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Solving the Stochastic Generation and Transmission Capacity Planning Problem Applied to Large-Scale Power Systems Using Generalized Shift-Factors

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Abstract: In this study, we successfully develop the transmission planning problem of large-scale power systems based on generalized shift-factors. These distribution factors produce a reduced solution space which does not need the voltage bus angles to model new transmission investments. The introduced formulation copes with the stochastic generation and transmission capacity expansion planning problem modeling the operational problem using a 24-hourly load behaviour. Results show that this formulation achieves an important reduction of decision variables and constraints in comparison with the classical disjunctive transmission planning methodology known as the Big M formulation without sacrificing optimality. We test both the introduced and the Big M formulations to find out convergence and time performance using a commercial solver. Finally, several test power systems and extensive computational experiments are conducted to assess the capacity planning methodology. Solving deterministic and stochastic problems, we demonstrate a prominent reduction in the solver simulation time especially with large-scale power systems.

Keywords: generalized distribution factors; stochastic programming; two-stage problem; hourly load modeling; large-scale power systems

1. Introduction

The capacity expansion planning (CEP) problem solves the following issues—(1) how much generation and transmission capacity requires the electrical power system, (2) when and what type of new power units and new transmission elements are needed and (3) where locate these new elements (generation units and transmission transformer and lines) to supply adequately the long-term energy of the customers.

1.1. Technical Literature Review

Assuming that the electricity market is centrally operated, for instance the Chilean situation, the CEP problem is formulated as a cost minimization problem considering both investment and operational components. Notice that the operational problem has been traditionally formulated using a linear optimal power flow.

Even though the expansion planning problem was carried out in 1957 [1], this optimization problem is presently figured out by innovative computational and/or decomposition techniques [2]. In the technical literature, different CEP methodologies have been applied to solve short- and medium-scale power systems adopting different objective functions, sets of decision variables and different constraints among other technical considerations [3].

The CEP formulation [4,5] solves the generation capacity expansion planning (GCEP) problem and the transmission capacity expansion planning (TCEP) problem simultaneously. In the CEP problem, the number of decision variables and constraints increases exponentially solving medium-scale power systems as well as modeling different energy customers estimations (load duration curves or an hourly data).

Traditionally, the CEP problem is divided using two optimization frameworks: (1) the GCEP problem and (2) the TCEP problem. This is accomplished not only because the investment infrastructure involved in each type of decision is different but also because the way of the new transmission decisions modify the transmission network approach. On the one side, the GCEP problem produces local modifications in the operational problem. On the other side, the TCEP problem makes important changes in the bus admittance matrix.

To solve the TCEP problem, the most popular mixed-integer linear problem (MILP) is the classical disjunctive Big M model [6]. Using the voltage bus angles as decision variables, the Big M formulation incorporates two groups of binary inequality constraints for modeling new transmission investments. The main disadvantage is the higher number of decision variables and constraints which depends on the electrical power system, the investment transmission candidates and the generation/load time-steps. Although the disjunctive model displays an improvement in linearity [7], this framework enables coping with stochastic problems solving medium- and large-scale power systems [8].

To solve the GCEP problem, Hinojosa et al. [9–11] accomplish several studies using traditional shift-factors without include the voltage bus angles to model the bulk transmission network [12,13]. Nevertheless, these linear factors cannot be applied directly to solve the TCEP problem because nonlinear terms appear in candidate transmission circuits.

To keep the power systems topology and generalized shift-factors (GSF) unchanged, the line outage is modeled with a virtual generation located in the sending bus m and a virtual load located in the receiving bus n. Using this GSF-based methodology, the power injection to bus m and withdrawal from bus n is canceled out each other with no impact on other transmission elements. Review reference [14] for a detailed information about the GSF applied to the security N-1 power flow problem.

In [15,16], authors formulate the TCEP problem taking into account GSF. These factors are calculated considering that existing and future transmission elements are included in the transmission network. The transmission investment decisions are added in the planning problem using the GSF-based methodology based on virtual power injections and a bilinear formulation. Applying a disjunctive technique, it is possible to transform one nonlinear power flow equation to two linear binary inequality constraints. Notice that this methodology eliminates the voltage bus angles as decision variables. Therefore, there is a very important reduction in decision variables and constraints.

In [15], the load of the customers is modeled in the multi-stage problem by only one yearly value. Besides, authors solve up to IEEE 118-bus power system using a deterministic approach. In our opinion, there is necessary to conduct several simulations in order to find out the performance of the GSF-based methodology. We propose to solve the multi-stage CEP problem applied to large scale problem as well as to model an hourly behaviour. Accordingly, we implement an hourly load modeling to obtain a resilient electrical power system that will be flexible and robust to withstand, for example, renewable power variability.

In the state-of-the-art, a few studies adequately model the long-term uncertainty in the planning problem because of the high computational complexity [17]. Although the idea of implementing stochastic-mixed integer programming is not new (over 15 years) [18], recently MILP improvements as well as high computational capacity have done possible to solve medium-scale power systems. For more information about uncertainty implementations, review references [2,10,11,16,17].

In this study, we carry out the stochastic CEP problem based on a two-stage MILP problem [19]. The mathematical formulation uses scenarios to represent uncertainty in the input variables. To obtain a flexible electrical power system, the load uncertainty is implemented by an hourly formulation using representative days for expansion decisions [20].

To accomplish effectively the stochastic CEP problem, it is necessary to include simultaneously both GCEP and TCEP formulations in order to evaluate performance and simulation time. We want to emphasize that there are no studies solving both planning problems [15,16]. Additionally, we conduct several analyses solving large-scale power systems.

1.2. Contributions

Based on the literature review, we could mention that development of new planning models is still an open issue in the power system field. The proposed CEP methodology has not been developed in the technical literature. Therefore, it would be very interesting to determine the performance of the methodology solving large-scale power systems. The following contributions are expected to accomplish in this study: (1) the stochastic CEP problem is solved using the GSF and (2) a very large-scale power system is successfully solved in a reasonable simulation time. In our opinion, the achieved stochastic formulation could bring better performance and practical advantages solving very large-scale problems.

This study is organized as follows. In Section 2, the CEP formulation is shown and applied to an example 4-bus power system. Section 3 presents results and comparisons using several power systems. Last, Section 4 concludes the study.

2. Introducing the Stochastic Capacity Expansion Planning Formulation

From a central planner point of view, the stochastic CEP problem is formulated as the minimization (1a) of the following costs: (1) generation CI^G (1b) and transmission CI^T (1c) investment costs; and (2) operational and unserved energy costs COP (1d). Notice that the operational problem is modeled using a DC-network through a linear optimal power flow and the transmission DC power losses are not included in the optimization problem.

The mathematical formulation uses scenarios to represent uncertainty. Therefore, the stochastic problem is carried out from a deterministic MILP formulation as the minimization of the expected value of the deterministic operational costs (1a) for E possible realizations of the uncertainty parameter with their respective probabilities of occurrence (Pr_e) [19].

In the two-stage CEP problem, the first stage variables, which are generation and transmission investment candidates, are decisions variables with uncertainty. These variables are the same for all different scenarios using non-anticipativity constraints [11]. The second stage variables, which are the power generation of each unit and virtual candidate power flows [16]), must supply each load scenario. Mathematically, the stochastic CEP problem is formulated as follows:

$$\min \sum_{t \in T} R_t \cdot [\, CI^G + CI^T + \sum_{e \in E} Pr_e \cdot \sum_{s \in S} H_s \cdot COP] \tag{1a}$$

where:

$$CI^G = \sum_{g \in G^B} (C_g \cdot (n_{g,t,e} - n_{g,t-1,e}) + OM_g \cdot n_{g,t,e}) \cdot P_g^M \tag{1b}$$

$$CI^T = \sum_{km \in L^B} C_{km} \sum_{y \in x_{km}^M} (x_{km,t,e,y} - x_{km,t-1,e,y}) \tag{1c}$$

$$COP = \sum_{g \in (G^E \cup G^B)} FC_g \cdot p_{g,s,t,e} + VoLL \cdot \sum_{g \in G^R} p_{g,s,t,e} \tag{1d}$$

The optimization problem is subject to the following technical constrains:

$$\sum_{g \in G^E} P_g^M + \sum_{g \in G^B} P_g^M \cdot n_{g,t,e} \geq D_{t,e}^M \cdot SR \tag{2}$$

$$\sum_{g \in (G^E \cup G^B \cup G^R)} h_s \cdot p_{g,s,t,e} = D^{total}_{s,t,e} \tag{3}$$

$$|\sum_{b \in B} GSF_{km,b} \cdot (p_{b,s,t,e} - D_{b,s,t,e}) + \sum_{km \in L^B} \sum_{y \in X^M_{km}} (GSF_{km,k} - GSF_{km,m}) \cdot \widetilde{f}_{km,s,t,e,y}| \leq F^M_{km} \qquad \forall\, km \in L^E \tag{4}$$

$$|\widetilde{f}_{km,s,t,e,y} - \sum_{b \in B} GSF_{km,b} \cdot (p_{b,s,t,e} - D_{b,s,t,e}) - \sum_{km \in L^B} \sum_{y \in X^M_{km}} (GSF_{km,k} - GSF_{km,m}) \cdot \widetilde{f}_{km,s,t,e,y}| \leq x_{km,t,e,y} \cdot F^M_{km} \qquad \forall\, km \in L^B \tag{5}$$

$$|\widetilde{f}_{km,s,t,e,y}| \leq (1 - x_{km,t,e,y}) \cdot M \qquad \forall\, km \in L^B \tag{6}$$

$$x_{km,t,e,y} \geq x_{km,t-1,e,y} \qquad \forall\, t \in T,\ t > 1 \tag{7}$$

$$n_{g,t,e} \geq n_{g,t-1,e} \qquad \forall\, t \in T,\ t > 1 \tag{8}$$

$$n_{g,t,e} - n_{g,t-1,e} \leq N^M_g \qquad \forall\, t \in T,\ t > 1 \tag{9}$$

$$\sum_{y \in X^M_{km}} (x_{km,t,e,y} - x_{km,t-1,e,y}) \leq X^M_{km} \qquad \forall\, t \in T,\ t > 1 \tag{10}$$

$$0 \leq p_{g,s,t,e} \leq P^M_g \qquad \forall\, g \in (G^E \cup G^R) \tag{11}$$

$$0 \leq p_{g,s,t,e} \leq P^M_g \cdot n_{g,t,e} \qquad \forall\, g \in G^B \tag{12}$$

$$n_{g,t,1} = \ldots = n_{g,t,e} \qquad \forall\, g \in G^B \tag{13}$$

$$x_{km,t,1,y} = \ldots = x_{km,t,e,y} \qquad \forall\, km \in L^B \tag{14}$$

$$0 \leq x_{km,t,e,y} \leq 1 \tag{15}$$

In (2), the planning reserve margin measures the amount of generation capacity available for unexpected generation/load events. Equation (3) represents the total load balance. Equations (4)–(6) accomplish a binary disjunctive formulation using GSF and virtual power flows [15,16]. Constraints (7) and (8) ensure coherence for both transmission and generation investment variables, respectively. Equations (9) and (10) limits the generation and transmission investment variables. Equation (11) shows the maximum power generation for existing power units. Equation (12) includes a disjunctive constraint to model investment generation capacity. Last, the non-anticipativity constraints for both generation and transmission variables are shown in (13) and (14), respectively. A binary variable models the candidate transmission elements (15).

For new transmission decisions, variable x_{km} represents km line status (ON/OFF) as follows: x_{km} = 1 indicates that a new line is built and x_{km} = 0 implies that there is no investment decision.

The Proposed Formulation Applied to an Example System (4-Bus)

In Figure 1, we present a 4-bus power system used to apply the deterministic CEP optimization problem. Transmission reactance and thermal limits (MW) are also included.

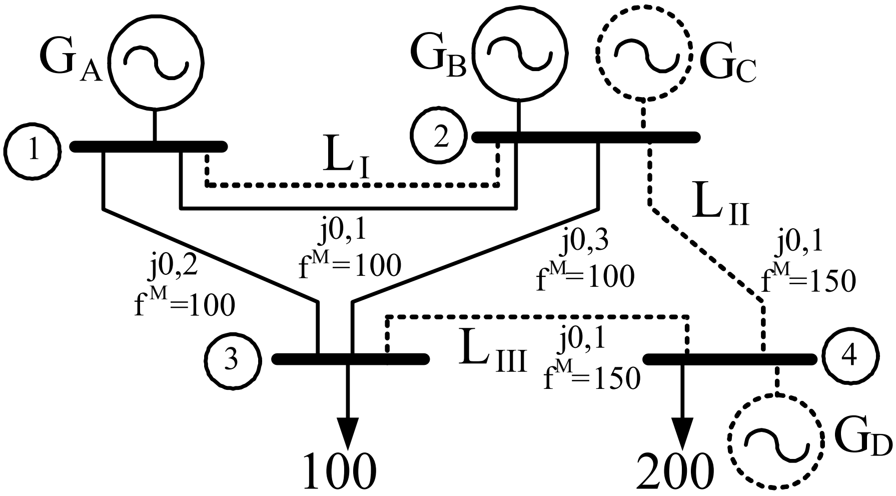

Figure 1. Power system example.

Table 1 gives information for existing and candidate power units. We also include investment and operational costs.

Table 1. New and future power generation data.

#	P_g^M (MW)	C_g ($)	FC_g ($/MWh)	N^M
G_A	250	existing unit	10	-
G_B	100	existing unit	14	-
G_C	150	3,000,000	16	2
G_D	110	2,500,000	8	3

Table 2 shows transmission investment data. Moreover, we can see investment cost for transmission future options.

Table 2. Investment data for the transmission system.

#	$(from)$	to	C_{km} ($)	X^M
L_I	1	2	10,000,000	1
L_{II}	2	4	6,000,000	2
L_{III}	3	4	5,000,000	3

These artificial data are employed to realize effects and insights solving the introduced methodology.

Table 3 shows the GSF matrix using bus 1 as the slack bus. Notice that shift-factor values consider both existing and future transmission elements.

Table 3. Generalized shift-factors for the power system example.

Line	bus_1	bus_2	bus_3	bus_4
1–2	0	−0.420	−0.317	−0.360
1–3	0	−0.159	−0.366	−0.283
2–4	0	0.034	−0.138	−0.069
2–4	0	0.062	−0.248	−0.324
3–4	0	−0.041	0.166	−0.117

For illustrative purposes, generation and transmission inequality constraints (4)–(6) and (12) are developed in this section. We use $M = 2\pi * S_{base}$; where S_{base} = 100.

Using (4) for the existing $1-2$, $1-3$ and $2-3$ transmission elements, the following constraints must be formulated (16), (17) and (18), respectively.

$$-203.45 \leq -0.420p_B - 0.420p_C - 0.360p_D + 0.420\widetilde{f}_{12,1} - 0.060\widetilde{f}_{24,1} - 0.060\widetilde{f}_{24,2} + 0.040\widetilde{f}_{34,1} + 0.040\widetilde{f}_{34,2} + 0.040\widetilde{f}_{34,3} \leq -3.45 \quad (16)$$

$$-193.10 \leq -0.159p_B - 0.159p_C - 0.283p_D + 0.159\widetilde{f}_{12,1} + 0.124\widetilde{f}_{24,1} + 0.124\widetilde{f}_{24,2} - 0.083\widetilde{f}_{34,1} - 0.083\widetilde{f}_{34,2} - 0.083\widetilde{f}_{34,3} \leq 6.90 \quad (17)$$

$$-127.59 \leq 0.034p_B + 0.034p_C - 0.069p_D - 0.034\widetilde{f}_{12,1} + 0.103\widetilde{f}_{24,1} + 0.103\widetilde{f}_{24,2} - 0.069\widetilde{f}_{34,1} - 0.069\widetilde{f}_{34,2} - 0.069\widetilde{f}_{34,3} \leq 72.41 \quad (18)$$

Using (5) and (6), we develop disjunctive constraints for the new transmission candidates as follows: (a) for line 1–2 (19) and (20); (b) for the first circuit of line 2–4 (21) and (22); (c) for the second circuit of line 2–4 (23) and (24); (d) for the first circuit of line 3–4 (25) and (26); (e) for the second circuit of line 3–4 (27) and (28); and (f) for the third circuit of line 3–4 (29) and (30).

$$|0.420p_B + 0.420p_C + 0.360p_D + 0.580\widetilde{f}_{12,1} + 0.060\widetilde{f}_{24,1} + 0.060\widetilde{f}_{24,2} - 0.040\widetilde{f}_{34,1} - 0.040\widetilde{f}_{34,2} - 0.040\widetilde{f}_{34,3} + 100x_{12,1}| \leq 103.45 \quad (19)$$

$$|\widetilde{f}_{12,1} - 628.32x_{12,1}| \leq 628.32 \quad (20)$$

$$|-0.062p_B - 0.062p_C + 0.324p_D + 0.062\widetilde{f}_{12,1} + 0.614\widetilde{f}_{24,1} - 0.386\widetilde{f}_{24,2} - 0.076\widetilde{f}_{34,1} - 0.076\widetilde{f}_{34,2} - 0.076\widetilde{f}_{34,3} + 150x_{24,1}| \leq 89.66 \quad (21)$$

$$|\widetilde{f}_{24,1} - 628.32x_{24,1}| \leq 628.32 \quad (22)$$

$$|-0.062p_B - 0.062p_C + 0.324p_D + 0.062\widetilde{f}_{12,1} - 0.386\widetilde{f}_{24,1} + 0.614\widetilde{f}_{24,2} - 0.076\widetilde{f}_{34,1} - 0.076\widetilde{f}_{34,2} - 0.076\widetilde{f}_{34,3} + 150x_{24,2}| \leq 89.66 \quad (23)$$

$$|\widetilde{f}_{24,2} - 628.32x_{24,2}| \leq 628.32 \quad (24)$$

$$|0.040p_B + 0.040p_C + 0.120p_D - 0.040\widetilde{f}_{12,1} - 0.080\widetilde{f}_{24,1} - 0.080\widetilde{f}_{24,2} + 0.720\widetilde{f}_{34,1} - 0.280\widetilde{f}_{34,2} - 0.280\widetilde{f}_{34,3} + 150x_{34,1}| \leq 6.90 \quad (25)$$

$$|\widetilde{f}_{34,1} - 628.32x_{34,1}| \leq 628.32 \quad (26)$$

$$|0.041p_B + 0.041p_C + 0.117p_D - 0.041\widetilde{f}_{12,1} - 0.076\widetilde{f}_{24,1} - 0.076\widetilde{f}_{24,2} - 0.283\widetilde{f}_{34,1} + 0.717\widetilde{f}_{34,2} - 0.283\widetilde{f}_{34,3} + 150x_{34,2}| \leq 6.90 \quad (27)$$

$$|\widetilde{f}_{34,2} - 628.32x_{34,2}| \leq 628.32 \quad (28)$$

$$|0.041p_B + 0.041p_C + 0.117p_D - 0.041\widetilde{f}_{12,1} - 0.076\widetilde{f}_{24,1} - 0.076\widetilde{f}_{24,2} - 0.283\widetilde{f}_{34,1} - 0.283\widetilde{f}_{34,2} + 0.717\widetilde{f}_{34,3} + 150x_{34,3}| \leq 6.90 \quad (29)$$

$$|\widetilde{f}_{34,3} - 628.32x_{34,3}| \leq 628.32 \quad (30)$$

Using (12) for candidate power generation units, constraint (31) model new power unit G_C and constraint (32) model new power unit G_D.

$$p_C - 250n_2 \leq 0 \quad (31)$$

$$p_D - 100n_4 \leq 0 \tag{32}$$

These mathematical constraints are presented to clearly explain the accomplished CEP problem.

The static planning problem is solved using the maximum load. Reserve requirements are not considered and the MILP gap is 0.0001%.

Using *Gurobi* (9.0.0) [21] to solve the CEP problem, the optimal cost is 33.7288$MM\$$ (1a). The investment planning problem includes one power unit at bus 4 and one transmission line $2-4$.

For the transmission planning problem, new line $2-4$ (investment decision $x_{24,1}$ = 1) means the virtual power flow is $\widetilde{f}_{24,1} = 0$ MW. The optimal solution does not require other transmission investments to supply the load of the customers. That is, $x_{12,1}$ = 0, $x_{24,2}$ = 0, $x_{34,1}$ = 0, $x_{34,2}$ = 0 and $x_{34,3}$ = 0. Additionally, *Guroby* displays the following virtual power flows: $\widetilde{f}_{12,1} = 100$ MW, $\widetilde{f}_{24,2} = 90$ MW, $\widetilde{f}_{34,1} = 40$ MW, $\widetilde{f}_{34,2} = 40$ MW and $\widetilde{f}_{34,3} = 40$ MW. Notice that these virtual power flows satisfy candidate transmission constraint (5) when decision variable is zero.

Figure 2 displays the power flow solution for existing and investment lines. We do not include all virtual investment power flows because these values are injected (generation) in the sending bus and withdrawal (load) in the receiving bus with no impact on the other transmission power flows.

It should be mentioned that the classical Big M formulation also obtains the same solution using a higher number of decision variables and constraints.

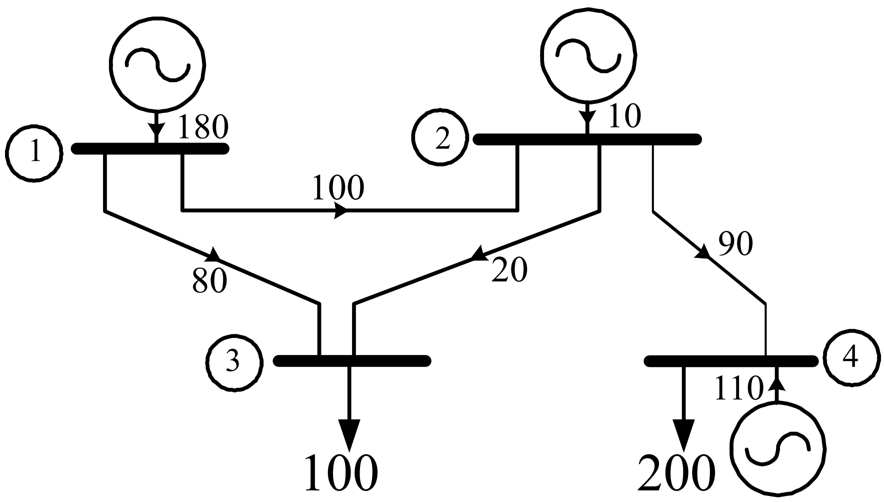

Figure 2. Power flow solution.

3. Simulation Results

In Figure 3, we present a diagram showing power system requirements as well as planning variables and parameters. Using this information, both generation and transmission decisions and hourly operational solution can be reported using the introduced formulation.

Two mathematical formulations are applied to solve the CEP problem—(1) the introduced formulation (GSF) and (2) the classical disjunctive Big M formulation (CD). The CD formulation validates optimality and performance solving the CEP problem. Several simulations are conducted using short- and large-scale electrical power systems to find out convergence and simulation time.

Matlab [22] (Version 2015b, UTFSM, Chile) develops the mathematical capacity planning formulation and *Gurobi* solves the optimization problem on a computer with the following characteristics: Intel Xeon E5-2630 (2.20 GHz) with a RAM of 32 GB.

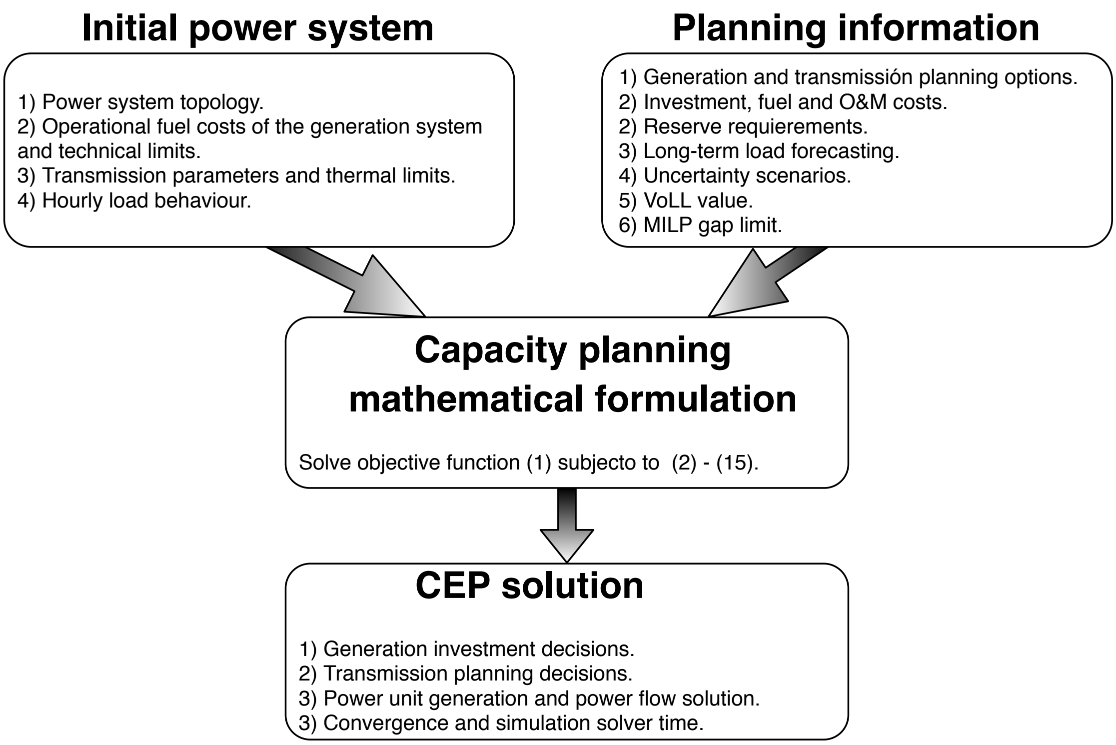

Figure 3. Capacity expansion planning framework.

3.1. Garver Test Power System

This classical power system [23] is traditionally used to present new planning formulations as well as to establish performance and optimality. Review reference [24] to obtain more information.

Table 4 shows generation data including buses where the power units are located, the maximum power generation, the investment and operation & maintenance costs, and the fuel cost for existing and future power units.

Table 4. Generator data for the Garver system.

#	Bus	P_g^M (MW)	C_g ($/MW)	OM_g ($/MW)	FC_g ($/MWh)
G_1	1	3 × 30	existing unit	-	14.08
G_2	1	1 × 60	existing unit	-	22.11
G_3	3	2 × 60	existing unit	-	25.95
G_4	3	2 × 120	300,000	9000	20.41
G_5	6	1 × 120	250,000	7500	25.95
G_6	6	2 × 240	350,000	10,500	14.08
G_7–G_9	2,4,5	999	virtual unit	-	$VoLL$

The CEP problem considers a reserve margin of 20%; that is 1.2 times the yearly peak load (760 MW). The $VoLL$ value is 10,000 [$/MWh]. For the multi-stage problem, the interest rate is 10%. For the MILP solver, the optimality gap is 0.0001%.

3.1.1. Static Deterministic Analysis

Applying the GSF-based formulation, the optimal cost is 475, 809, 470.91$; the transmission investment cost is 110 *MM*$ and the generation investment cost is 247.2 *MM*$. The yearly operational cost is 118.6095 *MM*$. The optimal solution does not display unserved energy.

The generation investment plan considers two units *G*4 (bus 3) and two units *G*6 (bus 6). Though the investment cost of power unit *G*5 is the lowest, that power unit is not needed to supply the load of the customers because of the highest fuel cost.

The power generation solution is p_1 = 90.0 MW, p_2 = 60.0 MW, p_3 = 70.61 MW, p_4 = 240.0 MW and p_6 = 299.40 MW. In the original TCEP problem [23], the maximum capacity at bus 6 is 600 MW. However, a lower power generation produced by *G*6 is enough to supply load demand.

The transmission investment plan incorporates three circuits in corridor x_{2-6} and one circuit in corridor x_{3-5}. This solution is well known in the technical literature [23,24].

Table 5 displays simulation results for both GSF and CD formulations. The authors include the number of decision variables (v), the number of equality and inequality constraints (Θ), the non-zeros elements (nZ), the objective function (OF) and the average solver time obtained by *Gurobi* (t_s) simulating 200–trials.

Results show that both formulations achieve the same optimal solution. Consequently, the introduced and the Big *M* formulations are equivalent using different solution spaces.

Table 5. Simulation results solving the Garver power system.

Model	v	Θ	nZ	OF	t_s (s)
GSF	236	123	8213	475,809,470.91	0.030
CD	269	132	778	475,809,470.91	0.021

Figure 4 displays the power flow solution. We do not include virtual power flows when there is no investment decision.

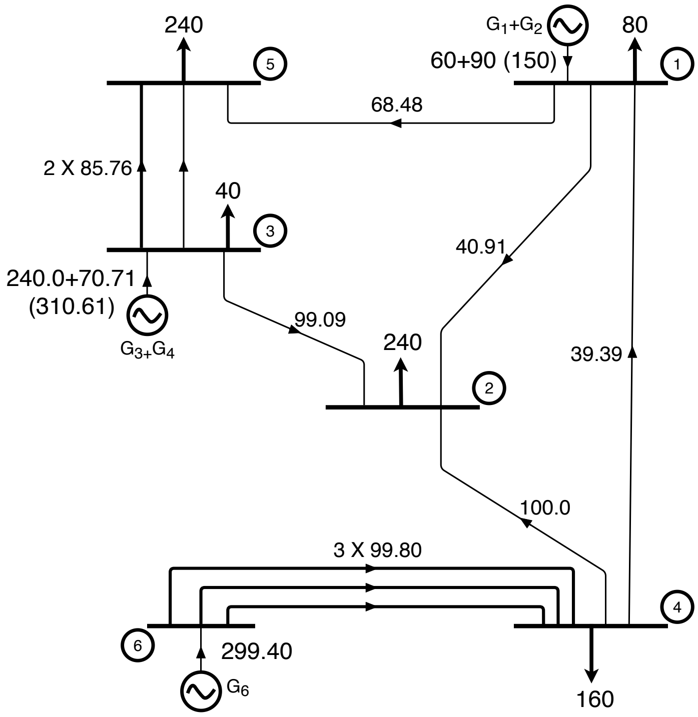

Figure 4. Power flow solution for the Garver power system.

3.1.2. Multi-Stage Deterministic and Stochastic Analyses Using an Hourly Load Modeling

Tenth investment years are incorporated to carry out the multi-stage CEP problem. Review reference [16] to have more information about the long-term load forecasting.

Figure 5 shows the hourly load behaviour. To formulate the multi-stage CEP problem, the yearly load demand is proportionally divided using these hourly data.

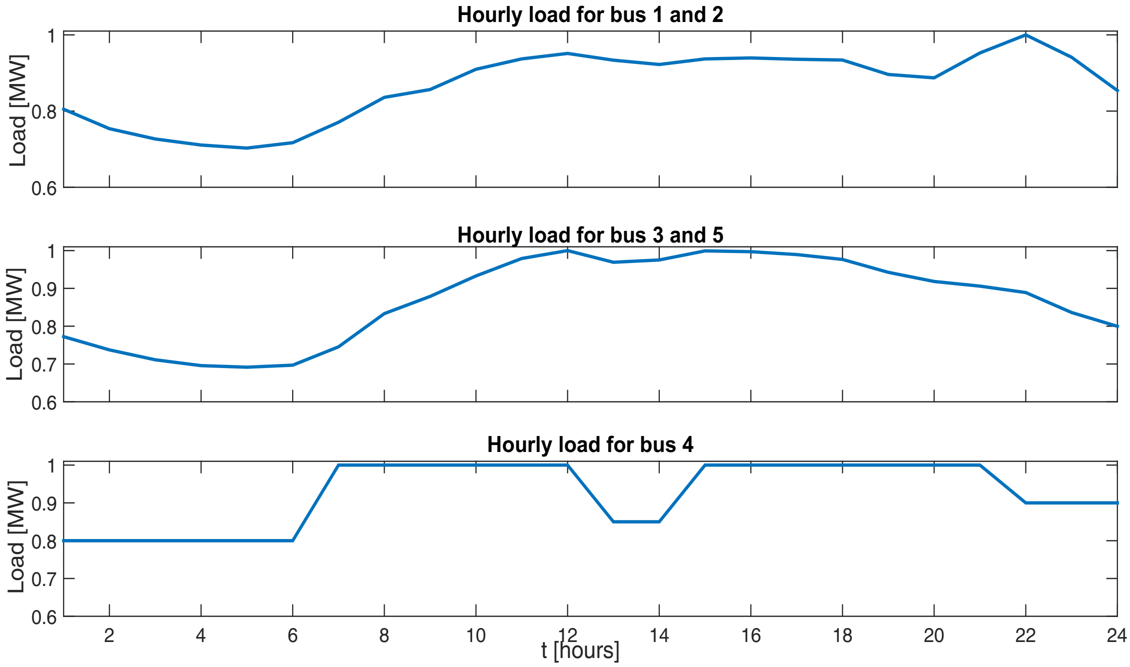

Figure 5. Hourly load demand for a typical day.

For the stochastic analysis, ten load scenarios $E = 10$ are created using the following factors: 1.0490, 0.9867, 0.9726, 0.9952, 0.9310, 0.9636, 1.0943, 0.8529, 1.0490, and 0.9404. To create five scenarios $E = 5$, we use the first five values. We assume each scenario has the same probability of occurrence with $Pr = 0.2$ for $E = 5$ and $Pr = 0.1$ for $E = 10$.

Table 6 shows simulation results performed by GSF-based formulation. We show the optimal objective function and the solver simulation time t_s achieved by the deterministic ($E = 1$) problem and the two stochastic ($E = 5$ and $E = 10$) problems. Results obtained by the Big M formulation (CD) are also included to verify optimality.

Table 6. Simulation data applied to the Garver system using the multi-stage analysis.

Model	E	v	Θ	nZ	OF	t_s (s)
GSF	1	56,908	16,895	1,959,394	621,509,628.5	979.55
CD	1	55,938	18,830	187,419	621,509,628.5	92.39
GSF	5	282,355	82,109	9,790,325	625,989,931.6	8793
CD	5	277,545	91,790	932,616	625,989,931.6	1043
GSF	10	564,153	163,624	19,578,388	629,964,367.3	35,692
CD	10	554,543	182,990	1,863,994	629,964,367.3	9801

For the deterministic problem, the generation planning solution includes two $G4$ units one in the second year (t_2) and one in the seventh year (t_7); and two $G6$ units one in the fourth year (t_4) and one in the ninth year (t_9).

The investment transmission solution considers one corridor of x_{3-5} in the eighth year (t_8); and three corridors of x_{2-6}, two in the fourth year (t_4) and one in the eighth year (t_8). Regarding the operational problem, there is no unserved energy.

Although the same optimal investment plan is also obtained by the stochastic formulation, there are differences in the expected operational costs.

In the GSF-based formulation, decisions variables increase. However, there is a little reduction in the number of constraints. Also, the non-zero elements increase. Notice that simulation time grows exponentially when different load scenarios are added in the optimization problem.

In these simulations, both formulations obtain the same solution. Review reference [25] to obtain more simulation results.

Even though the lower simulation time is obtained by the traditional Big M formulation, we conduct more analyses not only to clarify advantages or disadvantages of the proposed formulation but also to determine the performance using large-scale power systems.

3.2. IEEE 300-Bus Test Power System

This study case (http://www.ee.washington.edu/research/pstca/) includes several voltages (345, 230, 115, 86, 66, 27, 20, 16.5, 13.8, 6.6, 2.3, and 0.6 kV). The power system contains sixty-nine generators and four hundred eleven transmission lines. Data for existing power units can be reviewed using MatPower [26].

To model the stochastic CEP problem, tenth investment long-term periods and five load scenarios are modeled using hourly data. For each load scenario, we select the hourly behaviour of bus 1 (Figure 5).

To determine generation and transmission candidates, we solve an optimal power flow using the maximum load of the customers. Future planning options are selected based on unserved energy and transmission congestion problems as follows:

1. For the power unit candidates, we include fifteen new power generation options with a maximum number of $N_g^M = 1$.
2. For the transmission network candidates, there are thirty investments options where each line has a maximum power flow of 390 MW and the maximum number of new elements is $X_l^M = 2$.

The CEP problem considers a reserve margin of 20%. The $VoLL$ value is 10,000 $/MWh, the interest rate is 10% and the MILP gap is 1%.

Table 7 shows results and time performance for both deterministic (one scenario) and stochastic (five scenarios) CEP formulations.

Table 7. Simulation results obtained for the IEEE 300-bus power system.

Model	E	v	Θ	OF	t_s (s)
GSF	1	43,470	312,900	52,024,910,450.4	4032
CD	1	214,110	627,690	52,024,910,450.4	87,797
GSF	5	217,350	1,567,500	257,487,249,145.5	272,240
CD	5	1,070,550	3,141,450	-	-

The optimal investment planning solution consist of thirty-one transmission elements and four new power generation units.

In the deterministic analysis ($E = 1$), the optimal cost is the same for both GSF and CD formulations. More importantly, we achieve the same CEP solution using a reduced solution space. With respect to the classical Big M solution space, the introduced formulation has 79.7% and 50.0% lower number of decision variables and constraints, respectively. The outstanding result is the lower simulation time obtained by the GSF-based formulation, which is 95.4%—21.8 times—faster in comparison with the CD formulation.

Considering 5 load scenarios and using the traditional Big M stochastic formulation, we are not able to obtain a reasonable solution after a simulation time of 3.13 weeks. At that time, the *Gurobi* gap is approximately 6%. Therefore, we decide to stop the computer simulation because the solution achieved by the GSF-based formulation had a MILP gap lower than 1% in 272,240 s (3.2 days).

The hourly optimal solutions show there is no load shedding in the electrical power system.

3.3. The PEGASE 1354-Bus Test Power System

This case accurately represents the size and complexity of part of the European high voltage transmission network. The network contains 1354 buses, 260 generators, and 1991 branches, and these lines operates at 380 and 220 kV. Data is created from the Pan European Grid Advanced Simulation and State Estimation (PEGASE) project [27]. Power system information can be reviewed using *MatPower*.

Considering not only infinite simulation time but also memory *Matlab* problems, we are able to model only the deterministic planning problem taking into account two investment periods. We include the end-of-time operational component using five years as well.

To determine generation and transmission candidates, we solve an optimal power flow with the maximum load.

For the planning generation plan, we propose twenty-eight power generation options with $N_g^M = 1$.

For the transmission network plan, there are one-hundred and seven candidate lines. These transmission elements have a thermal limit of 500 MW and the maximum number of transmission candidates is $X_l^M = 2$.

Because *Matpower* does not include fuel variable costs, we use random numbers provided by a uniform discrete distribution function using as limits 40 and 100 $/MWh.

In the mathematical formulation, the reserve margin is 20%, the *VoLL* value is 10,000 $/MWh, the interest rate is 10% and the optimality gap is 3%. The hourly load behaviour is based on bus 1.

In this very large-scale problem, the *Guroby* solver time is approximately three weeks using the GSF-based formulation. We want to highlight that the traditional Big M formulation does not carry out the CEP optimization problem because of memory issues.

Using *Matlab* to formulate the planning problem, the accomplished CEP methodology is able to solve up to 1354-bus power systems using a 24-hourly load behaviour. Considering higher power system dimension and memory issues, stochastic generation and transmission planning problems should be modeled and simulated by decomposition techniques such as the Benders approach [28].

4. Conclusions

This study introduced an efficient planning methodology to formulate large-scale transmission networks based on generalized shift-factors. Several analyses were conducted to determine effects and insights solving stochastic planning problems with different uncertainty scenarios. Although the traditional disjunctive Big M formulation displays an improvement modeling the transmission planning problem, this methodology only had a better performance solving small-scale power systems. With reference to the stochastic CEP problem, the Big M formulation did not obtain a high quality solution applied to the IEEE 300-bus test power system in comparison with the solution achieved by the developed formulation. Furthermore, the Big M formulation was not able to model the deterministic CEP optimization problem applied to the PEGASE 1354-bus power system because of *Matlab* memory issues.

The outstanding result of this study is the best simulation time obtained by the generalized shift-factors formulation because the optimization problem was modeled using a lower number of decision variables and constraints without sacrificing optimality. Results lead to the conclusion that the proposed stochastic formulation could be applied to formulate real large-scale power systems. Even though this methodology reduced the computational burden of stochastic planning methodologies, we were limited by *Matlab* memory capacity to solve up to 1354-bus power systems. However, we are evaluating to implement the achieved formulation using *Python*.

Considering the best simulation time and practical advantages obtained by the proposed formulation, the research group will include operational generation Unit Commitment constraints to the stochastic CEP problem.

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