A Comparative Analysis of a Power System Stability with Virtual Inertia

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Abstract: The paper investigates the stability of a power system with synchronverters. A synchronverter is a control strategy for voltage source converters that introduces virtual inertia by mimicking synchronous machines. The authors picked a commonly known IEEE 9 bus and IEEE 39 bus test case systems for the test case studies. The paper presents the power system’s modal analysis with Voltage Source Converters (VSCs) controlled as synchronverters, vector control, or Rate of Change of Frequency-based Virtual Synchronous Generator, thus comparing different approaches to VSC control. The first case study compares selected control algorithms, the IEEE 9 bus system, with one VSC in the paper. The results demonstrate the benefits of synchronverters over other control strategies. The system with synchronverters has a higher minimal damping ratio, which is proven to be the case by numerical simulations. In the second case study, the effects of virtual inertia placement were investigated. The computations showed that placement is indeed important, however, the control strategy is as important. Besides, the system with synchronverters exhibits better stability characteristics. The paper demonstrates that the application of synchronverters is feasible and can meet the demand for algorithms that bring the benefits of virtual inertia.

Keywords: power system stability; virtual inertia; synchronverter; small-signal stability; power system modeling; renewables

1. Introduction

Power systems have dramatically changed in the last decade. The penetration of Renewable Energy Sources (RESs) combined with plans to phase out nuclear power plants in many countries shapes the industry. Those developments brought new dilemmas to power system operators and planners. A major one is how to deal with low grid inertia. There is no doubt that inertia plays a significant role in short-term frequency control. Since it naturally stores energy, which is used during a spike in a power imbalance in the grid. The importance of inertia for frequency regulation is demonstrated in Figure 1.

Therefore, many people in the industry are trying to solve the upcoming problem with control of a low inertia grid. Some researchers proposed to change the requirements on Rate of Change of Frequency (ROCOF) as a simple solution. This approach was adopted in Ireland [1]. There are other proposals, one that stands out the most is an introduction of virtual inertia. For instance, Hydro-Québec started to worry about low inertia in 2003 and set up requirements for power plant operators [2]. Similar ideas were proposed across the Atlantic by ENTSO-E in 2017 [3]. However, this document does not set mandatory requirements but instead introduces National Power System operators’ guidelines in Europe. Furthermore, it outlines some examples of Canada that already require frequency response by wind power plants. Reference [2] probably preceded others in setting up virtual inertia guidelines.

The most cost-effective solution to a low inertia grid is to make RES power plants behave like synchronous generators. The fundamental idea is to control VSC so it will respond to a disturbance like a synchronous generator. Intensive research of that topic
yielded many different control strategies. The authors primarily divide them into two categories grid following and grid forming. The grid-following control for synchronization requires a stiff grid reference. Vector control is typical grid-following control that requires Phase Locked Loop (PLL) for synchronization with the grid [5]. Different proposed control strategies emulate inertia response, however, some still do it through PLL-provided measurement of frequency [6,7]. The authors call such control strategies based on frequency measurement Rate of Change of Frequency-based Virtual Synchronous Generators or in abbreviation ROCOF VSG. Some of such proposed topologies [8] utilize simple power-based calculations. Others build on vector control and introduce voltage reference signal that emulate inertia response [9,10]. The other control approach is to emulate synchronous generators and make a VSC essentially a grid-forming device. Synchronous generator based topologies are synchronverters [11,12] and Virtual Synchronous Machine (VISMA) [13,14]. The advantage that emulation of inertia brings that it becomes a tunable parameter in the system. The rotating mass of the rotor gives the inertia of the generator. The energy storage limits virtual inertia on the other hand that Renewable Energy Source has. Usually, it is a capacitor or a battery.

Figure 1. Time intervals of frequency response during a contingency. Reprinted from [4].

Existing research papers mainly focused on analyzing the stability of a synchronverter connected to simple networks without comparing them to other common control strategies. For instance an interesting studies [15,16] on the application of synchronverters to damp oscillations grid did not discuss other possible control algorithms that can be an alternative to synchronverters. Additionally, in these papers, the dynamics of the DC bus is not considered. Furthermore, paper [17] investigates the interaction between grid following and grid forming devices. However, the test case system is focused on the stability of power converters, not on the power system dynamics with several generators. Thus, the main contribution of the paper to the field of power system stability is the following. The research focuses on the dynamics and stability of a power system with multiple generators. The paper compares different VSC control strategies that represent the main categories outlined by the authors. The chosen control strategies are vector control, ROCOF VSG [9] and synchronverters [11]. The paper presents the comparative analysis of the investigated control topologies in IEEE 9 bus and IEEE 39 bus benchmark systems. For the comparison, the authors conducted modal analysis and numerical simulations to verify the conclusions. In the first case, the authors analyze the IEEE 9 bus system’s stability with only one Photovoltaic Power Plant. In the second case where the test case system is large, authors investigate the effects of placement of VSCs with different control strategies. The conclu-
sion emerged from the placement analysis that the system’s performance depends not only on the placement of virtual inertia but on the control strategy that implements it.

2. Mathematical Models of the Devices

The section provides mathematical models of the devices connected to the power system. Furthermore, it explains some assumptions that have been made in the modeling. Additionally, it briefly describes how the devices can be connected to the network.

2.1. Synchronous Generator: Two-Axis Model

The two-axis model or 4th order model is used in the power system model described in the book [18]. This model closely enough approximates the generator’s behavior even though it neglects the dynamics of one damping winding in \( d \) and one in \( q \) axis. Furthermore, the stator transients are neglected [19]. This introduces some errors, however, the results are more conservative. Thus it is not a problem for stability studies. For the \( i \)th generator, the model can be stated as follows.

\[
T_{d,0,i} \frac{dE_{d,i}}{dt} = -E_{d,i} - (x_{d,i} - x'_{d,i})i_{d,i} + E_{f,d,i}
\]  

\[
T_{q,0,i} \frac{dE_{q,i}}{dt} = -E_{q,i} + (x_{q,i} - x'_{q,i})i_{q,i}
\]  

\[
\frac{d\delta_{i}}{dt} = \omega_{0}(\omega_{i} - \omega_{s})
\]  

\[
J_{i} \frac{d\delta_{i}}{dt} = M_{\text{mech},i} - E_{q,i}i_{q,i} - E_{d,i}i_{d,i} - (x_{d,i} - x'_{d,i})i_{d,i}i_{q,i} - T_{FW}
\]  

Stator algebraic equations:

\[
E_{q,i} = R_{s,i}i_{q,i} + x'_{d,i}i_{d,i} + v_{q,i}
\]  

\[
E_{d,i} = R_{s,i}i_{d,i} - x'_{q,i}i_{q,i} + v_{d,i}
\]

- \( E_{q,i}, E_{d,i} \) transient voltage in \( d - q \) axis
- \( E_{f,d,i} \) field voltage
- \( i_{q,i}, i_{d,i} \) current in \( d - q \) axis
- \( J_{i} \) inertia constant
- \( \omega_{i} \) generator angular velocity
- \( \delta_{i} \) load angle
- \( \omega_{0} \) nominal angular velocity
- \( \omega_{s} \) synchronous angular velocity
- \( T_{FW} \) damping torque to emulate the effect of damping windings

2.2. VSC Electric Part Model

Hereafter the circuit equations of the Voltage Source Converter are stated. Figure 2 shows the converter’s line diagram with an L-type filter. A further assumption is to neglect DC side switching losses since they are not relevant for grid dynamics/interaction. Thus it simplifies the dynamics of the DC side of the VSC.

\[
\frac{m_{d}v_{dc}}{2} = L_{f} \frac{di_{d,i}}{dt} - \omega_{s}L_{f}i_{q,i} + R_{f}i_{d,i} + v_{d,i}
\]  

\[
\frac{m_{q}v_{dc}}{2} = L_{f} \frac{di_{q,i}}{dt} + \omega_{s}L_{f}i_{d,i} + R_{f}i_{q,i} + v_{q,i}
\]  

\[
C_{dc} \frac{dv_{dc}}{dt} = \frac{p_{pv} - p_{vsc}}{v_{dc}}
\]
\[ p_{\text{esc}} = \frac{m_{q}v_{dc}i_{d} + m_{q}v_{dc}i_{q}}{2} \]  

\[ L_{f,i} \quad R_{f,i} \quad m_{q,i}, m_{d,i} \quad C_{dc} \quad v_{dc} \quad i_{q,i}, I_{d,i} \quad p_{pv} \quad p_{\text{esc}} \quad v_{d,i}, v_{q,i} \]

Figure 2. VSC schematic diagram.

2.3. Synchronverter Model in \(d-q\) Reference Frame

The paper [11] introduced the notion of a synchronverter, a converter that mimics a synchronous generator. Figure 3 shows the block diagram of the synchronverter. This section provides the mathematical model in \(d-q\) frame of a synchronverter connected to the grid.

Figure 3. Synchronverter block diagram.

The main idea behind the synchronverter control strategy is that it has, to some degree, mimic the behavior of a synchronous machine [20]. Hence, the mathematical model
of synchronverters in the $d-q$ frame should be similar to the synchronous generator equations. Assuming that the virtual machine has only three winding in $d-q$ frame:

$$v_d = R_d i_d + \frac{d\psi_d}{dt} - \omega\psi_q$$  \hspace{1cm} (11)$$

$$v_q = R_q i_q + \frac{d\psi_q}{dt} + \omega\psi_d$$  \hspace{1cm} (12)$$

$$v_f = R_f i_f + \frac{d\psi_f}{dt}$$  \hspace{1cm} (13)$$

The relations between fluxes and currents are following:

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_f \end{bmatrix} = \begin{bmatrix} -L_{dd} & 0 & L_{fd} \\ 0 & -L_{qq} & 0 \\ -L_{df} & 0 & L_{ff} \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \\ i_f \end{bmatrix}$$  \hspace{1cm} (14)$$

where $L_{fd}$ is the magnetic coupling inductance between filed circuit and $d$ stator axis. Furthermore, in synchronverters, the assumption is that there is only one-way magnetic coupling i.e., from rotor to stator. Therefore, $L_{df} = 0$ and $L_{dd} = L_f$, $L_f$ being leakage inductance in a real machine or it is filter inductance, in the case of the synchronverter. Additionally, the derivative of the field current is assumed to be 0. With the assumption that $R_d = R_q = R_s$, $R_s$ is filter resistance Equations (11)–(13) can be simplified as follows

$$v_d = -R_s i_d + \omega_r L_f i_q - L_f \frac{di_d}{dt}$$  \hspace{1cm} (15)$$

$$v_q = -R_s i_q - \omega_r L_f i_d - L_f \frac{di_q}{dt} + \omega_r \psi_f$$  \hspace{1cm} (16)$$

where $\psi_f = L_{fd} i_f$

Clearly (15) and (16) resemble the equations of the VSC electrical part (7) and (8). Thus modulations signals become $m_d = 0$ and

$$m_q = \omega_r \psi_f$$  \hspace{1cm} (17)$$

The swing equation emulating the mechanical part:

$$\int \frac{d\omega_r}{\omega_r} = \frac{p_{mech}}{\omega_r} - \frac{p_{synch}}{\omega_r} - \frac{D}{\omega_r - \omega_n}$$  \hspace{1cm} (18)$$

where power injected by synchronverter $p_{synch}$ in a synchronverter rotating frame is calculated considering assumptions (10) and (17). Thus,

$$p_{synch} = \frac{\omega_r \psi_f v_{dc}}{2} i_q$$  \hspace{1cm} (19)$$

Virtual mechanical power input—$p_{mech}$ is set to control the DC bus voltage [21,22]. Thus, the equations for the controller are:

$$p_{mech} = K_{p,p}(v_{dc} - v_{dc,ref}) + p_{reg,pi}$$  \hspace{1cm} (20)$$

$$\frac{dp_{reg,pi}}{dt} = K_{i,p}(v_{dc} - v_{dc,ref})$$  \hspace{1cm} (21)$$

where $K_{p,p}$ and $K_{i,p}$ that are proportional and integral constants respectively
The virtual load angle of the synchronverter is computed similarly to a synchronous machine:

$$\frac{d\delta}{dt} = \omega_r - \omega_s$$  \hspace{1cm} (22)$$

As Figure 3 shows that in PCC the synchronverter can control reactive power injection and/or voltage. Assuming that the synchronverter is set to be P-V node then

$$\frac{d\psi}{dt} = K_{i_d}(v_{ref} - v_{pcc}) + K_{i_q}(q_{ref} - q)$$  \hspace{1cm} (23)$$

where \(v_{pcc}\) is the voltage amplitude in PCC and \(K_{i_d}\) is integral constant of voltage regulator.

### 2.4. Vector Control of VSC

Voltage Source Converters are usually equipped with vector control. Vector control block diagram is shown in Figure 4. This control strategy allows decoupling the control of active and reactive power injections using Park transformation (A1). In power system transient stability studies, the effects of some parts are neglected, for example, losses in the converter. The mathematical model is provided here in addition to the equations of the converter. Here the calculation of modulation of voltage vector is provided. It should be noted that the control objective in the authors’ models is the voltage in PCC, not the reactive power injection. Furthermore, the chosen model utilizes a PI regulator for DC voltage control and an Integral (I) regulator for PCC voltage. Additionally, for the voltage shift’s compensation due to filter resistance and reactance, the feedforward shift is used for modulation [23]. Synchronization with the grid is provided by synchronous reference frame PLL (SRF-PLL). A thorough derivation of equations is not necessary in this case since it is a commonly known control strategy.

**Figure 4.** Vector control block diagram.

$$m_d = v_d - \omega_L i_L + K_{p,i_d}(i_{d,ref} - i_d) + M_d$$  \hspace{1cm} (24)$$

$$m_q = v_q + \omega_L i_L + K_{p,i_q}(i_{q,ref} - i_q) + M_q$$  \hspace{1cm} (25)$$

$$\frac{dM_d}{dt} = K_{i_d}(i_{d,ref} - i_d)$$  \hspace{1cm} (26)$$

$$\frac{dM_q}{dt} = K_{i_q}(i_{q,ref} - i_q)$$  \hspace{1cm} (27)$$

$$\frac{dp_{reg,pi}}{dt} = K_{i,c}(v_{dc} - v_{dc,ref})$$  \hspace{1cm} (28)$$
\[
\frac{di_{d,ref}}{dt} = K_{i,p}(p_{reg,pi} + K_{p,dc}(v_{dc} - v_{dc,ref}) - p_{vsc}) 
\]
\[
\frac{di_{q,ref}}{dt} = K_{i,v}(v_{ref} - v_{pcc}) 
\]
\[
\frac{di_{pll}}{dt} = k_{i,pll}(v_q - v_q^{ref}) 
\]
\[
\frac{dv_{dc}}{dt} = I_{vll} + k_{p,pll}(v_{c,q} - v_{ref,q}) 
\]
\[
\frac{d\omega_{PLL}}{dt} = I_{PLL} + k_{p,PLL}(v_{c,q} - v_{reference,q}) 
\]
\[
\frac{d\theta_{PLL}}{dt} = \omega_{PLL} - \omega_s 
\]

Note, currents regulator constants are the same, thus \(K_{p,ij} = K_{i,qi}\) and \(K_{i,ij} = K_{i,qi}\).

2.5. ROCOF VSG

The third examined strategy is somewhat between vector control and synchronverters. It also provides an inertia response to disturbance. However, the inertia control response is built on top of vector control. There are different topologies that emulate inertia response by using frequency measurement. Authors name such control approach a Rate of Change of Frequency Virtual Synchronous Generator or in abbreviation ROCOF VSG. This particular topology was proposed in the paper [9]. Figure 5 shows the block diagram of the examined control strategy. Fundamentally the mathematical model of this topology is similar to vector control with only one difference. The DC bus voltage reference consists of two signals. The first is the voltage bus reference, and the second is the virtual inertia response signal that modifies the DC bus reference. Thus to build mathematical model of a VSC control as ROCOF VSG in the Equations (25)–(33) DC bus voltage reference signal is going to be:

\[
v_{dc,ref} = v_{dc,nominal} + V_{J,ref} + D_p(\omega_{PLL} - \omega_n) 
\]

The control signal of virtual inertia response emulation is as follows:

\[
\frac{dV_{J,ref}}{dt} = (H_p\dot{\omega}_{PLL} - V_{J,ref}) / T_j 
\]

where \(H_p\) emulate inertia and \(D_p\) damping constants. The time delay \(T_j\) simulates the gradual power injection from the kinetic energy of a synchronous generator. Thus, ROCOF VSG provides a power response based on frequency change. It should be noted that this control strategy is not grid forming in itself. It does support frequency but still uses PLL to track the frequency of the grid. Therefore, ROCOF VSG is very sensitive to the ability of PLL to track frequency correctly without oscillations.

![Figure 5. ROCOF VSG (Rate of Change of Frequency Virtual Synchronous Generator) block diagram.](image-url)
2.6. Network Model

In transient stability studies, conventionally, the dynamics of lines are neglected. Since in comparison to time constants of machines, the propagation in lines happens almost instantly. Therefore the network equations become algebraic equations. The general form of a power system mathematical model has the following form:

\[
\frac{dx}{dt} = f(x, y, t) \tag{36}
\]

\[0 = g(x, y, t) \tag{37}\]

where \(x\) is the state vector, which depending on model, may contain variables of current injections, excitation controllers, turbine governors etc. Additionally, \(y\) vector usually corresponds to voltages.

The general form of network algebraic equations that describe power flow is following

\[Y \cdot V = I(x, V) = 0 \tag{38}\]

where \(I(x, V)\) respects variable impedance loads. However, for simplifications often the constant impedance loads are used in stability studies \([18,24,25]\). Thus, equations can be simplified

\[Y \cdot V = I(x) \tag{39}\]

Figure 6 shows the relation between different reference frames. In the authors’ implementation, all devices are modeled in their own rotating frame for more modular code, and the current injections are transformed using Park transformation Equation (A1) (Appendix A). The transformation between different \(d - q\) rotating frames can be made directly by simplifying transformation matrices. Let’s examine \(i\)th generator with angular velocity \(\omega_{r,i}\). The network has synchronous velocity \(\omega_s\), sometimes it is calculated as a fictitious center of inertia reference or fixed to one of the generators. Consequently, machine electrical angle is \(\theta_r = \int \omega_r dt\) or \(\theta_r = \omega_st + \delta_i\). As Figure 6 shows to transform a vector from machine \(d - q\) reference frame to two network \(d - q\) reference frame. Firstly, the Inverse of the park transformation Equation (A1) where angles are machine electrical angle must be applied. Then from \(abc\) natural frame back to network \(d - q\) frame, the park transformation Equation (A1) with grid electrical angle is applied. It can be written more strictly in matrix form:

\[\vec{f}_{grid} = T_{dq,grid} \cdot T_{dq,i}^{-1} \cdot \vec{f}_{rotor} \tag{40}\]

Assuming that zero component can be neglected, i.e., the symmetrical network:

\[
\begin{bmatrix}
    f_{d,i}^\text{grid} \\
    f_{q,i}^\text{grid}
\end{bmatrix} = \begin{bmatrix}
    \cos(\delta_i) & -\sin(\delta_i) \\
    \sin(\delta_i) & \cos(\delta_i)
\end{bmatrix} \begin{bmatrix}
    f_{d,i}^\text{rotor} \\
    f_{q,i}^\text{rotor}
\end{bmatrix} \tag{41}
\]
3. Stability Comparison

In this section, the authors compare a power system’s stability with a photovoltaic power plant controlled as a synchronverter, or vector control strategy, or ROCOF VSG. Additionally, a brief description of the methodology that was used for comparison is provided. For the case studies, the authors picked a commonly known IEEE 9 bus and IEEE 39 bus test systems [26]. IEEE 9 bus is complex enough but still can be easily understood. Therefore, for the IEEE 9 bus test system, the authors chose to carry out modal analysis and numerical simulations. The modal analysis provides an insight into system dynamics, and transient stability verifies whether the system can remain stable after big disturbances such as faults. For the case study of VSCs in the IEEE 39 bus test system, authors using modal analysis compare the quality of control for different configurations of the network. This test system allows demonstration of the effects of placement of virtual inertia since it is much larger. The mathematical models of the devices were implemented in Wolfram Mathematica [27]. Wolfram Mathematica combines powerful symbolic capability with all possible numerical tools for solving algebraic differential equations.

3.1. Modal Analysis

For small-signal stability analysis the general form of power system mathematical model (36) and (37) is linearized. Hence, the linear form of the model is as follows:

\[ \Delta \dot{x} = A \cdot \Delta x + B \cdot \Delta v \]  

(42)

where \( \Delta x \) is the state vector of the power system, \( A \) is the system matrix corresponding to devices constants and \( B \) is the input matrix that correlates with node voltages. Algebraic equations describe the power flow in the network.

\[ \Delta i = Y_N \cdot \Delta v \]  

(43)

where \( Y_N \) network admittance matrix and \( \Delta i \) is the vector of current injections.

The current injections of the devices are the following:

\[ \Delta i = C_D \cdot \Delta x + D_D \cdot \Delta v \]  

(44)

where \( C_D \) and \( D_D \) is matrix corresponding to individual devices. Thus, algebraic equations can be simplified:

\[ Y_N \cdot \Delta v = C_D \cdot \Delta x + D_D \cdot \Delta v \]  

(45)

Then the solution for the vector of voltages is following:

\[ \Delta v = (Y_N - D_D)^{-1} C_D \cdot \Delta x \]  

(46)

Hence, the system of differential-algebraic equations become the system of only differential equations:

\[ \Delta \dot{x} = A \cdot \Delta x + B(Y_N - D_D)^{-1} C_D \cdot \Delta x \]  

(47)

Consequently, \( A_{sys} = A + B(Y_N - D_D)^{-1} C_D \) is new system matrix with reduced algebraic equation. Thus, the small-signal stability can be analyzed by finding the eigenvalues of \( A_{sys} \). Furthermore, the damping ratio \( \zeta_i \) of individual eigenvalues might be evaluated using the following formula:

\[ \zeta_i = \frac{\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \]  

(48)

where the assumed form of the \( i \)th eigenvalue is \( \lambda_i = \sigma_i + j\omega_i \). The eigenvalues of the system matrix obtained by previous computations are then used for comparing the stability of examined strategies. The Lyapunov indirect method allows gaining insights into the system dynamics without directly solving differential equations. The ability of a linear
system to return to the steady-state after disturbance is defined by \( \max(\Re(\lambda)) \). Furthermore, the damping ratio of eigenvalues predicts whether the oscillations provided by the eigenvalue persist only for a short time after disturbance. Fundamentally, it is a relation between exponential decay of the amplitude and the oscillations given by the complex part of the eigenvalue. Moreover, as part of the modal analysis, the authors calculated the participation factors for individual modes. A more thorough explanation can be found in Sauer’s book [18], or Kundur’s book [24]. The participation matrix is calculated as follows:

\[
p_{k,i} = \frac{|w_{k,i}| |v_{k,i}|}{\sum_{k=1}^{n} |w_{k,i}| |v_{k,i}|} \tag{49}
\]

where \( w_{k,i} \) and \( v_{k,i} \) are components of left and right eigenvectors corresponding to \( i \)th eigenvalue. Inherently the sum of all participation factors for individual modes is equal to unity.

3.2. Case Study: IEEE 9 Bus

Western Electricity Coordinating Council three-machine system (Figure 7) is a classic test case system for stability studies. The model includes three machines, nine buses, and three loads. The authors chose to use constant impedance loads in the models. Furthermore, the machines are equipped with IEEE type 1 exciter and Steam turbine governor model (can be found in Sauer’s book [18]). Parameters of the system can be found in Appendix B.

In this test case, the PS with only one VSC connected to node 6 was analyzed. The PV power production was 0.6 pu. Firstly, the power system mathematical model was put together using equations in Section 2. After that, the equations were linearized, and the reduced system matrix was obtained. Computed eigenvalues of the system with synchronverters, vector control and ROCOF VSG are shown in Tables 1–3 correspondingly. Please note, only variables with a participation factor greater than 0.2 are displayed. Furthermore, the participation factor matrices are visualized in Figures A1–A3 (Appendix D). The eigenvalues with the smallest damping ratios \( \xi \) are highlighted in bold font in the tables. For the system with synchronverters, the damping ratio \( \xi_{\text{min}} = 0.0298 \), which was
the best result of all examined strategies. Vector control had the lowest $\xi_{\text{min}} = 0.0062$ damping ratio. The application of ROCOF VSG improved it, resulting in $\xi_{\text{min}} = 0.0112$. Figure 8 visualizes some of the results for demonstration of eigenvalues placements. In the case of synchronverters, most eigenvalues were closer to the x-axis, which means that the damping of these frequencies was higher than in the case of vector control or ROCOF VSG. Furthermore, the dashed lines (black for synchronverters, red for ROCOF VSG, and blue for vector control) show the lines corresponding to the smallest damping ratio. Additionally, in the case of synchronverters, the $\max(\Re(\lambda))$ was greater than for other control strategies. The maximum real part of the eigenvalues is a good indicator of the quality of control that provides information about the system’s ability to return to a steady-state after disturbance. For the ROCOF VCG and vector control, the eigenvalue with $\max(\Re(\lambda))$ was introduced by PLL.

**Table 1.** The eigenvalues and participation factors of the system with synchronverters.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Variable</th>
<th>PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{1,2} = -72.53 \pm 1265.85j$</td>
<td>$i_{q,s}, V_{dc}$</td>
<td>0.5, 0.2</td>
</tr>
<tr>
<td>$\lambda_{3,4} = -170.41 \pm 305.73j$</td>
<td>$\delta_{s}, \omega_{s}$</td>
<td>0.32, 0.27, 0.22</td>
</tr>
<tr>
<td>$\lambda_{5,6} = -0.64 \pm 12.86j$</td>
<td>$\omega_{3}, \delta_{2}$</td>
<td>0.38, 0.39</td>
</tr>
<tr>
<td>$\lambda_{7,8} = -5.5 \pm 7.95j$</td>
<td>$E_{q,d}, V_{r,2}$</td>
<td>0.4, 0.4</td>
</tr>
<tr>
<td>$\lambda_{9,10} = -5.34 \pm 7.93j$</td>
<td>$E_{q,d}, V_{r,3}$</td>
<td>0.38, 0.39</td>
</tr>
<tr>
<td>$\lambda_{11,12} = -5.25 \pm 7.86j$</td>
<td>$E_{q,d}, V_{r,1}$</td>
<td>0.39, 0.41</td>
</tr>
<tr>
<td>$\lambda_{13,14} = -0.25 \pm 8.56j$</td>
<td>$\omega_{2}, \delta_{2}$</td>
<td>0.29, 0.37</td>
</tr>
<tr>
<td>$\lambda_{15,16} = -1.15 \pm 6.69j$</td>
<td>$p_{\text{reg},\nu}, V_{dc}$</td>
<td>0.37, 0.32</td>
</tr>
<tr>
<td>$\lambda_{17} = 0.76$</td>
<td>$P_{s,v,2}, P_{s,v,3}$</td>
<td>0.27, 0.38</td>
</tr>
<tr>
<td>$\lambda_{18,19} = -5.26 \pm 0.45j$</td>
<td>$E_{d,2}, E_{d,3}, P_{s,v,3}$</td>
<td>0.2, 0.24, 0.25</td>
</tr>
<tr>
<td>$\lambda_{20} = -5.11$</td>
<td>$P_{s,v,1}$</td>
<td>0.79</td>
</tr>
<tr>
<td>$\lambda_{21} = -3.82$</td>
<td>$E_{d,2}, E_{d,3}$</td>
<td>0.39, 0.44</td>
</tr>
<tr>
<td>$\lambda_{22} = -3.23$</td>
<td>$E_{d,1}$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_{23} = -2.37$</td>
<td>$M_{\text{mech},1}, M_{\text{mech},2}$</td>
<td>0.44, 0.45</td>
</tr>
<tr>
<td>$\lambda_{24} = -2.32$</td>
<td>$M_{\text{mech},3}$</td>
<td>0.56</td>
</tr>
<tr>
<td>$\lambda_{25,26} = -1.12 \pm 0.82j$</td>
<td>$\omega_{1}$</td>
<td>0.23</td>
</tr>
<tr>
<td>$\lambda_{27,28} = -0.39 \pm 0.97j$</td>
<td>$E_{q,1}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda_{29,30} = -0.44 \pm 0.75j$</td>
<td>$E_{q,1}, R_{f,1}$</td>
<td>0.25, 0.2</td>
</tr>
<tr>
<td>$\lambda_{31,32} = -0.43 \pm 0.49j$</td>
<td>$E_{q,3}, R_{f,3}$</td>
<td>0.26, 0.31</td>
</tr>
<tr>
<td>$\lambda_{33} = -0.09$</td>
<td>$\psi_{3}$</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Figure 8. Eigenvalues of the IEEE 9 bus system with VSC connected to Node 6. Black dashed line shows $\xi_{\text{min}} = 0.0298$ of the system with synchronverters. Red dashed line corresponds to $\xi_{\text{min}} = 0.0112$ for ROCOF VSG control. Blue dashed line corresponds to $\xi_{\text{min}} = 0.0062$ for vector control.
Table 2. The eigenvalues and participation factors of the system with vector control.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Variable</th>
<th>PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{1,2} = -34.72 \pm 738.89j$</td>
<td>$i_{d,vsc}, V_{dc}$</td>
<td>0.49, 0.5</td>
</tr>
<tr>
<td>$\lambda_3 = -74.59$</td>
<td>$i_{q,vsc}$</td>
<td>0.92</td>
</tr>
<tr>
<td>$\lambda_{4,5} = -0.17 \pm 27.3j$</td>
<td>$\omega_{p}, \theta_{p}$</td>
<td>0.49, 0.49</td>
</tr>
<tr>
<td>$\lambda_{6,7} = -0.61 \pm 12.84j$</td>
<td>$\omega_{3}, \delta_3$</td>
<td>0.38, 0.39</td>
</tr>
<tr>
<td>$\lambda_{8,9} = -5.5 \pm 7.95j$</td>
<td>$E_{f,2}, V_{t,2}$</td>
<td>0.39, 0.4</td>
</tr>
<tr>
<td>$\lambda_{10,11} = -5.33 \pm 7.92j$</td>
<td>$E_{f,3}, V_{t,3}$</td>
<td>0.37, 0.38</td>
</tr>
<tr>
<td>$\lambda_{12,13} = -5.24 \pm 7.84j$</td>
<td>$E_{f,4,1}, V_{t,1}$</td>
<td>0.38, 0.39</td>
</tr>
<tr>
<td>$\lambda_{14,15} = -0.12 \pm 8.48j$</td>
<td>$\omega_2, \delta_2$</td>
<td>0.31, 0.4</td>
</tr>
<tr>
<td>$\lambda_{16} = -5.76$</td>
<td>$P_{sv,2}, P_{sv,3}$</td>
<td>0.26, 0.36</td>
</tr>
<tr>
<td>$\lambda_{17,18} = -5.26 \pm 0.45j$</td>
<td>$E_{d,2}, E_{d,3}, P_{sv,3}$</td>
<td>0.21, 0.23, 0.25</td>
</tr>
<tr>
<td>$\lambda_{19} = -5.16$</td>
<td>$P_{sv,1}$</td>
<td>0.76</td>
</tr>
<tr>
<td>$\lambda_{20} = -4.28$</td>
<td>$M_{d}$</td>
<td>0.53</td>
</tr>
<tr>
<td>$\lambda_{21} = -3.23$</td>
<td>$E_{d,1}$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_{22} = -3.04$</td>
<td>$E_{d,2}, E_{d,3}, M_{d}$</td>
<td>0.25, 0.28, 0.3</td>
</tr>
<tr>
<td>$\lambda_{23} = -2.37$</td>
<td>$M_{mech,1}, M_{mech,2}$</td>
<td>0.41, 0.46</td>
</tr>
<tr>
<td>$\lambda_{24} = -2.31$</td>
<td>$M_{mech,3}$</td>
<td>0.59</td>
</tr>
<tr>
<td>$\lambda_{25,26} = -1.03 \pm 0.95j$</td>
<td>$\omega_1$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\lambda_{27,28} = -0.42 \pm 1.15j$</td>
<td>$E_{q,1}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda_{29,30} = -0.15 \pm 1.17j$</td>
<td>$i_{d,ref}, M_{d}$</td>
<td>0.49, 0.48</td>
</tr>
<tr>
<td>$\lambda_{31,32} = -0.44 \pm 0.75j$</td>
<td>$E_{q,1}, R_{f,3}$</td>
<td>0.21, 0.28</td>
</tr>
<tr>
<td>$\lambda_{33,34} = -0.43 \pm 0.49j$</td>
<td>$E_{q,3}, R_{f,3}$</td>
<td>0.24, 0.29</td>
</tr>
<tr>
<td>$\lambda_{35} = -0.07$</td>
<td>$i_{q,ref}$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\lambda_{36} = -0.02$</td>
<td>$P_{reg,pi}$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_{37} = -0.02$</td>
<td>$I_{pil}$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3. The eigenvalues and participation factors of the system with ROCOF VSG.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Variable</th>
<th>PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{1,2} = -8.27 \pm 739.62j$</td>
<td>$i_{d,vsc}, V_{dc}$</td>
<td>0.49, 0.5</td>
</tr>
<tr>
<td>$\lambda_{3,4} = -1.25 \pm 26.25j$</td>
<td>$\omega_{p}, \theta_{p}$</td>
<td>0.44, 0.45</td>
</tr>
<tr>
<td>$\lambda_{5,6} = -11.41 \pm 13.03j$</td>
<td>$i_{q,vsc}, M_{d}$</td>
<td>0.44, 0.4</td>
</tr>
<tr>
<td>$\lambda_{7,8} = -0.67 \pm 12.91j$</td>
<td>$\omega_{3}, \delta_3$</td>
<td>0.39, 0.4</td>
</tr>
<tr>
<td>$\lambda_{9,10} = -5.49 \pm 7.95j$</td>
<td>$E_{f,2}, V_{t,2}$</td>
<td>0.38, 0.38</td>
</tr>
<tr>
<td>$\lambda_{11,12} = -5.32 \pm 7.91j$</td>
<td>$E_{f,3}, V_{t,3}$</td>
<td>0.32, 0.33</td>
</tr>
<tr>
<td>$\lambda_{13,14} = -5.24 \pm 7.82j$</td>
<td>$E_{f,4,1}, V_{t,1}$</td>
<td>0.32, 0.33</td>
</tr>
<tr>
<td>$\lambda_{15,16} = -0.31 \pm 8.56j$</td>
<td>$\omega_2, \delta_2$</td>
<td>0.31, 0.39</td>
</tr>
<tr>
<td>$\lambda_{17} = -5.85j$</td>
<td>$P_{so,2}, P_{so,3}$</td>
<td>0.3, 0.32</td>
</tr>
<tr>
<td>$\lambda_{18,19} = -5.26 \pm 0.44j$</td>
<td>$E_{d,2}, E_{d,3}, P_{so,3}$</td>
<td>0.21, 0.22, 0.23</td>
</tr>
<tr>
<td>$\lambda_{20} = -5.17$</td>
<td>$P_{so,1}$</td>
<td>0.81</td>
</tr>
<tr>
<td>$\lambda_{21} = -4.99$</td>
<td>$V_{f,ref}$</td>
<td>0.98</td>
</tr>
<tr>
<td>$\lambda_{22} = -3.29$</td>
<td>$E_{d,1}$</td>
<td>0.37, 0.39</td>
</tr>
<tr>
<td>$\lambda_{23} = -3.23$</td>
<td>$E_{d,2}, E_{d,3}$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_{24} = -2.37$</td>
<td>$M_{mech,1}, M_{mech,2}$</td>
<td>0.4, 0.48</td>
</tr>
<tr>
<td>$\lambda_{25} = -2.31$</td>
<td>$M_{mech,3}$</td>
<td>0.6</td>
</tr>
<tr>
<td>$\lambda_{26,27} = -1.01 \pm 0.96j$</td>
<td>$\omega_1$</td>
<td>0.28</td>
</tr>
<tr>
<td>$\lambda_{28,29} = -0.44 \pm 1.17j$</td>
<td>$E_{q,1}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\lambda_{30,31} = -0.02 \pm 1.17j$</td>
<td>$i_{d,ref}, M_{d}$</td>
<td>0.45, 0.45</td>
</tr>
<tr>
<td>$\lambda_{32,33} = -0.44 \pm 0.75j$</td>
<td>$E_{q,1}, R_{f,1}$</td>
<td>0.22, 0.28</td>
</tr>
<tr>
<td>$\lambda_{34,35} = -0.43 \pm 0.49j$</td>
<td>$E_{q,3}, R_{f,3}$</td>
<td>0.24, 0.29</td>
</tr>
<tr>
<td>$\lambda_{36} = -0.07$</td>
<td>$i_{q,ref}$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\lambda_{37} = -0.02$</td>
<td>$P_{reg,pi}$</td>
<td>1</td>
</tr>
<tr>
<td>$\lambda_{38} = -0.02$</td>
<td>$I_{pil}$</td>
<td>1</td>
</tr>
</tbody>
</table>
The most severe disturbance that occurs in PS is a three-phase fault. Therefore, the authors ran a simulation with the following scenario. The three-phase fault occurred on line 8–9. After the fault was cleared, the faulted line was disconnected. Clearing times $t_{cl} = 0.1$ s and $t_{cl} = 0.2$ s were considered. The simulation results for the load angle of the third machine are presented in Figures 9 and 10. For the $t_{cl} = 0.1$ s, as Figure 9 shows vector control and ROCOF VSG had higher load angle deviation from steady-state value than synchronverters. Besides, the ROCOF VSG expectedly was slightly better than vector control. For $t_{cl} = 0.2$ s, the system with vector control was unstable. ROCOF VSG allowed higher load angle deviation and had prolonged oscillations than synchronverters. It was expected because as Table 3 shows, ROCOF VSG had lower damping and greater $\max(\Re(\lambda))$. Therefore, the overall dynamic stability of the power system was increased by applying synchronverter control. The synchronverter increased the system’s total inertia, thus generators sped up longer due to higher system inertia, which resulted in lower load angle deviation from steady-state value. Since vector control did not provide additional inertia the generator loses synchronism for short-circuit with $t_{cl} = 0.2$ s. ROCOF VSG improved the system’s performance compared to vector control, but the resulting dynamics of the system had higher load angle deviations and oscillations than the system with synchronverters.

![Figure 9. IEEE 9 bus: Load angle of 3 generator during 3 phase fault for $t_{cl} = 0.1$ s.](image)

![Figure 10. IEEE 9 bus: Load angle of 3 generator during 3 phase fault for $t_{cl} = 0.2$ s.](image)

### 3.3. Case Study: IEEE 39 Bus

New England 10 machine system (Figure 11) is a well-known benchmark test system. This test case provides a fairly complex and close to the real power system model. In
the model, two VSC were considered, and the effects of control strategy on the system’s stability were investigated. Parameters of the system can be found in Appendix C.

![New England 10-machine, 39 bus system, Adapted from [28].](image)

The primary focus of the case study was to analyze the effect of placement of VSCs in different nodes. The following scenarios were considered. Two PV power plants connected through VSCs were placed in different nodes, excluding nodes connected to generators. Each power plant produced 0.8 pu. The following control quality criteria were computed: as in the previous case i.e., minimal damping and $\max(Re(\lambda))$. Both allowed us to gain insights into the dynamics of the system without solving the differential equations numerically. Figures 12–14 show that by using synchronverters in most cases the minimal damping ratio was improved. It should be noted that in some node pairs, the results are missing since this configurations had unstable eigenvalues, thus were excluded from calculation. The application of ROCOF VSG also improved damping but overall to a smaller degree than synchronverters.

![IEEE 39 bus: Damping ratios for VSCs with vector control connected to different nodes.](image)
Figures 13–17 show the results of computation of $\max(\Re(\lambda))$ for the IEEE 39 bus system with different control strategies. This quality of control index shows that the synchronverter improved the system’s ability to return to steady-state. Furthermore, it also shows that indeed the placement of virtual inertia was important and different configurations of the PS could result in substantial differences in the index value. On the contrary, for vector control, the calculated values of the $\max(\Re(\lambda))$ were the same for all considered configurations. It should be noted that for ROCOF VSG, the results were similar to vector control. That is explained by PLL primarily dictating the results for these control strategies. To summarize, the quality of control not only depended on the placement of virtual inertia [29] but on the control algorithm that it implemented. The results presented in this paper show that the quality of control could be affected by the control strategy for virtual inertia as much as placement.
4. Conclusions

The paper presents a thorough comparison of power system stability with different control strategies of VSCs. In the paper, three different control strategies were compared using modal analysis and numerical simulations. The first case study results proved that the synchronverter provides better damping and lower $\max(\Re(\lambda))$, which shows that it can reach steady-state faster. Furthermore, numerical simulations (Figure 9) demonstrated that Power System with synchronverters performs better during three-phase faults than PS with vector control or ROCOF VSG. ROCOF VSG also improves transient stability.
comparing to vector control and allows the system to withstand longer clearing times, but the synchronverter does it better, as results provided in Figure 10 show. In the second case study, the effects of placement were analyzed in IEEE 39 bus benchmark system. The results of the analysis yielded that for vector control and VSG ROCOF, the configuration of the network does not change \( \max(Re(\lambda)) \). However, for synchronverters, this parameter is very changed with the placement of VSCs. Furthermore, the presented results that synchronverters are better than other control strategies in almost every possible configuration in terms of damping and \( \max(Re(\lambda)) \). The overall conclusion is that the control strategy that provides virtual inertia is as important as the placement of VSCs.

To summarize, the presented cases demonstrate that the power system with synchronverters performs better than the one with vector control or ROCOF VSG. From the point of modal analysis, the stability is improved by the introduction of synchronverters. Furthermore, transient stability performance is indeed enhanced by the application of a synchronverter. In the authors’ opinion, synchronverters are a compelling control strategy that introduces virtual inertia to the system and meets the modern power system’s requirements. Additionally, virtual inertia becomes a tunable parameter that gives more flexibility to power system operators.

**Author Contributions:** Conceptualization, L.V. and Z.M.; methodology, L.V.; software, L.V.; validation, Z.M.; formal analysis, L.V. and Z.M.; investigation, L.V.; resources, Z.M.; writing—original draft preparation, L.V.; writing—review and editing, L.V.; visualization, L.V.; supervision, Z.M.; project administration, Z.M.; funding acquisition, Z.M. Both authors have read and agreed to the published version of the manuscript.

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**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Abbreviations**
The following abbreviations are used in this manuscript:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>VSC</td>
<td>Voltage source converter</td>
</tr>
<tr>
<td>VSG</td>
<td>Virtual Synchronous Generator</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>PF</td>
<td>Participation Factor</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>PS</td>
<td>Power System</td>
</tr>
<tr>
<td>PV</td>
<td>Photovoltaic</td>
</tr>
<tr>
<td>ROCOF</td>
<td>Rate of Change of Frequency</td>
</tr>
<tr>
<td>SRF</td>
<td>Synchronous Reference Frame</td>
</tr>
<tr>
<td>RES</td>
<td>Renewable Energy Source</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
</tbody>
</table>

**Appendix A. Park Transformation**

\[
T_{dq} = \frac{2}{3} \begin{bmatrix}
\cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\
-\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\] (A1)
Appendix B. IEEE 9 Bus: System Parameters

Machine parameters: $H_1 = 23.64$, $H_2 = 6.40$, $H_3 = 1.50$, $x_{d1} = 0.146$, $x_{d2} = 0.8958$, $x_{q1} = 1.3125$, $x'_{d1} = 0.0608$, $x'_{d2} = 0.1198$, $x'_{q1} = 0.1813$, $x_{q2} = 0.0969$, $x_{q3} = 0.8645$.

IEEE 9 bus: Matrix of Participation factors for the system with synchroneverter.

Appendix C. IEEE 39 Bus: System Parameters

VSC parameters: $S_n = 100$ MVA, $L_f = 9.737$ mH, $R_f = 147.402$ µOhm, $C_{dc} = 722$ mF, $V_{dc} = 40$ kV.

Synchronverter parameters: $J = 0.2$, $D_p = 0.15$, $K_{i,v} = 0.15$, $K_{p,p} = 57.32 \cdot 10^{-4}$, $K_{i,p} = 859.87 \cdot 10^{-4}$.

Vector Control: $K_{i,v} = 0.64$, $K_{p,d} = 38.2 \cdot 10^{-4}$, $K_{i,q} = 0.1433$, $K_{i,p} = 1.43$, $K_{p,dc} = 3.82$.

ROCOF VSG: $D_p = 0.1$, $T_j = 0.2$, $H_p = 5$. All other parameters similar to vector control.

Appendix D. Figures

Figure A1. IEEE 9 bus: Matrix of Participation factors for the system with synchroneverter.
Figure A2. IEEE 9 bus: Matrix of Participation factors for the system with vector control.

Figure A3. IEEE 9 bus: Matrix of Participation factors for the system with ROCOF VSG.

References


