td>
</tr>
</tbody>
</table>

4. Optimal Trajectory Planning

In this section, we established a model of bucket force during shoveling by applying Coulomb’s passive earth pressure theory in order to formulate the energy consumption during scooping. The trajectory is finally determined by optimizing the minimum energy consumption and the theoretical value is verified by the EDEM simulation.

As for the planning of the optimal trajectory, Hu Tiehua of Tsinghua University proposed a method of reducing the resistance and the energy consumption by reducing the unnecessary energy consumption of the movement [19]. The specific method is to let the blade edge plane only cut the pile while loading without squeezing or pushing the bulk material.

Using the image processing technology in MATLAB R2015a (Mathworks), the slope boundary function value is extracted, and a straight-line fitting is performed to solve the angle of repose. Opening and closing, which are two mathematical morphology methods, are used to process the bulk material pile image. Finally, the mean value solved by the two methods represents the angle of repose. The specific steps are shown in Figure 14. Firstly, the image is gray-scale processed, then binarized, next, the boundary is extracted and finally, the available slope is intercepted. The slope is fitted with MATLAB,
where \( k \) and \( b \) are the slope and intercept of the straight line in Figure 15. The indexes 1 and 2 represent the opening and closing morphology:

\[
y_{1,2} = k_{1,2}x_{1,2} + b_{1,2}
\]  

(6)

In summary, \( \beta \) is the angle of repose in the following:

\[
\beta = \frac{\tan^{-1}(k_1) + \tan^{-1}(k_2)}{2}
\]

(7)

4.1. Shoveling Trajectory Optimization

According to the path characteristics of the coordinated shoveling, when the trajectory of the front end of the bucket blade is substantially parallel to the slope of the pile, the energy consumption of the shoveling work is the least. Then, the optimized trajectory based on the coordinated shoveling method is shown in Figure 16.
which is the horizontal insertion depth. In phase 2, the boom moves forward with the LHD, causing the bucket edge to move in a direction that is consistent with the angle of repose. In the last phase, the flipping cylinder cooperates with the lifting cylinder to lift the bucket upwards away from the pile.

The volume of the preset shoveling material is the capacity of the bucket, which means the full bucket rate is 100%. Where, \(d_w\) is the width of the bucket, \(l\) is the length of insertion depth, \(L\) is the length of hypotenuse and \(h\) is the distance of lifting, as shown in Figure 17.

\[
V_0 = \frac{d_w[(L\sin(\beta) + lh + hL\cos(\beta))]}{2}
\]  
(8)

Figure 16. The optimized trajectory based on the coordinated shoveling method: (a) Schematic diagram of three main phases; (b) Parameter map of shoveling process.

In Figure 16a, the shoveling process is divided into three main phases. In Figure 16b, phase 1, the bottom of the bucket is parallel to the ground, the dump bucket and the lift cylinder remain stationary, the LHD moves forward at a constant speed and the depth of the inserted pile is the length of the A–B which is the horizontal insertion depth. In phase 2, the boom moves forward with the LHD, causing the bucket edge to move in a direction that is consistent with the angle of repose. In the last phase, the flipping cylinder cooperates with the lifting cylinder to lift the bucket upwards away from the pile.

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\[
V_0 = \frac{d_w[(L\sin(\beta) + lh + hL\cos(\beta))]}{2}
\]  
(8)

Figure 17. Drawing of the whole shoveling process.

4.2. Optimized Trajectory Energy Analysis

The shape of the bucket is appropriately simplified when analyzing the force of the bucket and only the main force portions during the shoveling work, such as the cutting edge and the bottom of the bucket, are retained.

When the lifting angle of the bucket is smaller than the angle of repose, the Coulomb passive earth pressure theory can be used to establish the shoveling model. The main assumption of Coulomb’s passive earth pressure theory is that the material triangle is regarded as a rigid sliding wedge without considering the internal stress state and the whole is in the static equilibrium state. The values of the variables used in this paper are shown in the Appendix A.
In the horizontal shoveling phase, the slip surface is forming, as shown in Figure 18. Coulomb passive earth pressure is obtained by the three-force balance intersection theorem.

\[ F_p = \frac{\sin(\omega + \delta_{tr})}{\sin(\omega + \delta_{tr} + \delta_{br})} G \]  \hspace{1cm} (9)

In this phase, the weight of the triangular wedge is:

\[ G = \frac{\gamma d_w l^2 \sin(\beta) \sin(\omega)}{2 \sin(\omega - \beta)} \]  \hspace{1cm} (10)

Substitute Equation (10) into Equation (9), then we can derive the result:

\[ F_p = \frac{\gamma d_w l^2 \sin(\beta) \sin(\omega + \delta_{tr})}{2 \sin(\omega - \beta) \sin(\omega + \delta_{tr} + \delta_{br})} \]  \hspace{1cm} (11)

Among them, \( G \) is the weight of the triangular material, \( F_p \) is the reaction force of the bucket, \( F_r \) is the frictional resistance between the materials, \( \omega \) the angle between the upper right slip surface and the horizontal plane, \( \delta_{br} \) is the friction angle between the material and the bucket, \( \delta_{tr} \) is the friction angle between the material particles, and \( \gamma \) is the unit weight of the pile. In the case of limit equilibrium, it is necessary to make \( F_p \) the extremum, that is:

\[ \frac{\partial F_p}{\partial \omega} = 0 \]  \hspace{1cm} (12)

The angle between the upper right slip surface and the horizontal can be obtained by:

\[ \frac{\sin(\omega) \sin(\omega + \delta_{tr})}{\sin(\omega - \beta) \sin(\omega + \delta_{tr} + \delta_{br})} = \frac{\sin(2\omega + \delta_{tr})}{\sin(2\omega + \delta_{tr} + \delta_{br} - \beta)} \]  \hspace{1cm} (13)

Further simplification [20] can be expressed as:

\[ \cot(\omega - \beta) = \frac{\sqrt{\sin(\delta_{br}) \sin(\delta_{tr} + \delta_{br})}}{\sin(\beta + \delta_{tr} + \delta_{br})} - \cot(\beta + \delta_{tr} + \delta_{br}) \]  \hspace{1cm} (14)

All variables except \( \omega \) in Equation (13) are known quantities, so the quantity of \( \omega \) can be obtained. It is indicated that the angle between the upper right slip surface and the horizontal plane is constant before the horizontal shoveling is not touched at the end of the bucket, which means the upper right slip surface is translated along the horizontal plane in the process.
In this phase, the coulomb passive earth pressure is:

\[ F_p = \frac{\gamma d w l^2 \sin(\beta)}{2} \left[ \cot(\alpha - \beta) \sin(\beta) \right] \left[ \cos(\beta + \delta) \sin(\beta + \delta) + \cot(\alpha - \beta) \sin(\beta + \delta) \right] \cos(\beta + \delta) + \cot(\alpha - \beta) \sin(\beta + \delta) \]  

(15)

By substituting Equation (13) into Equation (14), the specific quantity of the Coulomb passive earth pressure can be obtained. Where, \( F_p \) is the resultant force of \( F_1, F_4, \) and \( F_5 \) in Figure 6. Therefore, during the shoveling process at this phase, the horizontal resistance along the trajectory is:

\[ F_f = F_3 + 2F_p \sin \delta \]  

(16)

The insertion resistance is:

\[ F_3 = K_0 H S g \cos \kappa \]  

(17)

where, \( H \) is the depth of the material at the upper end of the cutting edge, \( S \) is the cross-sectional area of the material on the cutting edge, \( g \) is gravitational acceleration, \( K_0 \) is the influence coefficient of the insertion resistance of the material and \( \kappa \) is the angle between the moving direction of the bucket and the horizontal direction.

The energy consumption of Phase 1 is:

\[ E_1 = \int_0^l F_f dl' \]  

(18)

According to the dense nuclear theory, if the bucket is deeply inserted into the pile it will form a dense core with a large resistance. In practice, to avoid this, most operators insert a portion of the length of the bottom of the bucket before proceeding.

Hisashi [7] concluded that the bucket is subjected to the least resistance in the direction of the angle of repose. In phase 2, shown in Figure 19, the Coulomb passive earth pressure is the same as the horizontal shovel stage and increased the weight of some bulk materials \( F_G \) and the gravity of the bucket \( G_b \).

\[ F_G = \int_0^L \gamma d w l \sin(\beta) dL' \]  

(19)

So, the resistance along the trajectory is:

\[ F_f = F_3 \cos \beta + F_p \sin(\delta + \beta) + (F_G + G_b) \sin \beta \]  

(20)

The energy consumption of phase 2 is:

\[ E_2 = \int_0^L F_f dL' \]  

(21)

In phase 3 (lifting phase), the bucket is primarily subjected to weight from the bulk materials.

\[ F_G = \gamma V_0 - \gamma d w \int_0^h (h - h') \tan(\beta) dh' \]  

(22)

The energy consumption of the lifting phase is:

\[ E_3 = G_b h + \int_0^h F_G dh' \]  

(23)

So, the energy consumption during the entire process is:

\[ E_s(l) = E_1 + E_2 + E_3 \]  

(24)
The limitations include geometric dimension constraints of trajectories and maximum height ($H_{\text{max}}$) constraints of LHD.

\[
\begin{align*}
V &= V_0 \\
I \sin \beta &= h \cos \beta \\
h + L \sin \beta &\leq H_{\text{max}}
\end{align*}
\]  
(25)

Substitute the numerical solution and take the extremum:

\[
\frac{dE_s(l)}{dl} = 0 
\]  
(26)

As for Equation (25), after subtracting the negative roots, we can get two roots $l_1 = 28.1$ mm and $l_2 = 115.0$ mm. Then, continue to find the derivative of Equation (24), and the following formula can be derived:

\[
\frac{d^2E_s(l_1)}{dl^2} > 0, \quad \frac{d^2E_s(l_2)}{dl^2} < 0 
\]  
(27)

$E_s(l_1)$ and $E_s(l_2)$ are the minimum and maximum value for energy consumption. Considering the full bucket rate and the maximum height that the bucket can lift, the verification interval is selected from 30 mm to 90 mm. The optimized trajectory ($l = 60$ mm) simulation process is shown in Figure 20.
Regarding the method simulation verification, this paper uses the combination of MATLAB and EDEM to solve energy consumption. We can get the discrete data points of the resistance of the bulk material particles to the bucket through the EDEM Analyst process. We should add the weight of the bucket to calculate the total energy consumption. Then, conduct fitting through MATLAB to make the curve equation approach as many discrete points as possible. Next, the energy consumption of the insertion depth can be obtained by the integral solution of the curve equation in Figure 21. Thus, we can get seven sets of data about energy consumption and insertion depth through simulation, fitting these data and making a fit function curve to determine the overall trend. From Figure 21, we can conclude insertion depth is proportional to peak resistance force while inversely proportional to the length of the whole excavation path, so there is an optimal solution. The energy consumption of the bucket of simulation can be obtained by integrating the resistance force curve equation in Figure 21.

![Figure 21](image)

**Figure 21.** Resistance force received by bucket of different insertion depths in the simulation.

From Figure 22, we find that the simulation curve is consistent with the trend of the theoretical, which is first decreased and then increased. Offset occurred at the minimum energy consumption point which was the target we wanted. The theoretical minimum point is about 30 mm and the simulated point is from 60 mm to 80 mm. The cause of the gap between the simulation and theoretical is that the bulk materials mass (volume) of scooping into the bucket in the simulation is not the same preset value as in the theoretical, it will fluctuate due to different insertion depth, whole excavation path and working time of shoveling. The weight of scooping bulk materials in the simulation is in fact smaller than in the theoretical. According to Equation (8), when the insertion depth $l$ is getting smaller, the length of the hypotenuse $L$ is getting larger, which causes the whole excavation path $(l + L + h)$ to be bigger than before. From Figure 23, we can see that the bulk material entering the bucket is a negative correlation to the length of the path under the same circumstance, because the shape of the bulk material pile changed to be more enormous in the longer path which caused more loss of bulk materials. So, the resistance of the simulation received by the bucket in smaller insertion depths is larger than the theoretical value which caused the minimum point to shift to the right and the finding is reliable to determine the optimal trajectory.
Regarding the method simulation verification, this paper uses the combination of MATLAB and EDEM to solve energy consumption. We can get the discrete data points of the resistance of the bulk material particles to the bucket through the EDEM Analyst process. We should add the weight of the bucket to calculate the total energy consumption. Then, conduct fitting through MATLAB to make the curve equation approach as many discrete points as possible. Next, the energy consumption of the insertion depth can be obtained by the integral solution of the curve equation in Figure 21. Thus, we can get seven sets of data about energy consumption and insertion depth through simulation, fitting these data and making a fit function curve to determine the overall trend. From Figure 21, we can conclude insertion depth is proportional to peak resistance force while inversely proportional to

Figure 22. Simulated and theoretical energy consumption.

Figure 23. Simulated volume of bulk materials in the bucket.

5. Conclusions

In this paper, EDEM simulations were performed on four commonly used methods of excavation: one-step shoveling method, step-by-step shoveling method, excavation method, and coordinated shoveling method. We analyzed the deformation of bulk material pile shape and the change in particle velocity during the scooping process. The coordinated shoveling method was selected out due to its maximum full bucket rate per unit time, relatively small resistance peak and second-highest full bucket rate.

We established a model of bucket reaction force during shoveling by applying Coulomb’s passive earth pressure theory in order to calculate the energy consumption. We formulated the connection between energy consumption and insertion depth. Under the same full bucket rate, which is 100%, the optimal trajectory of shoveling was ultimately determined by optimizing the insertion depth to the minimum energy consumption. The theoretical value of insertion depth was about 30 mm and the simulation verification value was from 60 mm to 80 mm. The overall trend of simulation energy consumption and insertion depth curve was the same as that of theory, which was first decreased and then increased, and offset occurred at the minimum point. Although the minimum value of the simulation does not match the theory well, the same trend of the two curves can explain the feasibility of the theory. Because the volume of material in the actual excavation simulation was less than in the theoretical, this caused the whole curve to shift to the right. So, the finding is reliable to determine the optimal trajectory.

Our next step is to further optimize the trajectory by optimizing the speed of the LHD.

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Appendix A

Parameters involved in the energy optimization process:

- The angle of repose: $\beta = 36.9^\circ$;
- Static friction angle between bulk material and bucket: $\delta_{br} = \tan^{-1} 0.4 = 21.8^\circ$;
- Static friction angle between bulk material particles: $\delta_{rr} = \tan^{-1} 0.9 = 42.0^\circ$;
- Width of bucket: $d_w = 0.103m$;
- Capacity of bucket: $V_0 = 5.38 \times 10^{-4} m^3$;
- Horizontal length of bucket bottom: $l_1 = 0.09m$;
- Gravitational acceleration: $g = 9.8 m/s^2$;
- Mass of bucket: $M_b = 1.745 kg$;
- Material unit weight: $\gamma = 21000 N/m^3$;
- Material insertion resistance coefficient of influence: $K_0 = 1.5$;
- The angle between the bucket motion direction and the horizontal direction: $\kappa = 0^\circ$;
- Cross-sectional area of the material above the cutting edge: $S = H d_w$;
- The depth of the material at the upper end of the cutting edge: $H_{max} = 0.2m$.

References


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