

Review

# From Primordial Seed Magnetic Fields to the Galactic Dynamo

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**Abstract:** The origin and maintenance of coherent magnetic fields in the Universe is reviewed with an emphasis on the possible challenges that arise in their theoretical understanding. We begin with the interesting possibility that magnetic fields originated at some level from the early universe. This could be during inflation, the electroweak, or the quark-hadron phase transitions. These mechanisms can give rise to fields which could be strong, but often with much smaller coherence scales than galactic scales. Their subsequent turbulent decay decreases their strength but increases their coherence. We then turn to astrophysical batteries which can generate seed magnetic fields. Here the coherence scale can be large, but the field strength is generally very small. These seed fields need to be further amplified and maintained by a dynamo to explain observed magnetic fields in galaxies. Basic ideas behind both small and large-scale turbulent dynamos are outlined. The small-scale dynamo may help to understand the first magnetization of young galaxies, while the large-scale dynamo is important for the generation of fields with scales larger than the stirring scale, as observed in nearby disk galaxies. The current theoretical challenges that turbulent dynamos encounter and their possible resolution are discussed.

**Keywords:** early universe; galactic magnetic fields; dynamo theory; magneto-hydrodynamics simulations

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## 1. Introduction

The universe is magnetized, right from the Earth, the Sun, and other stars to disk galaxies, galaxy clusters, and perhaps also the intergalactic medium (IGM) in voids. In nearby disk galaxies, magnetic fields are observed to have both a coherent component of order a few micro Gauss, ordered on scales of a few to ten kilo parsecs (kpc) and a random component with scales of parsecs to tens of parsecs [1–3]. In these galaxies, both stars and the gas in the interstellar medium (ISM) are in a thin disk supported against gravity by their rotation. It is not clear what the strength and structure of magnetic fields in the other major type of galaxies, the ellipticals, are. This is perhaps related to the fact that normal ellipticals have much lower active star formation and lack the requisite cosmic ray electrons for producing significant synchrotron emission. There is tentative evidence that even young galaxies, which are several billion years younger than the Milky Way, host ordered micro Gauss strength magnetic fields [4–6]. Magnetic fields of similar strengths and coherence are detected even in the hot plasma filling the most massive collapsed objects in the universe, rich clusters of galaxies [7]. There is also indirect evidence for a lower limit of order  $10^{-16}$  G to the magnetic field contained in the intergalactic medium of large scale void regions between galaxies [8,9] (see, however, [10]). This strength refers to a coherence scale of a Mpc, and the field needs to be stronger if the coherence scale is smaller. The origin and maintenance of cosmic magnetism is an outstanding question of modern astrophysics. We focus here on galactic magnetic fields and trace their origin and maintenance from the early to the present day universe.

Magnetic fields of the observed strength need to be constantly maintained against turbulent decay, the turbulence either being self generated by the Lorentz force or driven by other forces. This is done by electromagnetic induction due to motions in a preexisting magnetic field. Such motions can induce an electric field with a curl which, by Faraday's law, can maintain the magnetic field. The resulting evolution of the magnetic field  $\mathbf{B}$  is governed by the induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \quad (1)$$

where  $\mathbf{V}$  is the fluid velocity and  $\eta$  the resistivity of the plasma. The first term in the induction equation describes the electromagnetic induction (the generation of electric field in a conductor moving across magnetic field), whereas the second term is responsible for its diffusion and resistive decay. If  $\eta \rightarrow 0$ , the magnetic flux through any surface moving with the fluid remains constant. The relative importance of induction versus resistance is measured by the dimensionless magnetic Reynolds number  $R_m = vl/\eta$ , where  $v$  and  $l$  are typical values for the fluid velocity and length scales, respectively. For inter-stellar turbulence, adopting  $v \sim 10 \text{ km s}^{-1}$  [11],  $l \sim 100 \text{ pc}$  [12], and Spitzer value for the resistivity  $\eta \sim 10^7 \text{ cm}^2 \text{ s}^{-1}$ , as applicable to ionized plasma at a temperature  $T \sim 10^4 \text{ K}$ , we have  $R_m \sim 3 \times 10^{19} \gg 1$ . From Equation (1) we see that one needs at least a seed magnetic field to be present before induction can amplify it. It turns out that most ideas of seed field generation lead to magnetic fields which are much smaller than observed. They need to then be amplified and maintained, a process called the dynamo. We review ideas for both these aspects.

## 2. Early Universe Origin

Seed magnetic fields could be a relic from the early Universe, arising during the inflationary epoch or in a later phase transition, when the electroweak symmetry is broken or when quarks gather into hadrons (for reviews see [13,14]). Indeed, if the evidence for weak, femto Gauss magnetic fields in the void regions is firmed up, an early universe mechanism would provide a natural explanation. Such a possibility can also help probing the physics of the early universe. In this section, we use the natural system of units in which the Planck constant, speed of light, and the Boltzmann constant are equal to 1.

In the expanding universe all length scales increase proportional to the expansion factor  $a(t)$ . Thus if the magnetic flux is frozen, the field strength at a time  $t$  decreases or redshifts as  $B(t) \propto 1/a^2(t)$ . Neglecting the effects of dissipation, its energy density then decreases as  $\rho_B(t) = B^2(t)/(8\pi) \propto 1/a^4(t)$ . The energy density of cosmic microwave background radiation (CMB),  $\rho_\gamma(t)$ , a relic of the hot 'big bang' beginning of the universe, also decreases with expansion in the same manner. This implies the approximate constancy of the ratio  $r_B = \rho_B(t)/\rho_\gamma(t)$  (approximate as particle annihilation at certain epochs can increase  $\rho_\gamma(t)$ ). This motivates characterizing the strength of the primordial field with either  $r_B$  or  $B_0$  the field strength at the present epoch, as a function of the scale  $L$  over which the field is averaged. A value of  $B_0 \sim 3.2 \text{ } \mu\text{G}$  corresponds to the field having the same energy density as the CMB today, or  $r_B = 1$ . Observations of CMB anisotropy or structure formation lead to upper limits of  $B_0$  at the nano Gauss levels assuming nearly scale invariant magnetic spectrum [13–17].

### 2.1. Generation During Inflation

The seeds for structures we see in the universe are thought to have originated during the inflationary epoch, from amplification of quantum vacuum fluctuations in the scalar field driving the rapid accelerating expansion of the universe. Inflation does have several useful features to generate coherent seed magnetic fields as well [18]. First, the rapid expansion during inflation stretches small scale wave modes to very large correlation scales corresponding to galaxies and larger. Second, such expansion dilutes any pre-existing charge densities to be negligible. Then the conductivity of the universe is negligible, there is no constraint from conservation of magnetic flux, and one can generate magnetic fields from a zero field. The idea is then to excite quantum fluctuations of the vacuum state

of the electromagnetic (perhaps more correctly hypermagnetic) field, when a given mode is within what is known as the Hubble radius, which then transits to random classical fluctuations as the mode is stretched well beyond the Hubble scale. Subsequently, when the universe reheats generating charge particles, the electric field is shorted and damped to zero, while the magnetic field part of what once was an electromagnetic wave is frozen into the resulting plasma.

This idea however faces one major difficulty. The conventional electromagnetic (EM) action  $S_{EM}$  is left invariant under a conformal transformation of the metric ( $g_{\mu\nu}$ ) given by  $g_{\mu\nu}^* = \Omega^2 g_{\mu\nu}$ . Moreover, the geometry of the Friedmann-Robertson-Walker (FRW) expanding universe itself transforms to its flat space version under a suitable conformal transformation. Then Maxwell equations and consequently the electromagnetic wave equation transform to what obtains in flat space-time. In such a case, EM wave fluctuations cannot be amplified in a FRW universe. The electromagnetic field still decays with expansion as  $1/a^2(t)$ , which is very drastic during inflation. Therefore significant inflationary magnetogenesis requires a mechanism for breaking conformal invariance of the electromagnetic action, so that the decrease of the field becomes milder, to say  $B \sim 1/a^\epsilon$  with  $\epsilon \ll 1$ . A variety of models where such a behaviour can be obtained have been suggested, one of them being to couple a scalar field  $\phi$  (perhaps the inflaton responsible for driving inflation) to the EM action as  $S = f^2(\phi)S_{EM}$  during inflation [18,19]. It turns out that in this model, one gets a scale invariant spectrum of magnetic fields for  $f \propto a^2$  or  $f \propto a^{-3}$ , with a present day amplitude [14]

$$B_0 \sim 0.6 \times 10^{-10} \text{G} \left( \frac{H}{10^{-5} M_{pl}} \right). \quad (2)$$

Here  $H$  is the Hubble expansion rate in energy units during inflation and  $M_{pl}$  the Planck energy. Thus, for specific evolutionary behaviour of the coupling function  $f$ , strong enough fields can be generated.

### 2.1.1. Constraints and Caveats

A number of constraints and caveats arise in models of inflationary magnetogenesis. First, a time dependent coupling  $f$  in front of the EM action implies that electric fields and magnetic fields evolve differently. For example in the model with  $f \propto a^{-3}$  electric fields increase rapidly with time even though the magnetic field remains almost constant. Then its energy density can begin to exceed the inflaton energy density causing a back reaction problem [20,21]. This does not happen in the model with  $f \propto a^2$ . However in the latter model the function  $f = f_i(a/a_i)^2$  increases greatly during inflation, from its initial value of  $f_i$  at  $a = a_i$ . When the interaction of the EM field with charged particles is taken into account, the value of  $f$  at the end of reheating,  $f_0$ , will renormalize the electric charge from  $e$  to  $e_N = e/f_0^2$ . Suppose we require the charge to take the present day value at the end of inflation, i.e.,  $f_0 = 1$ . Then  $f_i \ll 1$  initially and thus  $e_N = e/f_i^2 \gg e$  at early times. Demozzi et al. [22] argued that the theory is not trustable in this case as the EM field is in a strongly coupled regime. Alternatively, suppose one started with a weakly coupled theory where  $f_i \sim 1$ . Then  $f_0 \gg f_i$  by the end of inflation and so the renormalized charge  $e_N \ll e$ . When inflation ends, the interaction of the electromagnetic field with the charges will then be extremely weak. A third potential problem raised by [23] is that the creation of charged particles by the generated electric fields, due to the Schwinger effect, can increase the conductivity so much that magnetic field generation freezes.

We have built models which attempt to address these issues by having a rising  $f$  during inflation followed by a decreasing  $f$  until reheating, but now predict a blue magnetic field spectrum  $d\rho_B/d \ln k \propto k^4$  ( $k$  is the comoving wavenumber) and require a low energy scale of inflation and reheating [24,25]. The spectrum is cut-off at the Hubble wavenumber of reheating. The field is also helical when one adds a parity breaking piece to the EM action [25]. In this case the field orders itself considerably as it decays (see below). We find that a scenario with reheating at a temperature of 100 GeV leads to present day field strengths of order  $B_0 = 4 \times 10^{-11}$  G with a coherence scale of 70 kpc.

## 2.2. Generation During Phase Transitions

As the universe expands and cools from very high temperatures, it goes through the electroweak (EW) phase transition (at  $T = T_c \sim 100$  GeV) and the quark-hadron (QCD) phase transition (at  $T_c \sim 150$  MeV). Significant magnetic field generation can take place in these phase transitions, especially if they are of first order. In this case, the transition to the new phase occurs in bubbles nucleating in the old phase. These bubbles expand and collide with each other until the universe transits completely to the new phase. In these bubble collisions battery effects can operate to generate a seed magnetic field which is further amplified by a dynamo due to the turbulence generated during bubble collisions [26]. The consequences of such a picture has been studied for both the EW phase transition [27] and the QCD phase transition [28,29]. More subtle effects have also been considered invoking gradients in the Higgs field during EW phase transition [30], linking baryogenesis with magnetogenesis [31,32], or using the chiral anomaly of weak interactions [33,34]. A brief review of some of these effects is given in [14]. The properties the magnetic field generated in all these models is uncertain but  $\rho_B$  can be a few percent of  $\rho_\gamma$ . The coherence scale of the field can be as small as a few tens of the thermal de-Broglie wavelength  $1/T$  upto a significant fraction  $f_c$  of the Hubble scale. For the EW phase transition, which occurs at a temperature of about 100 GeV, the proper Hubble scale is of order a cm, and thus the comoving coherence scales will then be of order  $10^{15} f_c$  cm [14]. For the QCD phase transition, which would occur at a temperature of  $T \sim 150$  MeV, the Hubble radius is  $\sim 6.4 \times 10^5$  cm, and the comoving coherence scale is of order  $(f_c/3)$  pc. Moreover, the present-day strength for a magnetic field which has say a fraction  $r_B = 0.01$  is  $B_0 \sim 0.3 \mu\text{G}$ .

### 2.2.1. Magnetic Field Evolution in the Early Universe

The small-scale magnetic fields generated in these phase transitions or in the inflationary models with blue spectrum [24,25] are strong enough to drive decaying magnetohydrodynamic (MHD) turbulence [35,36]. The magnetic field energy density then decreases faster than the  $(1/a^2)$  dilution due to expansion. However, the field coherence scale simultaneously also increases with the decay. Note that in the radiation dominated universe, MHD equations reduce to their flat space-time version provided one uses a conformally transformed fields, for example  $B_* = Ba^2$ , conformal time  $\tau = \int dt/a(t)$ , and comoving spatial coordinate  $\mathbf{x} = (\mathbf{r}/a(t))$  ( $\mathbf{r}$  is the proper spatial coordinate). Moreover, the plasma in the early universe is an excellent electrical conductor, but its viscosity increases whenever the mean free path of a particle species (like the neutrino or photon) grows to be comparable with the coherence scale of motions. In epochs when viscosity dominates, the peculiar velocity induced by the Lorentz force becomes damped and hence does not in turn distort the field, freezing its evolution. In all other epochs, the Lorentz force induced velocity leads to decaying MHD turbulence.

In the case of decay of fluid turbulence in flat space-time, a general feature is that of preservation of large scales (larger than the coherence scale) during the decay, and then the evolution of energy and coherence scale depends on the energy spectrum on such large scales [37]. The case of nonhelical magnetic field decay appears to be more complicated. Numerical simulations find that the comoving magnetic energy density,  $E_M \propto (B_*^2/8\pi)$  decays slower than for pure hydro turbulence, as  $E_M \propto \tau^{-1}$  and undergoes an inverse transfer of energy with the coherence scale  $L_c(\tau)$  increasing as  $\tau^{1/2}$  [38,39]. If the field is fully helical, magnetic helicity conservation constrains the decay and further slows it down to  $E_M \propto \tau^{-2/3}$ , while  $L_c$  increases faster as  $\tau^{2/3}$  [36,40]. If the field is partially helical its decay is to begin with as in the nonhelical case but conserving helicity. This makes the field eventually fully helical after which they decay more slowly like in the fully helical case. For the radiation dominated universe with  $a(t) \propto t^{1/2}$ , we also have  $\tau \propto t^{1/2} \propto a(t)$ . Thus a power law decay in conformal time is still a power law decay in physical time (though slower). When matter starts dominating, the transformation law to the flat-space time MHD equations are different [36] and the relevant time co-ordinate becomes  $\tilde{\tau} = \int dt/a^{3/2}$ . Since the expansion  $a(t) \propto t^{2/3}$  in the matter dominated era,  $\tilde{\tau} \propto \ln(t)$  and any power law decay for the comoving magnetic field in  $\tilde{\tau}$  becomes only a logarithmic decay in real time. Therefore turbulent decay of the field almost freezes after matter domination.

### 2.2.2. Predicted Field Strengths and Coherence Scales

These ideas have been put together by several authors [13,14,24,25,36,38,41] to estimate  $B_0$  and  $L_0$ , the field strength and coherence scale respectively at the present epoch, of magnetic fields which can undergo nonlinear evolution. First, a general constraint relating  $B_0$  and  $L_0$  can be found from the following criterion, that the field decays to a strength where the Alfvén crossing time across  $L_0$  equals the present age of the universe [36]. This gives

$$B_0 \approx 5 \times 10^{-12} \text{G} \left( \frac{L_0}{\text{kpc}} \right). \quad (3)$$

The field strength  $B_0$  itself can be estimated assuming the scaling laws for turbulent decay starting from generation (when  $a = a_g$ ,  $T = T_g$ ) to end of matter radiation equality ( $a = a_{eq}$ ,  $T \sim 1$  eV). As discussed above, we assume the comoving field strength changes negligibly thereafter. For nonhelical fields assuming the possibility of inverse transfer [38], gives  $B_0 = (a_{eq}/a_g)^{-1/2} B_g$ , where  $B_g$  is the comoving magnetic field  $B_*$  at generation. The scale factor ratio can be related to the temperature ratio using entropy conservation during the radiation era. This gives  $aTg^{1/3}$  being constant with expansion, where we denote by  $g$  the degrees of freedom of relativistic particles. We assume  $g \sim 100$  at the epoch of generation and  $g \sim 4$  at the matter-radiation equality. This gives

$$B_0 \sim 6 \times 10^{-13} \left( \frac{r_B}{0.01} \right)^{1/2} T_{100}^{-1/2} \text{G}, \quad L_0 \sim 0.1 \text{kpc} \left( \frac{r_B}{0.01} \right)^{1/2} T_{100}^{-1/2} \quad (4)$$

where  $T_{100} = T_g/(100 \text{ GeV})$  and the coherence scales is obtained from using  $B_0$  in Equation (3). For the case where the field is only partial helical, with  $h_g$  the initial helical fraction, we have  $B_0 = B_g(a_{eq}/a_h)^{-1/3}(a_h/a_g)^{-1/2} = B_g(a_{eq}/a_g)^{-1/3}(a_g/a_h)^{1/6}$ . Here  $a_h$  is expansion factor and  $\tau_h$  the corresponding epoch, when the decay of the field makes it fully helical. The initial helical fraction is defined as the ratio of initial helicity  $H_g$  by the maximal helicity  $H_{max}$  for a given energy, which for a peaked magnetic spectrum is  $H_{max} \simeq B_g^2 L_c(\tau_g)$  [36]. We note that the initial helicity  $H_g$  is nearly conserved while energy decays. Therefore the fractional helicity subsequently scales as  $h \simeq H_g/(E_M(\tau)L_c(\tau)) = h_g(\tau/\tau_g)^{1/2}$  and becomes unity when  $(\tau_g/\tau_h) = (a_g/a_h) = h_g^2$  or when  $(a_g/a_h)^{1/6} = h_g^{1/3}$ . Hence  $B_0 = B_g(a_{eq}/a_g)^{-1/3} h_g^{1/3}$  and putting in numbers,

$$B_0 \sim 10^{-10} \left( \frac{r_B}{0.01} \right)^{1/2} T_{100}^{-1/3} h_g^{1/3} \text{G}, \quad L_0 \sim 20 \text{kpc} \left( \frac{r_B}{0.01} \right)^{1/2} T_{100}^{-1/3} h_g^{1/3}. \quad (5)$$

The above estimates agree reasonably with that of Banerjee and Jedamzik [36] from their detailed simulations of the magnetic field decay.

Thus primordial magnetic fields surviving from the early universe could account for the lower limits to magnetic field in voids coming from the  $\gamma$ -ray observations of blazars emitting in the TeV energies. Their field strengths and coherence scales could even be such as to influence other physical processes in the universe.

It is important to mention the following caveat with the  $\gamma$ -ray constraints. For this we recall how the constraint is obtained. Firstly it is argued that high energy TeV photons from a blazar interact with eV photons in the intergalactic space to produce a beam of relativistic electron-positron ( $e_{\pm}$ ) pairs, after travelling a distances of order tens of Mpc, typically into the void regions. This  $e_{\pm}$  beam inverse Compton scatters the ambient CMB photons to GeV energies such that one should see a GeV  $\gamma$ -ray halo around every TeV blazar, which is not detected. This null result can be explained if the beam gets sufficiently spread out due to deflection of electrons and positrons in opposite directions by an intergalactic magnetic field, which leads to the lower limit on such fields. However, there is ongoing debate as to whether the  $e_{\pm}$  beam traversing through the intergalactic medium, loses its energy due to plasma instabilities at a rate faster than the inverse Compton rate [10,13,42–46]. In such a case, one would not see a GeV halo around the blazar even if there were no magnetic fields in the voids

and consequently no lower limit on the intergalactic field would be obtained. Irrespective of the final outcome of this debate, the fact that one can potentially probe such a weak intergalactic magnetic field from  $\gamma$ -ray astronomy is very exciting. Of course, such a field need not be primordial, but could arise from the pollution of magnetic fields from galactic outflows [47,48], but the volume filling factor of such outflows is uncertain.

An important challenge for magnetogenesis scenarios involving phase transitions, is the requirement that they ideally be of first order. The EW or QCD phase transition are first order only in extensions to the standard model of particle physics [49–51]. In the standard model, the EW and QCD phase transitions are ‘crossover’ transitions, with thermodynamic variables changing continuously but significantly in a narrow range of temperature around the critical temperature  $T_c$  [52,53]. Magnetogenesis models which involve phase transitions of first order in the early universe and/or which generate strong magnetic fields with a blue power spectrum, like in the inflationary magnetogenesis models of [24,25], can lead to a significant stochastic gravitational wave background. This can be probed by space gravitational wave detectors like LISA in the future [54–57].

### 3. Astrophysical Batteries and Seed Magnetic Fields

The Universe is charge neutral but positive and negative charged particles have different masses—a feature which is at the root of many astrophysical battery mechanisms.

#### 3.1. Biermann Batteries

For example suppose a pressure gradient is applied to a fully ionized hydrogen plasma. Pressure depends on number density and temperature and if these are the same for electrons and protons, the force on these fluid components will also be identical. However the electrons, being much lighter than protons, will be accelerated much more than the protons. This relative acceleration leads to an electric field,  $E = -\nabla p_e / en_e$ , which couples back positive and negative charges so that they move together, obtained by equating the electron pressure gradient  $-\nabla p_e$  with the electric force  $-en_e E$ . Here  $n_e$ ,  $p_e = n_e kT$ , and  $T$  are respectively the number density, pressure, and temperature of the electron fluid, and we have assumed that protons are much more massive and so do not move. If this thermally generated electric field has a curl, from Faraday’s law, magnetic fields can grow from zero. Adding this electric field in Ohm’s law and taking the curl gives a modified induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) - \frac{ck_B}{e} \frac{\nabla n_e}{n_e} \times \nabla T. \quad (6)$$

We see that Equation (6) now contains a *source* term such that magnetic fields can be generated from initially zero fields. This source is nonzero if the density and temperature gradients,  $\nabla n_e$ , and  $\nabla T$  are not parallel to each other, and the resulting battery effect is known as the Biermann battery. It was first proposed as a mechanism for the generation of stellar magnetic fields [58,59], but has subsequently found wide applications to the cosmological context as well [60,61].

For example during reionization of the universe by star bursting galaxies and quasars, the temperature gradient is normal to the ionization front. However density gradients are determined by arbitrarily laid down density fluctuations, which will later collapse to form galaxies and clusters, and which need not be correlated to the source of the ionizing photons. The source term in Equation (6) is then nonzero and magnetic fields coherent on the scale of the density fluctuations, or galactic and larger scales can grow. This will be amplified further during the collapse to form galaxies and one expects a seed magnetic field in galaxies  $B \approx 10^{-21}$  G [60]. Direct numerical simulations of cosmic reionization [62] have confirmed such a scenario, and find a magnetic field ordered on Mpc scales, with a mass weighted average  $B \sim 10^{-19}$  G at a redshift of about 5.

The Biermann battery can also operate in oblique cosmological shocks which arise during the formation of galaxies and large scale structures to generate magnetic fields [61]. For partially ionized hydrogen, with uniform ionization fraction  $\chi$  and all species having the same temperature,

$p_e = \chi p / (1 + \chi)$  and  $n_e = \chi \rho / m_p$ . Here  $p$  is the total fluid pressure. Defining  $\Omega_B = e\mathbf{B}/m_p c$ , Equation (6) reduces to the same form as the induction equation but now for  $\Omega_B$  with a source term  $(\nabla p \times \nabla \rho) / (\rho^2 (1 + \chi))$ . This source term, without the extra factor  $-(1 + \chi)^{-1}$ , corresponds to the baroclinic term in the vorticity equation for  $\Omega = \nabla \times \mathbf{V}$ , where the Lorentz force is neglected. Thus provided viscosity and resistivity are neglected,  $\Omega_B (1 + \chi)$  and  $-\Omega$  satisfy the same equation, and if they were both zero initially, they will always be equal later, i.e.,  $e\mathbf{B}/m_p = -\Omega / (1 + \chi)$ . Taking the vorticity associated with spiral galaxies,

$$|\mathbf{B}| \approx 10^{-19} \text{G} \left( \frac{\Omega}{10^{-15} \text{s}^{-1}} \right). \quad (7)$$

Direct numerical simulations were used by Kulsrud et al. [61] to calculate the vorticity build up in structure formation shocks, which using Equation (7) then translates into a seed magnetic field of  $B \sim 10^{-21}$  G in regions about to collapse into galaxies at redshift  $z \sim 3$ .

### 3.2. Battery Due to Interaction with Radiation

The difference between the masses of positive and negative charges can also lead to battery effects when an ionized plasma interacts with radiation. Indeed electrons are more strongly coupled with radiation than the protons, because the Thomson cross section for its scattering off photons is larger, being inversely proportional to the mass of the charged particle. Due to this photon-electron/proton scattering asymmetry, during recombination, both vorticity and magnetic fields are generated in the second order of perturbations. The strength of these seed fields are again of tiny  $B \sim 10^{-30}$  G on Mpc scales up to  $B \sim 10^{-21}$  G at parsec scales [63–66]. Moreover, during reionization of the universe, the radiative force from a source is larger on electrons than protons, accelerating the electrons more, again generating an electric field which couples them back together. Due to the inhomogeneity of the intergalactic medium this electric field will have a curl leading to magnetic field generation. This field is estimated to be between  $10^{-23} - 10^{-19}$  G on coherence scales between hundreds of kpc to pc, respectively [67,68].

### 3.3. Plasma Effects

During cosmological structure formation, the infall kinetic energy of the intergalactic medium (IGM) is expected to be converted into thermal energy through many shocks. The densities in the IGM are small with  $n \sim 2 \times 10^{-7} (1 + z)^3 \text{cm}^{-3}$ , and therefore Coulomb collisions may not be strong enough to form these shocks, and one may need other means for particle collisions. Plasma instabilities like the Weibel instability [69,70], which occur when there are counter streaming plasma motions, generate small scale magnetic fields, which then effectively scatter particles. The idea that these fields provide seed magnetic fields has been explored by several authors [71,72]. Such a plasma instability has typical growth times  $\tau_p \sim (v/c)^{-1} (1/\omega_i)$ , where  $v$  is the upstream velocity,  $\omega_i = (4\pi n_i e^2 / m_i)^{1/2}$  the plasma frequency, and 'i' can represent electrons or ions, with coherence scales corresponding to the species skin depth  $c/\omega_i$ . These timescales are so small even for ions,  $\tau_p \approx 6 \times 10^2 v_2^{-1} n_{-5}^{-1/2}$  s, compared to astrophysical timescales that the instability would rapidly saturate. Here  $v_2 = v / (10^2 \text{km s}^{-1})$  of order 1–3 is a typical inflow velocity for galaxies which will have velocity dispersions of the same order and  $n_{-5} = n / (10^{-5} \text{cm}^{-3})$  the IGM density at redshifts of  $z \sim 4-5$ , given its  $(1 + z)^3$  scaling.

Particle in cell simulations show that saturation occurs when the field grows to a small fraction  $\epsilon_B$  of the kinetic energy density of the inflowing plasma. Then the gyro radius of ions becomes smaller than the skin depth, whereby particles will get strongly deflected and so not counter stream. The resulting magnetic fields at saturation can be strong with  $B \sim 3 \times 10^{-9} \text{G} (\epsilon_B / 10^{-3})^{1/2} v_2 n_{-5}^{1/2}$ , but correlated on the very small ion-skin depth  $10^{-8} n_{-5}^{-1/2}$  pc [73,74]. The long time survival of this shock generated field is unclear. Moreover, averaged over galactic scales they can only provide a tiny seed field for the dynamo (see Section 3.5).

### 3.4. Seed Fields from Stars and Active Galactic Nuclei (AGN)

A seed magnetic field for the galaxy can also be provided by ejection of stronger magnetic fields from stars and active galaxies which have a much shorter dynamical time scale and form before the bulk of the galactic interstellar medium gets magnetized [75–78]. These processes can give fairly large seed magnetic fields of order a nano Gauss or larger. Of course in this case the dynamo has to operate efficiently in stars and AGN, and faces the challenges that we describe later in this review. There is also the issue of how magnetized plasma ejected from these objects is mixed with the originally unmagnetized interstellar medium in a protogalaxy, and how this affects its strength and coherence scales.

### 3.5. Large-Scale Seed Magnetic Field from Small Scale Fields

In several contexts that we have discussed, the generated seed magnetic field even if strong, has a much smaller coherence scale than that of galaxies. In order to estimate the seed this provides for the galactic dynamo one has to determine the long wavelength (small wavenumber  $k$ ) tail of the corresponding 1-dimensional magnetic power spectrum  $M(k)$ . For hydrodynamic turbulence, both a 1 dimensional velocity power spectrum  $E(k) \propto k^2$  (called the Saffman spectrum) and  $E(k) \propto k^4$  are possible [37]. It has been argued that  $M(k) \propto k^4$  for the magnetic case using  $\nabla \cdot \mathbf{B} = 0$  and the analyticity of the power spectrum [79]. To elucidate the conditions required for this, we proceed as follows:

The magnetic correlation function in Fourier space  $\hat{M}_{ij}(k)$  is the Fourier transform of the real space correlation function. Contracting the indices, assuming statistical isotropy and homogeneity, the 3-D magnetic spectrum  $M_{3d}(k)$  is given by

$$M_{3d}(k) = \frac{1}{2} \int w(r) e^{ik \cdot r} d^3r = 2\pi \int \frac{d}{dr} [r^3 M_L] \frac{\sin(kr)}{kr} dr = 2\pi \int \frac{d}{dr} [r^3 M_L] [1 - k^2 r^2/6 + \dots]. \quad (8)$$

Here we have used the fact that for a statistically isotropic and homogeneous magnetic field  $w(r) = \overline{\mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r})} = 1/r^2 d(r^3 M_L)/dr$  and  $M_L(r)$  is the longitudinal correlation function [80,81]. The last step in Equation (8) has made a small  $kr$  expansion of  $\sin(kr)$ . The first term in the expansion in Equation (8) is  $r^3 M_L$  evaluated at infinity, and goes to zero if  $M_L$  falls off faster than  $1/r^3$ . Then the next term dominates at small  $k$ , provided the resulting integral is non zero, which it would be in general for  $M_L(r)$  falling off sufficiently rapidly. Then  $M_{3d}(k) \propto k^2$  and so the 1-d spectrum  $M(k) \propto k^2 M_{3d}(k) \propto k^4$ . On the other hand, if the magnetic field correlator  $M_L(r)$  falls off as  $1/r^3$  due to the persistence of long range correlations, then the first term in the integral does not vanish and instead, goes to a constant. Then  $M_{3d}(k) \rightarrow \text{constant}$  as  $kr \rightarrow 0$ , hence  $M(k) \rightarrow k^2$ . The first case would hold for example when the field is in randomly oriented magnetic field rings (or flux tubes), while the latter case will be obtained if one generates instead randomly oriented current rings. So both cases of  $M(k) \propto k^2$  (random current rings) and  $M(k) \propto k^4$  (random B flux rings) would seem possible depending on the origin of the field. For a spectrum  $M(k) \propto k^n$ , the power per logarithmic interval in  $k$  space scales as  $kM(k) \propto k^{n+1}$ , and hence the magnetic field smoothed over a volume of size  $l = 1/k$  scales as  $B_l \propto l^{-(n+1)/2}$ .

Suppose the field is coherent on a small scale  $l$ , has strength on this scale  $B_l$ , and the spectrum goes as  $M(k) \propto k^n$  for  $kl \ll 1$ , an estimate of the power on a large scale  $L \gg l$  is given by  $B_L \sim B_l (l/L)^{(n+1)/2}$ . For example, in case of the Weibel instability generated field of Section 3.3, with  $B_l \sim 3 \times 10^{-9}$  G at  $l \sim 3 \times 10^{10}$  cm, taking  $L = 1$  kpc, we get  $B_L \sim 10^{-25}$  G even for the  $n = 2$  case. On the other hand if supernovae seed fields of  $B_l \sim 10^{-6}$  G on scales of 100 pc, on a larger galactic scale of say  $L = 3$  kpc the seed field would be  $B_L \sim 6 \times 10^{-9}$  G for  $n = 2$  case and  $B_L \sim 2 \times 10^{-10}$  G, which are fairly strong seed magnetic fields for a dynamo to act on.



#### 4. Turbulent Dynamos and Their Challenges

Turbulence or random motions, which is prevalent in all systems from stars to galaxy clusters, is thought to be crucial for amplification of seed magnetic fields to the observed levels, a process called “turbulent dynamo”. Turbulence is driven mostly by supernovae in the galactic interstellar medium [82], although during the formation of a galaxy by collapse from the IGM, accretion shocks and flows along cold streams could also be important [83–87]. In disk galaxies, shear due to the differential rotation also plays an important role in the dynamo amplification process. Turbulent dynamos are conveniently divided into two classes, the fluctuation or small-scale and mean-field or large-scale dynamos. This split depends respectively on whether the generated field is ordered on scales smaller or larger than the scale of the turbulent motions. Here we briefly outline their role in galactic magnetism focusing on the challenges that they present. Much of our current understanding of these dynamos come from their analysis using statistical methods or direct numerical simulations (DNS). We shall focus more on some conceptual issues here.

##### 4.1. Fluctuation or Small-Scale Dynamos

The fluctuation dynamo is generic to sufficiently highly conducting plasma which hosts random motions, perhaps due to turbulence. First, in such plasma, magnetic flux through any area moving with the fluid is conserved. Moreover, in any turbulent flow, fluid parcels random walk away from each other and so magnetic field lines get extended. Consider a flux tube with plasma of density  $\rho$ , magnetic field  $B$ , area of cross section  $A$  and linking fluid elements separated by length  $l$ . Flux conservation implies  $BA = \text{constant}$ . Mass conservation in the flux tube gives  $\rho Al = \text{constant}$ , which implies  $B/\rho \propto l$ . Thus if  $l$  increases due to random stretching and  $\rho$  is roughly constant, then  $B$  increases. This of course comes at the cost of  $A \propto 1/\rho l \propto 1/B$  decreasing, the field being concentrated on smaller and smaller scales till resistivity becomes important at a scale  $l_B$ . An estimate for this resistive scale gives  $l_B \sim lR_m^{1/2}$ , got by balancing the decay rate due to resistive diffusion,  $\eta/l_B^2$ , with growth rate due to random stretching  $v/l$ . Here  $v$  and  $l$  are the velocity and its coherence scale respectively of turbulent eddies. As  $R_m$  is typically very large in astrophysical systems, the resistive scale  $l_B \ll l$ .

What happens when resistive dissipation balances random stretching can only be addressed by a quantitative calculation. The first such calculation was due to Kazantsev [88], who considered an idealized random flow which is  $\delta$ -function correlated in time. For such a flow one can write an exact evolution equation for the two-point magnetic correlator, which has exponentially growing solutions, or is a dynamo, when  $R_m$  exceeds a modest critical value  $R_c \sim 100$ . The growth rate is a fraction of the eddy turn over rate  $v/l$ , and at this kinematic stage, the field is shown to be concentrated on the scale  $l_B$ . From the idealized Kazantsev model, it also turns out that  $R_c$  is larger and the growth slower for compressible flows compared to the incompressible case [89–91]. Moreover, for Kolmogorov turbulence where the flow is multi-scale, ranging from the outer scale to the small scales where viscosity dominates, the fastest amplification is by the smallest supercritical eddy motions. In the galactic ISM, the kinematic viscosity  $\nu$  is typically much larger than the resistivity  $\eta$ , and then growth would be expected to occur first due to dynamo action by the smaller viscous scale eddies [92,93]. For the interstellar turbulence with an outer scale of turbulence  $l \sim 100$  pc and velocities  $v \sim 10$  km s<sup>-1</sup>, we expect  $R_m \gg R_c$  and a growth time scale  $l/v \sim 10^7$  years even by the largest eddies. This time scale is so much smaller than ages of even young high redshift galaxies, say a few times  $10^9$  years old, that the fluctuation dynamo is expected to rapidly grow even weak seed magnetic fields to micro Gauss levels. Moreover, as smaller eddies can grow the field faster, significant amplification occurs even earlier. However as  $l_B \ll l$ , the field in the growing phase is extremely intermittent and concentrated on the small resistive scales. The big challenge is then whether these fields can become coherent enough to explain for example observations of the Faraday rotation inferred in young galaxies.

This growth of random magnetic fields due to the fluctuation dynamo has been verified by direct numerical simulations of driven turbulence, albeit in the idealized setting of isothermal plasma, for both subsonic and supersonic flows [94–101]). Such simulations however have a modest values

of  $R_m/R_c \sim 10\text{--}20$ . The basic expectations of the idealized Kazantsev model during the kinematic phase are qualitatively verified. The field grows exponentially and is concentrated initially on the resistive scales. It is also found that the small-scale dynamo is less efficient for compressible compared to solenoidal forcing, as it generates less vorticity [100,102,103]. Importantly, the DNS can now also follow the field evolution in to the nonlinear regime when Lorentz forces act to saturate the dynamo. By the time the dynamo saturates, the coherence length of the field increases to be a fraction of order  $1/3\text{--}1/4$  the scale of the driving, at least when the magnetic Prandtl number  $\text{Pr}_m = \nu/\eta$  is of order unity [94,96–98,104]. These DNS have resolutions from  $512^3$  upto  $2048^3$ . More modest resolution ( $256^3$ ) DNS with large  $\text{Pr}_m$  but small fluid Reynolds number  $\text{Re}$  found the magnetic energy spectrum to be still peaked at the resistive scale  $l_B$  even at saturation [95]. It is difficult to directly simulate the case expected in the interstellar medium, of both a large  $R_m/R_c$  and large  $\text{Re}$ , as one then has to resolve both the widely separated resistive and viscous dissipation scales. Clearly the saturated state of the fluctuation dynamo deserves further study, especially in this highly turbulent and  $\text{Pr}_m \gg 1$  regime.

We have also directly determined Faraday rotation measures (RMs), in simulations of the fluctuation dynamo with various values of  $R_m$ , fluid Reynolds number  $\text{Re}$  and up to rms Mach number of  $\mathcal{M} = 2.4$  [97,101,105]. At dynamo saturation, for a range of parameters, we find an rms RM contribution which is about half the value expected if the field is coherent on the turbulent forcing scale. Interestingly, in subsonic and transonic cases, the general sea of volume filling fields, dominates in determining the strength of RM. The rarer, strong field structures, contribute only about 10%–20% to the RM signal, indicating that perhaps the coherence of the generated fields is associated with more typical volume filling magnetic field regions. However, when the turbulence is supersonic significant contributions to the RM also comes from strong field regions as well as moderately over dense regions. How exactly the field orders itself during saturation is at present an open problem.

One may wonder if magnetic reconnection is important for dynamo action. We note that the reconnection speed, depends inversely on the magnetic field strength even when it is efficient. Thus it would be too long compared to the dynamo growth rate to be relevant during the kinematic stage of the dynamo. It could however play a role once the field becomes dynamically important. Some interesting aspects of a reconnecting flux rope dynamo have been explored in [106]. In nearly collisionless plasmas like galaxy clusters, plasma effects could set transport properties even for weak fields and small scale dynamo action in such a context is just beginning to be explored [107,108].

Simulations of galaxy formation from cosmological initial conditions have also showed evidence for amplification by the fluctuation dynamo, over and above the result of amplification by flux freezing during the compressive collapse to form the galaxy [87,109–112]. One of the main limitations of such cosmological simulations is the resolution; that it will be very difficult to capture both the galactic scale and the dissipative scales, to predict correctly the rate of growth of magnetic energy and the coherence scale of the saturated field. Intriguingly, some of the direct simulations of SNe driven turbulence, which have possibility of a multiphase medium, do not yet show a strong fluctuation dynamo [113–115], although they do show large-scale dynamo action (except for [116]).

All in all, one expects energy of random, intermittent magnetic fields to generically grow rapidly in the turbulent ISM of galaxies. This turbulence could be driven by supernovae in disk galaxies. Galactic disks would then host significant fields, and a line of sight going through the disk could have a significant RM [97,101]. This can partly explain the statistical detection of excess RM in MgII absorption systems [4,5], which are thought to be associated with young galaxy disks at redshifts  $z \sim 1$ . However, the abundance of these systems gives evidence that the MgII absorption arises not only in line of sights through the disk, but also in extended gaseous halos [117]. Thus one would need the halo to be also magnetized and produce a significant RM. This could occur through outflows from the disk which also carry cold magnetized "clouds". More work is required to firm up such a speculation.

#### 4.2. Mean-Field or Large-Scale Dynamos and Galactic Magnetism

Remarkably, when turbulence is helical, magnetic fields on scales larger than the coherence scale of the turbulence can be amplified. In any rotating, stratified system like the ISM of a disk galaxy random motions driven by supernovae do become helical due to the Coriolis force, with one sign of helicity in the northern hemisphere and the opposite sign in the southern hemisphere. Such helical turbulent motions of the plasma draw out toroidal fields in the galaxy into a twisted loop generating poloidal components (called the  $\alpha$ -effect). Differential rotation of the disk shears the radial component of the poloidal field to generate back a toroidal component (the  $\omega$ -effect). These two can combine to exponentially amplify the large-scale field provided that the generation terms can overcome an extra resistivity due to the turbulence. This is quantified by a dimensionless dynamo number being supercritical. Turbulent resistivity also allows the mean-field flux to be changed.

Quantitatively, in mean-field dynamo theory, the total magnetic field is split as  $\mathbf{B} = \overline{\mathbf{B}} + \mathbf{b}$ , the sum of a mean (or the large-scale) field  $\overline{\mathbf{B}}$  and fluctuating (or the small-scale) field  $\mathbf{b}$ . A similar split of the velocity field gives  $\mathbf{V} = \overline{\mathbf{V}} + \mathbf{v}$ . The mean is defined by some form of averaging on scales larger than the turbulence coherence scale, ideally but not necessarily satisfying Reynolds rules for such averaging. These rules are [118]:  $\overline{\mathbf{B}_1 + \mathbf{B}_2} = \overline{\mathbf{B}_1} + \overline{\mathbf{B}_2}$ ,  $\overline{\overline{\mathbf{B}}} = \overline{\mathbf{B}}$ , so  $\overline{\mathbf{b}} = 0$ ,  $\overline{\mathbf{B}\mathbf{b}} = 0$ ,  $\overline{\mathbf{B}_1 \mathbf{B}_2} = \overline{\mathbf{B}_1} \overline{\mathbf{B}_2}$  and averaging commutes with both time and space derivatives. The induction equation Equation (1) then averages to give

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{V}} \times \overline{\mathbf{B}} + \mathcal{E} - \eta \nabla \times \overline{\mathbf{B}}). \quad (9)$$

Here a new term quadratic in the fluctuating fields arises, the mean electromotive force (EMF),  $\mathcal{E} = \overline{\mathbf{v} \times \mathbf{b}}$ . To express this in terms of the mean fields themselves presents a closure problem, even when Lorentz forces are not yet important. The simplest such closure, which is valid when the correlation time  $\tau$  is small compared to  $l/v$  gives  $\mathcal{E} = \alpha_K \overline{\mathbf{B}} - \eta_t \nabla \times \overline{\mathbf{B}}$ , where the turbulent motions are also assumed to be isotropic. Here  $\alpha_K = -\frac{1}{3} \tau \langle \mathbf{v} \cdot \boldsymbol{\omega} \rangle$  with  $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ , depending on the kinetic helicity of the turbulence and is the  $\alpha$ -effect mentioned above while  $\eta_t = \frac{1}{3} \tau \langle v^2 \rangle$  is a turbulent diffusivity and depends on the kinetic energy of the turbulence. In disk galaxies we also have a  $\overline{\mathbf{V}} = r\Omega(r)\boldsymbol{\phi}$  corresponding to its differential rotation with frequency  $\Omega$  along the toroidal direction  $\boldsymbol{\phi}$ . The mean-field dynamo equation (9) with this form for  $\mathcal{E}$  and  $\overline{\mathbf{V}}$ , has exponentially growing solutions provided a dimensionless dynamo number has magnitude  $D = |\alpha_0 S h^3 / \eta_t^2| > D_{crit} \sim 6$  [75,119,120]. Here  $h$  is the disk scale height and  $S = rd\Omega/dr$  the galactic shear,  $\alpha_0$  a typical value of  $\alpha$ , and we have defined  $D$  to be positive. This condition can be satisfied in disk galaxies and the mean field typically grows on the rotation time scale,  $\sim 10^8 - 10^9$  years. A detailed account of mean-field theory predictions for galactic dynamo theory and its comparisons to observations is done by other authors in this volume. We focus on the challenges for this general paradigm, in our view.

##### 4.2.1. Magnetic Helicity Conservation

The first potential difficulty, which has already received considerable attention, arises due to the conservation of magnetic helicity in the highly conducting galactic plasma. Magnetic helicity is usually defined as  $H = \int_V \mathbf{A} \cdot \mathbf{B} dV$  over a 'closed' volume  $V$ , with  $\mathbf{A}$  the vector potential satisfying  $\nabla \times \mathbf{A} = \mathbf{B}$ . It is invariant under a gauge transformation  $\mathbf{A}' = \mathbf{A} + \nabla \Lambda$  only if the normal component of the field on the boundary to volume  $V$  goes to zero. Magnetic helicity measures the linkages between field lines [121,122], is an ideal invariant and is better conserved than total energy in many contexts, even when resistivity is included. The mean-field dynamo works by generating poloidal from toroidal field and vice-versa and thus automatically generates links between these components, and thus a large-scale magnetic helicity. To conserve the total magnetic helicity, corresponding oppositely signed helicity must then be transferred to the small-scale field, which as we shall see is done by the turbulent emf  $\mathcal{E}$ .

In fact, when helical motions writhe the toroidal field to generate a poloidal field, an oppositely signed twist must develop on smaller scales, to conserve magnetic helicity. For the same magnitude of magnetic helicity on small and large scales, the Lorentz force  $(\mathbf{J} \times \mathbf{B})/c$  is generally stronger on small-scales (since  $\mathbf{J}$  the current density has two more derivatives compared to the vector potential which determines magnetic helicity). Thus Lorentz forces associated with this twist helicity can unwind the field while turbulent motions writhe it. According to closure models like the Eddy damped quasi linear Markovian (EDQNM) approximation [123] or the  $\tau$  approximation [81,124,125], Lorentz forces then lead to an additional effective magnetic  $\alpha$ -effect,  $\alpha_M = \frac{1}{3}\tau\overline{\mathbf{j} \cdot \mathbf{b}}/4\pi\rho$ , with the total  $\alpha = \alpha_K + \alpha_M$ . The generated magnetic  $\alpha_M$  opposes the kinetic  $\alpha_K$  produced by the helical turbulence and quenches the  $\alpha$ -effect and the dynamo, making it subcritical, much before the large-scale field grows strong enough to itself affect the turbulence. For avoiding such quenching, small-scale helicity must be shed from the galactic interstellar medium. In principle resistivity can dissipate small-scale magnetic helicity but this takes a time longer than the age of the universe! For large-scale dynamos to work small-scale helicity must be lost more rapidly, through magnetic helicity fluxes [81,122,126].

Magnetic helicity being a topological quantity, one may wonder how to define its density and its flux! A Gauge invariant definition of helicity density was given by Subramanian and Brandenburg [127] using the Gauss linking formula for the magnetic field [121,128]. They proposed that the magnetic helicity density  $h$  of a random magnetic field  $\mathbf{b}$  is the density of correlated links of the magnetic field [127]. This definition by construction involves only the random field  $\mathbf{b}$ , works if this field has a small correlation scale compared to the system scale, and is closest to the helicity density defined using the vector potential in Coulomb gauge. An evolution equation can then be derived for this density of helicity which now also involves a helicity flux density  $\overline{\mathbf{F}}$  [127],

$$\frac{\partial h}{\partial t} + \nabla \cdot \overline{\mathbf{F}} = -2\mathcal{E} \cdot \overline{\mathbf{B}} - 2\eta \overline{\nabla \times \mathbf{b} \cdot \mathbf{b}}. \quad (10)$$

This equation involves transfer of magnetic helicity from large to small scales by the turbulent emf along the mean field ( $-2\mathcal{E} \cdot \overline{\mathbf{B}}$  term), the dissipation by resistivity ( $-2\eta \overline{\nabla \times \mathbf{b} \cdot \mathbf{b}}$ ) and the spatial transport by the helicity flux ( $\nabla \cdot \overline{\mathbf{F}}$ ). In the absence of such a flux, and in the steady state we see that  $\mathcal{E} \cdot \overline{\mathbf{B}} = -2\eta \overline{\nabla \times \mathbf{b} \cdot \mathbf{b}}$  and so the emf along the field, which is important for the dynamo, is resistively suppressed for  $R_m \gg 1$ . Even in the time dependent case, as the  $\overline{\mathbf{B}}$  builds up,  $h$  also grows and produces an  $\alpha_M$  which cancels  $\alpha_K$  to suppress the net  $\alpha$  effect. In the presence of helicity fluxes however,  $h$  can be transported out of the system allowing mean-field dynamos to work efficiently [126,129,130].

One such flux is simply advection of the gas and its magnetic field out of the disk, i.e.,  $\overline{\mathbf{F}} = h\overline{\mathbf{v}}$  [127,129]. Several other types of helicity fluxes have been calculated like the Vishniac-Cho flux depending on shear and the mean field [81,131] and a flux involving inhomogeneous  $\alpha$  [132]. A diffusive flux  $\overline{\mathbf{F}} = -\kappa\nabla h$  was postulated by [133] and subsequently measured in DNS [134]. A new type of helicity flux which depends on purely an inhomogeneous random magnetic field and rotation or shear has been worked out by Vishniac [135], and could be potentially important to drive a large-scale dynamo purely from random fields in the galaxy, but has not yet been studied in detail. Both the diffusive flux and the later Vishniac flux have been derived from the irreducible triple correlator contribution to  $\overline{\mathbf{F}}$  by [136] using a simple  $\tau$ -closure theory, but they also find several other terms which cannot be reduced to either of these forms. A detailed study of magnetic helicity fluxes still remains one of the important challenges of the future.

As an interesting application of these ideas, Chamandy et al. [137] solved the mean-field dynamo equation incorporating both an advective flux and a diffusive flux in Equation (10). Advection can be larger from the optical spiral, where star formation and galactic outflows are expected to be enhanced. The helicity fluxes allow the mean-field dynamo to survive, but stronger outflow along spiral arms led to a relative suppression of mean field generation there and an interlaced pattern of magnetic and gaseous arms as seen in the galaxy NGC6946 [138]. Interestingly a wide spread magnetic spiral only results if the optical spiral is allowed to wind up and thus here we are constraining spiral structure

theory using magnetic field observations [137,139]! In another direction, the cosmic evolution of large-scale magnetic fields during hierarchical clustering in the universe to form galaxies, has also been extensively explored [140,141].

#### 4.2.2. Mean-Field Dynamo in Presence of the Fluctuation Dynamo

We have discussed possibilities of both fluctuation and mean-field dynamos in the turbulent interstellar medium. However random magnetic fields due to the fluctuation dynamo grow much faster on time scale of  $10^7$  years, at least a factor 10 faster than the mean-field. Lorentz forces can then become important to saturate the field growth much before the mean field has grown significantly. Will, then, these strong fluctuations make mean field theory invalid? Can the large-scale field then grow at all? Earlier work [93] suggested that perhaps the intermittency of the small-scale dynamo generated field on saturation still allows the Lorentz force to be sub dominant in the bulk, and thus allow large-scale field growth. Bhat et al. [142] examined this issue using direct simulations of magnetic field amplification due to fully helical turbulence in a periodic box, following up earlier work on the kinematic stage by [143]. Turbulence was forced at about 1/4 th the scale of the box, so that in principle both scales smaller and larger than forcing can grow. Initially all scales grow together as a shape invariant eigen function dominated by power on small-scales. This behaviour is akin to what happens in fluctuation dynamos. But crucially on saturation of small scales due to the Lorentz force, larger and larger scales continue to grow, and come to dominate due to the mean-field dynamo action. Finally system scale fields (here the scale of the box) develop provided small-scale magnetic helicity can be efficiently removed, which in this simulation is due to resistive dissipation. Recent work by Bhat et al. [144] in fact now finds evidence for two stages of exponential growth, the sequential operation of both the small-scale dynamo, and as it saturates, a quasi-kinematic large-scale dynamo, which is indeed exciting! This issue of how the small- and large- scale dynamos come to terms with each other deserves much more attention including a better analytic understanding.

### 5. Final Thoughts

We have traced briefly the generation of magnetic fields right from the early universe to their subsequent amplification by turbulent dynamos in the later universe. Several challenges remain to be addressed in each of the processes that were discussed. Apart from the issues already raised, early universe mechanisms need to be put in the context of particular particle physics models. As far as the dynamo, their saturation behaviour and how coherent the resulting fields become still raises intriguing questions. The observational future appears bright. A key objective of the Square Kilometre Array (SKA) is to elucidate the origin of cosmic magnetism. The determination of a large number of RMs and their modelling will likely yield rich dividends [145,146]. Of particular interest will be to probe magnetic fields in the high redshift universe and the field in intergalactic filaments which could reflect more pristine conditions. Surprisingly, Gamma-ray observations of TeV blazars have suggested lower limits at femto Gauss levels to the magnetic field in the IGM associated with large scale voids. Such weak magnetic fields are difficult to detect by other techniques and so it would be worthwhile to continue such studies. Gravitational wave astronomy, especially the detection of a stochastic background, could also help to probe phase transitions and associated magnetogenesis in the early universe. Clearly study of cosmic magnetism will continue to be fascinating.

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