

Review

An Investigation into the Energy-Efficient Motion of Autonomous Wheeled Mobile Robots

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Abstract: In recent years, the use of electric Autonomous Wheeled Mobile Robots (AWMRs) has dramatically increased in transport of the production chain. Generally, AWMRs must operate for several hours on a single battery charge. Since the energy density of the battery is limited, energy efficiency becomes a key element in improving material transportation performance during the manufacturing process. However, energy consumption is influenced by the navigation stages, because the type of motion necessary for the AWMR to perform during a mission is totally defined by these stages. Therefore, this paper analyzes methods of energy efficiency that have been studied recently for AWMR navigation stages. The selected publications are classified into planning and motion control categories in order to identify research gaps. Unlike other similar studies, this work focuses on these methods with respect to their implications for the energy consumption of AWMRs. In addition, by using an industrial Self-Guided Vehicle (SGV), we illustrate the direct influence of the motion planning stage on global energy consumption by means of several simulations and experiments. The results indicate that the reaction of the SGV in response to unforeseen obstacles can affect the amount of energy consumed. Hence, energy constraints must be considered when developing the motion planning of AWMRs.

Keywords: navigation; motion planning; dynamic; energy efficiency; autonomous wheeled mobile robots; self-guided vehicles



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1. Introduction

Robots are intelligent machines that sense, process, communicate, and perform multiple tasks through preprogramming. Therefore, they are replacing humans in different activities, such as human-centered intelligent robots, which cover all robot capabilities [1]. Moreover, robots are capable of operating in various environments, including in ground, air, water, and underwater surroundings [2]. Autonomous Wheeled Mobile Robots (AWMRs) are ground vehicles that make their own decisions and perform actions without operator intervention. Nowadays, the number of AWMR is significantly increasing, since they are crucial to different applications in both industry and for service providers [3]. These applications are performed in both indoor and outdoor environments and include activities such as object transportation [4] and power substation inspection [5]. In addition, they are very popular because of their ability to operate appropriately in applications with low mechanical complexity [6]. Nowadays, robotics and automation technologies are becoming more accessible, and can be helpful even in small and medium industries. Moreover, the fourth industrial revolution is aiming for autonomous production methods [7]. Therefore, usage of AWMR will increase, since they can move freely in static or dynamic environments. However, they need to operate for long periods of time on a single battery charge with

heavy payloads. Additionally, charging the battery pack requires several minutes or even hours, reducing their availability to perform missions. Recent studies on improvements to AWMR energy efficiency and consumption can be categorized in terms of whether they are related to software or hardware aspects. Although several advances have been reported regarding hardware improvements, only a few works have attempted to carry out studies on the software aspect. For instance, an AWMR can be designed to generate an energy-efficient path [8], perform more tasks in consideration of its remaining energy [9], and estimate important uncertainty parameters that can affect energy consumption [10].

Navigation is the main component of AWMR software. It is used to generate acceptable, safe, and smooth motion in accordance with a given mission. It guides the AWMR from a starting point to a goal point by following a path, avoiding obstacles, and using a multivariable cost function [11]. Carabin et al. [12] reviewed several papers related to energy optimization methods. However, they did not consider AWMRs, and only arm robots and automatic systems were discussed. Moreover, papers related to navigation techniques have been investigated comprehensively [11,13,14]. However, this article specifically surveys the latest papers considering energy as the main constraint of navigation stages and studying the effects of navigation stages on AWMRs' total energy consumption. In this work, the selected publications are classified into planning and motion control stages.

The remainder of this article is organized as follows. The second section presents a review of papers about energy efficiency and the reduction of energy consumption in the navigation stages of AWMRs. The third section describes simulations and experiments related to motion planning for Self-Guided Vehicles (SGVs). The fourth section proposes solutions for achieving energy-efficient motion. The fifth section discusses the importance of the effect of energy consumption on the obstacle avoidance problem, and this is followed by a conclusion.

2. Energy Efficiency in Navigation Stages: A Review

AWMRs can be classified into different categories. Depending on the motion constraints, AWMRs are divided into holonomic and nonholonomic platforms. Nonholonomic motion has some limitations with respect to moving in any direction, because the number of controller inputs is less than the dimensions of the configuration space. Sometimes, the AWMR is considered to be a particle in order to simplify the design of algorithms that are independent of the AWMR's motion constraints. With respect to the drive mode, AWMRs can be divided into differential, bicycle, tricycle, car, synchronous, omnidirectional, and tracked vehicles [2].

In most reported papers, the AWMR navigation module has five main stages, as shown in Figure 1. The process starts with perception and continues through to the motion control as the last stage for performing proper actions. However, the data flow (from sensors) can be a little different depending on the AWMR environment (indoor or outdoor) or the mission. The perception stage determines the essential information of the robot's surroundings that is necessary to perform a motion behavior. Sensors are indispensable components that allow the AWMR to perceive its environment.

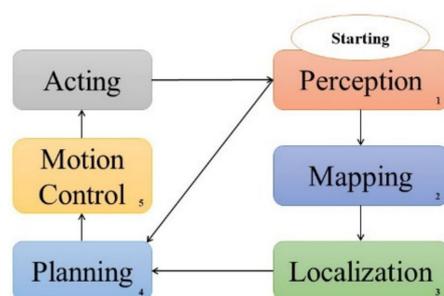


Figure 1. Flow diagram of the stages of the AWMR navigation process.

The effects of processing and sensing energy consumption were surveyed in [15]. Although this stage can influence energy consumption, it has not been studied deeply. For instance, the AWMR may not need to use all of its sensors at the same time. It can apply different sensors with in accordance with the required precision of the perception stage and switch between them at suitable times [16]. After the robot has discovered its surrounding area, the mapping stage models the environment. In other words, it integrates the information collected using the sensors into a representation. Consequently, the localization stage determines the location of the AWMR. Although both stages are vital during missions, they have a low impact on energy consumption. However, using suitable algorithms can decrease processor loads. Planning and motion control are stages that directly affect motion. Hence, they are comprehensively defined in the following sections. Moreover, the authors classify recent papers regarding energy in navigation stages into planning and motion control stages in order to identify research gaps.

2.1. Planning

After the AWMR's position in an environment has been located, the planning stage generates an appropriate path and motion in consideration of a variety of criteria. Based on the prior information related to the environment, the planning stage can be classified into two categories: global path planning and local path planning. Several comprehensive reviews on planning methods and algorithms have been reported [17–19].

2.1.1. Global Path Planning

Global path planning generates the whole path from the start to the goal point. Environmental information such as free spaces and obstacles is contained in the map, so global planning can be performed offline. Additionally, global planning considers the geometric characteristics of static obstacles in order to plan an appropriate path [20]. The most widely reported path search methods for global planning are A*, D*, and artificial intelligence algorithms [17]. Designing a global path in consideration of the energy source level is important [21]. The modified Newton algorithm was used by Duleba and Sasiadek [22] for nonholonomic energy-efficient path planning. This method assumed that the robot consumed less energy by finding the shortest path from a given location to a targeted position. However, the method was not able to accurately predict the amount of energy consumed from the battery, and it is well known that a number of different parameters can lead a short path to consume excessive energy. Changes in rolling resistance and density of obstacles are among such parameters.

Exploration missions can take a long time, and thus energy efficiency is a key aspect in successfully covering a wide area. Mei et al. [23] studied an approach to minimize energy consumption during the exploration of known and unknown environments. The method used was orientation-based target selection, which means that subsequent missions were selected depending on the robot's orientation. In addition, the path from the current position of the robot to the next target was designed in order to avoid repeated target coverage. Benkrid et al. [24] proposed energy-efficient exploration with multi-robot coordination in unknown environments. This method considered three criteria when calculating each robot's motion energy consumption: (i) energy needed for traveling between two goal points, (ii) stopping, and (iii) turning at determined states. In accordance with the given constraints, each robot selected an appropriate path in order to explore the environment. Wang et al. [25] designed a path planning method that took into consideration the robot's remaining energy in order to be sure that the robot would be able to perform the mission and return to the recharging station.

Liu and Sun [26] studied an optimal path planning method for a two-wheel differentially driven robot. The method used the A* global path planner. Furthermore, they considered a cost function in order to obtain the most energy-efficient path. Additionally, the cubic Bezier curve was used to smooth the generated path. They improved the designed method by creating an energy consumption model [27]. The Dubins method, which refers

to the shortest curve connecting two points, was used for a two-wheeled differential drive robot [28]. The study initially used the same method as Liu and Sun [29] to generate a global path. Subsequently, sharp turns were considered as waypoints. Finally, the Dubins method was used to smooth the sharp turns. Moreover, kinetic energy was used to develop the robot energy model.

Combining the A* algorithm with an energy-related cost function, the work reported in [30] defined a heuristic function for a three-wheeled omnidirectional mobile robot. An energy consumption model was developed based on [29,31] that included kinetic energy and friction as the main energy losses. The energy criteria were based on previous robot orientation to pass the nodes towards the next goal. After a suitable path had been found, the optimal η^3 -Spline parameters were used to create a smoother path. To decrease the travel time between waypoints, the method found the optimal velocity profile at the motion control stage by using sequential quadratic optimization. Valero et al. [32] proposed a time-efficient global path planning in known environments with static obstacles for a car-like mobile robot. The optimization problem was solved using quadratic sequential optimization techniques. Furthermore, the influence of energy consumption on the defined path was evaluated. Sathiya et al. [33] used evolutionary algorithms to generate a multi-objective optimal path for a Wheeled Mobile Robot (WMR). An elitist non-dominated sorting genetic algorithm and heterogeneous multi-objective differential evolution were applied. The algorithms helped to find trajectories with minimum traveling time and actuator effort in terms of two criteria for the global optimization functions.

The terrain in outdoor environments severely influence robot energy consumption because of frequent inclination changes and bad soil-wheel friction. The problem of finding the optimal energy path for robots traversing steep terrains has been surveyed [34]. Some inclination angles require a lot of power, and thus the robot is not able to generate enough energy to pass the terrain. Ganganath et al. discussed energy-efficient global path planning on uneven terrains in outdoor environments. They proposed a Z* heuristic search algorithm with an energy cost function while taking physical constraints into consideration [35]. This method generated zigzag paths in order to overcome impermissible traversal headings. In addition, they designed the Constraints Satisfying A* (CSA*) search algorithm to find the shortest path in consideration of the energy constraints implied by uneven terrains [36]. Energy-efficient path planning for Skid-steered Autonomous Ground Vehicles (SAGV) was surveyed by Sharma et al. [37]. SAGV wheels are fixed in a straight line relative to the body of the machine, so they do not have a steering mechanism [38]. A Sampling-Based Model Predictive Optimization (SBMPO) scheme [39] was suggested to estimate the energy requirements during the path planning process. Chuy et al. applied the energy model of the vehicle to a single surface that was dependent on the linear velocity and turning radius terms [40]. Pentzer et al. [41] developed the most recent method for addressing multiple surface types by using the kinematic model of a SAGV to estimate energy usage.

None of the above studies addressed explicit energy measurements. Furthermore, most of them did not assume the robot to possess a perfect energy model. For realistic path planning with energy constraints, a deep understanding of the correct estimation of energy consumption is mandatory. In addition, none of these studies reported the impact of energy consumption on the kino-dynamic performance of the robot.

2.1.2. Motion Planning

Local path planning, also called motion planning [20], generates an online trajectory. It defines trajectory segments along the path generated by the global planner. In addition, it considers the kinematic and dynamic constraints of the AWMR, as well as sudden changes in nearby areas, like obstacles. The type of robot reaction when facing an obstacle can enormously influence energy consumption. Figure 2 presents the classification of important path search methods for motion planning [17].

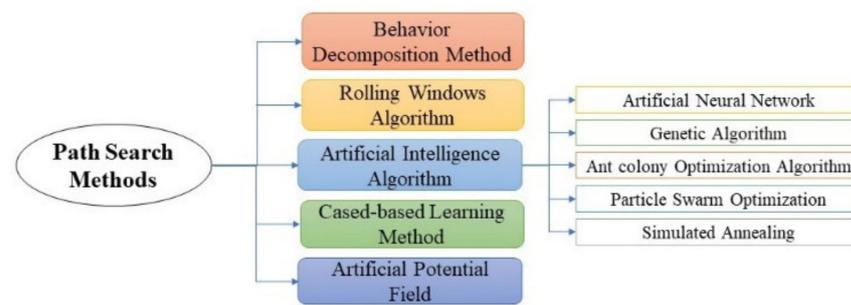


Figure 2. Path search methods in the motion planning stage.

Since static and dynamic obstacles can populate the navigation environment, the selection of an appropriate obstacle avoidance function is essential in motion planning. In addition, most global planners are unable to take into account dynamic or unforeseen obstacles during the trajectory global planning stage. Therefore, the motion planning module must determine a collision-free path in real-time. Mei et al. [42] studied an energy-efficient motion planning method for a three-wheeled omnidirectional mobile robot. The robot's task was to automatically clean the floor in an open area without obstacles. The energy model was built using a six-degree fitting function on the basis of experimental velocity data and only considering the kinematic model. The method was proposed for a specific type of robot, and could not be used for other types. Estimation and prediction rules can help robots to achieve appropriate obstacle avoidance reactions. Llamazares et al. [43] proposed an Approximate Inference Control framework (AIC) based on an energy consumption model for planning local paths. The model was based on overcoming inertia, road grade, tire friction, and aerodynamic loss. The AICO displayed an improvement in energy efficiency of more than 10% compared to the best results obtained using the classic algorithms. However, the assumption of the robot's linear speed may not be practical, because the robot must perform various actions in real missions.

In [44], the Dynamic Window Approach (DWA) was combined with a cost function based on energy consumption. An omnidirectional robot was considered in a partially dynamic environment. The energy consumption model was based on the model introduced in [29], which was improved by including electrical, frictional, and acceleration energy. The proposed method was modified by defining a new energy model and representing a new cost objective in order to reduce power consumption [45]. Alajlan et al. [46] proposed the use of a multi-sensor method for a WMR. To this end, infrared reflective sensors were applied for edge detection, and infrared measuring sensors, an ultrasonic sensor, and a camera were applied for obstacle detection, creating an integrated framework. Moreover, the constraints included energy consumption, time, and distance traveled. The generation of the shortest and most energy-efficient trajectory was the main criterion of the objective function that was used to avoid collision. During a mission, the method evaluated several possible trajectories and selected the least costly one.

This section examined recently published paper on planning that are related to energy. Table 1 provides a detailed analysis of the surveyed papers. The letters 'N' and 'Y' stand for "no" and "yes", respectively. Additionally, Table 1 describes each paper on the basis of whether static or dynamic obstacles were used, whether experiments were conducted along with simulations, whether an energy model was defined, and the type of AWMR used. Furthermore, it briefly explains the methods used. Although various methods have been developed to optimize global path planning, very few papers have used them to consider the constraint of energy efficiency. The motion planning aspect includes both kinematic and kinetic constraints. Accordingly, it has an enormous effect on energy consumption. However, there are few papers in the literature that address local path planning while taking energy consumption into consideration. In addition, an effective energy model should include proper planning that takes the energy efficiency into account. The surveyed papers have partially defined the AWMR energy model, but none has considered all of

the uncertainty parameters, or all of the geometric, kinematic, and dynamic constraints. A more precise energy model could assist with the generation of an appropriate energy efficiency path. The steps to define energy-efficient motion should result in:

1. The creation of an energy consumption model (ECM) that includes the uncertainty parameters, as well as the geometric, kinematic, and kinetic constraints, of AWMRs.
2. The definition of a cost function on the basis of the energy criteria and with respect to the ECM.
3. The execution of the path and trajectory in accordance with the defined cost function.

Table 1. Overview of papers on planning related to energy, with a summary of the methods used.

Paper	Static Obstacles	Dynamic Obstacles	Real-Time Experiment	Defining New Energy Model	AWMR Type	Method
[22]	N	N	N	N	Nonholonomic	Newton algorithm
[23]	Y	N	N	N	Particle	Decreasing the trajectories
[24]	Y	N	Y	Y	Nonholonomic	Using multi-robot coordination for exploration
[25]	N	N	Y	Y	Particle	Tabu-search-based
[26]	Y	N	Y	Y	Nonholonomic	Adding energy constraint to A* planner
[27]	Y	N	Y	Y	Nonholonomic	Adding energy constraint to A* planner
[28]	Y	N	Y	Y	Nonholonomic	Adding energy constraint to A* planner
[29]	Y	N	N	Y	Nonholonomic	Dubins method
[30]	Y	N	N	N	Holonomic	Integrating heuristic function to A* planner
[32]	Y	N	Y	N	Nonholonomic	Using optimization algorithm
[33]	Y	Y	N	Y	Nonholonomic	Using evolutionary algorithms
[35]	N	N	Y	N	Nonholonomic	Using zigzag-like path patterns
[36]	N	N	N	N	Particle	Integrating energy cost to A* planner
[37]	Y	N	N	Y	Nonholonomic	Using sampling-based model predictive optimization
[41]	N	N	Y	Y	Nonholonomic	Using sampling-based model predictive optimization
[42]	N	N	Y	Y	Holonomic	Using the six-degree polynomial for the energy cost function
[43]	Y	Y	Y	Y	Nonholonomic	Using approximate inference control framework
[44]	Y	Y	Y	N	Holonomic	Adding energy cost to DWA
[45]	Y	Y	Y	Y	Holonomic	Adding energy cost to DWA
[46]	Y	Y	Y	N	Nonholonomic	Multi-sensor path planning

2.2. Motion Control

Motion control is the last stage of the navigation process. It executes the correct velocity, acceleration, and torque required to follow the path and trajectory that were generated during the planning stage. It moves the robot in a controlled manner. The trajectory of the executed velocity and acceleration in the motors has a direct effect on energy consumption. Barili used a constant acceleration rate and limited frequent velocity changes in order to reduce power consumption [47]. However, the method assumed that

motion took place in straight lines and ignored the power consumption required by angular velocity changes. Brateman et al. [48] proposed an energy-saving method by scheduling motor speed and the processor frequency while preventing collisions. This approach was extended in [49] by generating a schedule by means of a genetic algorithm. Kim et al. [50] suggested three steps for the velocity control of efficient binary search algorithms for the WMR. The method considered the dissipation of practical energy demand in motors. They proposed optimal control theory to obtain optimal an velocity trajectory for a differential-driven WMR with a fixed total time of travel [51]. The cost function was the actual energy consumption of batteries, including motor armature resistance loss, kinetic energy, and viscous friction. However, rotational velocity was ignored, and only translational velocity changes on a straight-line path were considered. In fact, these assumptions limit the applicability of this method.

They continued their study on translational trajectory planning of three-wheeled omnidirectional mobile robots (TOMR) [52]. They surveyed both translational and rotational velocity trajectory planning for TOMR [53]. The algorithm was based on a dynamic model of the robot. The minimum energy rotational velocity trajectory was founded using Pontryagin's minimum principle. Furthermore, the minimum energy translational velocity trajectory was obtained using a novel algorithm based on the linearity condition of the state transition of TOMR. In [54], the authors proposed a method that considered a trajectory that expressed a curved-line path with the self-rotation motion of the robot. The paper developed a dynamic simulation using dynamic actuators. Tokekar et al. [55] studied a forward-only car-like robot, powered by direct current (DC) motors in a flat, obstacle-free environment. The problem of obtaining energy-efficient velocity profiles with/without limitations with respect to maximum velocity was studied for a given path. Consequently, they proposed an extended method in which the problems of achieving an energy-efficient path and velocity profiles were studied simultaneously using a discretized graph search algorithm [56].

Designing a trajectory tracking controller is challenging because of the dynamic impacts of the inertia and actuators of a mobile robot on acquiring the appropriate linear and angular velocities. In [57], a Model Predictive Control (MPC) method was used for the bicycle drive model of car-like robots. Firstly, a trajectory tracking MPC controller was designed using a nonlinear control law. Secondly, the cost function of energy consumption due to electric propulsion was used to find an energy-efficient trajectory. Wang et al. [58] designed a feedback controller to optimize predefined path tracking by considering energy efficiency for an indoor carrier robot. They developed a new dynamic model by using the dynamics of the actuator along with kinematic models of an omnidirectional robot.

The authors in [59] proposed energy-optimal path planning for tracking a moving target using a four-wheeled holonomic vehicle. The analytical simulation was performed in an environment with static obstacles, and the target possessed predictive linear motion. The artificial potential method (APF) and optimal control theory were used for path planning and energy-efficient motion, respectively. H. Kang et al. [60] discussed dynamics-based control methods in order to obtain energy-optimal trajectories between two endpoints. Their method used analytical mechanics to combine the contact kinematic between the wheels and the ground. In addition, it analyzed the robot dynamics with the assistance of the Gibbs-Apple method to create motion equations. Once the motion equations were known, two different techniques (servo constrained-based and differential flatness-based) were used to generate the input for tracking control. Serralheiro and Maruyama [61,62] obtained the optimal velocity trajectory for a nonholonomic robot by considering time efficiency as well as energy efficiency. Therefore, the method established a relation between the total energy optimization and traversal time by generating a penalty coefficient. Additionally, a convex optimization was applied instead of a nonlinear one to estimate suitable traversal time and total energy.

Table 2 provides an overview of papers on motion control related to energy. It describes each paper on the basis of experimental validation, energy model consideration, and the

algorithms used. Linear optimal control algorithms and simple controllers were mostly used for energy efficiency problems. However, none were able to address goals regarding AWMR uncertainty parameters such as mass, center of mass, and moment of inertia. This condition necessitates other control methods such as robust [63], adaptive [64], and fuzzy control [65] in order to be able to consider time-varying parameters.

Table 2. Overview of papers on motion control related to energy with a summary of the methods used.

Paper	Experiment	Energy Model	Method
[47]	Y	N	Clogging frequent velocity changes
[48]	Y	N	Scheduling motor speed and processor frequency
[49]	Y	N	Modifying [38] with genetic algorithms
[50]	N	Y	Three-step velocity control
[51]	Y	Y	Using optimal control theory to achieve optimal velocity trajectory
[52]	N	Y	Using optimal control theory to achieve optimal velocity trajectory
[53]	Y	Y	Combining Pontryagin's minimum principle and a novel algorithm to determine the velocity trajectory
[54]	Y	Y	Combining Pontryagin's minimum principle and a novel algorithm to determine cornering trajectory planning
[55]	Y	Y	Using dynamic programming method to obtain velocity profile
[56]	Y	Y	Using discretized graph search algorithm to find velocity profile
[57]	N	N	Using a Model Predictive Control for trajectory tracking
[58]	Y	Y	Using a Robust Feedback Controller for trajectory tracking
[59]	N	N	Using Pontryagin's minimum principle for target tracking
[60]	N	N	Using optimization algorithm based on the Ritz approximation
[62]	N	N	Convex optimization tools

3. Effect of Motion Planning on Energy Consumption

Among the navigation stages, motion planning is able to directly affect energy consumption, depending on the performed motion primitives. Following the global path while also avoiding unforeseen obstacles is the main task of this stage. To avoid a collision, several strategies can be employed. For instance, the robot can stop until the path is obstacle-free and then again start the movement, or it can change its path in various ways. The selection of each maneuver can change the velocity profile, the travel distance, etc. We study the obstacle avoidance problem to show the effect of the local motion planning stage on energy consumption. For this purpose, some simulations and experiments were performed using the DWA method and a manual control strategy. The goal is not to compare the two methods, but to understand and further highlight the role of motion planners in energy consumption. In the DWA approach, the platform is autonomously controlled by the machine (navigation module), while in the manual control mode, a human controls the platform motion to follow the trajectory that was generated by the global planner. In addition, a Self-Guided Vehicle (SGV) was selected to perform the simulations and experiments. Depending on the autonomy level in industrial robots, SGV is an improved model of an autonomous guided vehicle (AGV) because it has the ability to respond to changes in the mission's environment such as unforeseen obstacles [21,66].

3.1. Dynamic Window Approach (DWA): An Overview

DWA is a well-known collision avoidance navigation algorithm that was proposed by Dieter Fox et al. [67]. Furthermore, DWA is an online reactive method, and its cost function has been extended several times in recent years [68,69]. To select safe and optimal translational (v) and rotational (w) velocities, the method directly generates their profiles by considering the dynamics of the robot and the range limitation of the velocity and acceleration. The main search space for suitable velocities is intersected by three subspaces:

the space of possible velocities in accordance with the robot kinematic constraints, V_s ; the space of admissible velocities that allows the robot to stop without colliding with an obstacle, V_a ; and the space of possible velocities in consideration of the robots limited accelerations, V_d :

$$V_r = V_s \cap V_a \cap V_d \quad (1)$$

where V_r is the search space of optimal velocities, which is selected by maximizing the following objective function:

$$G(v, w) = \alpha * h(v, w) + \beta * d(v, w) + \gamma * v_F(v, w) \quad (2)$$

where h measures the alignment of the robot with the target direction, d is the distance to the closest obstacle, and v_F is the robot's forward velocity. α , β , and γ are tunable constant weights. Hence, the DWA method generates a lot of possible online local paths and then selects the most appropriate one on the basis of the objective function. Finally, the most suitable velocity for achieving the local path is executed.

3.2. Energy Consumption Model

An appropriate energy model is needed to analyze the effect of obstacle avoidance on energy consumption. Therefore, the energy model proposed by Wahab et al. [31] for differential drive robots was used. Their proposed model includes five main sections, which make up the total energy consumption model of the robot. The sections are discussed in more detail in the following. The DC motor's information was obtained from its datasheet and experiments. Additionally, the superscripts R and L refer to the right and left motors.

3.2.1. DC Motors

Energy consumption by DC Motors (E_{DC}) can be defined as

$$E_{DC} = \int ((I_a^R)^2 R_a^R + (I_a^L)^2 R_a^L) dt \quad (3)$$

where I_a is the armature currents, and R_a is the armature resistance of the DC motor.

3.2.2. Friction

The energy losses due to friction (E_F) are obtained through

$$E_F = \int \mu mg((v(t) + bw(t)) + (v(t) - bw(t))) dt \quad (4)$$

where v and w are the linear and angular velocities of the robot, μ is the coefficient of rolling friction, m is the robot mass, g is the gravity, and b is the axle length of the robot.

3.2.3. Kinetic Energy

The energy losses of the robot motion (E_K) are expressed by

$$\begin{aligned} v &= r(w^R + w^L)/2 \\ w &= r(w^R - w^L)/2b \\ E_K &= \int \left(\frac{1}{2} (mv(t)^2 + Iw(t)^2) \right) dt \end{aligned} \quad (5)$$

where w^R and w^L are rotational velocities of the DC motors, r is the wheel radius, and I is the robot's moment of inertia.

3.2.4. Electronics

The energy losses of the on-board electronics (E_E) were achieved by

$$E_E = \int (I_{elec} V_{elec}) dt \quad (6)$$

where I_{elec} and V_{elec} are the amount of current withdrawn by the electronics and their supply voltage, respectively.

3.2.5. Gear Friction

The energy losses of friction (E_G) in gearhead motors can be described as follows:

$$E_G = \int \left((P_{mech}^R - \eta_g^R P_{mech}^R) + (P_{mech}^L - \eta_g^L P_{mech}^L) \right) dt \quad (7)$$

where η_g is the efficiency of the gear, and P_{mech} is the motor's output mechanical power, which is obtained by the torque (τ):

$$\begin{aligned} P_{mech}^R &= \tau^R \omega^R \\ P_{mech}^L &= \tau^L \omega^L \end{aligned} \quad (8)$$

where the torque (τ) is displayed as:

$$\begin{aligned} \tau^R &= K^R I_a^R \\ \tau^L &= K^L I_a^L \end{aligned} \quad (9)$$

where K is the torque constant.

Finally, the sum of the above energy values creates the energy consumption model of the robot (E_{Total}):

$$E_{Total} = E_{DC} + E_K + E_F + E_E + E_G \quad (10)$$

3.2.6. Experimental Validation of the Energy Consumption Model

To validate the energy consumption model of the SGV, the measured and simulated power profiles were compared. The measured power was obtained by multiplying the measured current and the battery voltage. The SGV started following a straight trajectory from rest and stopped after 28 s. In addition, there was a rotational trajectory in the middle of the mission. Therefore, the longitudinal and rotational speed profiles followed a trapezoidal speed profile, as shown in Figure 3. Furthermore, the power consumption was obtained by using the vehicle model and the measured power. Although there is a small difference, the obtained energy (which is the integral of the power during the entire motion) is accurate enough to demonstrate the effect of the local motion planner on the energy usage.

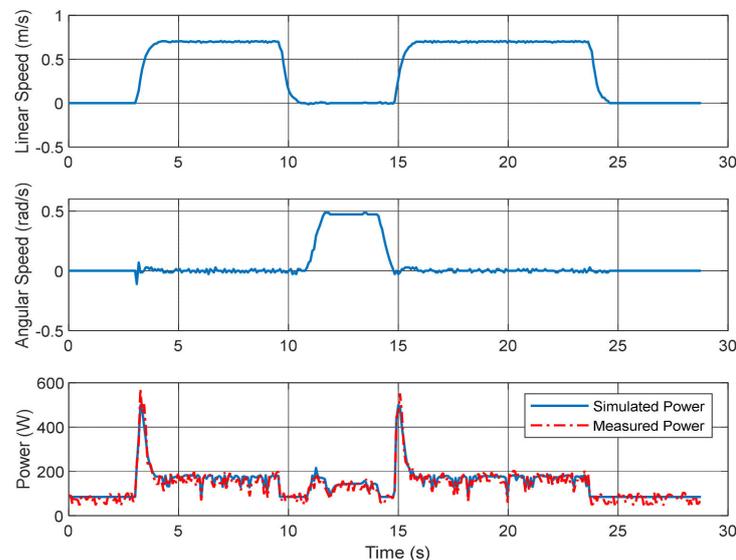


Figure 3. Comparison between simulated and measured power profiles.

3.3. Simulation

Gazebo is a powerful 3D simulator for calculating physics, generating sensor data, and providing convenient interfaces for making a specific robot. In addition, Gazebo is able to simulate a specific environment with all details. Therefore, Gazebo was used to create the 3D SGV model and the industrial environment taking into consideration the various types of dynamic and static obstacles. Figure 4 displays the SGV model in Gazebo, with properties $m = 90$ kg, $r = 0.1$ m and $b = 0.8$ m, which are the same as for the real SGV used in the Experiments section. It includes two LiDARs to perceive the environment and obstacles. Moreover, three small robots were created as unexpected obstacles to cross the SGV's path. Robot Operating System (ROS) was used as the navigation stack for the simulations as well as the experiments. ROS is a software development kit that helps with the creation of robot applications such as drivers, algorithms, and node creation and destruction for various operations. Hence, the co-operation with ROS and Gazebo was used to create an ideal environment in which to perform the simulations.

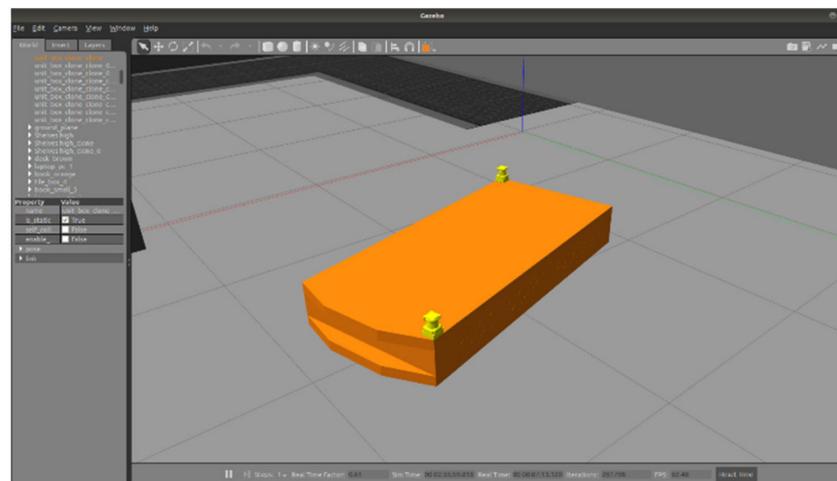


Figure 4. SGV model in Gazebo.

For simplicity, the global planner was kept generating the same straight trajectory between the start position and the goal position using a simple map of the navigation environment, with no obstacles located on the trajectory. Before starting the motion, several obstacles were included in the configuration space, meaning that the local planner would have to perform obstacle avoidance maneuvers. Three scenarios were defined to include these unforeseen obstacles between the start and goal positions, as described below:

1. Three unforeseen static obstacles
2. Two unforeseen dynamic obstacles
3. Two unforeseen static obstacles and one unforeseen dynamic obstacle

The A* method was selected as the global path planner to generate straight global trajectories between the start and target positions. Hart et al. [70] proposed the A* algorithm in 1968. It is a heuristic function-based algorithm for appropriate global path planning. It calculates the heuristic function value at each node in the work area. Afterwards, it finds the optimal solution that possesses zero probability of collision in order to generate an optimal path [71]. The local planner receives a straight trajectory (between the start position and the goal position) from the global planner. During all simulations, the path generated by the global planner was the same. Only the motion planners (with the task of avoiding unforeseen obstacles) were different. The local motion planning was carried out using the manual control (manual obstacle avoidance) and DWA methods for all three scenarios. For the manual tests, a joystick was defined in the ROS Navigation Stack as a controller in order to follow the global path generated by A* while avoiding unforeseen

obstacles. Moreover, a lot of simulations were performed in various scenarios with the use of different DWA parameter values in order to select appropriate tunable weights.

The values of the main parameters and the simulation results are presented in Table 3 for the three simulation scenarios. On the basis of this table, the maximum linear and angular speeds were the same for both methods. In the scenarios, the total energy consumption of manual control was less than that when using DWA algorithm. This shows that energy consumption is dependent on the motion and reaction of the SGV. In addition, these results suggest that even if the global planner generates an optimal trajectory, the way it is followed ultimately impacts energy consumption. Therefore, it is important to be aware of the potential energy requirement when performing any obstacle avoidance maneuvers.

Table 3. Summary of the simulation results.

Scenario Number	Local Path Planning Method	Maximum Linear Speed (m/s)	Maximum Angular Speed (rad/s)	Time (s)	Total Energy Consumption (J)	Energy Consumption Difference between DWA and Manual (%)
1	DWA	0.6	0.5	20.1	3720	−6.1 %
	Manual	0.6	0.5	18.6	3490	
2	DWA	0.6	0.5	21.4	3825	−7.5 %
	Manual	0.6	0.5	20.2	3535	
3	DWA	0.6	0.5	20.9	3880	−6.2 %
	Manual	0.6	0.5	18.9	3640	

3.4. Experiments

To experimentally validate the simulation conducted in the previous part, several experiments were carried out using an industrial SGV. This platform has a differential drive (Figure 5a) and consists of a Mini PC with the Ubuntu 16.04 LTS operating system and an ROS Navigation Stack. The SGV has encoders in the wheels to estimate the position and velocity, and two LiDARs to perceive environmental information. The three scenarios described in the simulation part were repeated for the experiments. Figure 5b shows the test environment with the SGV and three small AWMRs as obstacles.

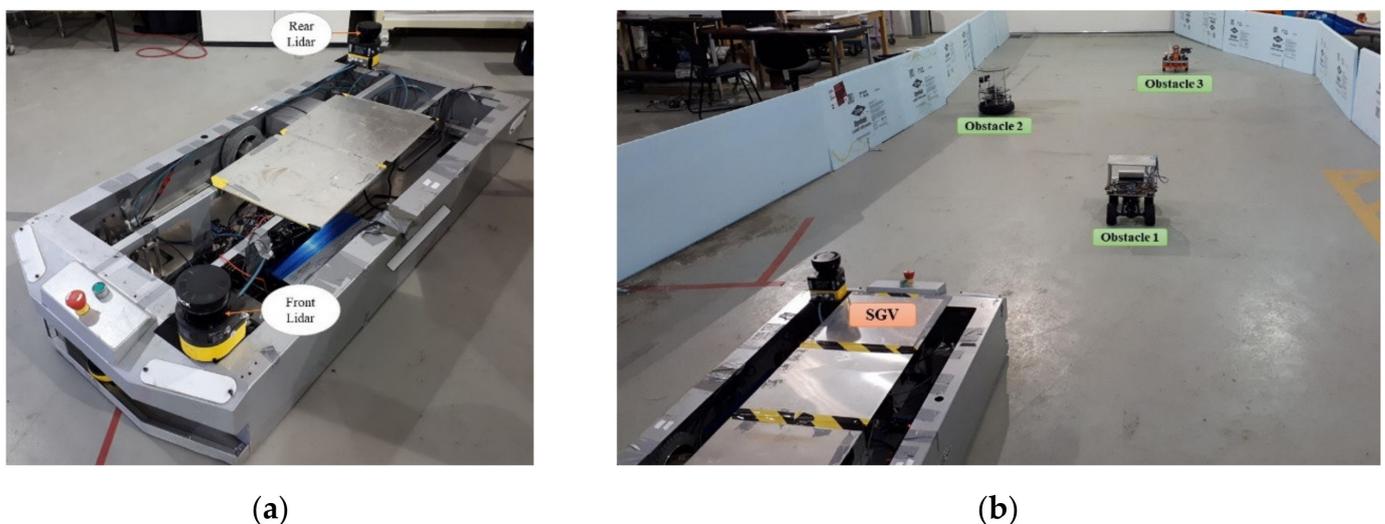


Figure 5. (a) The Self-Guided Vehicle (SGV); (b) the experimental environment.

Figure 6a displays the local planner trajectories for three different scenarios in the experiments. The green and yellow rectangles show the start and target points. Additionally, obstacles are indicated by the black rectangle, and the motion trajectories of dynamic obsta-

cles are illustrated by dotted (marked) lines. The straight black lines show the trajectory designed by the global planner, the red trajectory indicates manual control and the blue one is for DWA. On the basis of the power consumption profiles in Figure 6b, there are more frequent changes when using the DWA method. These changes were executed because of the sudden reaction of the SGV in response to unforeseen obstacles. Table 4 illustrates the results of both the DWA method and manual control in the real-time experiment. The maximum linear and angular speeds were the same for all scenarios. Although the time taken for each test was less than 23 s, the energy consumptions of the two methods were different. Hence, the type of SGV reaction can affect energy consumption. In addition, these results suggest that it is possible to modify the DWA planner.

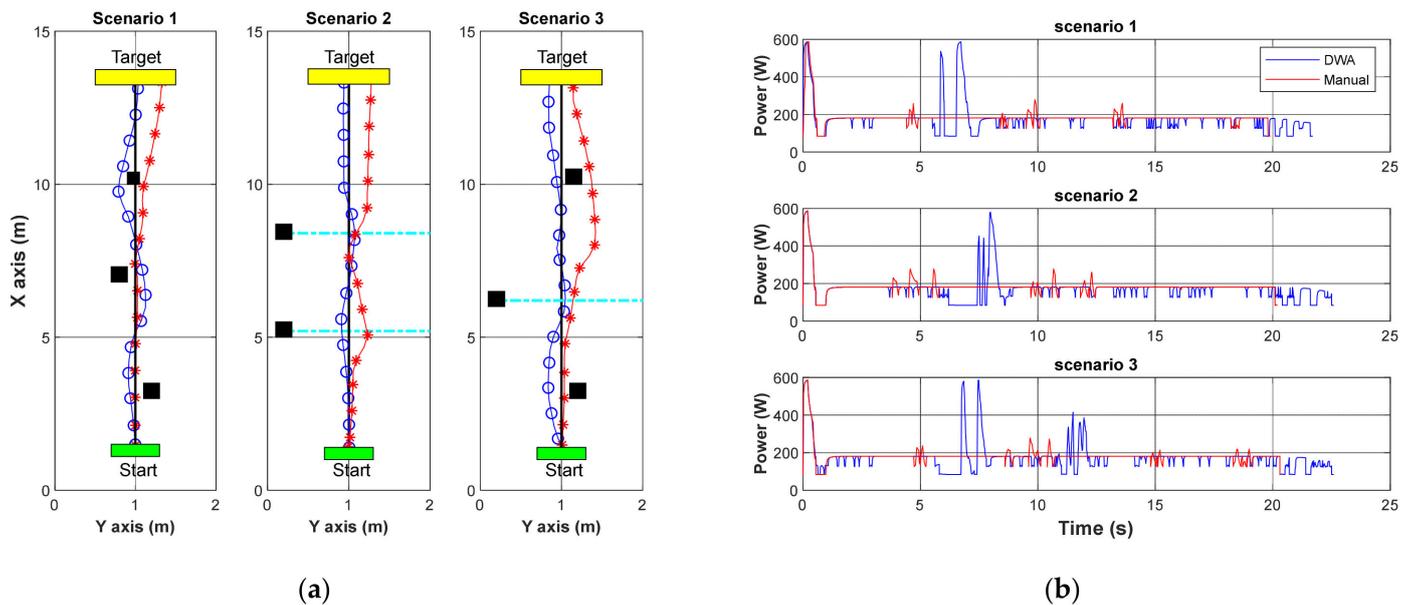


Figure 6. (a) SGV's paths in the experiments. (Red and blue trajectories show the paths generated by manual control and DWA, respectively); (b) SGV's power consumption in the experiments.

Table 4. Summary of the experimental results.

Scenario Number	Local Path Planning Method	Maximum Linear Speed (m/s)	Maximum Angular Speed (rad/s)	Time (s)	Total Energy Consumption (J)	Energy Consumption Difference between DWA and Manual (%)
1	DWA	0.6	0.5	21.6	3816	−4.9%
	Manual	0.6	0.5	19.9	3636	
2	DWA	0.6	0.5	22.6	3888	−8%
	Manual	0.6	0.5	20.2	3600	
3	DWA	0.6	0.5	22.6	3960	−5.7%
	Manual	0.6	0.5	20.3	3744	

4. Energy-Efficient Motion

On the basis of the results in the previous section, the motion planner method can be improved to cause the robot to consume less energy. Therefore, two solutions for addressing the problem are discussed in the following. To develop an algorithm to generate energy-efficient motion, a precise energy model is necessary. The model must include all the dynamic parameters of the robot that affect energy consumption. Then, it can be used as a constraint.

4.1. Adding Energy Constraint

On the basis of (2), the DWA selects a suitable path by using the cost function. However, the function does not include the energy constraint. Therefore, the new constraint can be added. The new cost function is:

$$G(v, w) = \alpha * h(v, w) + \beta * d(v, w) + \gamma * v_F(v, w) + \sigma * E(v, w) \quad (11)$$

where σ is a tunable constant weight. Moreover, E is the total energy consumption of the robot, which is a function of linear and angular speeds. Therefore, when the DWA generates many executable short paths online, it can predict which one will consume less energy by using the added parameter for the cost function. The proposed method can be used in other motion planning algorithms that include cost functions.

4.2. Variable Weights

The four constant weights in (11) play the main roles in DWA. They were selected on the basis of trial and error. Therefore, they are not optimal. The planning part can work in various ways depending on the aim and mission. However, Figure 7 shows in detail how the planning part operates in this work [72]. Therefore, the motion planning receives (i) navigation information from sensors, (ii) a globally designed path from the global planner, and (iii) information on the robot's surroundings, such as unforeseen obstacles, from the local cost map, which is called an online map. By collecting the information and receiving the global path, motion planning generates the appropriate linear and angular speeds to move the vehicle. Moreover, the global cost map provides the whole map of the environment, which is called an offline map. The three mentioned types of information change during the mission, and a DWA with constant weights does not consider parametric uncertainties and disturbance. Therefore, the robot is not able to carry out energy-efficient motion under all conditions. To solve this problem, the DWA's tunable weights need to adapt to changed situations. Two methods are proposed to find the optimized values of the weights in the following.

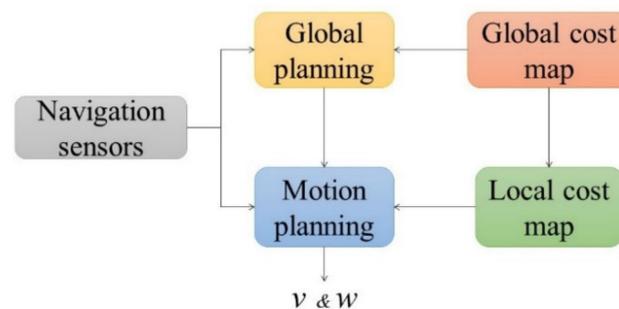


Figure 7. Flow diagram of the planning stage.

4.2.1. Fuzzy Logic Control

This method needs data. Therefore, some simulations and experiments need to be performed using different weights in order to collect data. Then, a fuzzy logic (FL) controller should be designed to use the collected data and the generated energy consumption model. The controller can find appropriate online weights that will cause the robot to consume less energy.

4.2.2. Reinforcement Learning

Applying a reinforcement learning (RL) algorithm in order to successfully interact with unforeseen obstacles within an environment can be helpful. The RL algorithm can be designed to use the actual behavior and the energy consumption model of the robot to

modify the weights. After some experiments, the RL algorithm will find the correct values for weights depending on the energy consumption constraints.

5. Discussion

The main aim of the various tests described in this paper was to show the effect of the obstacle avoidance algorithm on the energy consumption of the SGV. In fact, the platform's strategy (or choice of the path) for avoiding obstacles has a significant impact on energy usage within long-duration missions. When the SGV is driven using electric traction, careful battery management is required for two reasons:

- First, the stored energy, which is limited
- Second, the reduction in the available energy for traction due to battery degradation (as the SGV is aging)

It was observed that when the SGV faced a challenge from a dynamic obstacle, it can decrease its speed until the obstacle frees the path, or stop and then start its motion again safely. The first plan consumes less energy compared to the second one, because any start induces acceleration, which ultimately causes the platform to consume more energy (see the power profiles in Figure 6b). Moreover, some SGVs are used to transport goods in industrial environments, and they must be able to avoid unforeseen obstacles while maintaining good stability. The position of the center of mass and the geometrical distribution of the inertia matrix can also affect energy consumption. Most papers have used classic global path planning methods by adding the energy cost function to achieve a suitable trajectory. In recent years, global planning algorithms have been substantially improved. They can be used with energy cost functions to optimize the generated path. Although the motion planning stage directly affects energy consumption, the link between them has not been adequately taken into consideration. Various classic motion planning methods can be improved by adding energy criteria. More recently, they have been modified with computer vision and machine learning methods [73–75]. However, no evidence of the progress has been published so far.

6. Conclusions

In accordance with the growing demand for autonomous systems and technologies, AWMRs (Autonomous Wheeled Mobile Robots) have become very popular. They are often based on electric traction and use batteries as their primary energy storage. They have a lot of abilities that can improve performance in different environments. However, their limited energy storage can reduce their efficiency. The algorithms in the navigation stages have direct effects on the energy consumption of AWMRs because they are the main components designing the motions carried out in order to achieve the task. After thoroughly reviewing published papers related to navigation stages, we realized that few works have explicitly considered energy requirements, especially with respect to the local planner (motion planner) stage. To show the influence of the motion planning algorithm on energy consumption, we performed several simulations and experiments using a SGV (Self-Guided Vehicle). The analyzed scenarios illustrate that the constraint of energy consumption must be considered when designing motion planning algorithms. This constraint helps the SGV to perform tasks with high energy efficiency. In addition, the results suggest that DWA (Dynamic Windows Approach), as a popular motion planning algorithm, could be improved to use less energy in the context of the task of obstacle avoidance. Therefore, solutions for creating an energy-efficient DWA were discussed.

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