

Article

When Photons Are Lying about Where They Have Been

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Abstract: The history of photons in a nested Mach–Zehnder interferometer with an inserted Dove prism is analyzed. It is argued that the Dove prism does not change the past of the photon. Alonso and Jordan correctly point out that an experiment by Danan et al. demonstrating the past of the photon in a nested interferometer will show different results when the Dove prism is inserted. The reason, however, is not that the past is changed, but that the experimental demonstration becomes incorrect. The explanation of a signal from the place in which the photon was (almost) not present is given. Bohmian trajectory of the photon is specified.

Keywords: past of the photon; Mach–Zehnder interferometer; Dove prism; photon trajectory

1. Introduction

This work describes peculiar behaviour of photons in the modification of the experiment of Danan et al. [1] proposed by Alonso and Jordan (AJ) [2]. In the Danan et al. experiment, photons were asked where exactly they have been inside a nested interferometer tuned in a particular way. The AJ modification makes photons to tell that they have been in a place in which, according to the narrative of the two-state vector formalism (TSVF) [3], they could not have been. Note that this work is only slightly related to the results presented by one of the authors (L.V.) at “Emergent Quantum Mechanics” that have been already published [4,5].

Textbooks of quantum mechanics teach us that we are not supposed to ask where the photons passing through an interferometer were. Wheeler [6] introduced the delayed choice experiment in an attempt to analyze this question. Vaidman [3] suggested a different approach. He proposed a definition according to which a quantum particle was where it left a trace and showed that the past of the particle can be easily seen in the framework of the TSVF [7] as regions of the overlap of the forward and backward evolving quantum states. Vaidman, together with his collaborators, performed an experiment demonstrating a surprising trace of the photons in nested interferometers [1] (see Figure 1). These results became the topic of a very large controversy [8–46].

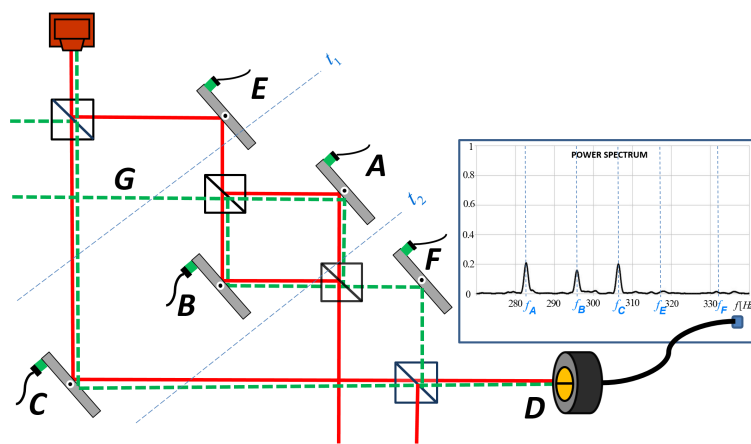


Figure 1. Nested Mach–Zehnder interferometer with inner interferometer tuned to destructive interference towards F . Although our ‘common sense’ suggests that the only possible path for the photon detected in D is path C , the trace was found also inside the inner interferometer supporting the TSVF proposal according to which the particle was present in the places where forward (red continuous line) and backward (green dashed line) evolving wavefunctions overlap. The latter is demonstrated by the results of the measurement by Danan et al. [1].

2. Alonso and Jordan Modified Interferometer

Here we analyze, in our view, the most interesting objection which was made by Alonso and Jordan [2]. They suggested inserting a Dove prism inside one of the arms of the inner interferometer (see Figure 2). They asked: “Can a Dove prism change the past of a single photon?”. Their analysis of this modified experiment was correct. Although the formalism suggested that the past of the photon remains the same as in the original experiment, i.e., the photon was present near mirrors C , A , B but not near mirrors E and F , the experiment should show, in addition to frequencies f_C , f_A , f_B , also the frequency f_E . This is in contradiction with the fact that the photons, according to Vaidman, were not present near mirror E .

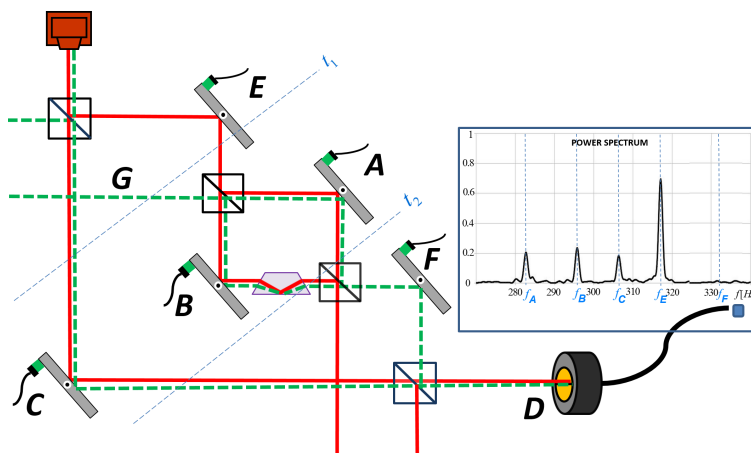


Figure 2. Nested Mach–Zehnder interferometer with a Dove prism inside the inner interferometer as suggested by Alonso and Jordan [2]. The region of the overlap of the forward and the backward evolving states remains the same, but predicted results of an experiment similar to [1] include a signal from mirror E where the photon was not supposed to be.

The experiment of Danan et al. was not a direct measurement of the trace left by the photons. The reason is that such direct measurement is very difficult, as it requires collecting data about the trace conditioned on detection of the photon by a particular detector. In the actual experiment, instead of measuring the trace on the external system (as in a recent experiment [47]), the trace was ‘written’ on the photons themselves, on the degree of freedom of their transverse motion. Observing this degree of freedom of post-selected particle replaced the coincidence counting in the experimental setup. Although indirect, the experiment [1] was correct. A local trace created at mirrors was read later on the quad-cell detector. We argue that introducing a Dove prism [2] spoils the experiment, making the signal at the quad-cell detector no longer a faithful representation of the trace created at mirror E .

Although the signal in the Danan et al. experiment was appearing as a particular frequency in the output of the quad-cell detector, the frequency was not an actual trace written on each photon. Wiggling with different frequencies was a trick that allowed in a single run to see records made at different mirrors. (It also improved significantly the signal-to-noise ratio, since noise had no preference for the frequencies of the wiggling mirrors.) The physical signal in the Danan et al. experiment (Figure 1) originated from the shift of the beam direction at a mirror. It corresponded to the transversal kick in the momentum δp_x . This momentum shift translated into a position shift of the beam, which was read in the quad-cell detector. The property which allowed to observe the trace was that the change δp_x in the transversal momentum had no change when the beam evolved towards port D from all mirrors and through all possible paths.

This is no longer the case when the Dove prism is introduced (Figure 2). For mirrors A and C , it is still true, since the modes do not pass through the Dove prism. For mirror B , there is a difference in that the Dove prism flips the sign of the signal. However, since we measure just the size of the signal, this change is not observable, and the peak at frequency f_B correctly signifies the presence of the photon in B . The only problem occurs with the mirror E . The beam from E reaches the detector through A and through B . The shifts are in opposite directions, so the reading position of the beam on the detector does not tell us what the shift of the transversal momentum in E was. Therefore, we should not rely on the result of the experiment with the setup of the Danan et al. experiment when the Dove prism is present.

Note that a simple modification will restore the results of the Danan et al. experiment even with the presence of the Dove prism. If the wiggling of mirrors is made such that the beam is shifted in the direction perpendicular to the plane of the interferometer, the Dove prism will not cause flipping of the direction of the shift and the peak at f_E will disappear.

3. The Trace Analysis

We have explained that the AJ modification of the Danan et al. experiment is not a legitimate experiment for measuring the presence (according the local trace definition) of the particle near mirror E . Still it is of interest to understand how a strong signal with frequency f_E is obtained in this modification. For this, we need a more detailed analysis of traces in the nested Mach–Zehnder interferometer (MZI) experiments.

We consider, for simplicity, an experiment in which only one particular mirror changes its angle at every run. The shift of the beam direction created at the mirror, characterized by the transversal momentum kick δp_x , leads to the shift of the beam position on the detector. This creates the signal: the difference in the current of the upper and the lower cells of the detector.

Let χ_0 be the original mode of the photons without shifts. The photons in a shifted beam will then be in a superposition of the original mode χ_0 and a mode χ_\perp , orthogonal to χ_0 :

$$|\chi'\rangle = \frac{1}{\sqrt{1+\epsilon^2}} (|\chi_0\rangle + \epsilon|\chi_\perp\rangle). \quad (1)$$

For small signals which appeared in the Danan et al. experiment, the momentum kick is proportional to the relative amplitude ϵ of the orthogonal mode [48]:

$$\delta p_x = 2\epsilon \text{Re} [\langle \chi_0 | p_x | \chi_\perp \rangle] + \mathcal{O}(\epsilon^2). \quad (2)$$

Note that, for a Gaussian beam (which is a good approximation of the beam in the experiment), higher order contributions do not appear [48].

What is important for our analysis is that χ_0 is symmetric with respect to the center of the beam in the transverse direction, while χ_\perp , which can be approximated as a difference between two slightly shifted Gaussians, is an antisymmetric mode. Indeed, in momentum representation, we have

$$\chi_0 \simeq \mathcal{N}_0 e^{-\frac{p_x^2 + p_y^2}{2\Delta^2}}, \quad \chi_\perp \simeq \mathcal{N}_\perp p_x e^{-\frac{p_x^2 + p_y^2}{2\Delta^2}}, \quad (3)$$

where Δ is the momentum uncertainty of the Gaussian beam, and $\mathcal{N}_0, \mathcal{N}_\perp$ are the normalization constants.

In the Danan et al. experiment (Figure 1), the trace of the photon was read as the shift of the beam on the detector. This shift is proportional to the strength of the trace quantified by the value of the relative amplitude ϵ of the orthogonal component. The original mode χ_0 and the orthogonal mode χ_\perp evolve towards port D from all mirrors and through all possible paths in an identical manner, so the position shift on the detector faithfully represents a locally created trace.

This is no longer the case when the Dove prism is introduced (Figure 2). For mirror B , there is a difference: mode χ_0 is unaffected by the presence of the prism, while mode χ_\perp flips the sign. The shift on the detector changes its direction. This change, however, is not observable in the experiment, since the frequency spectrum is sensitive only to the size of the signal. The observable difference appears for mirror E . There are two paths from E to the output port D , one passing through mirror A and another passing through mirror B . The original symmetric mode χ_0 would reach D undisturbed both on path A and on path B , while the orthogonal mode χ_\perp would reach D undisturbed on path A but with a flipped sign on path B . When combined, there exists a phase difference π between path A and path B , which leads to destructive interference of the original symmetric mode and constructive interference of the orthogonal antisymmetric mode at the output port towards mirror F . As a result, out of the modes of the light reflected by the mirror E , only the mode χ_\perp reaches D .

If we send the photon only in path A , and do not move mirror A , only mode χ_0 reaches the detector. Adding a small rotation of mirror A will lead to appearance of mode χ_\perp with relative amplitude ϵ . If, instead, in an undisturbed interferometer, we send the photon only in path E , and nothing will reach the detector. A small rotation of mirror E will lead to appearance of mode χ_\perp on the detector and only mode χ_\perp . This mode by itself does not lead to a shift of the center of the beam on the detector. In the experiment, the photon is in a superposition of two states, one coming from path C and the other from path E . From path C , we get mode χ_0 with the same amplitude as it comes from path A . It is the interference of mode χ_0 coming through C and mode χ_\perp coming through F on the surface of the detector that yields the shift of the center of the beam. The resultant shift is larger than the shift created by the same rotation of mirror A because, first, the intensity in E is twice the intensity in A so the amplitude of the mode χ_\perp created at E is larger than the amplitude of χ_\perp created at A , and, second, the amplitude is not reduced at the second beam splitter of the inner interferometer as it happens for the mode created at A , due to the constructive interference of χ_\perp mode in the inner interferometer with the Dove prism. This explains the larger signal observed at f_E .

4. Do the Photons Have Any Presence in E ?

Our analysis above shows that the experiment with the Dove prism does not contradict Vaidman's proposal [3] demonstrated in the Danan et al. experiment, and explains using standard quantum mechanics the appearance of the signal at frequency f_E . Thus, it provides a satisfactory reply to Alonso

and Jordan. However, it will also be of interest to explain the predicted results of Danan's setup with the Dove prism using Vaidman's approach.

Let us quote the Danan et al. Letter [1]:

"The photons themselves tell us where they have been. And the story they tell is surprising. The photons do not always follow continuous trajectories. Some of them have been inside the nested interferometer (otherwise, they could not have known the frequencies f_A, f_B), but they never entered and never left the nested interferometer, since otherwise they could not avoid the imprints of frequencies f_E and f_F of mirrors E and F leading photons into and out of the interferometer."

With the Dove prism present, however, we do get frequency f_E . How can it happen if the photons were not in E as we argued here? Let us analyse the situation, in which only mirror E changes its angle by a small amount leading to the superposition (1) of the modes of the photon.

We start by repeating the analysis of the setup without the Dove prism in the framework of the TSVF [7]. After passing the mirror E , at time t_1 , the forward evolving state is (see Figure 2)

$$|\Psi\rangle_{t_1} = \sqrt{\frac{2}{3(1+\epsilon^2)}} |E\rangle (|\chi_0\rangle + \epsilon|\chi_\perp\rangle) + \frac{1}{\sqrt{3}} |C\rangle |\chi_0\rangle, \quad (4)$$

where we split which path and the mode degrees of freedom of the photon. The forward evolving state, at time t_2 , in the middle of the interferometer is then

$$|\Psi\rangle_{t_2} = \frac{1}{\sqrt{3(1+\epsilon^2)}} (|A\rangle + i|B\rangle) (|\chi_0\rangle + \epsilon|\chi_\perp\rangle) + \frac{1}{\sqrt{3}} |C\rangle |\chi_0\rangle. \quad (5)$$

Since in the experiment we use photon degrees of freedom for the measurement, we do not postselect on a particular state but rather on a space of states corresponding to all modes reaching detector D . Thus, strictly speaking, there is no definite backwards evolving state. However, we can use a standard 'trick' [4], in which we consider a hypothetical additional verification measurement of the mode state after the postselection on the path D . We verify that the state which we calculate will surely be there, and this verification measurement, together with the path post-selection, defines the backward evolving state.

The wave packets from A and B destructively interfere toward F even when mirror E is slightly rotated, so the only mode reaching D is coming from C , which is χ_0 . Therefore, the backward evolving state starts from $\langle D|\langle\chi_0|$, which in the middle of the interferometer turns into

$$\langle\Phi|_{t_2} = \frac{1}{\sqrt{3}} (\langle A| - i\langle B| + \langle C|) \langle\chi_0|. \quad (6)$$

There is here destructive interference of the backward evolving quantum state toward E , so, at time t_1 , the backward evolving state is

$$\langle\Phi|_{t_1} = \frac{(\sqrt{2}\langle G| + \langle C|) \langle\chi_0|}{\sqrt{3}}. \quad (7)$$

Thus, the weak value of the projection operator $\mathbf{P}_E = |E\rangle\langle E|$ at E is

$$(\mathbf{P}_E)_w = \frac{\langle\Phi|\mathbf{P}_E|\Psi\rangle_{t_1}}{\langle\Phi|\Psi\rangle_{t_1}} = 0. \quad (8)$$

Therefore, at time t_1 the photons have no presence in E , not even a "small" presence.

With the Dove prism inside, this is no longer the case. Instead of (5), we obtain

$$|\Psi'\rangle_{t_2} = \frac{1}{\sqrt{3(1+\epsilon^2)}} [(|A\rangle + i|B\rangle)|\chi_0\rangle + \epsilon(|A\rangle - i|B\rangle)|\chi_\perp\rangle] + \frac{1}{\sqrt{3}} |C\rangle|\chi_0\rangle. \quad (9)$$

The wave packets from A and B destructively interfere towards F for mode χ_0 , while the mode χ_\perp interferes constructively towards F . As a result, the backward evolving state (given the proper hypothetical measurement) starts approximately as

$$\frac{1}{\sqrt{1+2\epsilon^2}} \langle D | (|\chi_0\rangle + \sqrt{2}\epsilon |\chi_\perp\rangle). \quad (10)$$

Evolving it backwards until time t_1 , we obtain approximately:

$$\langle \Phi' |_{t_1} = \frac{1}{\sqrt{3(1+2\epsilon^2)}} [(\sqrt{2}\langle G | + \langle C |) \langle \chi_0 | + \sqrt{2}\epsilon (\langle C | + \sqrt{2}\langle E |) \langle \chi_\perp |]. \quad (11)$$

The Dove prism does not change the forward evolving state at t_1 , so, even with the Dove prism, the state is still given by (4). Calculating now the weak value of projection on E yields

$$(\mathbf{P}_E)_w = \frac{\langle \Phi' | \mathbf{P}_E | \Psi \rangle_{t_1}}{\langle \Phi' | \Psi \rangle_{t_1}} \simeq 2\sqrt{2}\epsilon^2. \quad (12)$$

The photon in the experiment with the Dove prism and the tilted mirror E does have some presence in E . Thus, there is no clear paradox in obtaining the frequency f_E that was present only in E in the framework of the TSVE.

One might wonder why there is no signal at f_F similar to that at f_E in spite of the apparent symmetry of the experiment in the time symmetric TSVE. When the mirror F is tilted instead of mirror E , inserting the Dove prism spoils the destructive interference of the backward evolving wave function towards E similarly to spoiling interference toward F by tilting mirror E . However, more careful analysis shows that the symmetry is not complete. Tilting mirror E also changes the effective backward evolving state, while tilting mirror F does not change the forward evolving state. See details in the next section.

5. Quantifying the Presence of Photons

The explanation of the peak at the frequency f_E which we wish to provide is that the photon has a small presence there, but the experimental records imprinted on the pre- and postselected photon reaching the detector are strong, so the size of the peak is similar to that of frequencies f_C , f_B , and f_A , where the photon presence is strong, but the record is weak. However, the second order in ϵ for the presence of the photon in E looks too small for this to be the case. In more detail, for mirrors A , B , and C , the presence of the photon is of order 1 while the strength of the record is of size ϵ . For mirror E , on the other hand, the presence characterized by the weak value of projection operator (12) is apparently only of size ϵ^2 . The size of the record of an interaction is characterized by the created relative amplitude of the orthogonal component (see [48]). In our case, the record created at E which reaches the detector D is represented by the orthogonal component $|\chi_\perp\rangle$ and it is the only component reaching the detector, since the symmetric component $|\chi_0\rangle$ is 'filtered out' by the inner interferometer. Thus, we can say that the size of the record created at E which reaches the detector is of order 1. This naive consideration tells us that the peak at f_E should be of order ϵ^2 while other peaks are of order ϵ , in contradiction with predicted results of the experiment which show that the peaks are of the same order.

It is true that the weak effects which depend only on the presence of the photon in E , such as the momentum transferred to the mirror E by the photon, are proportional to $(\mathbf{P}_E)_w$, but the presence of a particle is defined according to *all* local traces it leaves (see Section 6 of [3]). In our case, the weak value

of the projection operator \mathbf{P}_E is not the correct parameter to quantify the presence of the particle. It is so when the pre- and post-selection is on spatial degrees of freedom only (see [48]). Here, however, due to the postselection on a subspace, effectively, we are required to consider an associated postselection on a particular mode, along with the well defined preselected mode. Let us define an operator O which connects between the mode $|\chi_0\rangle$ and the mode $|\chi_\perp\rangle$, possessing the eigenvalues ± 1 for the states $|\pm\rangle = (|\chi_0\rangle \pm |\chi_\perp\rangle) / \sqrt{2}$. For the experiment without the Dove prism, the weak value of local variable OP_E still vanishes, but when the Dove prism is present, we have

$$(OP_E)_w = \frac{\langle \Phi' | OP_E | \Psi \rangle_{t_1}}{\langle \Phi' | \Psi \rangle_{t_1}} \simeq 2\sqrt{2}\epsilon. \quad (13)$$

Therefore, the presence in E is found to be of the order ϵ rather than ϵ^2 , which is obtained when we naively quantify the presence by $(\mathbf{P}_E)_w$. This explains why we obtain the signal from mirror E of the same order as from other mirrors.

The weak value of local operator of order ϵ explains the signal, but, according to the definition of the full presence of photon in a particular place, we require an order 1 weak value of some local variable. In view of this, we have only 'secondary presence' [10] of the photon in E in the present case.

Now, when mirror E is tilted, we get $(OP_F)_w \simeq 2\epsilon$, indicating that the presence of the photon is of order ϵ also at mirror F . Nonetheless, we do not get the peak at f_F similar to that at f_E by tilting mirror F as well as E . The reason for this is that the record of the interaction reaching the detector from the tilting mirror F is of order ϵ and not of the order 1 as for the signal from mirror E . Note that, when only mirror F is tilted, we have $(OP_F)_w = 0$.

We have shown that the results of the interference experiment with a nested interferometer and a Dove prism inside it can be explained in the framework of the recently proposed approach [3]. We get signals from mirrors A , B , and C because the photon presence there is of order 1 and the trace recorded on the photon itself is of order ϵ . A similar signal is obtained from mirror E where the presence of the photon is of the order ϵ , but recorded trace is of order 1.

The signal in E should disappear if the mirror will be wiggled in the perpendicular direction. If only this mirror is wiggled and everything else is not, then there will be exactly zero presence at E . If all mirrors are wiggled as in the experiment [1], then the presence will be of order of ϵ , but the record will also be of order ϵ , so the signal will be too small to observe. It will be of interest to perform a nested Mach–Zehnder interferometer experiment with wiggling mirrors and the Dove prism to demonstrate these effects.

6. Bohmian Trajectory

Before concluding, let us analyse this nested interferometer in the framework of the Bohmian interpretation of quantum mechanics [49]. While Bohr preached to not ask where the particle inside the interferometer was, Wheeler suggested a 'common sense' proposal based on classical intuition. While we have suggested relying on the weak trace that the particle leaves using the TSVE, Bohm has a proposal for a deterministic theory which associates a unique trajectory for every particle. In a particular case of nested interferometer which we consider, with or without Dove prism, the particle detected in D has a well defined trajectory (see Figure 3). Note that it corrects an erroneous trajectory in Figure 2 of [50]. The simplest way to understand why Bohmian trajectory must be as shown is the observation that Bohmian trajectories do not cross [51]. The probability to reach detector D is only $1/9$, while the probability to be in path A is $1/3$. Thus, every Bohmian trajectory which reaches D had to pass through A .

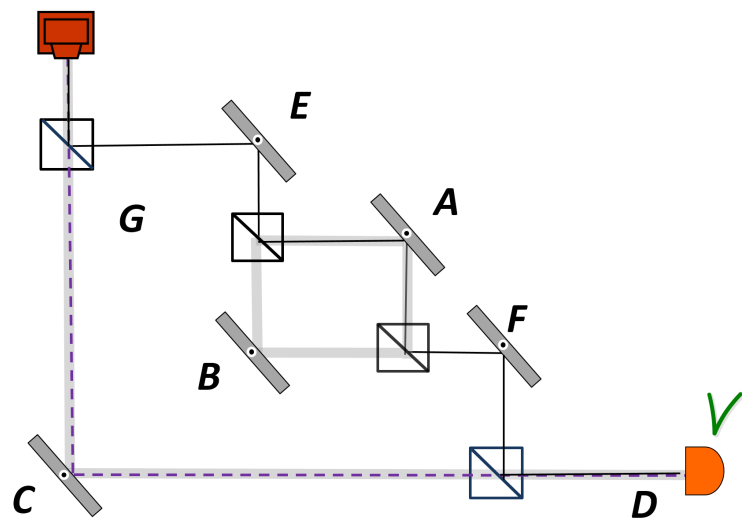


Figure 3. Nested Mach–Zehnder interferometer tuned to destructive interference towards F when a single photon is detected in D . The dashed line represents a common sense proposal by Wheeler, the thick gray line describes the past according to Vaidman’s proposal as places where the particle leaves a weak trace, the continuous line represents the Bohmian trajectory.

Bohmian trajectories are entities beyond the standard quantum theory. One of us (L.V.) had the privilege to spend a day of discussions with David Bohm (Charleston, SC, 1989). I remember telling him what I liked in his theory: a consistent deterministic theory of everything, a candidate for a final theory. However, he completely dismissed this approach. For him, it was nonsense to look for a final theory. He explained to me that his theory is just another step in an infinite search for a better understanding of nature. He was certain that quantum theory is not the last word, and for finding a deeper and more precise theory, quantum theory has to be reformulated. His theory was a counter example to the widespread belief generated by the von Neumann no-go theorem that it would be impossible to extend quantum mechanics consistently by adding hidden variables. Indeed, it opened new horizons for research.

7. Conclusions

Unless a quantum particle is described by a well localized wave packet, the standard quantum theory cannot tell us where the particle was. Vaidman [3] proposed the definition of where a quantum particle was according to the local trace it left: the particle was in a place where the trace is of the order of the trace a single localized particle would leave. In the Danan et al. experiment, photons told us where they have been (according to the trace definition) in a specially tuned nested interferometer. The AJ modification of this experiment, i.e., placing a Dove prism in one of the arms of the inner interferometer did not change significantly the past of the photons, but photons told a different story: they were also near mirror E in spite of the fact that, according to Vaidman’s narrative, they were not present there. We conclude that the photons were lying about their presence in E , in the sense that, although the trace they left there was much smaller than the trace that a localized photon would leave, the signal provided by the photons was large as if they had fully been present in E .

How could the photons produce the signal with frequency f_E which was larger than any other signal? In the original and the modified experiments, local traces were not observed. Instead, locally created traces were ‘written’ on the transversal degree of freedom of the photon itself. In the original experiment, the transversal degree of freedom was not distorted until it reached the detector, so these local traces were faithfully read by the detector. In the modified experiment, the Dove prism influenced the transversal degree of freedom spoiling the faithful readout of local traces by the detector. In fact,

AJ mentioned such an interpretation in [2] as one of the options: “one possible response to this result is that we have improperly read off the past of the photon by letting it suffer further interactions with the environment before reading the weak trace after it was written, so our weak measurement was a bad one for inferring the past of the photon.”

Apart from the explanation of the experimental results by the presence of the particle defined through the weak trace, Danan et al. presented a simpler argument of the presence of the photon in A , B and C . The detected photons had to be there because they brought to the detector information which was only there. However, the same should hold for the modified experiment: the particles had to be in E because they brought information about f_E that was present only in E . Sections 4 and 5 explain how it happens in spite of the fact that the trace left by the particles at E was very small. It was small, but not exactly zero, as in the original experiment when only mirror E was wiggling. The Dove prism did change the past of the photons a little.

Introducing a Dove prism not only spoiled faithful transmission of the transverse degree of freedom of the photon to the detector, it also made the inner interferometer extremely sensitive for the misalignment of the input beam. The strength of the signal in the experiment was proportional to the relative amount of the orthogonal component created by local interaction. This component was the asymmetric mode with which the Dove prism passed in full through the inner interferometer, while the reference, the symmetric mode, did not pass at all due to the destructive interference. This explains how a small presence of the photons in E caused a strong signal with frequency f_E .

Note that the Bohmian trajectory did pass through E . However, it also passed through F , although no frequency f_F was observed. It is well known, starting from ‘surrealistic trajectories’ [51], that we cannot view quantum particles as acting locally in their Bohmian positions (see also [52]).

We have observed that introducing the Dove prism into an inner interferometer of the Danan et al. experiment creates a tiny presence of the photons in E . However, we argue that from this we should not tell that the Dove prism changes the past of a photon in the nested interferometer proposed in [3]. In fact, the origin of the presence of the photons can be found in the disturbance of the mirror E . The weak value of any local operator at E is strictly zero in an ideal interferometer where no mirror is tilted, even if the Dove prism is there.

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