Retraction

Retraction: Ahmed, A. et al. Dual Solutions in a Boundary Layer Flow of a Power Law Fluid over a Moving Permeable Flat Plate with Thermal Radiation, Viscous Dissipation, and Heat Generation/Absorption. *Fluids* 2018, 3, 6, doi:10.3390/fluids3010006

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This article [1] published in *Fluids* will be marked as retracted. The Editorial Board have carefully looked over the paper and found that many features of the analysis in the paper are misleading. We provide details of four such instances below.

1. Many authors have previously considered “dual solutions” (examples being Merkin [2] and Magyari et al. [3]), but the second solution has always been unstable and is therefore unobtainable in practice. It can be a nice mathematical result and of interest to those who study the structure of solutions to nonlinear ordinary differential equations (ODEs). However, many authors have followed the original analyses for multiparameter problems with gradually increasing numbers of parameters. There is therefore only limited novelty in [1] and its results contain too many parameters for a research paper.

2. Dual solutions can only be defined and computed if the governing equations are ODEs. The authors have reduced their system to ODE form, but the system here should be a set of parabolic partial differential equations (PDEs). The problem being solved is non similar (with PDEs), as opposed to being self-similar (with ODEs). The authors have thus solved the boundary layer equations incorrectly. This is due to the fact that the coefficients in the boundary layer equations are not constants, as assumed by the authors, but are actually functions of the streamwise coordinate, x . Therefore single x -derivatives arising from the advection terms should be retained. This is quite standard for non-similar boundary layers and may be found in many papers dating back to the 1970s.

3. The authors have not correctly applied the boundary layer theory. Whilst the streamwise diffusion terms are absent (which always occurs), there are other terms whose orders of magnitude are unknown. If an overall Reynolds number had been defined (which is the starting point for a rigorous boundary layer theory, but was not done in [1]), then all the terms in the governing equations can be assigned orders of magnitude based upon an asymptotically large Reynolds number, which must be large, otherwise there would be no boundary layer. So, for example, it is not possible to tell how large the viscous dissipation term is in Equation (4) compared with the y -diffusion term. The same is true for the heat source and radiation terms. Usually, assumptions would be made about these orders of magnitude and clear statements made about whether they can be retained in the boundary layer equations. This has not happened in [1] and is a serious deficiency.

4. The x -dependent parameters turn out to be problematic in the present problem. Given that the Prandtl number is defined in the paper as infinite at the origin, Equation (16) reduces to $\theta'' = 0$, and therefore cannot be solved with both boundary conditions being satisfied. This precise situation arose in Rees and Pop [4] where Equation (11c) could not be solved by satisfying both boundary conditions simultaneously (see the section of [4] entitled, *Asymptotic Analysis near the Leading Edge*,

specifically the first two paragraphs of that section). Thus a two-layer structure has to be defined as part of an asymptotic analysis. It is only when this has taken place that one can proceed with the full PDE simulation.

We have come to the conclusion that if the physical situation corresponding to what the authors attempted to solve could exist, then the boundary layer problem would still be non similar. In addition to that, it would have a double layer structure near the leading edge, the analysis of which would be an essential prerequisite for starting a non-similar numerical solution. The authors have made a poor assumption by treating some functions of x as constants in order to obtain a system of ODEs, and have then attempted to determine solutions which, if the ODE system were correct, would nevertheless be unstable in practice. The correct method of analysis would have to follow of that given in [4]. We are not convinced that the boundary layer approximation has been made correctly.

We very much regret that these issues had not been addressed prior to publication. We would like to offer our apologies to readers of *Fluids* and wish to thank the Editorial Board Members who brought it to our attention.

References

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