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The Gibbs Paradox, the Landauer Principle and the Irreversibility Associated with Tilted Observers

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Abstract: It is well known that, in the context of General Relativity, some spacetimes, when described by a congruence of comoving observers, may consist of a distribution of a perfect (non–dissipative) fluid, whereas the same spacetime as seen by a "tilted" (Lorentz–boosted) congruence of observers may exhibit the presence of dissipative processes. As we shall see, the appearance of entropy-producing processes are related to the high dependence of entropy on the specific congruence of observers. This fact is well illustrated by the Gibbs paradox. The appearance of such dissipative processes, as required by the Landauer principle, are necessary in order to erase the different amount of information stored by comoving observers, with respect to tilted ones.

Keywords: tilted spacetimes; irreversibility; dissipative procceses

1. Introduction

"Irreversibility is a consequence of the explicit introduction of ignorance into the fundamental laws." M. Born Observers play an essential role in any physical theory. This is is particularly true in Thermodynamics and in General Relativity.

Indeed, in this latter theory, it is well known that a variety of line elements may satisfy the Einstein equations for different (physically meaningful) stress-energy tensors (see [1–12] and references therein). This ambiguity in the description of the source may be related, in some cases, to the arbitrariness in the choice of the four–velocity in terms of which the energy-momentum tensor is split.

The above-mentioned arbitrariness, in its turn, is related to the well-known fact that different congruences of observers would assign different four-velocities to a given fluid distribution. In this case, we have in mind the situation when one of the conguences corresponds to comoving observers, whereas the other is obtained by applying a Lorentz boost to the comoving observers.

For example, in the case of the zero curvature Friedmann–Robertson–Walker (FRW) model, we have a perfect fluid solution for observers at rest with respect to the timelike congruence defined by the eigenvectors of the Ricci tensor, whereas for observers moving relative to the previously mentioned congruence of observers, it can also be interpreted as the exact solution for a viscous dissipative fluid [4]. It is worth noting that the relative ("tilting") velocity between the two congruences may be related to a physical phenomenon such as the observed motion of our galaxy relative to the microwave background radiation [9].

Thus, zero curvature FRW models as described by "tilted" observers, will exhibit a dissipative fluid and energy–density inhomogeneity, as well as different values for the expansion scalar and the shear tensor, among other differences, with respect to the "standard" (comoving) observers (see [4] for a comprehensive discussion on this example).

The same phenomenon appears in the tilted versions of the Lemaitre–Tolman–Bondi (LTB) [13–15] (see [16]), the Szekeres spacetimes [17,18] (see [19]), and in many other circumstances (see [20–25] and references therein) .

At this point, we should mention that in the past it has been argued that dissipative fluids (understood as fluids whose energy-momentum tensors present a non-vanishing heat flux contribution), are not necessarily incompatible with reversible processes (e.g., see [26–28]).

In the context of the standard Eckart theory [29], a necessary condition for the compatibility of an imperfect fluid with vanishing entropy production (in the absence of bulk viscosity) is the existence of a conformal Killing vector field (CKV) χ^α such that $\chi^\alpha = \frac{V^\alpha}{T}$ where V^α is the four-velocity of the fluid and T denotes the temperature. In the context of causal dissipative theories, e.g., [30–35], the existence of such CKV is also necessary for an imperfect fluid to be compatible with vanishing entropy production (see [16]).

However, a much more careful analysis of the problem readily shows that the compatibility of reversible processes and the existence of dissipative fluxes becomes trivial if a constitutive transport equation is adopted, since in this latter case such compatibility forces the heat flux vector to vanish as well. In other words, even if ab initio the fluid is assumed to be imperfect (non–vanishing heat flow vector), the imposition of the CKV and the vanishing entropy production condition may cancel the heat flux once a transport equation is assumed (see [36] for a detailed discussion on this point).

In other words, in the presence of a CKV of the kind mentioned before, the assumption of a transport equation whether in the context of the Eckart–Landau theory, or a causal theory, implies that a vanishing entropy production leads to a vanishing heat flux vector. Therefore, under the conditions above, the system is not only reversible but also non-dissipative.

Furthermore, since neither LTB nor the Szekeres spacetimes admit a CKV, we may safely conclude that the heat flux vector appearing in these cases is associated with truly (entropy-producing) dissipative processes.

The main purpose of this work is to explain the origin of such processes.

2. Comoving and Tilted Observers

Let us consider a congruence of observers which are comoving with a dissipationless dust distribution, then the four-velocity for that congruence, in some globally defined coordinate sytem, reads

$$v^\mu = (1, 0, 0, 0). \tag{1}$$

In order to obtain the four-velocity corresponding to the tilted congruence (in the same globally-defined coordinate system), one proceeds as follows.

We have first to perform a (locally defined) coordinate transformation to the Locally Minkowskian Frame (LMF). Denoting by L^ν_μ the local coordinate transformation matrix, and by $\bar{v}^\alpha$ the components of the four velocity in such LMF, we have:

$$\bar{v}^\mu = L^\mu_\nu v^\nu. \tag{2}$$

Next, let us perform a Lorentz boost from the LMF associated with $\bar{v}^\alpha$, to the (tilted) LMF with respect to which a fluid element is moving with some, non-vanishing three–velocity.

Then the four-velocity in the tilted LMF is defined by:

$$\tilde{v}_\beta = \Lambda^\alpha_\beta \bar{v}_\alpha, \tag{3}$$

where Λ^α_β denotes the Lorentz matrix.

Finally, we have to perform a transformation from the tilted LMF, back to the (global) frame associated to the line element under consideration. Such a transformation, which obviously only exists locally, is defined by the inverse of L^ν_μ, and produces the four-velocity of the tilted congruence, in our globally defined coordinate system, say V^α.

Let us now consider a given spacetime, which according to comoving observers, is sourced by a dissipationless dust distribution, so that the energy momentum-tensor reads

$$T^{C}_{\mu\nu} = \mu_C v_\mu v_\nu, \tag{4}$$

where C stands for comoving and μ_C denotes the energy density, as measured by the comoving observers.

However, for the tilted congruence we may write

$$T^{T}_{\alpha\beta} = (\mu_T + P)V_\alpha V_\beta + P g_{\alpha\beta} + \Pi_{\alpha\beta} + q_\alpha V_\beta + q_\beta V_\alpha, \tag{5}$$

where T stands for tilted, and μ_T, q_α, P and $\Pi_{\alpha\beta}$ denote the energy density, the heat flux, the isotropic pressure, and the anisotropic tensor, as measured by the tilted observers.

Obviously, both energy–momentum tensors are exactly the same, since the metric is the same and therefore the Einstein tensor is the same. However, the way in which the energy–momentum tensor is split is not the same. This simple fact opens the possibility (for tilted observers) to obtain an energy–momentum tensor which describes a quite different picture from the one obtained by comoving observers. In the same order of ideas, it should be emphasized that the kinematical variables (four–acceleration, expansion scalar, shear tensor, vorticity tensor), being defined in terms of the four–velocity, will also differ from their values as measured by the comoving observers.

Among the differences appearing in the tilted congruence, with respect to the comoving one, there is one which raises the most intriguing question, namely: how it is possible that tilted observers may detect irreversible processes, whereas comoving observers describe an isentropic situation?

As we shall see, the answer to the above question is closely related to the fact that the definition of entropy is highly observer-dependent, as illustrated, for example, by the Gibbs paradox.

3. The Gibbs Paradox, the Landauer Principle and the Definition of Entropy

Entropy is a measure of how much is not known (uncertainty). Also known, although usually overlooked, is the fact that physical objects do not have an intrinsic uncertainty (entropy) (see [37] for an enlightening discussion on this issue).

The "subjective" nature of the concept of entropy is clearly illustrated by the Gibbs paradox. In its simplest form, the paradox appears from the consideration of a box divided by a wall in two identical parts, each of which is filled with an ideal gas (at the same pressure and temperature). Then, if the partition wall is removed, the gases of both parts of the box will mix.

Now, if the gases from both sides are distinguishable, the entropy of the system will rise, whereas if they are identical there is no increase in entropy. This leads to the striking conclusion that irreversibility (and thereby entropy), depends on the ability of the observer to distinguish, or not, the gases from both sides of the box. In other words, irreversibility would depend on our knowledge of physics [38], confirming thereby our previous statement that physical objects are deprived of intrinsic entropy. It can only be defined **after** the number of states that can be resolved by the measurements are established. The anthropomorphic nature of entropy has been brought out and discussed in detail by Jaynes [39].

Let us now turn back to our comoving and tilted observers.

If a given physical system is studied by a congruence of comoving observers, this implies at once that the three–velocity of any given fluid element is automatically assumed to vanish, whereas for the tilted observers this variable represents an additional degree of freedom. In other words, the number of possible states in the latter case is much larger than in the former one.

Since, for the comoving observers, the system is dissipationless, it is clear that the increasing of entropy, when passing to the tilted congruence, should imply the presence of dissipative (entropy producing) fluxes, in the tilted congruence.

It is instructive to take a look on this issue from a different perspective, by considering the transition from the tilted congruence to the comoving one. According to the Landauer principle,

[40] (also referred to as the Brillouin principle [41–45]), the erasure of one bit of information stored in a system requires the dissipation into the environment of a minimal amount of energy, the lower bound of which is given by

$$\triangle E = kT \ln 2, \tag{6}$$

where k is the Boltzmann constant and T denotes the temperature of the environment.

In the above, erasure is just a reset operation restoring the system to a specific state, and is achieved by means of an external agent. In other words, one can decrease the entropy of the system by doing work on it, but then one has to increase the entropy of another system (or the environment).

Thus, the Landauer principle is an expression of the fact that logical irreversibility necessarily implies thermodynamical irreversibility.

Now, when passing from the tilted to the comoving congruence, a decrease of entropy occurs, but we have no external agent, and therefore such a decrease of entropy is accounted for by the dissipative flux observed in the tilted congruence (we recall that in the comoving congruence the system is dissipationless).

The point is, that passing from one of the congruences to the other, we usually overlook the fact that both congruences of observers store different amounts of information. Here resides the clue to resolve the quandary mentioned above, about the presence or not of dissipative processes, depending on the congruence of observers, that carry out the analysis of the system.

Before concluding this section, it is necessary to make three remarks:

- The main issue discussed in this work, namely: the presence or not of dissipative processes, depending on the congruence of observers, that carry out the analysis of the system, will remain for any theory of gravity. However, specific details of the dissipative processes observed by the tilted observers, will depend on the theory of gravity under consideration.
- The discussion about the entropy budget of the universe is of the utmost relevance (see [46] and references therein), because its increase is associated with all possible irreversible processes, on all scales. However, in that reference, as well as in the references therein, the issue under consideration is the estimate of entropy as observed by *one given* congruence of observers. The main point of our work is to stress how (and why) any of these estimates, changes when it is evaluated by *different* congruences of observers.
- It goes without saying that, in the context of a covariant theory of gravity (such as General Relativity), a covariant definition of entropy should be invoked. Such a definition can be found in the context of different relativistic dissipative theories (see for example [30–35]). However, we have not made use of them in the text, which explains why we did not refer to this particular issue.

4. Conclusions

We may summarize the main issues addressed in this paper in the following points:

- Uncertainty (entropy) is highly dependent on the observer.
- Comoving and tilted observers store different amounts of information.
- According to the Landauer principle, erasure of information is always accompanied by dissipation (there is a price for forgetting).
- The detection of dissipative processes by tilted observers in physical systems which are described by comoving observers, such as perfect fluids, becomes intelligible in the light of the three previous comments.

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