Trade-Offs for the Optimal Energy Efficiency of Road Transportation: Domestic Cases in Developing Countries

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Abstract: The increase in domestic transportation in developing countries may adversely affect the energy efficiency of road transportation due to effective productivity and carbon dioxide emissions (CO₂). When evaluating quantitatively the countries on the efficiency frontier, poor efficiency can still be seen sometimes due to the slack available in undesirable output measures. This paper uses desirable and undesirable output variables, such as passenger-kilometers (PKM), tones-kilometers (TKM), and carbon dioxide (CO₂), to compute the weakly efficient decision-making units (DMUs). The data envelopment analysis (DEA) technology is used to assess the efficiencies of the decision-making units (DMUs), which are countries in our case. Then, the trade-off method with efficient binding surfaces is used to attain the optimal efficiencies of the weakly efficient DMUs. The marginal rates aid this trade-off analysis. Resultantly, such marginal trade-offs do not deteriorate the efficiency of the DMUs below the frontier line. We calculate the maximum change (margin) in a specific variable amount when another variable’s amount is changed. Thus, such a computation gives us different margins, with which each output variable can be traded off to bring a DMU further toward the closest optimal point possible. The marginal trade-off can help the managers and policymakers in effective decision-making, and it is further recommended to address efficiency damages (by the undesired outputs).

Keywords: data envelopment analysis; trade-offs; efficiency; road transportations

1. Introduction

The improvement of transportation is most effectively judged by its energy consumption and productivity (i.e., passengers-kilometer and tone-kilometer) [1]. When it comes to indirect measures, it becomes crucial to assess optimal efficiency. Some of these measures are desirable, and others are undesirable. In the case of direct measuring variables, when the decision-making unit (DMU) is weakly efficient in practice, it may be shown as efficient in analysis; see Akbar et al. [2]. To provide a practical solution, and to address the weak efficiency of the DMUs, it is necessary to find an adequate margin that can be a trade-off to improve such DMUs, so that they can further reach the closest efficiency point (possible optimal point).
Data envelopment analysis (DEA) is one of the methods to calculate such margins. Golany and Tamir [3] provide an extended DEA method to evaluate trade-offs based on the Dantzig–Wolfe decomposition algorithm. Cooper et al. [4], similar to the model of Rosen et al. [5] (pp. 213–221), went for efficient managerial operations; they found valid trade-off margins by using the marginal rates and advanced the elasticity of substitutions. Their work is different from conventional methods, which can use positive and negative slack values. Again, Khoshandam et al. [6] (pp. 411–412) referred to the work of Cooper et al. and Asmild et al. [7] and used the concept of marginal rate to propose a new procedure for trade-off calculations in the presence of non-discretionary (ND) factors.

The concept of the trade-off is nothing new. It has been widely established and used in industrial operations to achieve efficiencies [8–10]. However, considering transportation energy literature (to the best of our knowledge), the missing elements are the clear quantitative trade-offs for the optimal solution. The global competition that has recently formed a trading race [11] has made domestic transportation, in stakeholder countries, vital in terms of energy efficiency. Additionally, the work related to the trade-off method is normally applied to the management of business operations of the banking industry, power plants, hospitals, etcetera [8,12–14]. Therefore, this article contributes to the existing literature in two ways. First, it fills the gap in transportation energy efficiency by using effective marginal values to perform quantitative trade-offs. Second, in this study, quantitative trade-offs have been applied to domestic road transportation of the weakly efficient DMUs to bring them to the optimal efficient boundary point.

This study used data from a previous study, which evaluated the transportation energy efficiency of 19 “Belt and Road” (BRI) stakeholder countries; see Akbar et al. [2]. This article first uses DEA technology, for DEA not only enables us to evaluate the current efficiencies of the DMUs, but also provides us with the slacks as an improvement indication among bad and good outputs. Resultantly, it was found that the apparent efficient countries are weakly efficient due to the slack values in an undesirable output, the carbon dioxide (CO₂). Then the marginal trade-off method between desired and undesired outputs is applied considering the ideal marginal range. This method is similar to what was proposed by Mirzaei et al. [15] and has been involved in the field of hydroelectric power plants. Thus, such a marginal trade-off must make a DMU achieving the optimal efficiency at the efficient frontier.

The paper is organized as follows. In Section 2, the literature further validates the study by discussing the trade-off methods found in transportation energy efficiency and the computation difficulties in qualitative trade-offs using DEA. Section 3 presents the model adopted for marginal trade-offs. Sections 4 and 5 detail the different trade-off scenarios in the presence of the desirable and undesirable outputs followed by the selected output variables and analysis. Section 6 concludes the study, along with limitations and future research recommendations.

2. Literature Review

Transportation plays an important role in the economic freedom (a tool to boost financial activities) of the developing countries [16]. Still, it also accompanies the carbon emission considered as an indirect cost [17]. There is a lot of variation in the research due to the perceived definition of the quantitative trade-off methods. Some use trade-offs as a tool to find an effective balance between efficiency-related variables, for example, see the recent study of Akbar et al. [18]. In contrast, others consider eliminating or decreasing the trade-offs margins for sustainable and efficient performance, for example, Grukas et al. [19], and Shahbazpour and Seidel [20]. Our work adheres to the former definition of trade-off. The method selection of trade-offs is also crucial, because it depends on the operational plans and gains. Therefore, we divide our literature into the following two parts. The first part validates our contribution to the existing literature of transportation energy efficiency, whereas the second part assesses the best fit of the DEA model for the quantitative trade-offs.
2.1. Trade-Off Methods for Transportation Energy Efficiency

The trade-off methods used for transportation energy efficiency are rather scarce in the literature. Considering the methods closely related to our work, Akbar et al. (2020) [21] detailed the concept of quantitative trade-offs for competitive transport operations. The correct types of trade-offs in the transportation sector are addressed, and practical implications are explained in their work using statistical methods of probabilities. Salehi et al. (2017) [22] undertook the problem of drivers’ assignment and transportation scheduling and used trade-offs between direct transport costs and indirect transport costs (i.e., CO₂). The same holistic approach of trade-offs by Olcer and Ballini [23], has been used to advance the cost–benefit analysis techniques considering efficient seaborne transportation. This decision-making model was also validated for rail and road transportation by the authors. Again, Kim et al. [24] have demonstrated the different approaches and used trade-offs as a decision support tool to mitigate CO₂ emissions. Their optimization approach authenticates the trade-off relation between logistic cost and CO₂ emissions. Another interesting study performed trade-offs between operational efficiency and energy consumption considering the schedule in the container terminal, which minimizes the task completion delay time and energy consumption; see He et al. [25].

To the best of our knowledge, other than above, there is a huge pile of literature on transportation growth and its effect on energy efficiency, considering other effective and applicable methods. The readers may study the refs. [26–36] (Djordjević and Krmac, 2019; Popovic et al., 2018; Shindina et al., 2018; Kot et al., 2017; Talbi, 2017; Talbi, 2017; Kasperowicz, 2015; Zhang et al., 2015; Cui and Li, 2014; Lipscy and Schipper, 2013; Chang et al., 2013; Ji and Chen, 2006).

2.2. Computational Difficulties in Trade-Off Analysis Using DEA

Although many studies based on DEA are aimed at its applicability in organizations’ performance measurement [37–45], minute work is found in the transportation production function models. In the decision-making process, the knowledge of the trade-offs in designing the operational policy for transportation energy efficiency is vital. For example, decision-makers are interested in knowing the excessive inputs which can be reduced to increase any particular output with a small amount of trade-off. And, if the inputs cannot be reduced, they may be only interested in the trade-off between two output variables to obtain the best results of the third output.

Many studies have penetrated in the concept of trade-offs. The slack-based measure (SBM) is being considered the best-fit DEA model to identify the slacks and then the way out to calculate the trade-off using this mathematical model. The non-zero inputs and outputs’ slacks are to occur to indicate efficiency. So the non-zero slack values are usually inefficient [46]. The original DEA model uses a weighted performance index to evaluate each DMU, where DMUs obtained at the efficient boundary is called an efficient frontier line. It has been used in many cases by using the DEA to determine good (expected output) performance [47]. Chang et al. [32] have used the DEA to perform trade-off-based measurements to calculate the environmental efficiency of China’s transportation system. In this way, they have found better performance, and room to produce high-efficiency greenhouse gases by reducing energy emissions and consumption. Bao et al. [48] have also used a DEA linear program with desirable outputs to calculate the efficiency level of the DMU, and broke the conventional practice based on the relaxation sorting method. Watto and Mugera [49] also estimate the groundwater utilization efficiency by adopting trade-off models in DEA to calculate efficient groundwater usage. The slacks of the technical efficiency revealed the result that the pipeline owners are more efficient than water buyers.

Cooper et al. [4] studied the modification of the basic additive DEA model and derived an effective compromising marginal replacement rate from slacks in the results. A study by Huang et al. [50] proposed a general method that calculates the rate of change from output to input along the effective surfaces of the DEA production set. Sueyoshi and Goto [51] calculated the effect of trade-offs on certain throughput using a production method in which a set of variables perform trade-offs among themselves. Moreover, they studied the trade-off rate (marginal replacement rate) with the availability
of the non-discretionary factors. Rosen et al. solved the problem of balanced trade-off values on the efficient boundary with a framework for calculating the trade-off between two variables in DEA.

The methods, as mentioned earlier, calculate a single measure of the trade-offs, i.e., only among two variables. When we change another variable, those compromise methods can foresee the largest possible compromise in a variable. However, a study by Mirzaei et al. (2016) [15] discusses trade-offs of this kind considering multiple output variables. Considering them, the new boundary point must be as close as possible to the original point. Hence, the surface of the efficient set can be defined with the help of marginal substitution, so that we can calculate different marginal rates for the decision-making units (DMUs), which are at the efficient point but identified as weekly efficient.

Considering the work of Cooper et al., Mirzaei et al., and Rosen et al. [4,5,15], the piecewise linear frontier in Data envelopment analysis (DEA) technology are indistinguishable at extreme ends, and trade-off (margins) values in one or more variables are only for small substitution. When trade-off is performed against any throughput, a specific change can be calculated in other throughputs using existing trade-off analysis. We can use the support surface bounding with an efficient point to define different trade-off balances. In this way, we obtain an optimal trade-off value for each efficient boundary point.

3. Basic Model for Trade-Off Balances

In mathematical terms, the marginal rate (MR) is used to calculate the margins with which trade-offs can be performed. Mathematicians are most interested in using efficient frontiers that support Pareto-effective surfaces and hyperplanes. In the multi-dimensional input and output space, you can use the DEA production possibility set to easily read the performance of the transport unit without losing any mathematical objectivity. By considering a general process, the marginal substitution rate is used, in which the input vector \( x \) and the output vector \( y \) are vectors and are above zero. Suppose the efficient boundary of this DEA technology is \( F(x, y) = 0 \); we can assume that

\[
\frac{\partial F(x, y)}{\partial x_i} < 0 \quad i = 1, \ldots, m
\]

\[
\frac{\partial F(x, y)}{\partial y_r} < 0 \quad r = 1, \ldots, s
\]

In addition, we assume that \( (x, y) \) is continuously differentiable. Let \( z_0 = x_0, y_0 \) be the efficient boundary; i.e., \( F(z_0) = F(x_0, y_0) = 0 \). By definition, the trade-off \( \tau \) amount of the throughput to another throughput at the efficiency boundary is as follows:

\[
\tau_{jk}^+ (z_0) = \lim_{h \to 0^+} \left( \frac{\delta_z F}{\delta z_k} \right)_{z_0} = \lim_{h \to 0^+} \frac{f_i(z_{01}, \ldots, z_{0l} + h, \ldots, z_{0m} + h)}{h}
\]

\[
\tau_{jk}^- (z_0) = \lim_{h \to 0^-} \left( \frac{\delta_z F}{\delta z_k} \right)_{z_0} = \lim_{h \to 0^-} \frac{f_i(z_{01}, \ldots, z_{0l} - h, \ldots, z_{0m} - h)}{h}
\]

Asmild et al. [7] gave a DEA-based program for calculating trade-off throughput \( j \) to throughput \( k \) from the right side of the efficient boundary point \( z_0 = x_0, y_0 \), as mentioned below:

\[
\tau_{jk}^+ (z_0) = \frac{z_j^o - z_j}{\delta_k}
\]
where \( z_{jo}^* \) is the optimal efficient point; \( h \) is the number (small and positive), and it is the solution to the following linear program:

\[
\begin{align*}
\text{max} & \quad z_{jo}^* \\
\text{s.t.} & \quad \sum_{l=1}^{n} \delta_l z_l \geq z_{lo} \quad l \neq j, k \\
& \quad \sum_{l=1}^{n} \delta_l z_{jo}^* = z_{kj}^* + h \\
& \quad \sum_{l=1}^{n} \delta_l = 1 \\
& \quad \delta_l \geq 1
\end{align*}
\]

In the above program, use \(-h\) instead to get the left edge replacement rate. In this process, the new efficient point gets calculated at the frontier boundary \( z_{jo}^* = (z_{1o}, \ldots, z_{ko}^* + h, \ldots, z_{mo}^* + h) \), when the \( z_{jo}^* \) is maximized. We can also use the other points closest instead of \( z_{jo}^* \). Now, two different marginal rates of substitution will be calculated, one under optimistic conditions and the other under pessimistic conditions.

3.1. Use of the Marginal Rates

Suppose there are optimistic conditions under which fundamental trade-offs are performed, so when a particular factor increases by a small amount or even decreases, it determines the extreme variation for the specific element of the output vector. To calculate the trade-offs, the following program needs to be solved:

\[
\begin{align*}
\text{max} & \quad z_{lo}' \\
\text{s.t.} & \quad z_{lo}' \in T \\
\text{where} & \quad z_{lo}' = (z_{1o}', \ldots, z_{ko}^* + h, \ldots, z_{mo}^* + h)
\end{align*}
\]

We can see a new vector above (whether it is an output or input vector). Equation (5) is a maximizing objective function, and it is optimistic. Therefore, it finds a trade-off (i.e., when another throughput changes by a small amount, the maximum value reaches a specific throughput). This model looks for an efficient boundary where the specified variable is maximized or minimized. However, the new point is an efficient frontier point in both cases, which is our requirement.

3.2. Illustrative Example with the Simple Data Set

We show this with the following simple example by taking seven DMUs—see Table 1—with two inputs \( x_1 \) and \( x_2 \), and one output \( y_1 \). Mirzaei et al. (2016) used this example with the help of the BCC model of Banker et al. [52], in their work, we can see that all the DMUs are efficient.

<table>
<thead>
<tr>
<th>Decision-Making Units (DMUs)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs ( x_1 )</td>
<td>0.9</td>
<td>0.5</td>
<td>1.1</td>
<td>0.2</td>
<td>2.2</td>
<td>2.8</td>
<td>3</td>
</tr>
<tr>
<td>and ( x_2 )</td>
<td>1.63</td>
<td>1.36</td>
<td>1.55</td>
<td>2.15</td>
<td>2.04</td>
<td>1.40</td>
<td>2.04</td>
</tr>
<tr>
<td>output ( y_1 )</td>
<td>0.65</td>
<td>0.35</td>
<td>0.65</td>
<td>0.55</td>
<td>1.2</td>
<td>0.8</td>
<td>1.3</td>
</tr>
</tbody>
</table>

The optimistic method gives a new point on the frontier facet ACE, where \((-x_{11}, -x_{21}, y_1') = (-1.136, -1.704, 0.75)\) and \( \tau_{13}^+ = 2.364 \), and \( \tau_{23}^+ = 0.745 \) is the change value of the first input.
This change occurs when a single output increases by 0.1 units. It is worth noting that \((-x_{11}, -x_{21}, y_1') = (-1.136, -1.704, 0.75)\) is the outcome of the following model:

\[
\begin{align*}
\min \quad & z_1^* + z_2^* \\
\text{s.t.} \quad & -0.9\delta_1 - 0.5\delta_2 - 1.1\delta_3 - 0.2\delta_4 - 2.2\delta_5 - 2.8\delta_6 - 3\delta_7 \geq -z_1^* \\
& -1.63\delta_1 - 1.36\delta_2 - 1.55\delta_3 - 2.15\delta_4 - 2.04\delta_5 - 1.4\delta_6 - 2.04\delta_7 \geq -z_2^* \\
& 0.65\delta_1 - 0.35\delta_2 - 0.63\delta_3 - 0.55\delta_4 - 1.2\delta_5 - 0.8\delta_6 - 1.3\delta_7 \geq 0.65 + 0.1 \\
& \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 = 1 \\
& \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7 \geq 0
\end{align*}
\] (6)

All points on the boundary will calculate a new trade-off (i.e., ABC, ABD, and ADE). Changing in the direction would give a unique point on a new efficiency frontier. For example, the points \((-x_{11}, -x_{21}, y_1') = (-0.815, -2.116, 0.75)\) are efficient points of ADE, with trade-offs \(\tau_{13}^- = -0.8462\) and \(\tau_{23}^- = -4.8615\). In DMU_A, the two trade-off ratios, \(\tau_{13}^+\) and \(\tau_{23}^+\), indicate that if we increase \(y_1\) from 0.65 to 0.75, and change the two inputs to 1.1364 and 1.7045, we will still stay at the efficient boundary. \(\tau_{13}^-\) and \(\tau_{23}^-\) mean that \(h = +0.1\); if both the inputs are changed to 0.8154 and 2.1162, respectively, both will remain on the effective boundary. Thus, the exciting fact is that in terms of two different efficient aspects, these two new points are very different. The point \((-x_{11}, -x_{21}, y_1') = (-0.815, -2.116, 0.75)\) is optimal because of the following program:

\[
\begin{align*}
\min \quad & -z_1^* + z_2^* \\
\text{s.t.} \quad & -0.9\delta_1 - 0.5\delta_2 - 1.1\delta_3 - 0.2\delta_4 - 2.2\delta_5 - 2.8\delta_6 - 3\delta_7 \geq -z_1^* \\
& -1.63\delta_1 - 1.36\delta_2 - 1.55\delta_3 - 2.15\delta_4 - 2.04\delta_5 - 1.4\delta_6 - 2.04\delta_7 \geq -z_2^* \\
& 0.65\delta_1 - 0.35\delta_2 - 0.63\delta_3 - 0.55\delta_4 - 1.2\delta_5 - 0.8\delta_6 - 1.3\delta_7 \geq 0.65 + 0.1 \\
& \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 + \delta_7 = 1 \\
& \delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7 \geq 0
\end{align*}
\] (7)

Two trade-off margins \(\tau_{13}^+, \tau_{23}^+, \text{ and } \tau_{13}^-\), \(\tau_{23}^-\) are calculated at an efficient point \(z_0 = -x_0, y_0\). For this purpose, the following three-step of Mirzaei et al. has been adapted:

Decide on the \(h\). This should be a small number for \(k\) throughput.

Secondly, we solve the two linear programs.

\[
\begin{align*}
z_1^+ = & \max \sum_{l \in M} d_1^{(1)} z_{l0}^* \\
\text{s.t.} \quad & \sum_{l = 1}^n \delta_l z_{l0}^* \geq z_{l0}^*, \quad l = 1, \ldots, m + s \\
& \sum_{l = 1}^n \delta_l = 1 \\
z_{l0}^* = & z_{l0}, \quad l \notin M, N \\
z_{l0}^* = & z_{l0} + h, \quad l \in N \\
\delta_1, z_{l0}^* \geq 0, \quad \forall t, l
\end{align*}
\] (8)

\[
\begin{align*}
z_1^- = & \max \sum_{l \in M} d_1^{(2)} z_{l0}^- \\
\text{s.t.} \quad & \sum_{l = 1}^n \delta_l z_{l0}^* \geq z_{l0}^*, \quad l = 1, \ldots, m + s \\
& \sum_{l = 1}^n \delta_l = 1 \\
z_{l0}^- = & z_{l0}, \quad l \notin M, N \\
z_{l0}^- = & z_{l0} + h, \quad l \in N \\
\delta_1, z_{l0}^- \geq 0, \quad \forall t, l
\end{align*}
\] (9)
Please note that \( d_1^{(1)} \) and \( d_1^{(2)} \) are constant numbers which are user-defined. \( M \) and \( N \) are two groups at the frontier boundary point \( z'_o \). See Equation (5).

Calculation of the trade-offs for negative and positive rates from the right are as follows:

\[
\tau^+_{jk}(z_o) = \frac{z'_p - z'_o}{h} \quad \forall j \in M, \, k \in N.
\]

(10)

The two trade-offs can be calculated by replacing \(-h\) with \( h\). Then, the program (8) and (9) obtains the projection at the frontier. Moreover, different trade-offs will result in a differently weighted vector \( d \).

The weighted vector \( d \) (user-defined) enables us to determine the efficient surface direction.

**4. Trade-Offs with Desirable and Undesirable Outputs**

This study documents that decision-makers are only focused on the efficient outputs of the transportation system without consideration of explicit inputs. The priorities in such an operational system are good and bad outputs when decision-makers are not much concerned about the consumption of the input factor. To consider a real-life application, we formulate the transportation energy systems, without inputs, for efficient trade-offs in the presence of good and bad outputs. The program presented below is the same as what has been reformulated as an alternative trade-off in DEA; read Mirzaei et al. [15].

Suppose there are \( n \) DMUs: \( j = 1, \ldots, n \), with two good output vectors \( y_{aj} = (y_{a1j}, \ldots, y_{asaj}) \geq 0 \) and \( y_{bj} = (y_{b1j}, \ldots, y_{bsbj}) \geq 0 \), and one bad output vector \( z_j = (z_{1pj}, \ldots, z_{pj}) \geq 0 \). By following Shepherd’s [53] assumption, the following linear production technology can be used to solve bad (undesirable) output in the transportation system without the explicit inputs.

\[
T_{WI} = \{(y_a, y_b, z) : \sum_{j=1}^{n} \delta_j y_{aj} \geq y_{aro}, \sum_{j=1}^{n} \delta_j y_{bj} \geq y_{rbo}, \sum_{j=1}^{n} \delta_j z_j \geq z, \sum_{j=1}^{n} \delta_j = \theta, 0 \leq \theta \leq 1, \delta_j \geq 0, \forall j \}. \quad (11)
\]

Suppose if we evaluate a country “o” for the possible reduction in the bad output, the following linear program can be used.

\[
\begin{align*}
\min & \quad \theta^* \\
\text{s.t.} & \quad \sum_{j=1}^{n} \delta_j y_{raj} \geq y_{raro}, \, r = 1, \ldots, s_r \\
& \quad \sum_{j=1}^{n} \delta_j y_{rbdj} \geq y_{rbo}, \, r = 1, \ldots, s_b \\
& \quad \sum_{j=1}^{n} \delta_j z_{pj} \geq \theta' z_{pao}, \, p = 1, \ldots, P \\
& \quad \sum_{j=1}^{n} \delta_j = \theta \\
& \quad 0 \leq \theta \leq 1, \, \delta_j \geq 0, \, j = 1, \ldots, n.
\end{align*}
\]

(12)

Now, using the reformulation procedure by Mirzaei et al. [15], we can compute trade-off margins, as in group \( N \), from the bad-output to the good-outputs as in group \( M \), at the frontier boundary. In first
step we have to choose a small number \(h\) (positive number) to solve the linear program problem at the second step, as follows:

\[
\begin{align*}
\text{max} & \sum_{p \in M} d^+_{p} z^+_{p} \\
\text{s.t.} & \sum_{j=1}^{n} \delta_{j}y_{r_{aj}} \geq y_{r_{ao}}, \ r = 1, \ldots, s_{a}, \ r \notin N \\
& \sum_{j=1}^{n} \delta_{j}y_{r_{aj}} \geq y_{r_{ao}} + h, \ r \in N \\
& \sum_{j=1}^{n} \delta_{j}y_{r_{bj}} \geq y_{r_{bo}}, \ r = 1, \ldots, s_{b}, \ r \notin N \\
& \sum_{j=1}^{n} \delta_{j}y_{r_{bj}} \geq y_{r_{bo}} + h, \ r \in N \\
& \sum_{j=1}^{n} \delta_{j}z_{pj} \geq \theta^* z_{po}, \ p = 1, \ldots, P, \ p \notin M \\
& \sum_{j=1}^{n} \delta_{j}z_{pj} \geq z^+_{p}, \ p \in M \\
& \sum_{j=1}^{n} \delta_{j} = \theta \\
& 0 \leq \theta \leq 1, \ \delta_{j} z^+_{p} \geq 0
\end{align*}
\]

\[\text{max} \sum_{p \in M} d^-_{p} z^-_{p} \]

\[\text{s.t.} \sum_{j=1}^{n} \delta_{j}y_{r_{aj}} \geq y_{r_{ao}}, \ r = 1, \ldots, s_{a}, \ r \notin N \]

\[\sum_{j=1}^{n} \delta_{j}y_{r_{aj}} \geq y_{r_{ao}} + h, \ r \in N \]

\[\sum_{j=1}^{n} \delta_{j}y_{r_{bj}} \geq y_{r_{bo}}, \ r = 1, \ldots, s_{b}, \ r \notin N \]

\[\sum_{j=1}^{n} \delta_{j}y_{r_{bj}} \geq y_{r_{bo}} + h, \ r \in N \]

\[\sum_{j=1}^{n} \delta_{j}z_{pj} \geq \theta^* z_{po}, \ p = 1, \ldots, P, \ p \notin M \]

\[\sum_{j=1}^{n} \delta_{j}z_{pj} \geq z^-_{p}, \ p \in M \]

\[\sum_{j=1}^{n} \delta_{j} = \theta \]

\[0 \leq \theta \leq 1, \ \delta_{j} z^-_{p} \geq 0 \]

Thirdly, we can calculate the positive and negative trade-off rates, as shown below. Again, to calculate the two trade-offs, simply replace \(-h\) with \(h\).

\[
\tau^+_{pk}(y_{ao}, y_{bo}, z_{o}) = \frac{z^+_{p} - z_{po}}{h}
\]

\[
\tau^-_{pk}(y_{ao}, y_{bo}, z_{o}) = \frac{z^-_{p} - z_{po}}{h}
\]

5. Application of the Model in the Real World

Given the domestic road transportation of the “Belt and Road” (BRI) countries, it can be observed from the results of Akbar et al. [2] that the “bad” output (CO\(_2\) emissions) is not separable from the good outputs—i.e., tones per kilometers (TKM) and passengers per kilometers (PKM). Reducing undesirable outputs will inevitably lead to a reduction in desirable output variables. In addition, it often happens that certain undesirable outputs are closely related (inseparable) to specific inputs. For example,
CO₂ emissions in the transportation industries have the maximum emissions rate among the other greenhouse gases, and it is proportional to the energy (oil and fuel) consumption.

5.1. Efficiency Measuring Variables

To illustrate the trade-offs in real-world cases, the data consist of 19 developing countries. Here, the data set of the year 2018 is composed of multiple output variables, because the inputs are assumed of no interest to managers in this particular case. Among the three indicators, two are regarded as good outputs, and one is the only bad output. The number of passenger trips per kilometer (PKM) in each country is considered the first good output, which is denoted as \( y_1 \), and the number of tons per kilometer (TKM) is the second desirable output, denoted as \( y_2 \). The only undesirable output, \( z_1 \), is the CO₂, which positively correlates with the two good outputs. These indicators also show the ecological efficiency of transport operations; see Table 2.

**Table 2.** Output variables for road transportation energy efficiency.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Unit of Measure</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_1 )</td>
<td>PKM</td>
<td>World Bank</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>TKM</td>
<td>World Bank</td>
</tr>
<tr>
<td>( z_1 )</td>
<td>CO₂ emissions</td>
<td>Our World in Data</td>
</tr>
</tbody>
</table>

5.2. Results and Discussion

The data set has been re-evaluated using the model program (12). The data have been converted into a natural logarithm form for better analysis. We can see that 4 out of 19 countries appear efficient (i.e., Bangladesh “BGD,” Cambodia “KHM,” Myanmar “MMR,” and India “IND”). The data set and the results are listed in Table 3, where column 6 shows the relative efficiency obtained using the model (12).

**Table 3.** The dataset of 2018 and the efficiency results without trade-offs (\( h = 0 \)).

<table>
<thead>
<tr>
<th>Sr.</th>
<th>DMU(_j)</th>
<th>Countries</th>
<th>PKM((y_1))</th>
<th>TKM((y_2))</th>
<th>CO₂((z_1))</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BGD</td>
<td>Bangladesh</td>
<td>9.214332</td>
<td>13.43234</td>
<td>18.2935</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>BEL</td>
<td>Belarus</td>
<td>8.734721</td>
<td>11.84053</td>
<td>17.93246</td>
<td>0.841333</td>
</tr>
<tr>
<td>3</td>
<td>MNG</td>
<td>Mongolia</td>
<td>6.880384</td>
<td>5.129899</td>
<td>17.22965</td>
<td>0.710685</td>
</tr>
<tr>
<td>4</td>
<td>KHM</td>
<td>Cambodia</td>
<td>3.806662</td>
<td>8.852951</td>
<td>15.88721</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>KGZ</td>
<td>Kyrgyzstan</td>
<td>3.7612</td>
<td>8.25892</td>
<td>16.1605</td>
<td>0.545924</td>
</tr>
<tr>
<td>6</td>
<td>MMR</td>
<td>Myanmar</td>
<td>8.333991</td>
<td>8.542471</td>
<td>17.04763</td>
<td>0.622848</td>
</tr>
<tr>
<td>7</td>
<td>TKM</td>
<td>Kazakhstan</td>
<td>7.757906</td>
<td>7.757906</td>
<td>18.10189</td>
<td>0.622848</td>
</tr>
<tr>
<td>8</td>
<td>TJK</td>
<td>Turkmenistan</td>
<td>3.332205</td>
<td>4.867534</td>
<td>15.55797</td>
<td>0.438998</td>
</tr>
<tr>
<td>9</td>
<td>UZB</td>
<td>Uzbekistan</td>
<td>8.364974</td>
<td>9.549452</td>
<td>18.41062</td>
<td>0.822167</td>
</tr>
<tr>
<td>10</td>
<td>KAZ</td>
<td>Kazakhstan</td>
<td>9.864799</td>
<td>8.313057</td>
<td>19.49428</td>
<td>0.71986</td>
</tr>
<tr>
<td>11</td>
<td>VNM</td>
<td>Vietnam</td>
<td>8.172447</td>
<td>9.796904</td>
<td>19.10794</td>
<td>0.571031</td>
</tr>
<tr>
<td>12</td>
<td>Pak</td>
<td>Pakistan</td>
<td>10.12274</td>
<td>13.33776</td>
<td>19.10786</td>
<td>0.811313</td>
</tr>
<tr>
<td>13</td>
<td>POL</td>
<td>Poland</td>
<td>9.155462</td>
<td>13.03737</td>
<td>19.60426</td>
<td>0.811313</td>
</tr>
<tr>
<td>14</td>
<td>MYS</td>
<td>Malaysia</td>
<td>7.615298</td>
<td>12.39327</td>
<td>19.35511</td>
<td>0.559786</td>
</tr>
<tr>
<td>15</td>
<td>THA</td>
<td>Thailand</td>
<td>8.991189</td>
<td>8.923191</td>
<td>19.61714</td>
<td>0.543572</td>
</tr>
<tr>
<td>16</td>
<td>RUS</td>
<td>Russia</td>
<td>11.77044</td>
<td>14.77017</td>
<td>21.24965</td>
<td>0.779435</td>
</tr>
<tr>
<td>17</td>
<td>IND</td>
<td>India</td>
<td>13.95513</td>
<td>14.70581</td>
<td>21.62617</td>
<td>1</td>
</tr>
<tr>
<td>18</td>
<td>DEU</td>
<td>Germany</td>
<td>11.28296</td>
<td>12.66592</td>
<td>20.49934</td>
<td>0.739192</td>
</tr>
<tr>
<td>19</td>
<td>CHN</td>
<td>China</td>
<td>13.43162</td>
<td>15.7142</td>
<td>23.00959</td>
<td>0.785696</td>
</tr>
</tbody>
</table>

Suppose \( M = \{1, 2\} \) and \( N = \{1\} \). We want to calculate the trade-off response (i.e., change in the \( z_1 \)) when applied to the first good output PKM \((y_1)\) and then the second good output TKM \((y_2)\).
It is important to note that for all effective countries, \( h = 0.5 \) and \( h = -0.5 \) produce feasible points. This means that the increase or decrease of 0.5 units (i.e., between 0.5 and \(-0.5\)) in good output can further improve the efficiency due to the reduction in the excessive CO\(_2\) emissions (which is an indirect measure). Additionally, this is, of course, while maintaining DMU’s efficiency point on the frontier boundary even if there is no improvement (i.e., no change).

Tables 4 and 5 show the results with two different values of \( h \) (i.e., \(+0.5\), \(-0.5\)) applied to the two good output variables PKM \((y_1)\) and TKM \((y_2)\). In each resultant table, the first column shows the new values of the CO\(_2\) \((z^+1)\), when the PKM \((y_1)\) or TKM \((y_2)\) is a trade-off. The third column shows the trade-off rates \( \tau^+1 \) in each table.

Now, we analyze the results of four efficient countries, namely, Bangladesh (BGD), Cambodia (KHM), Myanmar (MMR), and India (IND). Each good output \((y_1\) or \(y_2)\) is increased individually (one at a time) by 0.5 unit; the excessive CO\(_2\) is either reduced or remains the same. Even if there is an increase in the already bloated bad output, it still does not take the DMU (i.e., Myanmar “MMR”) off the efficient frontier due to the selection of the feasible trade-off margins \((h = +0.5, -0.5)\); see Figure 1.

![Figure 1. Trade-off response to bad output CO\(_2\) \((z^1)\) with respect to PKM \((y_1)\) and TKM \((y_2)\).](image)

**Table 4.** Resulting trade-off when \( h = 0.5 \) and \( d = (1, 1) \).

<table>
<thead>
<tr>
<th>Sr.</th>
<th>DMU (_j)</th>
<th>( z^+_1 )</th>
<th>( \tau^+_11 )</th>
<th>( z^+_2 )</th>
<th>( \tau^+_12 )</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BGD2018</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>KHM2018</td>
<td>3.8304</td>
<td>0</td>
<td>3.6066</td>
<td>(-0.2238)</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>MMR2018</td>
<td>3.8312</td>
<td>(-0.3013)</td>
<td>4.1325</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>IND2018</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 5.** Resulting trade-off when \( h = -0.5 \) and \( d = (1, 1) \).

<table>
<thead>
<tr>
<th>Sr.</th>
<th>DMU (_j)</th>
<th>( z^-1 )</th>
<th>( \tau^-11 )</th>
<th>( z^-2 )</th>
<th>( \tau^-12 )</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>BGD2018</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>KHM2018</td>
<td>3.8304</td>
<td>0</td>
<td>4.0715</td>
<td>0.2411</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>MMR2018</td>
<td>4.4562</td>
<td>0.3237</td>
<td>4.1325</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>IND2018</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Tables 4 and 5 show that there is no room available for a trade-off in Bangladesh (BGD) and India (IND), and they are already at the optimal level. However, considering Cambodia (KHM), if we change the value by \( h = 0.5 \), then the increase in \( y_1 \). from its current value is 3.806 to 4.306, with \( d = (1, 1) \); see Table 3. The average \( CO_2 \) emissions value \( (z_1) \) remains the same (i.e., when \( h = 0 \), 3.8304) with respect to \( y_1 \), but it reduces to 3.6066 for \( y_2 \), with trade-off rate \( \tau_{12}^+ = -0.2238 \). In Myanmar (MMR), if we increase 0.5 units in its available factor \( y_1 \) (from 8.3339 to 8.8339; see Table 3), the excessive bad output \( z_1 \) is decreased to 3.8312 with \( \tau_{11}^+ = -0.3013 \). Nonetheless, if the same is applied for \( y_2 \) again there is no change in bad output \( CO_2 \) (i.e., 4.1325), with \( \tau_{12}^- = 0 \); see Table 4. It is interesting to note that the DMUs Cambodia (KHM) and Myanmar (MMR) are further improved, which were found as weakly efficient in the first place.

Now, let us consider \( h = -0.5 \), and the results are given in Table 5. Again, there is no room for trade-offs in Bangladesh (BGD) and India (IND). Consider Cambodia (KHM) again; if the factor is reduced from its original value of 8.8529 to 8.3529 \( (h = -0.5) \), and \( d = (1, 1) \), then \( y_1 \) appears the same as when \( h = 0 \), i.e., 3.8304. Besides, with the trade-off \( \tau_{12}^- = 0.2411 \), the average bad output \( CO_2 \) increases to 4.0715. It is worth noting that when we ran the program against different non-negative numbers of \( d \), it still led to similar results.

We have also applied the same program to the inefficient countries to see the ideal value of \( h \), and we found that at \( h = +0.5 \) change in \( y_1 \) (PKM), there is an improvement in almost all the inefficient countries also; see Figure 2.

![Figure 2. The optimal marginal value of h with trade-off experiments.](image)

6. Conclusions

From different perspectives, the quantitative calculation of the trade-off margins is very troublesome. In management studies, existing trade-off methods use theoretical logic, assumptions, and hypotheses to conceptualize the maximum change; that is, when we change one variable with a throughput, another variable will have an impact. However, we put a practical quantitative example of this powerful concept of the marginal trade-off in the transportation operations’ context. The marginal trade-off appeared to be the most effective method. It extends the concept of performance trade-offs to bring an optimal efficiency among the decision-making units (DMUs) at the efficient frontier.

DEA production technology is used to analyze the trade-off behavior among the output variables. This has enabled us to analyze the results of several outputs, which is different from the traditional practice of measuring trade-offs (i.e., only between two variables). It is possible to get different trade-off margins for each efficient frontier point. In this study, we perform marginal trade-offs with the margin
We applied these trade-offs on two good outputs (namely, passengers-kilometers $y_1$ and tone-kilometers $y_2$) to see the efficiency brought the bad output variable CO$_2$ ($z_1$). We established that at $h = +0.5$ in output $y_1$, the optimal efficiency can be achieved for the weakly efficient DMUs. Moreover, considering the overall efficiency, this trade-off ($h = +0.5$) has increased the efficiency of the majority of the DMUs without losing the relative efficiency of the unchanged DMUs. The obtained results prove that marginal trade-offs can bring even those DMUs to the optimal point (closest efficient point on the frontier boundary), which appeared to be weakly efficient in the slacks. Since they are on or very close to the frontier line already, the further improvement in the efficiency (of the weakly efficient countries) bring the DMUs to the optimally efficient point.

It is interesting to note that, when we applied different values within the selected marginal range (i.e. $+0.5$, $−0.5$), the results were the same. In comparison with the recent related work of cost efficiency or technical efficiency problems [2,15,54], this research follows the suggested direction of the multiple output estimation. However, we do not take the desirable outputs only but the undesirable output (effecting the cost indirectly) is also been considered. And the conditional DEA concept also gets contributed with this work.

The results obtained using real numerical data proves the applicability of this method in different transportation energy efficiency assessments and improvements. However, this study is limited to output variables (two good and one lousy variable). Future researches can be expanded by considering more output variables or more comprehensive data (including input variables), such as, applying marginal trade-offs to input variables and examine the behavior of output variables to find better effective possibilities. The marginal trade-off can help the managers and policymakers in effective decision-making, and it is further recommended to address efficiency damages (by the undesired outputs).

**Author Contributions:** U.A. and M.A. collected, designed, and analyzed the experiments. M.A.K., É.Z.T.N. and J.O. contributed to authenticating the analysis tool and results. U.A. wrote the paper. All authors have read and agreed to the published version of the manuscript.

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**Conflicts of Interest:** The authors declare no conflict of interest.

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