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Article

Performance Optimization of Generalized Irreversible Refrigerator Based on a New Ecological Criterion

Jie Xu *, Liping Pang and Jun Wang

School of Aviation Science and Engineering, Beihang University, Beijing 100191, China;
E-Mails: panglipingbuaa@gmail.com (L.P.); wangjun@buaa.edu.cn (J.W.)

* Author to whom correspondence should be addressed; E-Mail: buaaxujie@gmail.com;
Tel.: +86-10-8233-9486; Fax: +86-010-8231-6654.

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Abstract: On the basis of the exergy analysis, a performance optimization is carried out for a generalized irreversible refrigerator model, which takes into account the heat resistance, heat leakage and internal irreversibility losses. A new ecological criterion, named coefficient of performance of exergy (*COPE*), defined as the dimensionless ratio of the exergy output rate to the exergy loss rate, is proposed as an objective function in this paper. The optimal performance factors which maximize the ecological objective function have been discussed. Numerical examples are given to explain the influences of heat leakage and internal irreversibility on the generalized and optimal performances. This new ecological criterion may be beneficial for determining the reasonable design of refrigerators.

Keywords: thermodynamic cycle; performance; refrigerator; ecological optimization

1. Introduction

Since the 1970s, many studies on identification of the performance limits and optimization of the thermodynamic cycles for refrigerators have been carried out based on various objective functions [1–25]. Most of these previous works have chosen the cooling load, power input, coefficient of performance, exergy output rate and entropy generation rate as optimization criteria. Angulo-Brown *et al.* [26] proposed an ecological objective function, $E' = P - T_L\sigma$, for finite-time Carnot heat engines, where T_L is the temperature of the cold heat reservoir, P is the power output and σ is the entropy generation rate. Yan [27] modified it to $E = P - T_0\sigma$, where T_0 is environment temperature. It is more reasonable to use

E as the criterion function than that proposed by Angulo-Brown, because E is the best compromise between power output, P, and power loss, $T_0\sigma$, when $T_L \neq T_0$.

The ideas used in heat engines can also be applied to study the performance of refrigeration systems. Based on the viewpoint of energy analysis, Chen *et al.* [28,29] defined an ecological function $E^{"} = R - \varepsilon_c T_H \sigma$ for endoreversible Carnot refrigerators working between two heat reservoirs (the heat sink and the heat source) with temperature T_H and T_L, where R is cooling load of the refrigerator, $\varepsilon_c = T_L / (T_H - T_L)$ is the reversible coefficient of performance (*COP*). Ust *et al.* [30,31] defined a new thermo-ecological objective function called ecological coefficient of performance (*ECOP*), which is defined as the ratio of cooling load to the loss of the availability, *i.e.*, $ECOP = R / T_0\sigma$. Furthermore, Chen *et al.* [32] proposed a unified ecological optimization objective for all thermodynamics cycles based on the view of exergy analysis as follows:

$$E = A / \tau - T_0 \sigma \tag{1}$$

where A / τ is the exergy output of the cycle, T_0 is the environment temperature of the cycle, and σ is the rate of the entropy generation of the cycle. When Equation (1) is examined, one can see that the objective function (E) may take negative values. At this condition, the exergy loss rate (second term in the E function) is greater than the exergy output rate (first term in the E function). Therefore, this objective function Equation (1) calls for a thermodynamic interpretation. For this purpose, we modify Equation (1) defined by Chen *et al.* [32] to the dimensionless ratio of exergy output rate to exergy loss rate (entropy generation rate), named coefficient of performance of exergy (*COPE*), *i.e.*, $COPE = (A/\tau)/(T_0\sigma)$. The *COPE* gives information about the exergy loss rate and entropy generation rate. Optimizations based on this new ecological criterion function have been performed. Numerical examples are presented to show the effects of heat leakage and internal irreversibility on the optimal performance of a generalized irreversible refrigerator.

2. Model and Basic Assumption

The Temperature-Entropy ($T - S$) diagram of the irreversible refrigerator is shown in Figure 1. The cycle operates between a heat sink at temperature T_H and a heat resource at temperature T_L. The working fluid in the cycle works at temperature T_{HC} and T_{LC}, respectively. Q_{HC} is due to the driving force of ($T_{HC} - T_H$) in the high-temperature heat exchanger, Q_{LC} is due to the driving force of ($T_L - T_{LC}$). Q_H is the net heat rates transferred to the heat source, and Q_L is the net heat rates transferred from the heat sink, *i.e.*, the cooling load(R). The relationship between T_{HC}, T_H, T_L, T_{LC} satisfies the following inequality:

$$T_{HC} > T_H > T_L > T_{LC} \tag{2}$$

The model for the generalized irreversible refrigerator and its surroundings to be considered in this paper is shown in Figure 2.

Figure 1. $T-S$ diagram for the generalized refrigerator model.

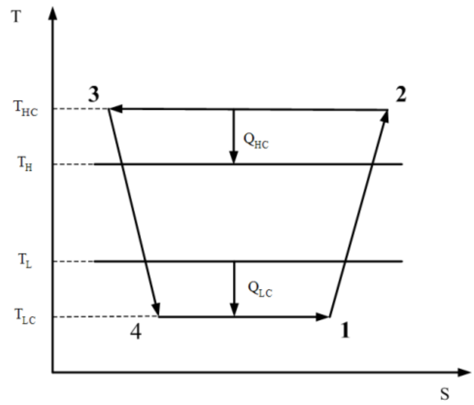
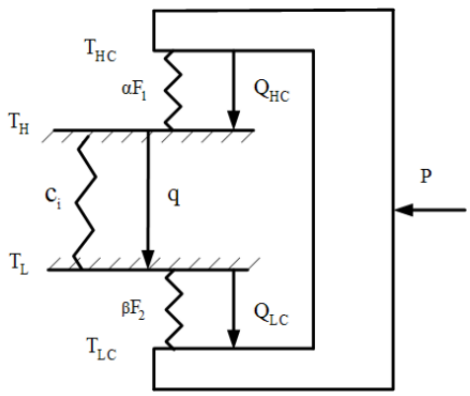


Figure 2. The model of the generalized refrigerator and its surroundings.



The following assumptions are made for this model [33]:

(1) The working fluid flows through the cycle at a steady state. The cycle consists of two isothermal processes and two adiabatic processes. All four processes are irreversible.

(2) The heat transfer surface areas ($F_1$and F_2) of the high- and low- temperature heat exchangers are finite. The total heat transfer surface area (F) of the two heat exchangers is assumed to be a constant:

$$F = F_1 + F_2 \tag{3}$$

(3) There exists heat leakage (q) from the heat sink to the heat source. Thus:

$$Q_H = Q_{HC} - q \tag{4}$$

$$Q_L = R = Q_{LC} - q \tag{5}$$

(4) There are various irreversibilities in the refrigeration cycle due to: (i) heat resistance between heat reservoirs and working fluid. (ii) heat leakage between heat reservoirs and (iii) miscellaneous factors such friction, turbulence and non-equilibrium activities inside the refrigerator. Therefore, a larger power input is needed when compared with an endoreversible refrigerator. In other words, the rate of heat rejected to the sink (Q_{HC}) of the generalized irreversible refrigerator is higher than that of an endoreversible one ($Q_{HC}^{'}$). To scale these irreversibilities, we introduce a constant factor, ϕ, to characterize the additional internal

miscellaneous irreversibility effect:

$$\phi = \frac{Q_{HC}}{Q'_{HC}} \geq 1 \tag{6}$$

The model described above is a more general one than the endoreversible [28] and irreversible [34–39] refrigerator models. If $q = 0$ and $\phi = 1$, the model is reduced to the endoreversible refrigerator [28]. If $q > 0$ and $\phi = 1$, the model is reduced to the irreversible refrigerator with heat resistance and heat leak losses[34–36]. If $q = 0$ and $\phi > 1$, the model is reduced to the irreversible refrigerator with heat resistance and internal irreversibilities [37–39].

3. Optimization for Irreversible Refrigerator

The second law of thermodynamics for an irreversible refrigerator requires that:

$$\frac{Q'_{HC}}{Q_{LC}} = \frac{T_{HC}}{T_{LC}} \tag{7}$$

Combining Equations (6) and (7) yields:

$$\frac{Q_{HC}}{T_{HC}} = \phi \frac{Q_{LC}}{T_{LC}} \tag{8}$$

Consider that the heat transfers between the refrigerator and its surroundings obey Newton's linear law:

$$Q_{HC} = \alpha F_1 (T_{HC} - T_H) \tag{9}$$

and:

$$Q_{LC} = \beta F_2 (T_L - T_{LC}) \tag{10}$$

We define a heat transfer surface area ratio (*f*):

$$f = F_1 / F_2 \tag{11}$$

The power input (*P*) to the refrigerator, according to the first law of thermodynamics, is:

$$P = Q_{HC} - Q_{LC} = Q_H - Q_L = Q_H - R \tag{12}$$

The coefficient of performance (ε) of the refrigerator is:

$$\varepsilon = \frac{Q_L}{P} = \frac{R}{P} \tag{13}$$

Equations (7)–(13) yield:

$$\varepsilon = \frac{R}{(R+q)(\phi \frac{T_{HC}}{T_{LC}} - 1)} \tag{14}$$

$$\sigma = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = \frac{Q_{HC} - q}{T_H} - \frac{R}{T_L} \tag{15}$$

Combining Equations (8)–(11) gives:

$$\frac{Q_{HC}}{R+q}=\frac{\phi T_{HC}}{T_{LC}}=\frac{f\alpha(T_{HC}-T_H)}{\beta(T_L-T_{LC})} \tag{16}$$

which then yields:

$$\frac{T_{HC}}{T_{LC}}=\frac{(f\alpha/\beta)T_H}{(\phi+f\alpha/\beta)T_{LC}-\phi T_L} \tag{17}$$

Combining Equations (3) and (9)–(11) gives:

$$T_{LC}=T_L-\frac{(R+q)(1+f)}{\beta F} \tag{18}$$

Substituting Equation (18) into Equations (16) and (17), we obtain:

$$\frac{T_{HC}}{T_{LC}}=\frac{T_H f(\alpha/\beta)}{(\phi+f\alpha/\beta)(T_L-(R+q)(1+f)/(\beta F))-\phi T_L} \tag{19}$$

and:

$$Q_{HC}=\frac{\phi T_H(R+q)}{T_L-(R+q)(1+f)(\phi/\alpha+f/\beta)/fF} \tag{20}$$

Substituting Equations (19) and (20) into Equations (14) and (15) derives the entropy generation rate (σ) and coefficient of performance (ε) of the generalized irreversible refrigerator cycle as follows:

$$\begin{aligned}\sigma&=\frac{Q_H}{T_H}-\frac{Q_L}{T_L}=\frac{Q_{HC}-q}{T_H}-\frac{R}{T_L}\\&=\frac{(R+q)\phi}{T_L-(R+q)(1+f)(\phi/\alpha+f/\beta)/fF}-\frac{R}{T_L}-\frac{q}{T_H}\end{aligned} \tag{21}$$

$$\varepsilon=[R/(R+q)]\{\phi T_H/[T_L-(R+q)(1+f)(\phi/\alpha+f/\beta)/(fF)]-1\}^{-1} \tag{22}$$

Based on the viewpoint of exergy analysis, the objective function of ecological optimization, proposed by Angulo-Brown [26] and modified by Yan [27], is:

$$E=P-T_0\sigma \tag{23}$$

where T_0 is the environment temperature.

The *ECOP* (ecological coefficient of performance) is defined by Ust *et al.* [31], as the ratio of power output to the loss rate of availability, *i.e.*,

$$ECOP=\frac{P}{T_0\sigma} \tag{24}$$

Based on the viewpoint of exergy analysis, Chen *et al.* [32] provide an ecological optimization objective for refrigerator cycles:

$$E=R[(\frac{T_0}{T_L}-1)-(1+\frac{1}{\varepsilon})(\frac{T_0}{T_H}-1)]-T_0\sigma \tag{25}$$

The *COPE* (coefficient of performance of exergy) is defined as the ratio of the exergy output rate to exergy loss rate (entropy generation rate), thus, *COPE* is a dimensionless ecological function. It is written as:

$$COPE = \frac{R[(\frac{T_0}{T_L} - 1) - (1 + \frac{1}{\varepsilon})(\frac{T_0}{T_H} - 1)]}{T_0 \sigma} \tag{26}$$

Combining Equations (21), (22) and (26) one obtains:

$$COPE = \frac{(Ra_1 + qa_2)[T_L - (R+q)(1+f)(\phi/\alpha + f/\beta)/fF] - (R+q)(T_0 - T_H)}{(R+q)T_0 - (Ra_1 + qa_2)[T_L/\phi - (R+q)(1+f)(\phi/\alpha + f/\beta)/fF]} \tag{27}$$

where $a_1 = \frac{T_0}{T_L} - 1$ and $a_2 = \frac{T_0}{T_H} - 1$.

Equations (21), (22) and (27) indicate that the entropy generation rate (σ), *COP*(σ) and *COPE* of the generalized irreversible refrigerator are functions of the heat transfer surface ratio (*f*) for given T_H, T_L, T_0, R, q, α, β, F, and ϕ. Taking the derivatives of σ, ε and *COPE* with respect to *f* and letting them equal to zero ($d\sigma/df = 0$, $d\varepsilon/df = 0$, $dCOPE/df = 0$) gives the same optimum surface area ratio:

$$f_a = (\beta\phi/\alpha)^{0.5} \tag{28}$$

The corresponding optimal entropy generation rate, the optimal *COP* and the optimal *COPE* are as follows:

$$\sigma = \frac{R+q}{T_L/\phi - (R+q)B} - \frac{R}{T_L} - \frac{q}{T_H} \tag{29}$$

$$\varepsilon = \frac{R[T_L/\phi - (R+q)B]}{(R+q)[T_H - T_L/\phi + (R+q)B]} \tag{30}$$

$$COPE = \frac{(Ra_1 + qa_2)[T_L/\phi - (R+q)B] - a_2(R+q)T_H}{(R+q)T_0 - [R(a_1+1) + q(a_2+1)](T_L/\phi - (R+q)B)} \tag{31}$$

where $B = \frac{[1 + (\alpha/\beta\phi)^{0.5}]^2}{\alpha F}$.

4. Discussion

(1) If $q = 0$ and $\phi = 1$, the model is reduced to the endoreversible refrigerator with external heat resistance as the sole irreversibility in the system, and Equations (29)–(31) become:

$$\sigma_1 = \frac{R}{T_L - RB_1} - \frac{R}{T_L} \tag{32}$$

$$\varepsilon_1 = \frac{T_L - RB_1}{T_H - T_L + RB_1} \tag{33}$$

$$COPE_1 = \frac{a_1(T_L - RB_1) - a_2 T_H}{T_0 - (a_1+1)(T_L - RB_1)} \tag{34}$$

where $B_1 = \frac{[1+(\alpha/\beta)^{0.5}]^2}{\alpha F}$.

(2) If $q>0$ and $\phi=1$, the model is reduced to the irreversible refrigerator with heat resistance and heat leak losses, and Equations (29)–(31) become:

$$\sigma_2 = \frac{R+q}{T_L-(R+q)B_1} - \frac{R}{T_L} - \frac{q}{T_H} \tag{35}$$

$$\varepsilon_2 = \frac{R[T_L-(R+q)B_1]}{(R+q)[T_H-T_L+(R+q)B_1]} \tag{36}$$

$$COPE_2 = \frac{(Ra_1+qa_2)[T_L-(R+q)B_1]-a_2T_H(R+q)}{(R+q)T_0-[R(a_1+1)+q(a_2+1)][T_L-(R+q)B_1]} \tag{37}$$

(3) If $q=0$ and $\phi>1$, the model is reduced to the irreversible refrigerator with heat resistance and internal irreversibilities, and Equations (29)–(31) become:

$$\sigma_3 = \frac{R}{T_L/\phi-RB} - \frac{R}{T_L} \tag{38}$$

$$\varepsilon_3 = \frac{T_L/\phi-RB}{T_H-T_L/\phi+RB} \tag{39}$$

$$COPE_3 = \frac{a_1(T_L/\phi-RB)-a_2T_H}{T_0-(a_1+1)(T_L/\phi-RB)} \tag{40}$$

(4) If $q>0$ and $\phi>1$, and in order to find the optimal coefficient of performance of exergy (*COPE*) for the generalized irreversible refrigerator, letting $dCOPE/dR=0$ yields:

$$R_m = \frac{Z_6}{2Z_4} \tag{41}$$

$$COPE_{\max} = \frac{Z_8Z_9-a_2T_HZ_{10}}{T_0Z_{10}-Z_9(Z_8-Z_9)} \tag{39}$$

where:

$$Z_1 = qT_H - qT_L/\phi + q^2B \tag{40}$$

$$Z_2 = q + \frac{q^2B}{T_H} - \frac{qT_L}{\phi T_H} \tag{41}$$

$$Z_3 = \frac{T_0T_L}{\phi} - \frac{T_0T_L}{\phi} \tag{42}$$

$$Z_4 = B(1-\frac{T_H}{T_L}) + qB^2(\frac{1}{T_H}-\frac{1}{T_L}) \tag{43}$$

$$Z_5 = (\frac{T_L}{\phi} - T_H - 2qB - 2RB)Z_2 - (1-\frac{1}{\phi}+qB(\frac{1}{T_L}+\frac{1}{T_H})+2RB\frac{T_0}{T_L})Z_1 \tag{44}$$

$$Z_6 = (Z_3^2-4Z_4Z_5)^{0.5} \tag{45}$$

$$Z_7 = Z_6 - Z_3 \tag{46}$$

$$Z_8 = \frac{a_1 Z_6}{2Z_4} + qa_2 \tag{47}$$

$$Z_9 = (\frac{T_L}{\phi} - qB - \frac{BZ_6}{2Z_4}) \tag{48}$$

$$Z_{10} = \frac{Z_6}{2Z_4} + q \tag{49}$$

5. Numerical Examples

5.1. Calculation Conditions

Assume that heat leakage from the heat sink at temperature T_H to the heat source at temperature T_L follows a linear law, which was first provided by Bejan [34], and it is given as:

$$q = C_i(T_H - T_L) \tag{53}$$

where C_i is the internal conductance of the refrigerator. Substituting Equation (53) into the above performance equations yields the corresponding optimal relationships.

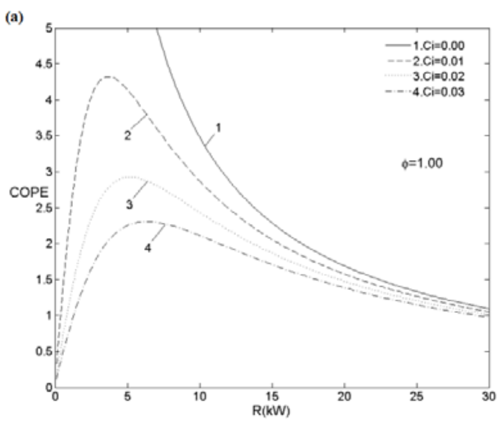
In the numerical calculation for the performance characteristics of the refrigerator, $\alpha = \beta$, $\alpha F = 4\text{kWK}^{-1}$, $T_H = 300\text{K}$, $T_L = 260\text{K}$, $T_0 = 290K$, $\phi = 1.00 - 1.30$ and $Ci = 0.00 - 0.03\text{kWK}^{-1}$ are set.

5.2. The Main Influencing Factors on COPE

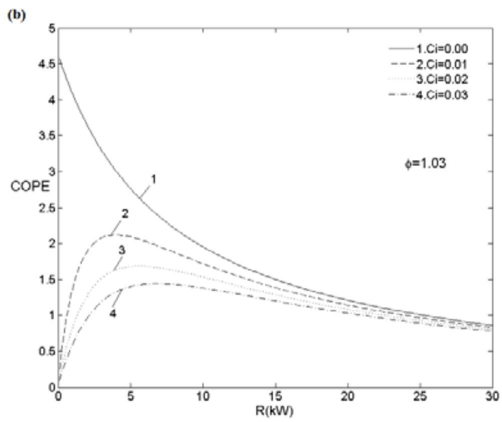
5.2.1. Cooling Load and Entropy Generation Rate

The variation of the *COPE* ecological function with respect to cooling load, $COPE - R$, and the variation of the *COPE* ecological function *versus* entropy generation rate, $COPE - \sigma$, for different values of ϕ and C_i are plotted in Figures 3 and 4.

Figure 3. The variation of the *COPE* ecological function with respect to the cooling load (R) for different values of ϕ and C_i. **(a)** $\phi = 1.00$ **(b)** $\phi = 1.03$ **(c)** $\phi = 1.07$ **(d)** $\phi = 1.10$

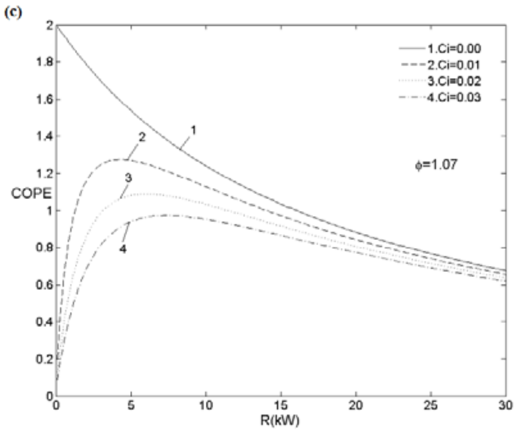


(a)



(b)

Figure 3. *Cont.*



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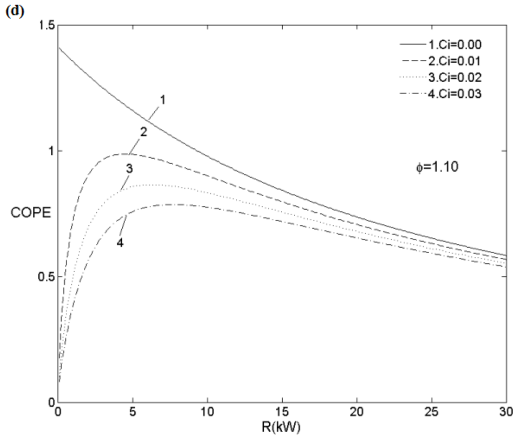
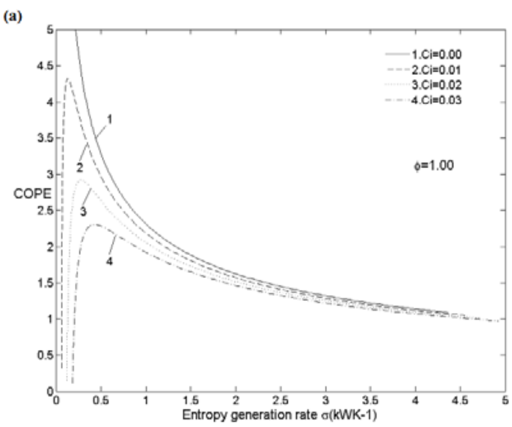
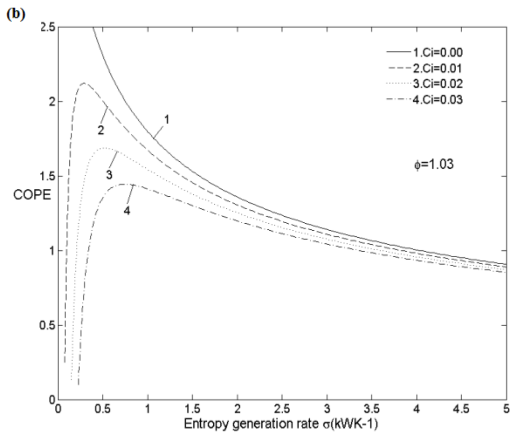


(d)

Figure 4. Variation of the *COPE* ecological function with respect to the entropy generation rate (σ) for different values of ϕ and C_i. **(a)** $\phi = 1.00$ **(b)** $\phi = 1.03$ **(c)** $\phi = 1.07$ **(d)** $\phi = 1.10$.



(a)



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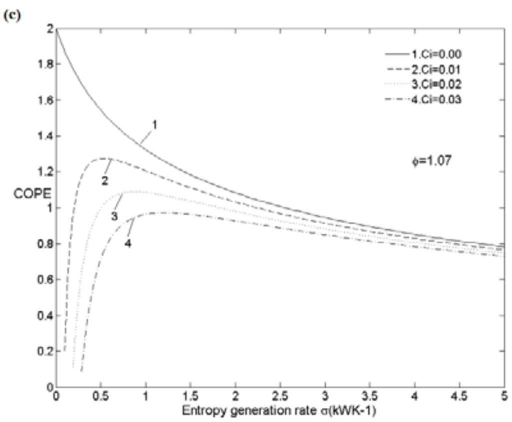

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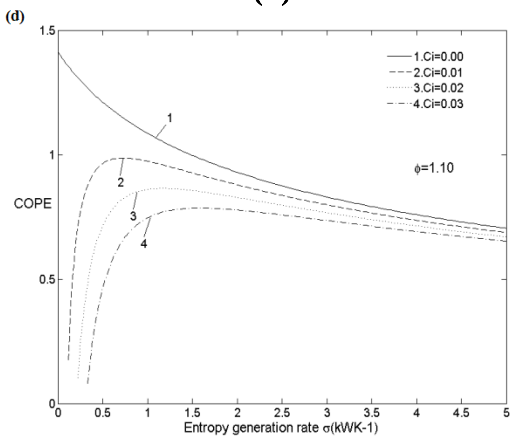


(d)

Figure 3 illustrates that the *COPE* is a monotonic decreasing function while $C_i = 0$. In other situation, the *COPE* reaches the maximum point while the cooling load is at the optimal value. The maximum of *COPE* declines from about 4.5 to less than 1.5 with the growth of irreversibility ϕ.

The details can be seen from Table 1. Figure 4 shows that the *COPE* function follows the similar trend with respect to the entropy generation rate (ϕ) as it *versus* the cooling load (*R*).

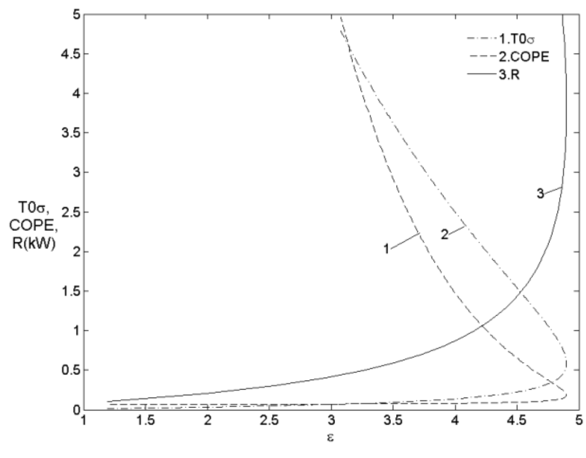
Table 1. Performance of irreversible refrigerators.

ϕ	**1.0**	**1.0**	**1.0**	**1.1**	**1.1**	**1.1**
C_i	0.01	0.02	0.03	0.01	0.02	0.03
$T_0\sigma_E$(kW)	0.14	0.27	0.43	0.73	1.17	1.58
ε_E	5.23	4.78	4.46	3.11	2.89	2.73
$COPE_{\max}$	4.31	2.92	2.30	0.98	0.8644	0.79
$ECOP_m$	27.87	18.77	14.76	6.19	5.39	4.88
R_m(kW)	3.68	5.2	6.356	4.51	6.34	7.72
ϕ	1.2	1.2	1.2	1.3	1.3	1.3
C_i	0.01	0.02	0.03	0.01	0.02	0.03
$T_0\sigma_E$(kW)	1.47	2.26	2.92	2.21	3.37	4.36
ε_E	2.22	2.08	1.97	1.73	1.62	1.55
$COPE_{\max}$	0.57	0.52	0.49	0.41	0.38	0.36
$ECOP_m$	3.49	3.16	2.93	2.43	2.24	2.10
R_m(kW)	5.00	7.05	8.58	5.376	7.53	9.15

5.2.2. The Coefficient of Performance (*COP*)

Exergy loss rate, *COPE*, cooling load *versus COP* characteristic with $\phi = 1.01$ and $C_i = 0.01\text{kWK}^{-1}$ is illustrated in Figure 5. It shows that there exists a maximum *COP* point while the corresponding exergy loss rate, *COPE*, and cooling load are $(T_0\sigma)_\varepsilon$, $COPE_\varepsilon$, and R_ε, respectively [32]. It can be seen from Figure 5 that the optional working field should be on the upper half of the characteristic curves, that is the region lied in $T_0\sigma > (T_0\sigma)_\varepsilon$, $COPE > COPE_\varepsilon$ or $R > R_\varepsilon$.

Figure 5. Variation of the exergy loss rate, *COPE* and cooling load (R) with respect to *COP* (ε) characteristics with $\phi = 1.01$ and $C_i = 0.01\text{kWK}^{-1}$.



5.2.3. The Irreversibility and Internal Conductance

One can observe from Figures 6 and 7 that with increase of the irreversibility ϕ or the internal heat conductance of the refrigerator C_i, the *COPE* function decreases monotonically. It also shows that as the cooling load increases, slope of the *COPE* function decreases.

Figure 6. Influence of ϕ on *COPE* with different values of cooling load (*R*).

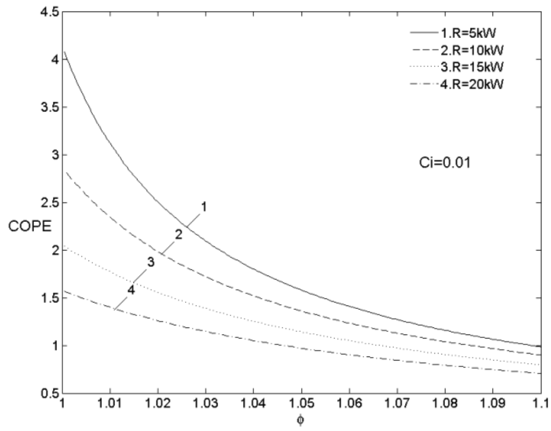
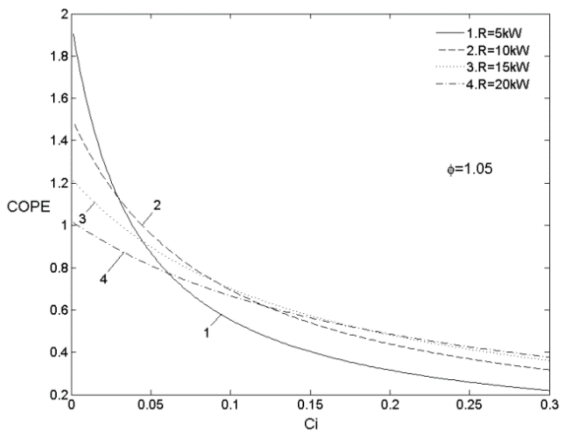


Figure 7. Influence of C_i on *COPE* with different values of cooling load (*R*).



5.3. Optimal Matching Criterion

The optimization of the dimensionless *COPE* ecological function provides a selectable design point which satisfies the condition mentioned above and gives the corresponding optimal selection. In order to judge design point performance, we choose another arbitrary working condition, for example the condition where the cooling load is 6 kW with $\phi = 1.2$ and $C_i = 0.01$. Comparing the chosen working point with the optimal one, the *COP* decreases about 1%, but the entropy generation rate increases about 21%.

The optimization of the *COPE* function makes the entropy generation rate of the cycle decrease significantly, and the *COP* reaches the maximum point when the cooling load is chosen at the optimal point. Therefore, it represents a compromise between the exergy output rate and the exergy loss rate (entropy generation rate) when the cooling load is a constant.

5.4. Comparison between COPE *and* ECOP

Ust proposed in [40] an ecological function *ECOP* for irreversible air refrigeration cycles as $ECOP = R / T_0\sigma$, where R is the cooling load and $T_0\sigma$ is the entropy generation rate of the cycle. In fact, for refrigeration cycles, the exergy output is not the cooling load [28,29]. The exergy output rate of refrigeration cycles should be written as:

$$A = Q_{\mathrm{L}}(T_0 / T_{\mathrm{L}} - 1) - Q_{\mathrm{H}}(T_0 / T_{\mathrm{H}} - 1) \tag{54}$$

which is just the first term of the right side of Equation (25). *ECOP* combines the cooling load R, which is a physical quantity based on the energy point, with the entropy generation rate (exergy loss rate) $T_0\sigma$, which is a physical quantity based on the exergy point. It cannot represent a best compromise between the exergy output rate, A, and the exergy loss rate, $T_0\sigma$, of the refrigeration cycles, based on the viewpoint of exergy analysis.

Therefore, we propose the *COPE* function as a unified ecological criterion. It has several advantages to analyze the performance of the refrigeration system. First, the *COPE* function would not take negative values as equations with the form like $E = R - T_0\sigma$. The dimensionless form of the *COPE* guarantees the objective function meaningful both mathematically and thermodynamically. Second, the *COPE* function is defined purely from the perspective of exergy. It can provide the best compromise between the exergy output rate to the exergy loss rate.

6. Conclusions

A thermo-ecological performance optimization of the generalized irreversible refrigerator models with finite heat transfer ability, heat leakage and internal irreversibilities based on a new objective function named coefficient of performance of exergy (*COPE*) has been carried out. Numerical examples are given to analyze the influences of heat leakage and internal irreversibilities on the optimal performance of the generalized irreversible refrigerator.

By optimizing the ecological function *COPE*, we find that: (1) the irreversibility and internal conductance have inverse proportion relationship with the ecological function *COPE*, that is, both of them contribute to exergy destruction; (2) when the cooling load increases, the effects of irreversibilities and heat leakage on *COPE* function become weaker; (3) there exists a maximum point of *COP* at about 5 for a given exergy loss rate or cooling load; (4) the optimization process gives a new criterion on how to match cooling load for given generalized irreversible refrigerator system.

Based on the viewpoint of exergy analysis, we have derived analytically the optimal cooling load, the optimum coefficient of performance and the minimum entropy generation rate. It also represents a compromise between the exergy output rate and the entropy generation rate. Furthermore, it gives a candidate option for refrigerator design. Thus, it is essential to investigate and optimize the ecological function *COPE* for the generalized irreversible refrigerator by taking account of both the internal irreversibilities and heat leakages.

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Conflicts of Interest

The authors declare no conflict of interest.

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