

Table S1. Main stand variables of the sample plots used to estimate Reineke's maximum stand density and maximum basal area.

Species	<i>n</i>		<i>N</i>	<i>Dg</i>	<i>H_{dom}</i>
Norway spruce	550	Mean	558	34.1	25.5
		SD	345	9.6	4.2
		Min	100	12.2	11.5
		Max	2350	66.1	41.2
European beech	158	Mean	574	29.0	22.5
		SD	327	9.7	5.7
		Min	100	13.7	12.5
		Max	1550	57.6	35.8

n: sample size; *N*: stem density (ha^{-1}); *Dg*: quadratic mean diameter (cm); *H_{dom}*: dominant height (m); SD: standard deviation.

Table S2. Main variables for the modelling of the annual past radial growth increment.

n: sample size; *APRI*: annual past radial growth increment ($\text{mm}\cdot\text{year}^{-1}$); *DBH*: diameter at breast height (cm); *BA*:

Species	<i>n</i>		<i>APRI</i>	<i>DBH</i>	<i>BA</i>	<i>BAL</i>	<i>BAL_{sp}</i>	<i>t</i>	<i>p</i>	<i>mi</i>
Norway spruce	22995	Mean	1.57	35.0	47.8	26.0	24.9	2118	860	0.979
		SD	1.30	15.4	22.5	20.1	19.9	544	207	0.029
		Minimum	-12.24	12.5	0.6	0.0	0.0	935	243	0.579
		Maximum	25.55	109.5	293.3	289.8	281.8	4112	1649	1.000
European beech	7105	Mean	1.33	30.6	40.7	24.1	16.8	2701	974	0.963
		SD	1.15	13.8	18.8	18.1	15.2	422	234	0.038
		Minimum	-8.28	12.5	0.7	0.0	0.0	1637	595	0.615
		Maximum	9.02	185.5	281.9	186.1	182.0	3838	1826	1.000

stand basal area ($\text{m}^2\cdot\text{ha}^{-1}$); *BAL*: basal area of the trees larger than the subject tree ($\text{m}^2\cdot\text{ha}^{-1}$); *BAL_{sp}*: basal area of the trees of the same species and larger than the subject tree ($\text{m}^2\cdot\text{ha}^{-1}$); *t*: sum of the daily mean temperature above 3°C per growing season ($\text{degree days}\cdot\text{year}^{-1}$); *p*: precipitation sum per growing season ($\text{mm}\cdot\text{year}^{-1}$); *mi*: mean daily moisture index per growing season (%); SD: standard deviation.

Table S3. Coefficients of the 3- and 2 parameter extended competition density rule estimated using nonlinear regression.

Species	<i>n</i>	Alternative	<i>a</i> ₀	<i>b</i> ₀	<i>a</i> ₁	<i>b</i> ₁	RSE
Norway spruce	550	3P	1.38E-06	3.345	0.729	-	4.626
		2P	-	3.144	0.719	-	4.628
European beech	158	3P	3.20E-05	0.468	-0.145	-	4.836
		2P	-	0.481	-0.145	-	4.831

n: sample size; RSE: residual standard error.

Table S4. Fixed effect parameter estimates for the Norway spruce and European beech annual past radial increment models.

Parameter	Coeff	Norway spruce			European beech		
		Value	SE	p-value	Value	SE	p-value
Intercept	β_{i0}	15.404	0.523	<0.0001	5.769	0.498	<0.0001
1/DBH	β_{i1}	-18.677	2.222	<0.0001	-12.319	1.073	<0.0001
ln(DBH)	β_{i2}	-0.790	0.089	<0.0001	-	-	-
ln(BAL/(ln(DBH+1))+1)	β_{i3}	-0.810	0.043	<0.0001	0.113	0.033	0.0008
ln(BA)	β_{i4}	-0.409	0.024	<0.0001	-0.536	0.039	<0.0001
t	β_{i5}	0.001	0.000	<0.0001	0.001	0.000	<0.0001
p	β_{i6}	0.000	0.000	<0.0001	-0.001	0.000	<0.0001
mi	β_{i7}	3.434	0.358	<0.0001	6.560	0.458	<0.0001
ln(BAL _{sp} +1)	β_{i8}	0.376	0.028	<0.0001	-0.239	0.017	<0.0001

SE: standard error.

Table S5. Sample size and goodness of fit statistics for the annual past radial increment models.

Species	<i>n</i> _{tree}	<i>n</i> _{obs}	RMSE
Norway spruce	33649	16172	0.704
European beech	10382	4631	0.690

RMSE: root mean square error.

Table S6. Statistics of the resulting GI.

	Norway spruce	European beech
Mean	1.000	1.004
SD	0.030	0.052
Minimum	0.724	0.718
1 st quantile	0.983	0.975
3 rd quantile	1.014	1.025
Maximum	1.190	1.331

SD: standard deviation.

Table S7. Sample size and goodness of fit statistics for the growth models using 3P proportions.

Species	<i>n</i> _{obs}	<i>n</i> _{plots}	RMSE
Norway spruce	1346	797	0.1854
European beech	546	322	0.1707

RMSE: root mean square error.

Table S8. Fixed effect parameter estimates for the growth models using different alternatives **(a)** 2P, **(b)** Direct-SNFI, **(c)** Direct-Charru and **(d)** goodness of fit statistics.

(a) 2P		Norway spruce			European beech		
Parameter	Coeff	Value	SE	p-value	Value	SE	p-value
Intercept	a_i	-5.959	1.360	<0.0001	-6.931	0.776	<0.0001
t	b_{i1}	4.30E-04	2.69E-05	<0.0001	5.68E-04	4.24E-05	<0.0001
p	b_{i3}	-2.44E-04	6.72E-05	0.0003	-8.44E-04	7.51E-05	<0.0001
mi	b_{i5}	7.450	1.286	<0.0001	5.343	0.515	<0.0001
ln(GI _i)	c_i	10.448	0.340	<0.0001	6.988	0.695	<0.0001
ln(Dg)	d_{i1}	-0.693	0.146	<0.0001	0.282	0.152	0.0641
ln(sd)	d_{i2}	0.610	0.020	<0.0001	0.685	0.029	<0.0001
P _i	e_i	3.416	1.471	0.0206	1.833	0.645	0.0049
mi x P _i	f_{i1}	-4.774	1.398	0.0007	-	-	-
ln(GI _i) x P _i	f_{i2}	-	-	-	2.030	0.917	0.028
ln(Dg) x P _i	f_{i3}	0.415	0.157	0.0085	-0.647	0.183	0.0005

(b) Direct-SNFI		Norway spruce			European beech		
Parameter	Coeff	Value	SE	p-value	Value	SE	p-value
Intercept	a_i	-4.478	1.437	0.0019	-7.960	0.762	<0.0001
t	b_{i1}	4.32E-04	2.69E-05	<0.0001	5.49E-04	4.25E-05	<0.0001
p	b_{i3}	-2.48E-04	6.73E-05	0.0003	-8.36E-04	7.54E-05	<0.0001
mi	b_{i5}	7.635	1.362	<0.0001	5.262	0.516	<0.0001
ln(GI _i)	c_i	10.445	0.340	<0.0001	7.208	0.652	<0.0001
ln(Dg)	d_{i1}	-1.233	0.153	<0.0001	0.654	0.146	<0.0001
ln(sd)	d_{i2}	0.613	0.020	<0.0001	0.682	0.029	<0.0001
P _i	e_i	1.947	1.550	0.2096	3.004	0.626	<0.0001
mi x P _i	f_{i1}	-4.958	1.477	0.0008	-	-	-
ln(GI _i) x P _i	f_{i2}	-	-	-	1.758	0.876	0.0461
ln(Dg) x P _i	f_{i3}	0.952	0.165	<0.0001	-1.027	0.177	<0.0001

(c) Direct-Charru		Norway spruce			European beech		
Parameter	Coeff	Value	SE	p-value	Value	SE	p-value
Intercept	a_i	-6.349	1.322	0.0019	-6.447	0.784	<0.0001
t	b_{i1}	4.37E-04	2.70E-05	<0.0001	5.54E-04	4.23E-05	<0.0001
p	b_{i3}	-2.51E-04	6.73E-05	0.0002	-8.47E-04	7.47E-05	<0.0001
mi	b_{i5}	7.249	1.249	<0.0001	5.306	0.512	<0.0001
ln(GI _i)	c_i	10.443	0.340	<0.0001	6.943	0.716	<0.0001
ln(Dg)	d_{i1}	-0.498	0.142	0.0005	0.136	0.156	0.3832
ln(sd)	d_{i2}	0.611	0.020	<0.0001	0.682	0.029	<0.0001
P _i	e_i	3.745	1.432	0.0092	1.398	0.657	0.0344
mi x P _i	f_{i1}	-4.529	1.360	0.0009	-	-	-
ln(GI _i) x P _i	f_{i2}	-	-	-	2.092	0.937	0.0266
ln(Dg) x P _i	f_{i3}	0.223	0.153	0.1467	-0.492	0.187	0.0091

(d) Goodness of fit statistics						
Species	n_{obs}	n_{plots}	2P	Direct-SNFI	Direct-Charru	
			RMSE	RMSE	RMSE	
Norway spruce	1346	797	0.1855	0.1854	0.1852	
European beech	546	322	0.1707	0.1699	0.1704	

SE: standard error; RMSE: root mean square error.

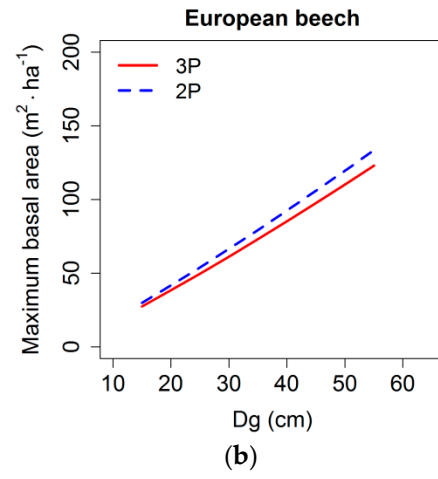
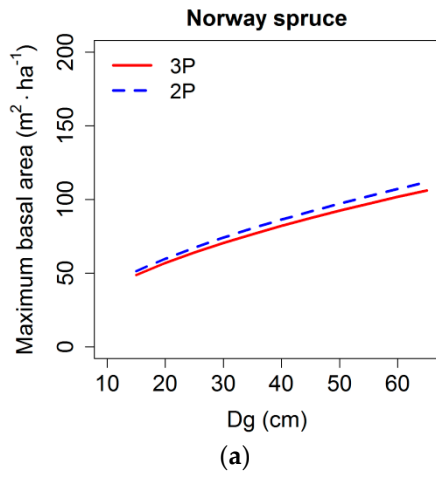


Figure S1. Maximum basal area over quadratic mean diameter fitted using two alternatives (3P, 2P). (a) Norway spruce, (b) European beech.

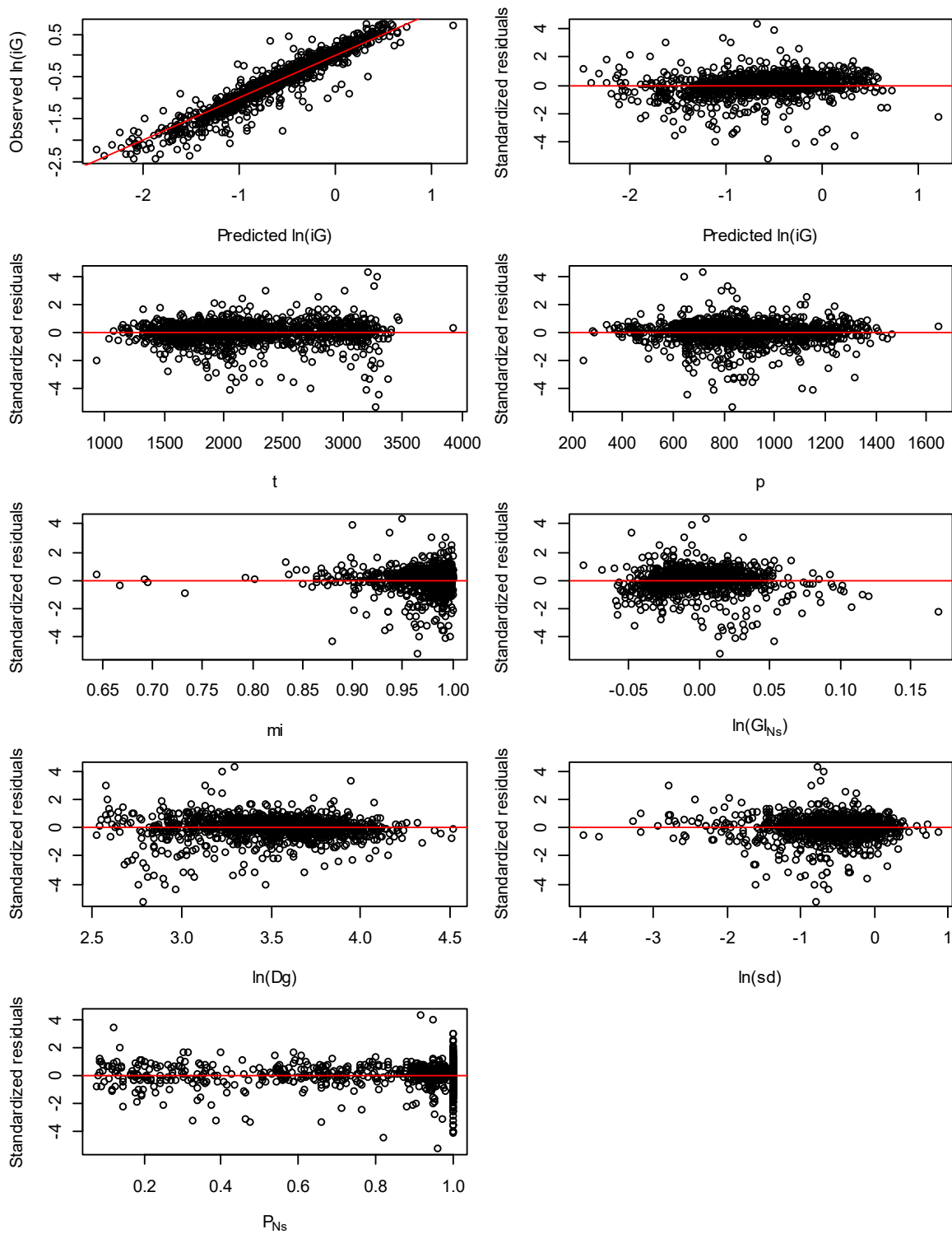


Figure S2. Predicted versus observed data plots and standardized residuals versus explanatory variables plots for the Norway spruce growth model.

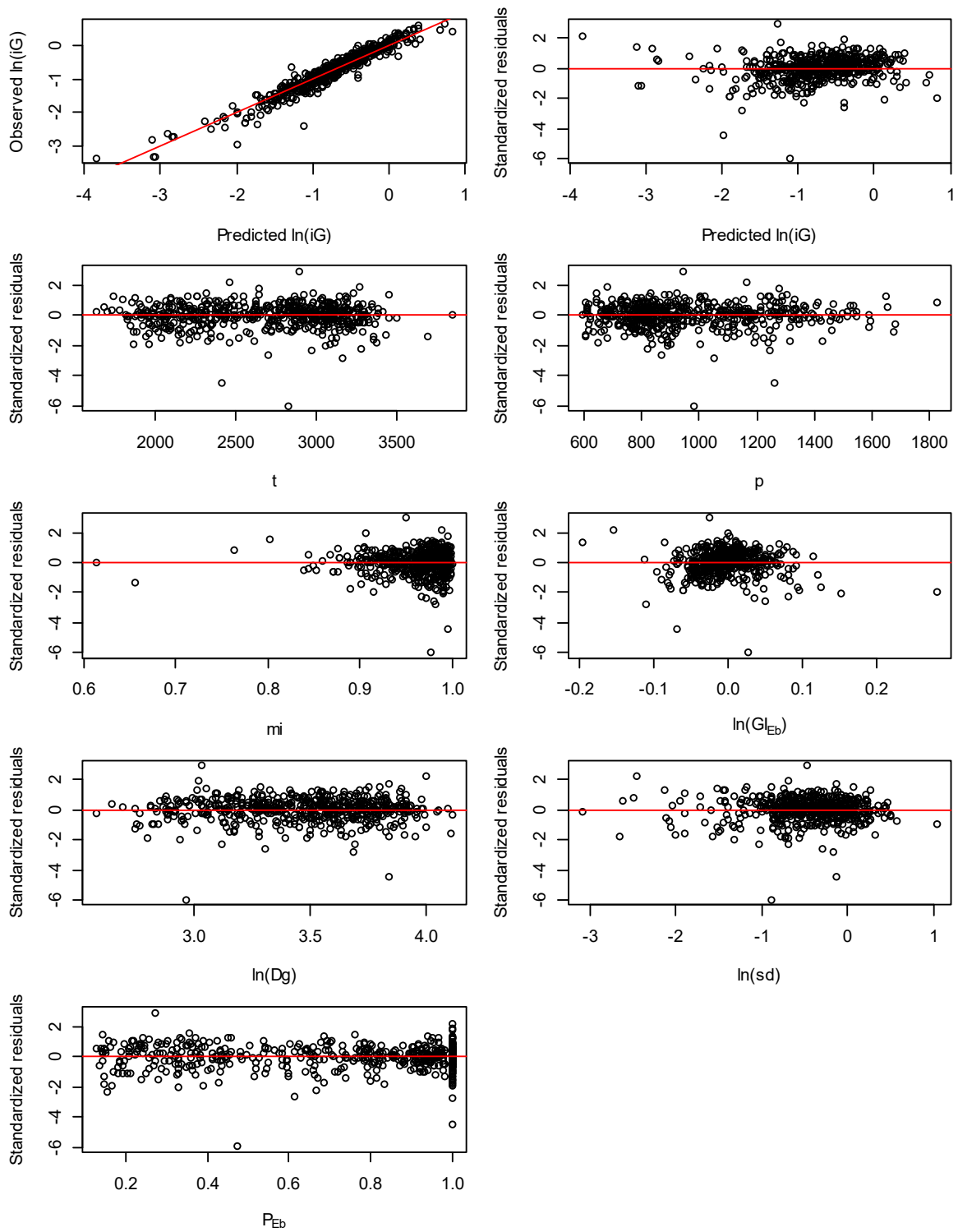


Figure S3. Predicted versus observed data plots and standardized residuals versus explanatory variables plots for the European beech growth model.

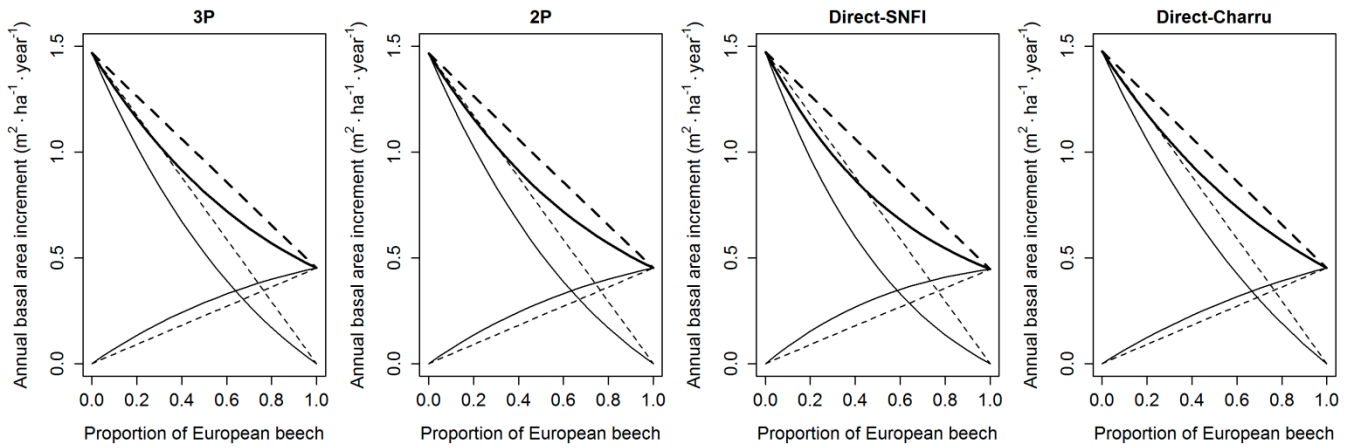


Figure S4. Replacement diagrams of the NFI site showing the lowest underyielding (using the 3P alternative). The basal area increment was predicted for $D_g = 35$ cm, $s_d = 0.65$, temperature sum = 3307 degree days·year⁻¹, precipitation sum = 655 mm·year⁻¹, moisture index = 0.8803, $GI_{Ns} = 1.055$, and $GI_{Eb} = 0.985$.

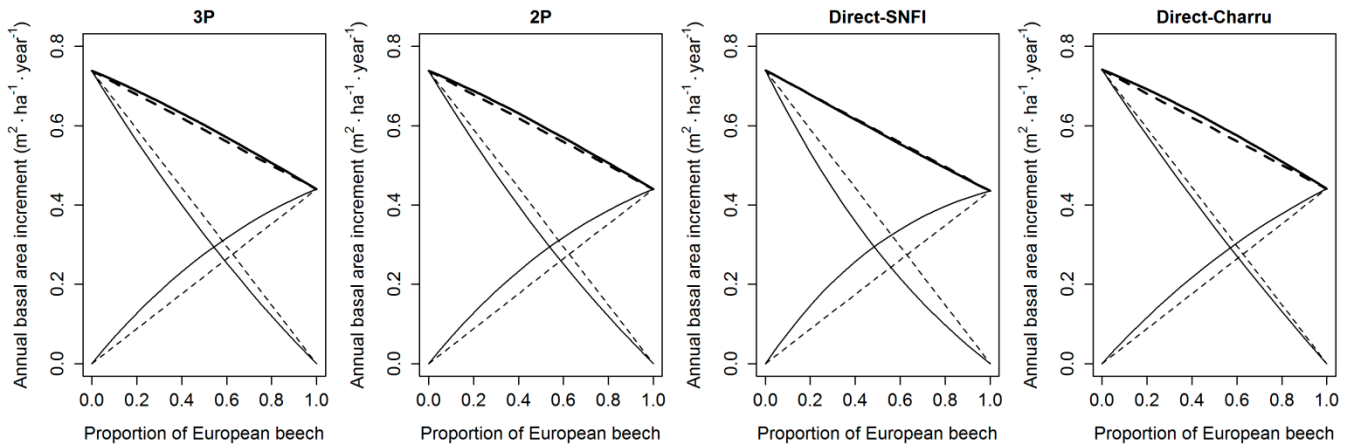


Figure S5. Replacement diagrams of the NFI site showing the mean overyielding (using the 3P alternative). The basal area increment was predicted for $D_g = 35$ cm, $s_d = 0.65$, temperature sum = 2952 degree days·year⁻¹, precipitation sum = 1165 mm·year⁻¹, moisture index 0.9686, $GI_{Ns} = 0.992$, and $GI_{Eb} = 1.000$.

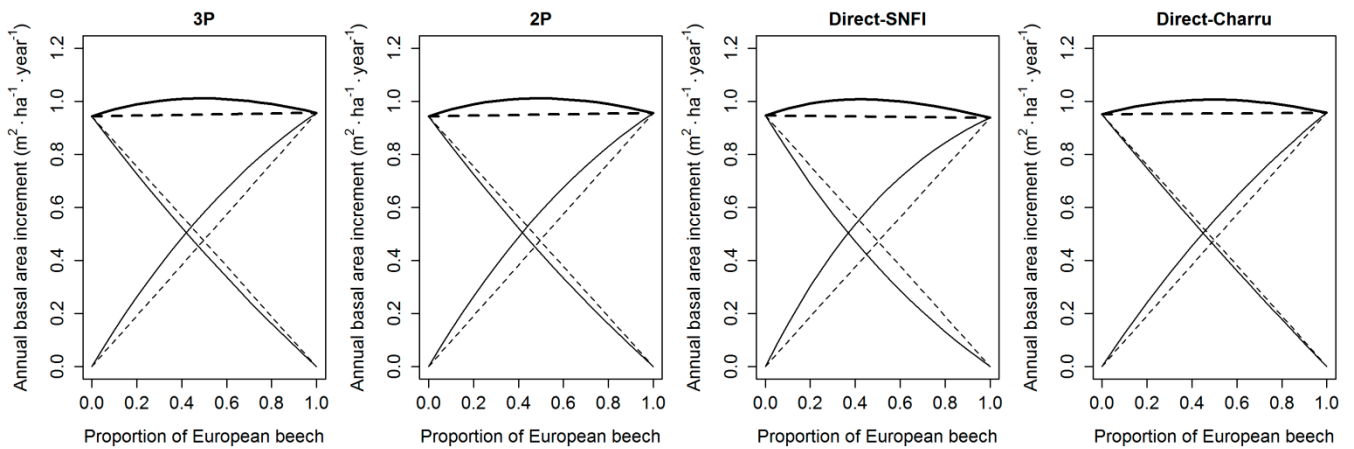


Figure S6. Replacement diagrams of a NFI site showing a transgressive overyielding. The basal area increment was predicted for $D_g = 35$ cm, $s_d = 0.65$, temperature sum = 3138 degree days·year⁻¹, precipitation sum = 781.9 mm·year⁻¹, moisture index = 0.9821, $GI_{Ns} = 0.995$, and $GI_{Eb} = 1.031$.