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Review

Influence of Vertical Loading on Behavior of Laterally Loaded Foundation Piles: A Review

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Abstract: The majority of installed offshore wind turbines are supported on large-diameter, open-ended steel pile foundations, known as monopiles. These piles are subjected to vertical and lateral loads while in service. In current design practice, interaction of vertical and lateral loads are not considered, rather piles are designed to resist vertical and lateral loads independently. Whilst interaction effects are widely studied for shallow foundations, the limited research on this topic for pile foundations often produces conflicting results. This paper reviews the research of the influence of vertical loading on the lateral response of pile foundations under combined loads, from the perspective of analytical research, numerical research, and experimental research from tests performed on *1*-g (gravitational acceleration) model, centrifuge, and full-scale piles. The potential reasons for the differences among the results of previous research are discussed. Some guidance for future research on the effect of vertical loads on the lateral response of piles is provided.

Keywords: monopile; vertical load; lateral response; interaction effects

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1. Introduction

The offshore wind sector has experienced significant growth in recent years driven largely by development and innovation in turbine capacity. The industry is seen as one of the most proven ways for society to transition away from carbon to renewable energy sources. The European Green Deal [1] sets an ambitious target of achieving a carbon-neutral continent by 2050 and offshore wind developments are currently the leading technology to assist in achieving this aim. Offshore wind already supplies over 11% of Europe's energy demand [2], with projections this will increase to 30% or more by 2030. To maintain momentum, turbine technology has been developing at a rapid pace, and many near shore sites have been exploited to date. Future developments are looking towards far offshore deeper water sites, which require significant engineering advances to overcome the novel challenges.

Over 87% of offshore wind turbines (OWTs) are founded on single, large diameter piles known as monopiles, which currently have typically diameters, D in the range 4–6 m, and embedded lengths, L between 20 m and 30 m [2,3]. In the present paper, the term "length" denotes embedded length unless otherwise stated. Increased turbine sizes and deeper water sites mean that in the near future piles with diameters of 10 m or more will be deployed. To date, OWTs have been considered as light, flexible structures, with lateral loading predominantly from wind and waves, and overturning moments due to the high load eccentricity [4,5]. With the evolution in new high capacity and heavier energy converters, vertical loading (V) is becoming more prominent, whilst the L/D ratios are reducing to values of ≈2–3 [6] (similar to shallow foundations), leading to renewed questions over vertical–lateral load interaction effects.

Previous research has produced conflicting findings on whether the presence of vertical loading on piles increases or reduces lateral displacements under corresponding lateral loads. A variety of mechanisms have been postulated in an attempt to explain the behavior observed in previous research. Understanding whether and how vertical loading reduces or increases lateral displacements is becoming increasingly important for accurate design of combined-loaded pile systems, such as those used to support emerging OWTs. This paper presents a review on the topic, and focuses on analytical research, numerical modeling, scaled, and full-scale field tests, and centrifuge modeling research that has been conducted by researchers to date in an attempt to develop an understanding of the primary mechanisms contributing to the problem. Elements of focus include the influence of soil type and density/stiffness, pile slenderness, the sequence of applied loads, boundary conditions of the pile, pile material, cross-sectional shape, and the relative magnitude of applied vertical to lateral loading. This review establishes the state-of-the-art in vertical-lateral load interaction for foundation piles and suggests elements of focus for future research to further understand the underlying behavior. Section 2 focuses on analytical research, Section 3 focuses on numerical research, Section 4 reviews full-scale pile tests, Section 5 details scaled *1*-g (gravitational acceleration) model tests, and Section 6 reviews recent centrifuge analyses. Finally, Section 7 presents a discussion of the main findings, and Section 8 concludes the study.

2. Analytical Research

The behavior of piles under combined vertical and lateral loading has been theoretically investigated by several researchers. Davisson and Robinson [7] developed an approximate procedure applicable to partially embedded piles under lateral, vertical, and moment loading. The solution procedure considers partially embedded piles as freestanding with an equivalent fixed-base below ground level, and considers both systems with a constant subgrade reaction modulus with depth and systems with a linearly increasing subgrade reaction modulus with depth. Using their model, results suggest that the application of vertical loads tends to magnify lateral displacement and bending moments experienced at the pile head. Similarly, Shakhirev and Yanyshev [8] showed that the presence of vertical loading (surcharge) leads to a reduction in lateral resistance of single piles, and note that the lateral displacement of flexible piles can increase by a factor of two or more in the presence of vertical loading. Using a model of a beam on an elastic Winkler foundation [9–12], Goryunov [13] analyzed the deformation behavior of a flexible tubular pile with length/diameter (L/D) of 48 under the action of a vertical load of 2 MN and a lateral load of 0.1 MN. In agreement with the aforementioned research, this study shows that at a given applied lateral load, the lateral displacement, and pile bending moment increase under increasing vertical loads. The model considered three densities for soil to characterize the Winkler springs; namely dense, medium dense, and loose soil, all underlain by dense soil. The generation of an additional secondary moment, namely the P-Delta influence, is postulated as the reason for the increase in lateral displacement. Figure 1 reproduces some of the results from this study and shows how lateral displacement and bending moment vary with applied vertical loading for a given lateral load. As evident, the lateral displacement and bending moment vary nonlinearly with increasing applied vertical loading.

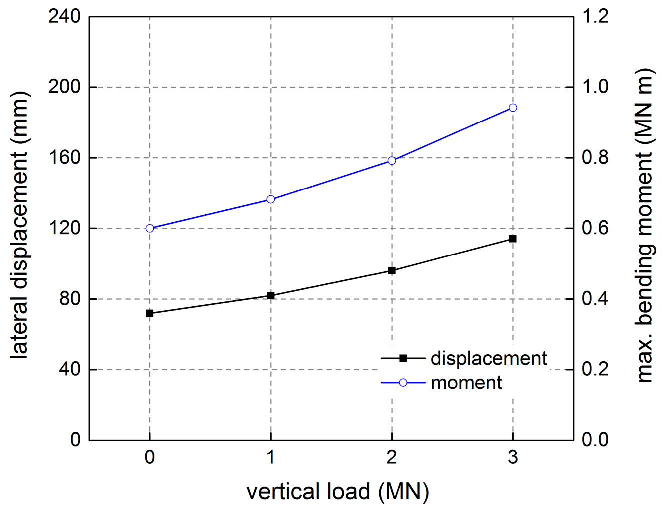


Figure 1. Influence of vertical loading on the lateral displacement and maximum bending moment of a laterally-loaded pile [13].

Reddy and Ramasamy [14] investigated the influence of pile head fixity on the resulting interaction effects between lateral and vertical loading on uniformly tapering circular piles. Three types of fixity were considered; namely fixed–fixed, fixed–free, and free–free, referring to the top and bottom boundary conditions of the pile, respectively. The pile behavior is characterized by two zones: the top plastic zone and bottom elastic zone. Analyses considered different combinations of lateral and vertical loading with varying tapers on the resulting pile head displacements and bending moments. In agreement with the findings of previous research, the application of increasing vertical loads increased the displacement and maximum bending moments at equivalent lateral loads. Additionally, the head displacement of the pile with fixed head and free base condition is considerably less than that with free head and free base under the same combination of vertical and lateral loads.

Valsangkar et al. [15] investigated interaction effects on combined loaded piles and presented generalized solutions, which take into account the elasto-plastic nature of both cohesive and cohesionless soils. For cohesive soils, the governing differential equation to be solved for the upper region of the pile (where ultimate soil resistance is reached) is shown in Equation (1), and the differential equation for the region where the soil resistance-lateral displacement (*p*-*y*) relationship is linear, is as shown in Equation (2).

$$\mathrm{EI}\frac{d^4y}{dx^4}+V\frac{d^2y}{dx^2}+t+t_1x=0,\quad 0\le x\le L_1 \tag{1}$$

$$\mathrm{EI}\frac{d^4y}{dx^4}+\frac{d}{dx}\left[V\left\{1-\varphi_1\left(\frac{x-L_1}{L}\right)\right\}\frac{dy}{dx}\right]+K_0y=0,\quad L_1\le x\le L \tag{2}$$

where y = lateral displacement of the pile, x = distance from the groundline, E = Young's modulus of the pile, I = second moment of area, V = vertical load, t and t_1 are coefficients of the ultimate soil resistance, L = pile length, L_1 = depth to which soil has entered plastic condition, and φ_1 = a coefficient that depends on skin friction mobilization. In the case of cohesionless soils, the governing differential equation for the top (plastic) zone is shown in Equation (3), and for the lower elastic zone is shown in Equation (4).

$$\mathrm{EI}\frac{d^4y}{dx^4}+V\frac{d^2y}{dx^2}+t_2x+t_3x^2=0,\quad 0\le x\le L_1 \tag{3}$$

$$\mathrm{EI}\frac{d^4y}{dx^4}+\frac{d}{dx}\left[V\left\{1-\varphi_2\left(\frac{x^2-L_1{}^2}{L^2}\right)\right\}\frac{dy}{dx}\right]+K_1xy=0,\quad L_1\le x\le L \tag{4}$$

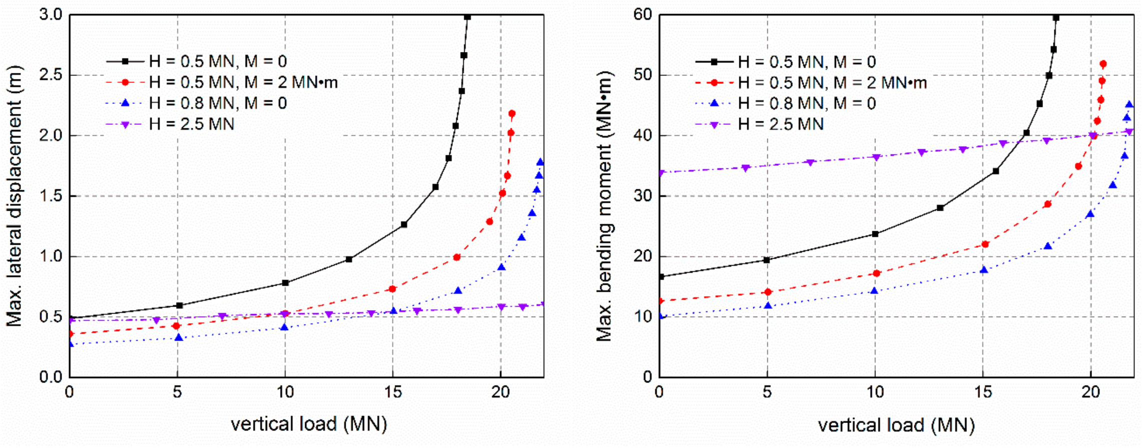
where t_2 and t_3 are coefficients of plastic resistance, φ_2 = vertical load variation coefficient. K_0 and K_1 are subgrade reaction moduli. The results indicated that the flexural behavior of combined loaded piles is influenced considerably by the nature of the plastic resistance mobilized, as well as variations in operational soil modulus and boundary conditions at the pile head. Broadly speaking, it was found that applying vertical loads increases the lateral displacement at given applied lateral loads.

Xingman [16] (referenced in Ming and Ming-Hua [17]) analyzed piles under inclined loads applied at ground level and suggested solutions for estimating the displacement and internal forces of these systems. They suggest that for piles under combined vertical and lateral loading, the principle of superposition no longer applies, essentially in agreement with the findings of previous research. Vertical loads tend to increase lateral displacement and bending moments due to the P-Delta effect. Wentian [18] derived the differential equation governing the deflection of piles with uniform cross-sectional properties under combined vertical and lateral loading within a Winkler-type [9–11] framework, as shown in Equation (5).

$$\mathrm{EI}\frac{\mathrm{d}^4\mathrm{y}}{\mathrm{dx}^4}+\mathrm{V}\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}+\mathrm{Ky}=0 \tag{5}$$

where EI = flexural rigidity of the pile, V is the vertical load, y is the lateral displacement of the pile, x is the distance from the groundline, and K is modulus of subgrade reaction. The solution of the equation enables displacement, rotation, bending, and shear behavior be calculated. The interaction effects between vertical and lateral loading were found to depend on the compressive strain of the cross-section in slender piles. Applying vertical and lateral loads, and using this model, the pile head lateral displacement and rotation were found to be twice and 1.78 times larger, respectively, than for the case where no interaction mechanism is considered. Han and Frost [19] investigated the influence of vertical loading on the lateral response of piles using the variational approach. In agreement with previous research, they found that applying vertical loads increases corresponding lateral displacements. Moreover, they suggest that the increase in lateral displacement is nonlinear with respect to the applied vertical loading, similar to the findings of Goryunov [13]. They suggest that under certain values of vertical load, the corresponding lateral displacements almost double their value compared with in the absence of vertical loading. Han and Frost [19] also considered the effect of boundary conditions on the responses and noted that the load-displacement behavior is highly dependent on the boundary condition of the pile in the ground. Similar to Reddy and Ramasamy [14], they note that the response is highly influenced by the pile top fixity condition. Ming and Ming-Hua [17] developed a laterally layered isotropic and elastic half-space soil model and analyzed the influence of vertical loads on lateral pile behavior. The model was calibrated against model tests. They report the results of laterally loaded free-headed aluminum pipe piles with varying geometrical properties, which were partially embedded in sand. This analysis considered many factors including variations in pile stiffness, length, and loading eccentricity. No firm conclusions were given on the influence of vertical loading on pile lateral response features in this work.

Li et al. [20] derived power-progression solutions for the displacement of single piles in single and multi-layer soils under combined vertical and lateral loading. The presence of gravity, pile-soil friction, external load distribution, and pile inclination are taken into account, as well as the P-Delta effect. Pile inclination has the greatest influence on the load-displacement characteristics of the pile from the calculation results presented. This paper agrees with the previous research and notes that when minor inclination is induced on the pile the corresponding displacements and rotations increase, as a result of the P-Delta effect. Zhang et al. [21] suggest a method to improve the design reliability of flexible single piles under combined loading, based on a subgrade reaction model. In their analysis, the ultimate soil resistance and subgrade reaction coefficient were assumed constant with depth. As with the previous research, the lateral displacement and bending moment increase with increasing vertical loads and load eccentricity under corresponding lateral loads, and decrease with increasing pile stiffness, subgrade reaction coefficient and soil yielding displacement. Figure 2 presents the results of the analyses from Zhang et al. [21] and shows the relationship between the lateral pile displacement and maximum bending moment under increasing vertical loads for varying lateral loads (H) and applied moments (M).



(**a**) (**b**)

Figure 2. Variation in lateral response features of piles under increasing vertical loads (reproduced from Zhang et al. [21]) (**a**) maximum lateral displacement, (**b**) maximum bending moment.

The results from the various analytical models discussed above can be summarized as follows:

- The findings agree that the presence of vertical loading causes an increase in lateral displacement under corresponding lateral loads, as per the P-Delta hypothesis.
- There is a general agreement that the pile head fixity has an effect on the results.
- The pile and soil stiffness also exhibit an influence, though the trend remains that vertical loads tend to exacerbate lateral displacements and lead to the generation of higher bending moments under applied lateral loads than in their absence.

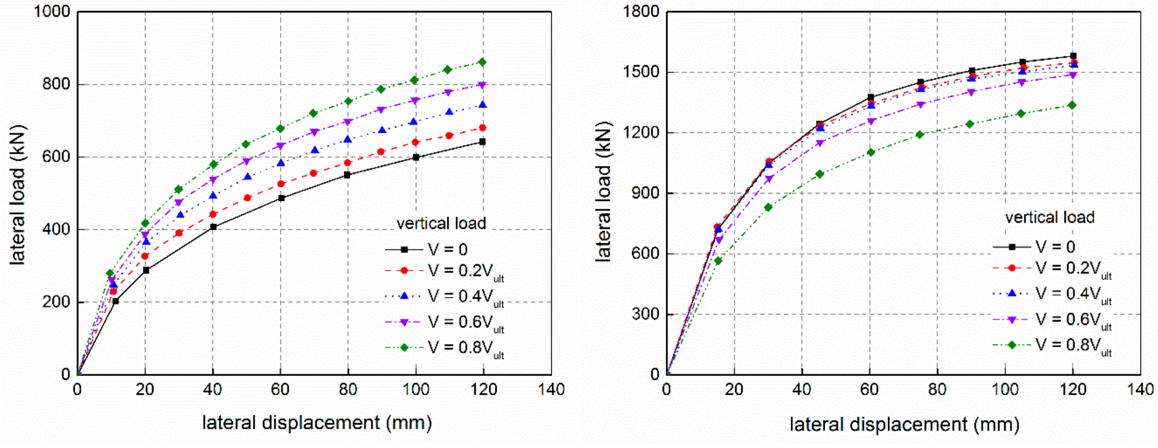
3. Numerical Research

The previous section reviews literature based on performing research using analytical models. The majority of the research produced similar findings, namely applying vertical loads to laterally loaded piles tends to exacerbate lateral displacements. In this section, literature that utilized numerical models to research the combined loaded pile problem is reviewed.

Most research to date on combined loading of piles uses numerical modeling, such as finite-element analysis. Poulos and Davis [22], and Anagnostopoulos and Georgiadis [23] were among the first to attempt to analyze the influence of vertical loading on pile lateral responses using two-dimensional finite-element analysis. Since pile-soil interaction is an inherently three-dimensional problem, two-dimensional finite-element analysis has some limitations. Recognizing this, other researchers have used three-dimensional finite-element analysis to study the lateral behavior of piles under lateral loading, for example [24–26]. Research on the interaction effects between vertical and lateral loading applied to piles using three-dimensional analyses are discussed in the following paragraphs.

Some of the numerical research produces findings that broadly agree with those of the analytical research (discussed in Section 2), specifically Davisson and Robinson [7], Shakhirev and Yanyshev [8], Goryunov [13], Reddy and Ramasamy [14], Xingman [16], Han and Frost [19], Li et al. [20], and Zhang et al. [21] that vertical loads tend to increase corresponding lateral displacements of laterally loaded piles. Madhav and Sarma [27] investigated the response of long piles in clay under both vertical and lateral loading using the finite-difference method. The analysis considered a pile under combined loading and compared it to the situation where only lateral loads are applied. The results suggest that the application of vertical loading causes an increase in pile top lateral displacement and bending moment along the embedded pile length. In the case considered, these response features doubled in value in the presence of vertical loading. Meera et al. [28] developed a generalized procedure to analyze and predict the flexural behavior of vertically and laterally loaded piles in liquefiable loose sand using the finite-difference method. Their analysis considered a 7 m long concrete pile with cross-sectional dimensions of 0.35 m × 0.35 m. Loading comprised vertical loads from a superstructure and lateral loading from wind, waves, and inertial and kinematic effects (from seismic actions). Vertical loading was found to have a significant effect on the lateral response of piles in liquefiable soils and should be explicitly considered in design. The presence of vertical loading was found to exacerbate lateral pile head displacements and bending moments. It is suggested that if the vertical load exceeds a threshold value, pile failure during liquefaction occurs not solely due to lateral spreading, but also due to vertical loading.

Klein and Karavaev [29] performed finite-element analysis on a pile that was subjected to both vertical and lateral loads. The analysis considered a geometrically nonlinear formulation of a reinforced concrete end-bearing pile, embedded 2 m into the ground, with a 2.5 m stick up, and with cross-sectional dimensions of 0.3 m × 0.3 m. Results suggested that the presence of vertical loading both increases and decreases the pile lateral bearing capacity (and resulting displacements) based on the soil type considered. For dense soil, vertical loading leads to an increase in lateral capacity (reduction in corresponding lateral displacements), while for weak soil, a decrease in lateral capacity (increase in displacements) was observed. Karthigeyan et al. [30] investigated combined load effects on concrete piles with cross-sectional dimensions of 1.2 m × 1.2 m embedded 10 m in both homogeneous clayey and sandy soils. A Von Mises constitutive model with associated flow rule and a Drucker–Prager constitutive model with non-associated flow rule were used to simulate the stress-strain behavior for the clayey and sandy soils, respectively. Combined loading was applied in two stages: firstly, vertical loading was applied, and secondly, lateral loads were applied while maintaining a constant vertical load. The presence of vertical loading was observed to cause an increase and a decrease in corresponding lateral displacements under given applied lateral loads, which depended on the soil type and how the loads were applied. For sandy soils, an increase in lateral capacity of 40% (decrease in lateral displacement) was observed, whereas for clayey soils, a decrease in lateral capacity (increase in displacement) of 20% was observed. For sandy soil, the increasing vertical load leads to an increase in vertical confining stress at various depths around the pile, which enabled larger lateral and shear stresses to develop along the pile's frictional interface. For the clayey soil, the reduced capacity is likely attributable to the early failure of interface resistance due to the presence of the vertical loading. The bending moments increased for both piles in sandy and clayey soils. In sandy soil, an increase in maximum bending moment of 30–35% was observed, whereas for clayey soil, the maximum bending moment increased by 10–15% for a pile with $L/D < 15$, and 30% for longer piles. Figure 3 shows the influence of vertical loading on the lateral response of piles in sandy and clayey soils, respectively, and Figure 4 presents the variation in lateral displacement along the pile in sandy soil for a certain lateral load.



(**a**) (**b**)

Figure 3. Lateral load-displacement response of piles with increasing applied vertical loads, reported in Karthigeyan et al. [30]: (**a**) sandy soil, (**b**) clayey soil.

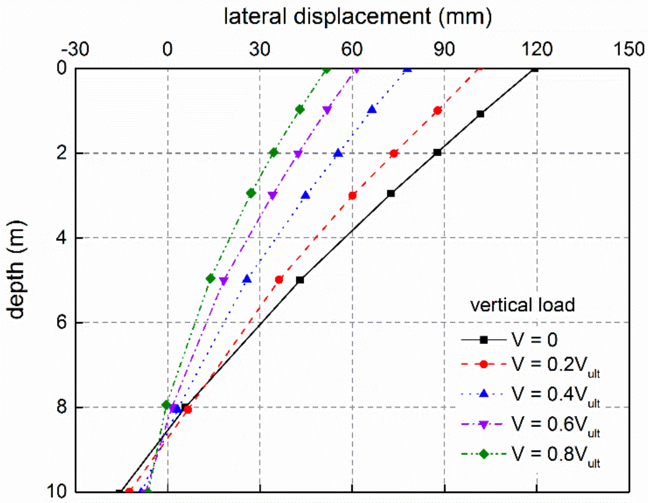


Figure 4. Variation in lateral displacement along a pile in sand for an applied lateral load of 641 kN [30].

Some numerical research disagrees with the findings of the analytical research. Karthigeyan et al. [31] studied the influence of vertical loading on laterally loaded pile behavior in sandy soils using a three-dimensional finite-element program GEOFEM3D [32]. The pile was treated as a linear-elastic material and the soil was idealized using a Drucker–Prager constitutive model with a non-associated flow rule. The pile had cross-sectional dimensions of 1.2 m × 1.2 m and was 10 m in length. Both loose and dense sand were considered. It was found that the lateral response of vertically loaded piles depends on many parameters including the sequence of load application, the shear strength of the soil, the boundary conditions of the pile, and the pile slenderness (L/D). Under a given applied lateral load, the pile lateral displacement decreased with increasing vertical load. When vertical and lateral loads are applied simultaneously, the effect of vertical loading is only apparent at greater displacement levels (there is less of an influence). When vertical loading is applied prior to lateral loading, the effect of the vertical load is considerable at all displacements, and reduces the corresponding displacements at each lateral load. The influence is furthermore more significant in dense sand than in loose sand, and is equally significant for piles with free head and fixed head conditions. As piles become longer, the effect of the vertical load becomes less. Achmus and Thieken [33] developed a numerical model to simulate and quantify the effects of combined loading on a pile installed in sand. The model simulated a closed-ended circular pile with diameter, $D = 1$ m, and length, $L = 20$ m. The pile and surrounding soil were discretized as 8-noded volume elements using the commercial software, ABAQUS, and an elasto-plastic material law with Mohr–Coulomb failure criterion and stress-dependent stiffness was adopted. Results of simulations conducted show that under a given applied vertical load, the lateral capacity was observed to increase (with corresponding reduction in lateral displacements), essentially in agreement with Karthigeyan et al. [31]. However, it was noted that the interaction is quite complex, and under certain loading conditions, the opposite effects can occur. Under combined vertical and lateral loading, it was initially observed that increasing vertical loads lead to an increased lateral stiffness. Once the vertical loads passed a threshold value, larger lateral displacements were observed. A parametric study showed that compact piles with low slenderness and rigid piles exhibited strong interaction effects under combined loading. It was concluded that the interaction effects on the lateral stiffness and capacity were small.

Taheri et al. [34] investigated, via numerical modeling, the influence of combined loading on a reinforced concrete pile installed in silty sand. Using ABAQUS, two piles with lengths 34.9 m and 7 m, respectively, were developed to investigate the effects incorporating pile slenderness. Both piles had the same diameter of 1.5 m. The modulus of subgrade reaction was estimated from back-analysis of the response. The soil behavior was modeled as a linear-elastic perfect-plastic Mohr–Coulomb model with non-associated flow rule, and the pile behavior varied using different stress–strain relationships for the concrete material [35]. The results of the analyses broadly agreed with Karthigeyan et al. [31], and showed that the lateral displacements reduced considerably due to the presence of vertical loading, for both the long and short piles. The magnitude of the reduction was observed to increase for higher values of lateral load.

The results of numerical modeling of the problem of combined loading on piles can be summarized herein:

- The research has revealed contradictory findings, which likely depend on soil type, modeling assumptions, constitutive laws, relative load magnitude, pile slenderness, boundary conditions, and other considerations such as load application sequence.
- There is no general consensus on whether vertical loading increases or decreases subsequent lateral displacements under applied lateral loads from the research in this section.

4. Full-Scale Pile Tests

Several authors reported field tests designed to shed further light on the governing mechanisms contributing to combined loaded pile behavior.

The majority of reported field tests suggest that the application of vertical loading decreases lateral displacements under subsequent applied lateral loads. This finding is directly in contrast to the majority of reported works on analytical models in Section 2. Evans [36] conducted a series of field tests on various types of piles subjected to lateral loading and combined vertical and lateral loading. The study considered steel H-piles, precast concrete piles, Raymond step-tapered piles, and tubular piles, installed vertically and as battered piles into loam soil (silty sand). The considered piles were installed between 10.2 m and 17.4 m. The results of a number of combined load tests are reproduced in Figure 5, which shows the lateral load-displacement response of two piles under both zero and 400 kN vertical loads.

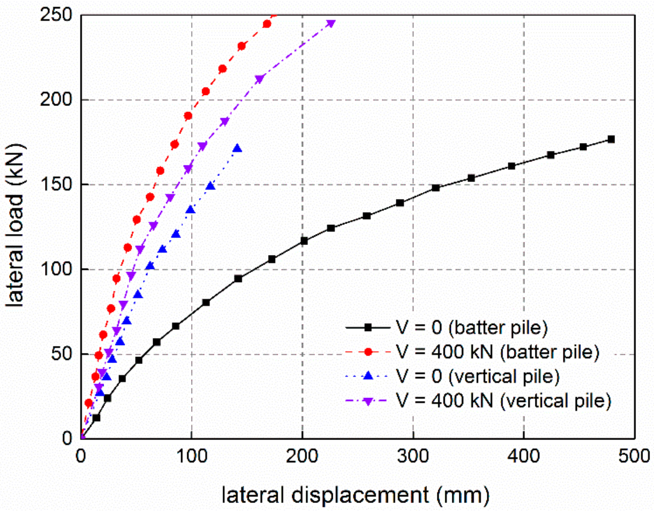


Figure 5. Lateral load vs. displacement of two piles under vertical loading (modified from Evans [36]).

The results in Figure 5 show that for a given pile, the action of applying a vertical load is to decrease the corresponding lateral displacements at equivalent applied lateral loads. Bartolomey [37] studied the behavior of single pre-stressed concrete piles under pure lateral and combined loading installed in clay. The square piles had a cross-section 300 mm × 300 mm, and varied between 5 m and 12 m in length. The results of the tests suggested that the lateral resistance of piles subjected to vertical loads increased by between 15–30%, as compared to the lateral resistance in the absence of vertical loads being applied. Increased lateral resistance leads to a reduction in corresponding lateral displacement, in agreement with the results of Evans [36]. In addition to the observed load-displacement behavior, piles that were subjected to pure lateral loading were observed to crack, as compared to those under combined loading. It is suggested that the presence of the vertical loading reduces the development of bending moments in the piles, so corresponding tension cracks are reduced.

Karasev et al. [35] performed field tests on single, cast-in-situ short concrete piles, with diameter 600 mm and length 3 m, installed in sandy soil. The behavior of the piles under combined loading was investigated, whereby vertical loads were first applied until a threshold settlement was achieved, and a lateral load was subsequently applied. The results indicated that the application of the vertical load leads to an increase in shear stresses along the pile wall, which increased the lateral resistance and subsequently leads to a decreased lateral displacement under equivalent lateral loads. The mechanism for the increased lateral resistance is likely due to changes in the mean stress level in the soil vicinity surrounding the pile. The results of a given load test from Karasev et al. [35] are reproduced in Figure 6, which shows that increasing the vertical loading decreased the subsequent lateral displacements. The points marked a–b in the plot are the results of removing the vertical load, which implies that the lateral displacements are restored somewhat after load removal, implying the increased lateral resistance from the vertical loading only applies while the pile is loaded.

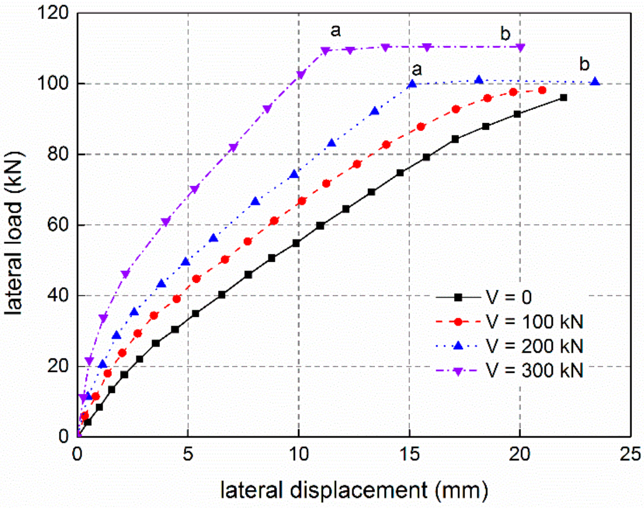


Figure 6. Lateral load vs displacement from combined loading tests (modified from Karasev et al. [35]).

Zhukov and Balov [38] performed combined loaded field tests on precast concrete piles, with square cross-sections 300 mm × 300 mm, installed in both homogeneous weak saturated clay and very stiff slump-prone soils. The embedment of tested piles were 2 m and 4 m, with loading eccentricities of 2.4 m to 2.5 m. The results of combined load tests suggested that lateral pile displacements increased under vertical loads in weak saturated soil, and decreased in stiff soils, suggesting soil stiffness has an inherent influence on the results. In the weak saturated soils, the lateral capacity was observed to decrease under vertical loading by approximately 15%, whereas in the stiff soil it increased by ≈1.9 times.

- The results of the documented field tests broadly agree that vertical loading leads to a decrease in subsequent lateral displacements, which directly disagrees with the predictions of the analytical models postulated in Section 2.
- The effect of plugging and partial plugging in the case of tubular piles on the observed behavior should be investigated to ascertain if this has an influence on the mechanism.

5. *1*-g Small-Scale Model Pile Tests

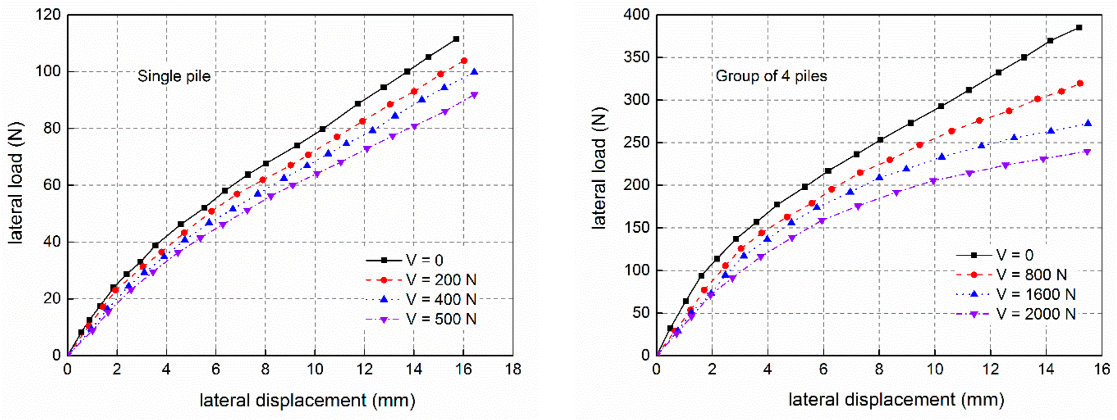
Due to the difficulty and costs in performing full-scale field trials, an alternative strategy adopted by various researchers is to perform scaled model testing. These offer a viable means to test a wider variety of conditions, with the limitation that the stress conditions are not similar to the full-scale situations.

Meyerhof and Sastry [39] performed combined loading tests on closed-ended steel pipe piles with a diameter 75 mm, and length of 990 mm, installed in loose sand. They suggest an expression for the load interaction, as shown in Equation (6).

$$\left(\frac{V_\theta \cos\theta}{V_{ult}}\right)^2 + \left(\frac{V_\theta \sin\theta}{H_{ult}}\right)^2 = 1 \tag{6}$$

where V_θ is the applied vertical load under an inclination angle θ to the vertical direction, and V_{ult} and H_{ult} are the ultimate vertical and lateral capacities, respectively. It is suggested that the ultimate vertical and lateral capacities in Equation (6) are dependent on the load inclination angle θ. Under the action of small vertical loads, a larger ultimate lateral capacity was observed than in the absence of vertical loading, similar to the findings of Achmus and Thieken [33]. It is suggested the application of vertical loading resulted in lower lateral displacements.

The opposite effect was observed in a study by Jain et al. [40], who performed combined load tests on fully and partially embedded long flexible single piles and pile groups. The model piles were fabricated from aluminum, with a diameter of 32 mm and length of 1000 mm. In their tests, the relative density of the sand sample equated to 78%. In order to apply the vertical loading without introducing any rotational fixity, a roller system was incorporated between the actuator and the pile. The results of their tests suggest that the application of vertical loading increases corresponding lateral displacements, which is in agreement with the analytical research described in Section 2. The results of their tests are reproduced in Figure 7, which shows the results for both the single pile and pile groups. The results of this analysis might provide insight related to the influence of boundary conditions on the results, as the use of the roller system to avoid additional rotational fixity may be partly responsible for the observed phenomena.



(a) (b)

Figure 7. Lateral load vs. displacement results from model pile tests (reproduced from Jain et al. [40]): (**a**) single pile, (**b**) pile group.

Similar to Jain et al. [40], Lee [41] and Lee et al. [42,43] also performed model pile tests to assess the influence of combined loading on piles installed in sand. In these studies, two kinds of instrumented circular piles were tested, both driven and non-displacement, in dry sand with varying densities. The model piles were stainless steel pipe piles, with a diameter of 30 mm, thickness 2 mm, and embedment 1200 mm. Increasing vertical loads were applied to investigate combined loading effects, varying among 0, 0.25, 0.5, and 0.75 times the ultimate vertical capacity. The tests were conducted in dense, medium dense, and loose sand samples. In each case, vertical loading was applied prior to application of lateral loads. The results indicate that vertical loading increases lateral displacements at corresponding lateral loads. When compared to the case of a pile under pure lateral loading, additional lateral displacements of approximately 8% of the pile diameter were observed under a vertical load of 0.75 times the ultimate capacity at the same lateral load level. Bending moments were also observed to increase substantially under combined loading. The results of these studies agree with the findings of the analytical research in Section 2. Furthermore, the effect of vertical loading was observed to be greater in the case of dense sand than for medium dense or loose sand, suggesting soil density potentially plays a role in the observed behavior.

Anagnostopoulos and Georgiadis [23] obtained inconclusive findings when they performed laboratory-scaled combined load tests on aluminum closed-ended piles (diameter = 19 mm, pile wall thickness = 1.5 mm, and embedment = 500 mm), installed in soft clay. Tests were performed on single piles under both vertical and lateral loads. Vertical loads were applied using dead weights. The pile-testing program involved applying a variety of vertical and lateral loads in different sequences. The results of the tests were not conclusive, as there appeared to be no dominant trend with limited effect observed. This is similar to the findings of Shahrour and Meimom [44], Trochanis et al. [45], and Abdel-Rahman and Achmus [46], who observed similar results through finite-element analyses. It is also similar to the findings of Sastry and Meyerhof [47], from model pile tests. Authors suggested that an alternative approach to study the potential interaction effects could be to use finite-element analyses and they suggest that approaches based on elastic half-space models or subgrade reaction methods would provide limited insight.

The main findings are summarized as follows:

- The majority of laboratory-scale studies agree with the analytical research that vertical loads exacerbate lateral displacements under equivalent lateral loads.
- This directly contradicts the majority of full-scale studies that suggest the opposite is true.
- There are several possible reasons for the disagreement, for example: the pile head boundary condition applied by loading actuators, the variation in in-situ stress conditions (between scaled and full-scale tests), and geometrical and material considerations.

6. *N*-g Centrifuge Model Pile Tests

Geotechnical centrifuge testing offers a unique opportunity to investigate full-scale behavior of geotechnical models at reduced scales, where the stresses experienced by the full-scale systems can be simulated on laboratory models [4,48–51]. Since the results of the *1*-g scaled tests discussed in the previous section tend to disagree with those of the full-scale tests discussed above, it is likely that stress differences play a role in observed behavior. A limited number of centrifuge tests have been performed to study the influence of vertical loading on lateral pile behavior.

Choo et al. [52] carried out centrifuge testing to investigate the lateral behavior of a 6 m diameter monopile with an embedment of 31 m (at prototype scale), used as a foundation for an offshore wind turbine. Different soil and rock characteristics were considered in the tests; two tests simulated a monopile installed in a dense sand and socketed into rock [53], while another two tests simulated monopiles installed in homogeneous sand with relative density of 85%. While combined loading was not explicitly considered, it is suggested that the self-weight of the monopile and extension rod acting as an additional vertical load may be one reason for the observed lateral displacements being larger than expected, when compared against those predicted by design codes [54].

Mu et al. [55] conducted a series of centrifuge tests to investigate the influence of vertical loading on the lateral response of monopiles installed in sand. The pile model had a prototype diameter of 6 m and length 50 m, and was closed-ended. Vertical and lateral loading was simultaneously applied, with vertical loads having a magnitude of between 10% and 90% the vertical pile capacity. It was found that the presence of vertical loading lead to a decrease in the lateral displacement of the monopile, the opposite result to that observed in Choo et al. [52]. Lu and Zhang [56] performed centrifuge testing on a free-headed monopile with prototype diameter 1 m and length 16.5 m, installed in saturated sand. A half-model arrangement was used whereby the pile displacements could be observed using a transparent window in the model. Vertical loads were added using dead weights applied to the pile top in the model. In terms of application sequence, vertical loads were first applied followed by lateral loads. It was observed that for a given applied lateral load, the lateral displacement decreased with increasing vertical load, in agreement with Mu et al. [55]. It was postulated that the presence of a vertical load densifies the soil near the pile, leading to a reduction in lateral displacements. On the other hand, the applied vertical loading leading to additional bending moments on the pile from the P-Delta effect may begin to play a role at higher values of displacement, as observed in Choo et al. [52]. It was concluded that the observed behavior may be a trade-off between both competing mechanisms.

7. Discussion and Recommendations

This paper reviewed literature that investigates the influence of vertical loading on the lateral response of piles, where authors undertook analytical research, numerical modeling, full-scale and laboratory-scale physical testing, and centrifuge modeling. The consensus on whether vertical loading increases or decreases lateral displacements under equivalent lateral loads remains inconclusive. Some of the reasons postulated for the results obtained are discussed and summarized in this section. Some recommendations for future research are provided, based on the issues identified.

The analytical research in Section 2 broadly agrees that vertical loading exacerbates lateral displacements due to the P-Delta effect, where the eccentricity caused by lateral displacements induces a further secondary moment, further increasing displacement [7,8,13–16,18–21]. This research also postulates that the boundary conditions of the pile may influence the nature of the results.

The numerical research in Section 3 presents contradictory findings, whereby some authors suggest that vertical loading increases lateral displacements under applied lateral loads [27,28], some research presents conflicting findings [29,30], and some research concludes that vertical loading reduces corresponding lateral displacements [31,33,34]. It is suggested by Klein and Karavaev [29] that soil density may be one reason for contradictory findings; in dense soil vertical loading may lead to a reduction in corresponding lateral displacements, whereas in weak soil, increased displacements might be observed. Karthigeyan et al. [30] also suggest soil type might have an influence, as well as the sequence of how loads are applied. They report that in sandy soils, a decrease in corresponding lateral displacements under vertical loading might be observed, whereas in clayey soils an increase might be observed. For the sandy soil, the increase in confining stress from the applied vertical load is potentially responsible for the development of shear stresses around the pile, which subsequently influence the lateral displacements (mean stress effects). In clay, early failure of interface resistance is possibly responsible for the conflicting behavior. Karthigeyan et al. [31] further suggest that boundary conditions and pile slenderness may play a part in the observed behavior.

The full-scale field tests mostly suggest that vertical loading leads to a reduction in corresponding lateral displacements [35–37], with one study [38] suggesting both results can occur. The mechanism responsible for the observed results is suggested to be that vertical loads lead to increased shear stresses along a pile wall, which lead to increased lateral resistance. The results of Zhukov and Balov [38] showed that pile lateral displacements increased in weak soils under applied vertical loading, and decreased in stiff soils, which is somewhat in agreement with the numerical research by Karthigeyan et al. [30]. Scaled *1*-g testing tends to disagree with the results of the full-scale field tests, except for Meyerhof and Sastry [39], who showed that vertical loads lead to a reduction in corresponding lateral displacements. This general disagreement is likely attributable to the difference in stress conditions. Most of the scaled *1*-g testing studies agree with the findings of the analytical research, that vertical loading increases lateral displacements [40–43]. The boundary conditions are suggested as one reason for the results obtained, similar to Karthigeyan et al. [31], and soil density is proposed as another factor. The scaled testing suffers the common drawback that the stress conditions are not equivalent to the full-scale situation, which may influence the results.

In centrifuge studies, some contradictory results were also obtained, with Choo et al. [52] suggesting vertical loading exacerbates lateral deflections; while Mu et al. [55] and Lu and Zhang [56] suggest lateral displacements decrease under vertical loading. The study in Choo et al. [52] could only provide a qualitative assessment on the influence of vertical loading, as this was not quantified directly in the study, and is therefore potentially less reliable than the remaining studies. One reason for the observed behavior proposed in Lu and Zhang [56] is that vertical load densifies the soil around the pile, leading to reduced lateral displacements, essentially in agreement with other studies who proposed similar mechanisms [30]. The results of the centrifuge studies are mostly in agreement with those of the full-scale field tests, which suggests that the matching stress conditions is an important element in the research, and potentially contributes to the governing mechanism.

In terms of the boundary conditions across all testing approaches, no definitive trend is observed about whether this governs the response or not (it likely plays a role, but other factors might overshadow this). For the analytical research, all models resulted in vertical loads increasing corresponding lateral displacements, even though pile boundary conditions varied from free-free to fixed-free. In the numerical research, piles with a free-free boundary condition resulted in cases where lateral displacements increased under vertical loading and decreased in other cases. Similar conflicting results occurred for the scaled *1*-g testing. It is therefore not directly possible to suggest how the pile boundary effects influenced the results, based on the reviewed works.

For the pile slenderness, L/D, flexible systems with $L/D > 10$ once again show no consistent trend in terms of the effect of vertical load increasing or decreasing lateral displacements (results depended more on the nature of the tests conducted). For rigid piles with $L/D < 10$ (applicable to offshore wind foundations), some studies report that vertical loading decreases corresponding lateral displacements but others report that both decreases and increases can occur. It is therefore difficult to conclude the nature of the influence pile slenderness plays in this respect, which has tangible ramifications for the offshore wind sector in particular.

From the reviewed works, it would generally appear that there is little agreement on the governing mechanisms contributing to the problem, with a variety of mechanisms likely occurring. In order to investigate the underlying mechanisms contributing to the observed behavior, centrifuge modeling is the most acceptable recourse, as it enables a large amount of testing with the ability to vary both soil and pile parameters, which limits the bias observed in full-scale field testing. Moreover, centrifuge testing enables tests to be performed at representative real-life stress conditions. It is recommended to undertake a comprehensive centrifuge program with piles of varying materials, lengths, and slenderness, installed in different soil types with varying stiffness profiles and properties, under a variety of boundary conditions. The installation method should also be varied, as this governs the in-situ stress condition, which is suggested to play a role in observed behavior of combined loaded piles. Installing a pile in-flight leads to a more realistic stress state than installing at *1*-g, for centrifuge tests. The load application sequence should be varied, as there is some evidence that the order of application of vertical and lateral loads may contribute to the observed response. Moreover, the relative magnitude of applied vertical and lateral loading ratio (V/H) should be varied, as it is likely that the governing mechanism for whether lateral displacements are increased or decreased will depend on which resistance mechanism is mobilized (increased shear resistances due to vertical loading may result in lower lateral displacements, whereas the P-Delta effect might overcome this at a certain lateral load magnitude resulting in increased displacements)—this requires careful attention. In terms of the boundary conditions, this is an issue for centrifuge modeling in general as the actuators used to apply loads may also inflict certain fixity to the test piles [4]. It is therefore recommended to develop mechanisms to enable application of lateral and vertical loads without causing additional rotational fixity, using ball-type connections and dead weights, as developed in Li et al. [4]. Furthermore, as the issues related to combined loading are becoming relevant for emerging wind turbines, the behavior of these systems with low L/D subjected to cyclic loading is also of interest, and has not received much attention in the literature. It would be of interest to assess if cyclic loading influences the nature of the responses obtained in combined loading systems, and is therefore recommended as future work.

The observations from the various reviewed works are summarized in Table 1.

Table 1. Summary of research findings.

Method	Reference	Effect of Vertical Load on Lateral Response	Soil Info.	Pile Info.	*L/D* (Rigid/Flexible)	Installation Method	Boundary Conditions	Load Application Sequence	Load Ratio (V/H)
Analytical	[7]	Increased	Sand/Clay	Concrete	-	Driven	Free-Free	-	-
	[8]	Increased	-	-	-	-	-	-	-
	[13]	Increased	Sand	Steel	48	Driven	Fixed-Free	-	Up to 30
	[14]	Increased	Sand	Tapering circular	-	Driven	Fixed-FixedFixed-Free Free-Free	-	-
	[15]	Increased	Sand/Clay	-	(Flexible)	-	Free-Free	-	-
	[16]	Increased	-	-	-	-	-	-	-
	[18]	Increased	-	Concrete	(Flexible)	-	Free-Free	H prior to V	-
	[19]	Increased	Dense/Loose	Various	20–80	-	Fixed-Free Free-Free	-	-
	[20]	Increased	Single/Multilayered	Concrete	24	Wished-in-place	Free-Free	Simultaneously	-
	[21]	Increased	Medium dense sand	Concrete	22	-	Fixed-Free Free-Free	H prior to V	Up to 40
Numerical	[27]	Increased	Clay	-	(Flexible)	Driven	-	-	-
	[28]	Increased	Loose sand	Concrete	20	-	Free-Free Fixed-Free	-	-
	[29]	Increased/Decreased	Dense/Weak soil	Concrete	≈7	Non-displacement	Free-Free	Simultaneously	-
	[30]	Increased/Decreased	Dense sand/Clay	Concrete	≈8	Non-displacement	Free-Free	V prior to H	Up to 3
	[31]	Decreased	Dense/Loose sand	Concrete	≈8	Non-displacement	Free-Free	V prior to H/Simultaneously	Up to 3
	[33]	Inconclusive	Medium dense sand	Concrete	20	-	Free-Free	Simultaneously	0 to ∞
	[34]	Decreased	Silty sand	Concrete	≈23 ≈5	Bored	Free-Free	V prior to H	Up to 8.8
Full-scale Field Tests	[36]	Decreased	Silty sand	Steel and concrete		Driven	-	-	-
	[37]	Decreased	Clay	Concrete	≈17–40	Non-displacement	-	-	-
	[35]	Decreased	Sand	Concrete	5	Non-displacement	Fixed-Free	V prior to H	Up to 3.75
	[38]	Increased/ Decreased	Weak/Stiff clay	Concrete	≈7–≈13	Driven	-	V prior to H	Up to 14
1-g Small-scale Model Pile Tests	[39]	Decreased	Loose sand/Soft clay	Steel	≈13	-	Fixed-Free	Simultaneously	Up to 2
	[40]	Increased	Dense sand	Aluminum	≈31	-	Fixed-Free Free-Free	-	-
	[41–43]	Increased	Sand	Steel	40	Driven/non-displacement	Fixed-Free	V prior to H	Up to 2.5
	[23]	Inconclusive	Soft clay	Aluminum	≈26	Jacked	-	V prior to H	Up to 1.23

Table 1. *Cont.*

Method	Reference	Effect of Vertical Load on Lateral Response	Soil Info.	Pile Info.	L/D (Rigid/Flexible)	Installation Method	Boundary Conditions	Load Application Sequence	Load Ratio (V/H)
N-g Centrifuge Model Pile Tests	[52]	-	Dense sand	Steel	≈5	Non-displacement	Free-FreeFree-Fixed	V prior to H	-
	[55]	Decreased	Dense sand	Aluminum	≈8	Non-displacement	-	V prior to H	Up to 4
	[56]	Decreased	Dense sand	Aluminum	16.5	Non-displacement	Free-Free	V prior to H	Up to 2.5

8. Conclusions

The influence of vertical loading on the lateral response characteristics of laterally loaded piles has been reviewed and discussed in this paper. The purpose of the review is to attempt to understand the reasons for the contradictory findings reported in published literature on whether vertical loading increases or decreases corresponding lateral displacements on laterally loaded piles. The research is timely due to the advent of emerging large capacity and heavy offshore wind generators, which subject pile foundations to combined lateral and vertical loading. The paper analyses research from a number of perspectives; analytical models, numerical research, full-scale and scaled pile tests, and centrifuge model tests. Specific focus is placed on soil type and density, pile L/D, load application sequence, pile boundary conditions, pile material, and applied load relative magnitude (V/H) in order to attempt to derive an understanding of the contributing issues. There is little agreement on what mechanisms govern the response characteristics from the reviewed literature, so some common discussion points were extracted with a view to identifying a plan of action for future research, and this is discussed in Section 7 of the paper. It is recommended that in order to develop a more rounded understanding of the relative importance of the various operating parameters, centrifuge tests should be conducted, which offer the most viable routeway to identify the interplay between mechanisms.

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