An Optimal Compensation Agency Model for Sustainability under the Risk Aversion Utility Perspective

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Abstract: This paper explores how to construct a fair and optimal compensation system between the principal and the agent in the face of financial compensation agency problems during a limited period in relation to the concept of sustainability. In the construction of the principal’s compensation system, the agent’s degree of operational financial effort will affect the overall revenue function for reaching sustainability. Both the principal and the agent have a maximum expected utility in the negative exponential pattern of the general hyperbolic absolute risk aversion (HARA) utility function that satisfies their respective objective functions. The proposed model and numerical analysis results prove that the compensation system for sustainability can provide a fair and optimal financial system, from a sustainability perspective. The main contribution of this study is the construction and development of an optimal compensation agency model for risk management, which is derived by considering the effect of risk aversion utility on revenue. The proposed model can provide a fair and feasible approach within the issue of compensation, from the viewpoint of sustainability, for an optimal compensation agency problem.

Keywords: financial; risk; management; sustainability; optimal compensation system; agency problem; HARA utility; risk management

1. Introduction

The purpose of this paper is to construct and develop an optimal agency problem, from a risk management perspective. The basic application model is derived by considering the influences of risk aversion, in terms of the social, economic, and environmental aspects for sustainability considerations. The mathematical model provides fair, reasonable, and feasible performance compensations for sustainability, to help the principal and agent make decisions concerning an optimal compensation agency problem. In the proposed analysis method, the agents organize the theoretical framework of economic responsibility and financial decisions in a rational, logical, and clear manner, based on risk aversion for the social, economic, and environmental responsibilities of sustainability. A fair and optimal compensation system is constructed for the compensation agency problem. In terms of the construction of the principal’s sustainability compensation system, the agent’s operational efforts influence the overall revenue of sustainable social responsibility, and its feasibility and applicability are proved using a sensitivity analysis.

It has become increasingly important to incorporate sustainability into company management, which is part of the company’s efforts for social recognition (Meyer and Rowan 1977). Sustainability is not just a concept; it is achieved through use and daily practice (Corvellec 2016; Finch et al. 2015). Common evolutionary change takes time to generate sustainable structural changes, and there is a need for technical, economic, and organizational forms of change (Köhler et al. 2019; Loorbach et al. 2010). Linnenluecke and Griffiths (2010) point out that the concept of sustainability is incorporated into the agency problem. The value of the agent is consistent with whether the organization can achieve its
goals; in other words, the sustainability of the company is very important (Dunfee 1991). Over the past several decades, sustainability has been a familiar concept (Lyon et al. 2018). Elements of the principal–agent model, combined with agency theory, can identify the cumulative value of risk, with concepts such as risk aversion and risk appetite, which is the pursuit of profit to avoid losses (Kahneman and Tversky 1979). A number of researchers have reported that the agent is less willing to take risks (Denya et al. 2005; Wiseman and Gomez-Mejia 1998; Martin et al. 2013). Gomez-Mejia et al. (2019) point out that when the risk appetites of the agents and the principals are different, the principals will conduct close monitoring and limit the incentive measures, which will have an impact on the agent’s efforts. The level of the principal’s risk-taking can explain why the principal has a stronger motivation and ability to supervise the agent (Anderson and Reeb 2003; Gomez-Mejia et al. 2003; Martin et al. 2016; Shleifer and Vishny 1986).

Wang et al. (2016) emphasized two main agency issues and held that, first of all, the separation of ownership and management rights is closely related, because the owners do not manage the business; therefore, the second problem arises, namely, the different interests between the two sides. According to Bassett et al. (2007), enterprise managers tend to solve information asymmetry in two ways: by monitoring and by increasing information disclosure. As companies are concerned about how to improve their sustainability, this has become a hot topic in both government and business (Lozano et al. 2015). In an earlier study, Welford (1995) suggested that companies must make transformations in their organizational culture when faced with social and environmental challenges, and the core concept of such a transformation is an important process if a company wants to move towards and develop an organizational culture of sustainability (Linnenluecke and Griffiths 2010). The question of how to construct a feasible strategy of sustainability is an important consideration for companies when making their decisions. In particular, the financial value of the economic aspect is an important pillar for companies to maintain sustainability (Lin et al. 2019).

2. Literature Review

Elkington (1997) developed the triple bottom line (TBL) model and pointed out that enterprises, companies, and countries should adhere to environmental, social, and economic responsibilities while pursuing profit maximization, in order to achieve sustainability. When profits are not completely dependent on the operations of companies, they can develop jointly with society and the environment to achieve a common prosperity; however, the opposite occurs if their responsibilities are neglected. Teece (2010) emphasized the value of creating a functional business model that is focused on organizational value creation and argued that sustainability can be extended to the measurement of the value of the social and ecological environment. The influence of existing business models on global economic and social sustainability has special significance in the context of sustainable development, and can generate new forms of governance (Schaltegger et al. 2016).

In addition to global economic and financial crises, the COVID-19 pandemic in 2020 has led to the enhanced sustainability value of companies under the maximum profit mode of business model behavior, in cooperation with principal–agent problems. When an agent sets a management strategy for the principal, the result usually affects the interests of the agent himself; thus, an important issue is how the ‘agent problem’ generates the appropriate contracts (Hart 1995). In addition, agency contracts affect the principal’s interests and cause a conflict of interests; in this case, an ‘agency dilemma’ such as a ‘moral hazard’ may occur, due to a conflict of interest between the principal and the agent (Hart and Hölmstrom 2010). In agency theory, interests are usually reflected in the recognition of monetary value. Assuming that the principal tends to be risk-neutral or risk-averse regarding the interests of the operations, whereas the agents tend to be risk-averse toward the cost of compensation and the degree of self-effort, compensation contracts must have both an insurance effect and an incentive effect (Jensen and Meckling 1976).
Arrow (1984) pointed out that the agency relationship always exists in economics and that the principal–agent relationship is an important part of almost all transactions. Assuming that both the principal and agent clearly understand each other’s strategies, they should consider how to make the best decisions under their own constraints for reaching optimal contracts, in order for contracted transactions to be realized. Spremann (1987) held that the agency theory focuses on cooperation with external influences and with asymmetric information. Typical cases show how the principal assists the agent in deciding the degree and type of his/her operational efforts, which means that the principal pays some rewards to the agent in exchange for certain decisions/actions/operational efforts of the agent, which has a motivating effect in practice. Hölmstrom (1979) stated that when the agent’s operational effort and output capability are observable, the principal can reduce the agent’s laziness through supervision. In this situation, the agent’s compensation is determined by a fixed salary that is not related to the output. In this theory, the research results define the operational efforts of insurance agents by distinguishing wealth from utility, finding that incentive efforts should be given to the agent; that is, when the principal earns profits, offering dividends to the agent is a feasible incentive scheme. In addition, Solow (1979) provided a series of analytical opinions on the important factors that affect the optimization of the agent’s operational effort, as well as the relationship between the agent’s operational efforts and the wage rate. At the same time, Solow put forward some opinions regarding the optimal design of the company scale, from the starting point of an optimal salary design that maximizes the principal’s profits.

Most recent studies of the agent’s compensation patterns are linked by a wealth value, such as monetary payment and equity linkage. Marini et al. (2018) proposed that if an agent manages the company with the mentality of low agent risk aversion, then the principals have to adopt a higher risk aversion mentality and a more efficient new technology requirement, in order to reduce the agent’s moral hazard. Schmidli (2017) held that agents act rationally and measure their wealth using a utility function, but no opinions were put forward about the different perceptions of agents and clients regarding the expected utility and risk aversion. Wu et al. (2016) argued that the economy, environment, and society cannot cover the concept of sustainable development, and proposed four aspects, namely, operation, flexibility, long term, and benefits, in order to explain the concept of sustainable development more completely. Global enterprises are increasingly linking agent compensation to sustainable development-related goals; for example, Berrone and Gomez-Mejia (2009) and Hong et al. (2016) found that sustainability-based agent compensation is positively related to corporate social performance, which indicates that a compensation contract is an effective incentive for agents to improve.

Dunfee (1991) emphasized that an employment contract is a social contract between an agent and a principal, and when both parties accept a contract, they expect certain norms and behaviors from each other; the consistency of the agent’s values is therefore important for the realization of the client’s goals and for the sustainable operation of the company. Devers et al. (2007) were of the view that a sustainable operating compensation policy is an incentive tool. Olson et al. (2018) suggested that a sustainable management compensation policy is to reward past behavior and influence future behavior, and this includes offering a fixed salary, a variable salary, and incentive measures. A study by Baraibar-Diez et al. (2019) showed that a sustainable operation compensation policy will affect the sustainable operation and will produce better non-financial performance. Kiron et al. (2017) found that 90% of principals think that sustainability is very important, whereas Quartey and Kotey (2019) pointed out the relationship between sustainable revenue and operational efficiency. Incentives for the agent increase the principal’s sustainable income (Sao Joao et al. 2019). Adjusting and optimizing the layout of the enterprise not only reduces the cost of a sustainable operation, but are also indispensable for the sustainable operation of an enterprise (Xue et al. 2019). Sustainability has different labels, depending on the research field or perspective (Lopez-Cabrales and Valle-Cabrera 2020). This study defines the proprietary terms of the agency model for sustainable operations in the available literature as follows:
(a) sustainability revenue—revenue after deducting the salary paid to the agent under the premise of sustainable operation; (b) sustainability payment contract—the agent signs a minimum contract with the principal on the premise of a sustainable operation; (c) sustainability salary compensation—the salary that the client must pay after deducting the agent’s effort costs from the revenue of the client, under a sustainable operation; and (d) sustainable operation costs—the cost that the principal must pay for the agent’s best operational efforts in a sustainable operation.

This study combines the expected utility with the generalized risk aversion hyperbolic absolute risk aversion (HARA) utility function as the basis for the development of a principal–agent model of the compensation problem. Franke et al. (2018) explained how the decision-making of the HARA utility function is affected by sustainability, finance, and the degree of risk aversion in the environmental aspect of uncertainty. This paper aims to determine the sustainability effort-compensation strategy for generalizing the HARA utility function (represented by a negative exponential function), i.e., within a limited range, the maximum sustainability effort and sustainability optimal compensation agency system can be measured according to the expected utility, as defined by the principal and the agent, respectively. The results of this study could fill the current gap in agency theory, whereby compensation does not show the difference in the expected utility of the principal and the agent, or the risk perceptions of the principal and the agent, and further provides companies with a better measurement of the value of social responsibility, when considering sustainability.

3. Research Method

Under the cognitive framework of sustainability between the principal and agent, this paper explores how to construct a fair and optimal compensation system of sustainability in the agency problem within a limited period. In the construction of a principal’s sustainability compensation system, this paper considers how to properly link the impact of the degree of the agent’s operational effort with the overall sustainable operation revenue function for meeting sustainability, and to link the impact of the self-paid effort cost function with the agent’s degree of operational effort. The major decision variables are the optimal agent’s operational effort coefficient \( \alpha^* \), the optimal sensitivity parameter \( \eta^* \) of the payment contract with the principal (fixed salary in the payment contract), and the optimal performance-to-revenue ratio \( \delta^* \) (variable compensation rate in the contract), on the condition that both the principals and the agents have the maximum expected utility in the negative exponential pattern of the HARA utility function, namely, that it satisfies their respective objective functions. The research results can serve as the basis for companies to construct a fair and optimal system for the consideration of sustainability. The main contribution of this paper is that, by constructing and exploring an optimal principal–agent problem from the perspective of risk management, it adopts the basic application model that considers the effect of the risk aversion utility. This can provide a reference for solving the principal–agent problem and performance issues in order to achieve sustainability, which means that principals can make a fair and feasible evaluation of the agent’s consideration of sustainability values and social responsibilities.

Based on the operational constructs of sustainability, this paper assumes that there is a principal–agent problem for professional managers in a company, and that the principal expects to establish a fair compensation system for sustainability, which includes performance evaluation and paying compensation for the agent. There is a certain functional relationship between the degree of the agent’s operational efforts and his/her self-paid costs, which can also affect the sustainability revenue and performance of the company. In their perception of monetary value, both principals and the agents have their own risk aversion utility relationship regarding the social, economic, and environmental aspects of sustainability. At the same time, under the sustainability compensation system of both parties is the sustainability to achieve the expected utility that corresponds to the principal’s net revenue and the expected utility of the agent’s optimal sustainability compensation
level. Both sides will pursue the salary system and the best degree of operational effort with the maximum expected utility to solve the problem. The optimal agent’s operational effort coefficient $\alpha^*$, the optimal sensitivity parameters $\eta^*$ of the principal’s sustainable operation payment contract, and the optimal performance-to-revenue ratio $\delta^*$ are considered to be the main decision variables. The main purpose of this paper is to construct a fair and reasonable compensation system that covers the social, economic, and environmental aspects of sustainability.

3.1. The Cognitive Mechanism of an Agent’s Compensation with Sustainability

After the agent considers the social, economic, and environmental aspects of sustainability, he/she will adopt the negative exponential utility function $U_A(C) = -\frac{e^{\gamma_A C}}{\gamma_A}$ with the risk coefficient $\gamma_A > 0$ (when $U_A(C) = C$; the model construction is restored to the original monetary values) in the HARA utility function, where $C$ is the specific indicator for this risk aversion utility function, which satisfies the social, economic, and environmental aspects of sustainability. If the principal’s sustainability compensation mechanism is to sign a sustainability reward system with the agent at the beginning of the period (time point 0), then the distribution of the sustainability business revenue $X$ after the first period (time point 1) is to satisfy the normal distribution change of the average revenue $a$ and the variation of the revenue $\sigma^2$; it meets $X \sim N(\alpha, \sigma^2)$, where $a \geq 0$, which denotes the coefficient of the agent’s degree of operational effort. The higher the $a$, the more effort the agent makes in terms of sustainability and his social responsibilities, and the better the performance of the business revenue indicator of sustainability. Assuming that the functional relationship $X = X_0(1 + a)$ is established, the agent’s degree of operational effort coefficient $a$ and insurance business revenue variance $\sigma^2$ are independent, and there is no mutual effect on the aspect of sustainability. The coefficient of the agent’s degree of operational effort shows that there is a functional correspondence between the average degree of sustainability operational effort and its sustainability social responsibility costs. Here, it is assumed that the cost function of the agent’s operational effort coefficient is $g(\alpha) = \beta_A \times \alpha^2$, where $\beta_A > 0$, which denotes the scale coefficient of cost payment for the degree of the operational effort coefficient, $\epsilon$ denotes the power coefficient reflected in the cost function for sustainability and social responsibility in the agent’s degree of operational effort, and $\epsilon \neq 0$, which can be a positive or negative fixed coefficient. This study takes a simplified hypothesis represented by $g(\alpha) = \beta_A \times \alpha^2$; that is, with $\epsilon = 2$ as an example.

The agent’s objective function is to find the optimal agent’s operational effort coefficient $\alpha = a^*$ under the maximum expected utility function, as generated by deducting the social responsibility and sustainability costs paid by the operational effort representative coefficient $g(\alpha)$ from the monetary reward paid by the principal through the compensation system $\mathbb{R}(\alpha)$. Therefore, $\alpha = a^*$ is:

$$\max_{\alpha} E[U_A(\mathbb{R}(X(\alpha)) - g(\alpha))] \quad (1)$$

which meets:

$$\mathbb{R}(X(\alpha)) - g(\alpha) = (\delta X(\alpha) + \eta) - \beta_A \alpha^2 \quad (2)$$

where the sustainability compensation system constructed by the principal is:

$$\mathbb{R}(X(\alpha)) = \delta X(\alpha) + \eta \quad (3)$$

where the sensitivity parameter of the principal’s sustainability payment contract $\eta \geq 0$ is the fixed salary coefficient, and the performance ratio of the revenue $\delta$ ($0 \leq \delta \leq 1$) is the ratio coefficient of the variable salary, as generated by the principal’s revenue $X(\alpha)$ from the sustainability under social responsibility.

For the purpose of avoiding a moral hazard, when a principal and an agent sign a principal–agent sustainability contract, they normally include the minimum constraint conditions for maintaining the contract. The maximum expected utility function value of
sustainability under the agent’s object function $\max_a E[U_A(\mathbb{R}(X(a)) - g(a))]$ must be no less than the agent’s minimum utility function value $U_A(R)$ of sustainability. The following is the negative constraint condition:

$$\max_a E[U_A(\mathbb{R}(X(a)) - g(a))] \geq U_A(R) \quad (4)$$

If the agent fails to meet the negative constraint condition, the sustainability contract may be invalidated or dismissed, without compensation.

3.2. The Principal’s Compensation System with Sustainability

Under the social, financial, and environmental aspects of sustainability, the principals must construct a reasonable sustainability compensation for the agents. In the above Equation (3), $\eta^*$ is the optimal sensitivity parameter of the sustainability payment contract with the principal (the fixed salary in the contract), and $\delta^*$ is the optimal performance-to-revenue ratio of the variable salary, as generated from the revenue of sustainable operations $X(a)$ (variable salary ratio in the contract, $0 \leq \delta^* \leq 1$). Similarly, under the social, financial, and environmental aspects of sustainability, the principal adopts the negative exponential utility function $U_P(C) = -e^{-\gamma P C}$ with a risk coefficient $> 0$ in the HARA utility function, where $C$ is also a specific indicator to satisfy this risk aversion utility function under the social, financial, and environmental aspects of sustainability, as described in the previous section.

The principal’s objective function is the maximum expected utility function of the net operating revenue $X(a^*) - \mathbb{R}(X(a^*))$, which is obtained by deducting the salary compensation $\mathbb{R}(X(a^*)) = \delta X(a^*) + \eta$ paid to the agent under the optimal agent’s operational effort coefficient $\alpha^*$ from the operating revenue $X(a^*)$ of sustainability under the optimal agent’s operational effort coefficient $\alpha^*$, that is:

$$\max_{\delta, \eta} E[U_P(X(a^*) - \mathbb{R}(\delta, \eta; a^*))] \quad (5)$$

which meets the optimal constraint formula of the sustainability salary compensation in Equation (3):

$$\mathbb{R}(X(a^*)) = \delta^* X(a^*) + \eta^* \quad (6)$$

where $\alpha^*$, $\eta^*$, and $\delta^*$ are simultaneously true optimal decision variables to be determined.

3.3. Optimal Compensation Agency Problems with Sustainability

If the sustainable operation compensation system in the sustainability payment contract by the principal $\mathbb{R}(X(a^*)) = \delta X(a^*) + \eta$ is inconsistent with the optimal sustainability compensation $\mathbb{R}(X(a^*))$ paid through the linkage relationship generated by the principal’s income $X(a^*)$, then, due to the occurrence of a moral hazard, the agent may not make the best operational efforts to implement the project plan when his/her degree of operational effort coefficient is $\alpha \neq \alpha^*$, in order to reduce the costs that must be paid to improve the degree of operational effort, i.e., $g(\alpha) < g(\alpha^*)$. In practice, the principal can regulate the contract with the agent, which means that the sustainability compensation system $\mathbb{R}(X(a^*)) = \delta X(a^*) + \eta$ shall be composed of the optimal fixed salary $\eta^*$ and a performance bonus $\delta^* X(a^*)$. The perceptions of the principal and the agent regarding the sustainability compensation system are consistent with Equation (6), as described in the previous section, i.e., $\mathbb{R}(X(a^*)) = \delta^* X(a^*) + \eta^*$, where the principal pays the optimal sensitivity parameter of sustainability contract $\eta^*$ (fixed salary by contract) and the optimal sustainability performance to income ratio $\delta^*$ (variable compensation ratio by contract). Here, parameters $\delta^*$ and $\eta^*$ are called sustainability contract sensitivity parameters.
3.3.1. Solution of Agent’s Optimization

If the agent’s sustainability contract compensation meets Equation (6), then he will adopt the optimal agent’s degree of the operational effort coefficient $\alpha^*$ of expected utility $E[U_P(X(a) - \mathbb{E}(X(a)))].$ In other words, the sustainability objective function is:

$$\max_{\alpha} E[U_A(\mathbb{E}(X(a)) - g(a))] = \max_{\alpha} \left[-\frac{1}{\gamma_A} e^{-\gamma_A(\delta X(a) + \eta - \beta_A \alpha^2)}\right]$$

(7)

with the moment generating function expressed as:

$$E[\omega X(a)] = \omega e^{\alpha} + \frac{1}{2} \sigma^2 \omega^2, \ X(a) \sim N(\alpha, \sigma^2)$$

(8)

where $\omega$ is a constant. We introduce Equation (8) into Equation (7) to obtain:

$$\max_{\alpha} E\left[-\frac{1}{\gamma_A} e^{-\gamma_A(\delta X(a) + \eta - \beta_A \alpha^2)}\right] = \max_{\alpha} \left\{-\frac{1}{\gamma_A} e^{-\gamma_A(\delta \alpha + \gamma A \beta_A \alpha^2)}\right\}$$

(9)

Under the solution of the optimal agent’s degree of operational effort coefficient, the maximum computation value of Equation (9) is equal to the minimum computation value of Equation (10):

$$\min_{\alpha} F(\alpha) \equiv \min_{\alpha} \left\{-\gamma_A \delta \alpha + \gamma A \beta_A \alpha^2\right\}$$

(10)

In Equation (10), the first-order derivative function is $F'(\alpha) = -\gamma_A \delta + 2\gamma A \beta_A \alpha = 0.$ The optimal agent’s degree of operational effort coefficient $\alpha^*$ for sustainability can be obtained as:

$$\alpha^* = \frac{\delta}{2 \beta_A}$$

(11)

where $\delta = \delta^*$ is the to-be-determined parameter, and $\delta^* > 0$ is the optimal sustainability performance to income ratio $\delta^*$ (variable compensation ratio by contract).

In Equation (10), the second-order derivative function is $F''(\alpha) = 2\gamma A \beta_A > 0.$ Therefore, Equation (11) is the optimal $\alpha^*$ value corresponding to the minimum value of Equation (7). At this time, the maximum expected utility of the agency $\max_{\alpha^*} E[U_A(\mathbb{E}(\alpha^*) - g(\alpha^*))]$ can be calculated as:

$$E[U_A(\mathbb{E}(\alpha^*) - g(\alpha^*))|\eta^*, \delta^*] = -\frac{1}{\gamma_A} e^{-\gamma A \eta^* + \frac{1}{2} \sigma^2 \delta^*^2 \gamma_A^2}$$

(12)

3.3.2. Solution of the Principal’s Optimization

Given the sustainable operation costs under the practice of sustainability, when an agent executes a project plan according to the aforementioned optimal agent’s operational effort coefficient $\alpha^*$, the sustainability revenue received by the corresponding principal is the objective function that pays the agent’s sustainability contract compensation $\mathbb{E}(\alpha^*)$ under the maximization of the expected utility $E[U_P(X(a) - \mathbb{E}(X(a)))],$ as derived from the principal’s net revenue $X(\alpha^*) - \mathbb{E}(\alpha^*),$ i.e.,:

$$\max_{\delta, \eta} \left\{E[U_P(X(\alpha^*) - \mathbb{E}(\delta, \eta; \alpha^*))]\right\} = \max_{\delta, \eta} \left\{-\frac{1}{\gamma_P} E\left[e^{-\gamma_P(X(\alpha^*) - \delta X(\alpha^*) - \eta)}\right]\right\}$$

$$= \max_{\delta, \eta} \left\{-\frac{1}{\gamma_P} e^{-\gamma_P(1-\delta)\alpha^* + \frac{1}{2} \sigma^2 \gamma_P(1-\delta)^2}\right\}$$

(13)

where $\alpha^* = \frac{\delta}{2 \beta_A}$ (see Equation (11)). In addition, according to the natural characteristics of Equation (6) and the optimal sensitivity parameters $\eta^*$ of the sustainability (fixed salary by the contract) of the principal’s sustainability payment contract, the expected utility
of the principal $E[U_P(X(a^*) - R(a^*)] is a decreasing function of the optimal sensitivity parameters $\eta^*$ of the contract—that is to say, it satisfies the following equation:

$$\frac{\partial E[U_P(X(a^*) - R(\delta^*, \eta^*, a^*))]}{\partial \eta^*} < 0$$  \hspace{1cm} (14)

Therefore, the principals intend to choose the optimal sustainability sensitivity parameter for the principal’s payment contract $\eta^*$; thus, the optimal agent’s participation constraint can be established, i.e.,:

$$\max_{\eta} E\left[U_A(R(\delta, \eta; a^*) - \beta_A(a^*)^2]\right] = U_A(R)$$  \hspace{1cm} (15)

When Equation (12) is imported into the above equation, the following equation can be obtained:

$$-\frac{1}{\gamma_A} e^{\gamma_A \eta^* + \frac{1}{2} \sigma^2 \delta^2 \gamma_A^2} = -\frac{1}{\gamma_A} e^{-\gamma_A R}$$  \hspace{1cm} (16)

With the second-order derivative function of Equation (12), $e^{-\gamma_A \eta^* + \frac{1}{2} \sigma^2 \delta^2 \gamma_A^2} > 0$, and Equation (16), the optimal sensitivity parameter of the principal’s payment contract $\eta^*$ can be obtained:

$$\eta^* = \frac{1}{2} \sigma^2 \delta^2 \gamma_A + R$$  \hspace{1cm} (17)

The determined decision parameter $a^*$ in Equation (11) and $\eta^*$ in Equation (17) are introduced to decide the optimal sustainability performance-to-earnings ratio $\delta^*$, which meets the following equation:

$$\max_{\delta} E[U_P(X(\delta; a^*, \eta^*) - R(a^*))] = \max_{\delta} \left\{ -\frac{1}{\gamma_P} e^{\gamma_P \eta^* - \gamma_P (1-\delta) a^* + \frac{1}{2} \sigma^2 \gamma_P (1-\delta)^2} \right\}$$  \hspace{1cm} (18)

After rearrangement, the above equation equals the solution of the following equation:

$$\min_{\delta} \left\{ \gamma_P \left( \frac{1}{2} \sigma^2 \delta^2 \gamma_A + R \right) - \gamma_P (1-\delta) \frac{\delta}{\beta_A} + \frac{1}{2} \sigma^2 \gamma_P (1-\delta)^2 \right\}$$  \hspace{1cm} (19)

From the second-order derivative function of Equation (19), $\gamma_P (\gamma_A + 1) \sigma^2 + (\frac{1}{\beta_A} + 1) \gamma_P > 0$, the first-order derivative function is equal to 0. After reorganization, the optimal sustainability performance-to-earnings ratio $\delta^*$ is:

$$\delta^* = \frac{\frac{1}{\gamma_P} \sigma^2 \gamma_A + \frac{1}{\beta_A}}{\sigma^2 (\gamma_A - 1) + \frac{1}{\beta_A}}$$  \hspace{1cm} (20)

**Lemma 1.** When the sustainability contract compensation paid by the principal $E[U_P(X(a^*) - R(a^*)] corresponds with Equation (6), the optimal sustainability contract compensation by the principal is $E[U_P(X(\delta^*; \eta^*, a^*) - R(a^*)) = \delta^* X(a^*) + \eta^* a^*$, where the optimal sensitivity parameter of the principal’s sustainability payment contract $\eta^*$ (fixed salary by the contract) and the optimal sustainability performance-to-earnings ratio $\delta^*$ (variable sustainability compensation ratio by contract) are $\eta^* = \frac{1}{2} \sigma^2 \delta^2 \gamma_A + R$ and $\delta^* = \frac{\frac{1}{\gamma_P} \sigma^2 \gamma_A + \frac{1}{\beta_A}}{\sigma^2 (\gamma_A - 1) + \frac{1}{\beta_A}}$. In this case, the optimal agent’s operational effort coefficient is $a^* = \frac{\delta^*}{\gamma_P} = \frac{\frac{1}{\gamma_P} \sigma^2 \gamma_A + \frac{1}{\beta_A}}{\sigma^2 (\gamma_A - 1) + \frac{1}{\beta_A}}$.

**Lemma 2.** Sensitivity Analysis

(1) The result of the first-order partial differential equation of $\eta^*$, as based on the result of Lemma 1, is: $\frac{\partial \eta^*}{\partial \sigma^2} = \frac{1}{2} \delta^* \gamma_A > 0$; $\frac{\partial \eta^*}{\partial \gamma_A} = \frac{1}{2} \sigma^2 \gamma_A > 0$; $\frac{\partial \eta^*}{\partial \beta_A} = \frac{1}{2} \sigma^2 \delta^* > 0$; $\frac{\partial \eta^*}{\partial \gamma_P} = 1 > 0$, which indicates that the impacts of $\delta^*, \eta^*, a^*$ and $R$ on $\eta^*$ have increasing function relationships;
(2) The result of the first-order partial differential equation of $\delta^*$, as based on the result of Lemma 1, is: if $\sigma^2 \geq (\langle \delta \rangle - \beta_A(\gamma_A - 1))$, then $\frac{\partial \delta^*}{\partial \gamma_A} \geq (\langle 0 \rangle)$; if $0.5 \geq (\langle \gamma_A \rangle - 1)$, then $\frac{\partial \delta^*}{\partial \gamma_A} \geq (\langle 0 \rangle)$.

$$\frac{\partial \delta^*}{\partial \gamma_A} = -\left(0.5\sigma^2 + \frac{1}{\beta_A}\right)(\sigma^2(\gamma_A - 1) + \frac{1}{\beta_A}) < 0$$

(3) The result of the first-order partial differential equation of $\alpha^*$, as based on the result of Lemma 1, is: if $\frac{\partial \delta^*}{\partial \alpha^*} = \frac{\partial \delta^*}{\partial \gamma_A}$, then the direction is the same as $\frac{\partial \delta^*}{\partial \alpha^*}$; if $\frac{\partial \delta^*}{\partial \gamma_A} = \frac{\partial \delta^*}{\partial \gamma_A}$, then the direction is the same as $\frac{\partial \delta^*}{\partial \alpha^*} - 0.5\alpha^* \geq (\langle 0 \rangle)$, then $\frac{\partial \delta^*}{\partial \alpha^*} \geq (\langle 0 \rangle)$.

(4) Based on Lemma 2: When the principal pays the sustainability compensation R and the agent’s operational effort remains unchanged, only the fixed sustainability salary provided by the principal increases with R, whereas the principal’s expectation value changes from positive to negative. This is because the agent cares about the monetary reward as an increase in sustainability revenue, which brings risks to the objective business, and the agent is also simultaneously affected by the sustainability risk aversion utility function of $\gamma_A$. Through the first-order partial differential equation of $\eta^*$, as based on the sensitivity analysis of Lemma 2, the effect of R on $\eta^*$ is a relation of an increasing function, which is in accordance with the sensitivity analysis of Lemma 2.

### 4. Numerical Examples of the Insurer–Professional Manager Problem

Assuming that the principal (insurer) employs an agent (professional manager) to professionally manage an insurance business, the insurer and the professional manager both expect to establish a fair and sustainable operating performance-based compensation system to pay the professional manager. Table 1 shows when the professional manager executes professional management, the perception of sustainability compensation comes from adopting the sustainability risk coefficient in the negative exponential form of the HARA utility function $\gamma_A \geq 0$; the assumed numerical example is $\gamma_A = 3$. The expected utility of the professional managers’ sustainable compensation is determined by the selected agent’s operational effort coefficient $\alpha$, among which the cost function relationship between the size of $\alpha$ and the sustainability revenue of the insurance business is $g(\alpha) = \beta_A \times \alpha^2$, and the operational effort and cost-related scale coefficient of the professional manager is $\beta_A$. Assuming $\beta_A = 1$ and the optimal agent’s operational effort coefficient $\alpha^* = 0.63$, then $g(\alpha^*) = 0.396$. It can be seen from Lemma 2 that the optimal agent’s operational effort coefficient $\alpha^*$ of the professional manager increases with the $g(\alpha^*)$ cost; that is, the first-order differential of $\alpha^*$ is higher than 0, indicating that the greater the operational effort paid by the professional manager, the higher the $g(\alpha^*)$ cost paid by him.

<table>
<thead>
<tr>
<th>$X_0$</th>
<th>$\gamma_A$</th>
<th>$\gamma_P$</th>
<th>$\alpha^*$</th>
<th>$\sigma^2$</th>
<th>$R$</th>
<th>$\delta^*$</th>
<th>$\eta^*$</th>
<th>EX</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<td>2</td>
<td>1</td>
<td>0.1</td>
<td>2</td>
<td>0.63</td>
<td>0.875</td>
<td>2.114</td>
</tr>
<tr>
<td>EU_A</td>
<td>EU_P</td>
<td>$g(\alpha)$</td>
<td>RX</td>
<td>$\uparrow\alpha_0$</td>
<td>$\uparrow P_0$</td>
<td>4</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_0$</th>
<th>$\gamma_A$</th>
<th>$\gamma_P$</th>
<th>$\alpha^*$</th>
<th>$\sigma^2$</th>
<th>$R$</th>
<th>$\delta^*$</th>
<th>$\eta^*$</th>
<th>EX</th>
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<td>10</td>
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<td>2</td>
<td>1</td>
<td>0.1</td>
<td>2</td>
<td>0.63</td>
<td>0.875</td>
<td>2.114</td>
</tr>
<tr>
<td>EU_A</td>
<td>EU_P</td>
<td>$g(\alpha)$</td>
<td>RX</td>
<td>$\uparrow\alpha_0$</td>
<td>$\uparrow P_0$</td>
<td>4</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

When $\gamma_A = 3$ generates $g(\alpha^*) = 0.396$, the operation compensation paid by the insurer to the professional manager is $RX = 16.377$, and the equation with the insurer is $R(X(\alpha^*)) = \delta^* X(\alpha^*) + \eta^*$, among which $\eta^* \geq 0$ is the fixed salary coefficient, and $0 \leq \delta^* \leq 1$ is the variable salary ratio coefficient, as generated from the insurance business revenue $X(\alpha^*)$ from sustainability. When the basic operating revenue is $X_0 = 10$, the expectation value of professional manager’s output $EX = X_0(1 + \alpha^*)$ is 16.3, and the professional manager’s utility is $EU_A = \uparrow\alpha_0 - \frac{1}{\gamma_A} \times e^{-\delta_A \times \delta^* \times EX + \gamma^* - g(\alpha^*)}$. Adding utility adjustment parameter $U\alpha_0 = 4$ to the numerical example does not directly affect the pattern of the utility curve. The calculation result shows that $EU_A = 4$ is the expected utility of the professional manager, whereas the insurer’s utility is $EU_P = UP_0 - \frac{1}{\gamma_P} \times e^{i\eta^* - g_P(1-\delta^*)} + e^{2\gamma_P(1-\delta^*)^2}$. Adding utility adjustment parameter $UP_0 = 40$ to the numerical example does not directly
affect the pattern of the utility curve. The results of the calculation show that \( EU_P = 8.898 \) is the expected utility of the insurer.

According to the abovementioned formulae, this paper conducts a model calculation and explores the differences caused by the operational efforts of professional managers, the sustainability risk utility function, and the business dispersion. This paper also discusses the match between the insurer’s sustainability compensation system and the professional manager’s operation compensation.

When \( U_A(C) = C \), the model construction is restored to the original monetary values. Table 2 is based on actual cases of undisclosed numerical data within a company. The data in this study are calculated after conversion. Due to research limitations, individual cases are imported for analysis in subsequent studies. At \( \alpha^* = 0.63 \), the optimal degree of effort of the agent, the scale of market returns \( R \) is constant, the income \( X_0 \) changes with the degree of effort \( \alpha \), and \( \delta \) changes with the income \( X_0 \), so the agent’s monetary reward is \( RX = (R(X_0)\alpha^*)\delta + \eta \). Table 3 shows the change in the agent’s monetary reward at different degrees of effort. At \( \alpha^* = 0.63 \), the agent obtains the highest monetary reward, with \( RX = 13.139, \eta = 2.114 \) and \( \delta = 0.875 \); at \( \alpha = 0.60 \), due to the decline in the degree of effort, the agent’s monetary reward \( X_0 = 8 \) reduces to \( RX = 10.178 \), the fixed salary remains unchanged \( \eta = 2.114 \), and the variable salary reduces to \( \delta = 0.84 \); whereas at \( \alpha = 0.65 \), the agent’s reward reduces to \( RX = 11.941 \), the fixed salary reduces to \( \eta = 1.866 \), and the variable salary reduces to \( \delta = 0.775 \). Under the relationship of monetary values, when the agent works harder, there is a higher risk that the monetary reward will not be obtained. The agent will have the highest monetary reward only when the degree of effort is optimal. This is consistent with the results of the agent’s best efforts in this study. This paper also suggests that the utility function of insurance companies dropped sharply after the introduction of increased rewards, and professional managers who would not give higher rewards to insurance companies ignore the impact of perpetual business risk and risk coefficient divergence, due to the limited effort cost scale. Insurers and professional managers have different perceptions of the sustainable business risks of both parties. Although insurance companies increase their salaries, professional managers will not ignore the risks because of their salaries, and their effectiveness does not increase. This study shows the effects of scale effort in relation to professional managers, after the import utility. However, the current research on agents is mainly based on the monetary measurement, and there has been no in-depth discussion on the effectiveness of both parties. The results provided by this research can fill the current research gap in the theory of agency compensation.

Table 2. Numerical examples of the relationship of monetary values.

<table>
<thead>
<tr>
<th>( X_0 )</th>
<th>( \beta_A )</th>
<th>( R )</th>
<th>( \alpha^* )</th>
<th>( \delta^* )</th>
<th>( \eta^* )</th>
<th>( RX )</th>
<th>( FC )</th>
<th>( VC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>2</td>
<td>0.63</td>
<td>0.875</td>
<td>2.114</td>
<td>13.139</td>
<td>0.105</td>
<td>0.551</td>
</tr>
</tbody>
</table>

Table 3. Some parameters changed for numerical examples of monetary values.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta_A )</th>
<th>( R )</th>
<th>( X_0 )</th>
<th>( \delta )</th>
<th>( \eta )</th>
<th>( RX )</th>
<th>( FC )</th>
<th>( VC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>1</td>
<td>2</td>
<td>8</td>
<td>0.84</td>
<td>2.114</td>
<td>10.178</td>
<td>0.105</td>
<td>0.504</td>
</tr>
<tr>
<td>0.63</td>
<td>1</td>
<td>2</td>
<td>10</td>
<td>0.875</td>
<td>2.114</td>
<td>13.139</td>
<td>0.105</td>
<td>0.551</td>
</tr>
<tr>
<td>0.65</td>
<td>1</td>
<td>2</td>
<td>10</td>
<td>0.775</td>
<td>1.866</td>
<td>11.941</td>
<td>0.093</td>
<td>0.503</td>
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</tbody>
</table>

Sensitivity Analysis

According to sensitivity analysis, when the sustainable operation risk aversion utility function of professional managers changes from \( \gamma_A = 3 \) to \( \gamma_A = 3.5 \), it does not affect the cost scale \( \beta_A \), but with the optimal sensitivity parameter (fixed salary) \( \eta^* \) of the sustainability payment contract with the principal, due to increased risk aversion, \( \alpha^* \) increases from 0.63 to 0.65. As the degree of operational effort increases, the cost scale function of the pro-
professional manager $g(\alpha^*)$ increases from 0.396 to 0.430, the optimum sensitivity parameter of professional managers’ agent payment contract (fixed salary) $\eta^*$ of sustainability increases from 2.114 to 2.123, and the expected value of the insurer’s $EU_P$ increases from 8.898 to 9.555. Due to the increase in the cost scale $g(\alpha^*)$, the overall compensation obtained by the professional manager $RX$ decreases from 16.377 to 16.035. However, if the sustainability risk aversion utility function changes from $\gamma_A = 3$ to $\gamma_A = 2.5$, while it does not affect the paid cost scale $\beta_A$, due to the decreased risk aversion, the optimal agent’s operational effort coefficient $\alpha^*$ decreases from 0.63 to 0.60, the cost scale function of the professional manager $g(\alpha^*)$ falls from 0.396 to 0.364, the professional managers’ optimal sensitivity parameter (fixed salary) $\eta^*$ in the principal’s sustainability payment contract reduces from 2.114 to 2.108, the expectation value of the insurer’s reduces from 8.898 to 8.441, and the overall compensation obtained by the professional manager $RX$ increases from 16.377 to 16.747, due to the decrease in the cost scale function $g(\alpha^*)$. This indicates that when the risk aversion utility function of professional managers increases, the operational effort of professional managers is affected by the sustainability aversion utility, and they try harder to raise the expectations of the insurers. However, because of the cost scale resulting from their efforts, the reward rate decreases, whereas their fixed salary increases moderately, in proportion to the degree of their efforts.

When the imported cost sale increases from 1 to 1.25, the optimal agent’s operational effort coefficient $\alpha^*$ increases from 0.63 to 0.83, due to the rise in the cost scale $\beta_A$, which leads to a further increase in the cost scale function of the professional manager’s $g(\alpha^*)$ from 0.396 to 0.86, the professional manager’s optimal sensitivity parameter (fixed salary) in the principal’s sustainability payment contract $\eta^*$ falls from 2.114 to 2.108, the professional manager’s optimal performance-to-revenue ratio (variable salary) $\delta^*$ decreases from 0.875 to 0.85, the principal’s expectation value decreases from 8.898 to 12.427, and the professional manager’s overall compensation $RX$ increases from 16.377 to 17.664. If the cost scale $\beta_A$ reduces from 1 to 0.75, then the optimal agent’s operational effort coefficient $\alpha^*$ of the professional manager will reduce from 0.63 to 0.44 due to the decrease in the cost scale $\beta_A$, and because of the reduction in the cost scale and degree of effort, the professional manager’s cost scale function $g(\alpha^*)$ decreases from 0.396 to 0.15, the professional manager’s optimal sensitivity parameter (fixed salary) $\eta^*$ of sustainability increases from 2.114 to 2.122, the professional manager’s optimal performance-to-revenue ratio (variable salary) $\delta^*$ increases from 0.875 to 0.9, the insurer’s expectation value decreases from 8.898 to 6.490, and the professional manager’s overall compensation $RX$ decreases from 16.377 to 15.180. It can be observed that when the cost scale $\beta_A$ and the optimal agent’s operational effort coefficient $\alpha^*$ increase, the professional manager’s optimal sensitivity parameter (fixed salary) $\eta^*$ and (variable salary) $\delta^*$ of sustainability both decrease; however, when the cost scale and degree of operational effort drop, the professional manager’s optimal sensitivity parameter (fixed salary) $\eta^*$ and the optimal performance-to-revenue ratio (variable salary) $\delta^*$ of the agent’s sustainability payment contract increase in the same direction.

When the risk coefficient (variance) $\sigma^2$ increases from 0.1 to 0.15, due to the higher dispersion degree, the optimal agent’s operational effort coefficient $\alpha^*$ of the professional manager increases from 0.63 to 0.69, the professional manager’s cost scale function $g(\alpha^*)$ increases from 0.396 to 0.488, the professional manager’s optimal sensitivity parameter (fixed salary) $\eta^*$ of the agent’s sustainability payment contract increases from 2.114 to 2.15, the professional manager’s optimal performance-to-revenue ratio (variable salary) $\delta^*$ decreases from 0.875 to 0.82, the insurer’s expectation value $EU_P$ decreases from 8.898 to 8.64, and the professional manager’s overall compensation $RX$ decreases from 16.377 to 16.20. When the observed $\sigma^2$ reduces from 0.1 to 0.05, the optimal agent’s operational effort coefficient $\alpha^*$ of the professional manager reduces from 0.63 to 0.56, the professional manager’s cost scale function $g(\alpha^*)$ reduces from 0.396 to 0.317, the professional manager’s optimal sensitivity parameter (fixed salary) $\eta^*$ of the agent’s sustainability payment contract reduces from 2.114 to 2.06, the professional manager’s optimal performance-to-revenue ratio (variable salary) $\delta^*$ increases from 0.875 to 0.93, the insurer’s expectation value $EU_P$
increases from 8.898 to 10.22, and the professional manager’s overall compensation $RX$ increases from 16.377 to 16.63. Obviously, the degree of dispersion $\sigma^2$ increases or decreases in the same direction as the optimal agent’s operational effort coefficient of professional managers and the optimal sensitivity parameter of the principal’s payment contract (fixed salary), whereas the optimal agent effort coefficient increases or decreases in the opposite direction to that of the optimal sustainability performance-to-revenue ratio (variable salary).

When the $R$ imported into the model increases from 2 to 2.3, the optimal agent’s operational effort coefficient of the professional manager remains the same, but the optimal sensitivity parameter of the principal’s sustainability payment contract (fixed salary) $\eta^*$ increases from 2.114 to 2.414; thus, the insurer’s expectation value $EU_P$ decreases from 8.898 to $-16.67$, whereas the professional manager’s overall compensation $RX$ increases from 16.377 to 16.677. If the $R$ is reduced from 2 to 1.7, then the optimal agent’s operational effort coefficient of the professional manager remains unchanged, but the optimal sensitivity parameter of the principal’s payment contract (fixed salary) $\eta^*$ decreases from 2.114 to 1.814, the insurer’s expectation value $EU_P$ increases from 8.898 to 22.931, and the professional manager’s overall compensation $RX$ decreases from 16.377 to 16.077. Thus, it can be deduced that, regardless of the increase or decrease in monetary compensation, the optimal agent’s operational effort coefficient of the professional manager and the cost scale remain unchanged; only the optimal sensitivity parameter of the principal’s sustainability payment contract (fixed salary) $\eta^*$ and $R$ change in the same direction, indicating that monetary compensation is not the only consideration for professional managers during agency interactions.

A sensitivity analysis of the study variables is shown in Table 4 below.

### Table 4. Sensitivity analysis.

<table>
<thead>
<tr>
<th>$\gamma_A$</th>
<th>$\alpha$</th>
<th>$\beta_A$</th>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$EU_A$</th>
<th>$EU_P$</th>
<th>$\sigma^2$</th>
<th>$EX$</th>
<th>$g(a)$</th>
<th>$\gamma_P$</th>
<th>$R$</th>
<th>$RX$</th>
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<tbody>
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<td>16.5</td>
<td>0.430</td>
<td>2</td>
<td>2</td>
<td>16.035</td>
</tr>
<tr>
<td>3</td>
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<td>2.114</td>
<td>4</td>
<td>8.898</td>
<td>0.1</td>
<td>16.3</td>
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<td>16.377</td>
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<table>
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<th>$\eta$</th>
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<th>$EU_P$</th>
<th>$\sigma^2$</th>
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<td>8.898</td>
<td>0.1</td>
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<td>0.396</td>
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<td>16.377</td>
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<th>$EU_P$</th>
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<table>
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<th>$\delta$</th>
<th>$\eta$</th>
<th>$EU_A$</th>
<th>$EU_P$</th>
<th>$\sigma^2$</th>
<th>$EX$</th>
<th>$g(a)$</th>
<th>$\gamma_P$</th>
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</table>

Whether the sensitivity analysis of the numerical column is in line with the Lemma is explained as follows.

According to Lemma 1, the calculation is made on the optimal sustainability payment contract compensation by the insurer in the numerical examples to obtain the optimal agent’s operational effort coefficient $\alpha^*$ of the professional manager. An analysis of the numerical column shows that the optimal agent’s operational effort coefficient of the professional manager and the cost scale they must pay are mutually influenced by each other, which is in line with, and matches, the sensitivity analysis result of Lemma 2.

Regarding the sensitivity analysis of Lemma 2, after the first-order partial differential of $\eta^*$ in the result of Lemma 1, we see that the influence of $\delta^*, \eta^*, \alpha^*$ and $R$ on $\eta^*$ are all increasing functions. A sensitivity analysis of the numerical column shows that, when the
optimal agent’s operational effort coefficients of professional managers increase, $\delta^*$, $\eta^*$ and $R$ increase in the same direction, which is consistent with Lemma 2.

Regarding the sensitivity analysis of Lemma 2, the first-order partial differential of $\delta^*$ is

$$\frac{\partial \delta^*}{\partial \gamma_A} = - \left( 0.5 \sigma^2 + \frac{1}{\sigma^2} \right) \left( \sigma^2 (\gamma_A - 1) + \frac{1}{\gamma_A} \right) < 0;$$

the variable salary is affected by the optimal agent effort coefficient of the professional manager; the lower the risk coefficient, the more concentrated the effort, and the higher the variable salary. The sensitivity analysis of the numerical column is consistent with that of Lemma 2.

5. Conclusions

In this study, we developed a fair and reasonable principal–agent optimal sustainability model as a principal–agent solution mechanism, which separates ownership from management rights for sustainability. The question of how to properly design the agent’s salary structure to maintain the sustainability corporate value is an important consideration in the incentive mechanism of performance management in the principal–agent mechanism. This study links the optimal match between the principal’s sustainability compensation payment and the agent’s operational effort by considering the sustainability of companies and combining past studies on the agency theory for the major purposes of this study. While discussing the optimal sustainability compensation structure of the principal and the agent, this paper introduces the negative exponential pattern of the HARA utility function to establish a model to test the proportional relationship between the optimal agent’s operational effort coefficient $\alpha^*$, the optimal sensitivity parameter of the principal’s sustainability payment contract (fixed salary) $\eta^*$, and the optimal performance-to-revenue ratio (variable salary) $\delta^*$ under the maximum revenue expected utility $EU_P$ of the principal and the salary income expectation $EU_A$ of the agent, which can provide the basis for judgment of the decision indicators of Lemma 1 and the sensitivity direction of the important parameters of Lemma 2. It also offers an important opportunity for companies to pursue sustainable development and to create value for principal–agent solutions through sustainability.

The academic and practical meaning of this study are as follows.

5.1. Academic Implications

Under the concept of sustainable development, and considering the professional governance of sustainability in companies, principals expect to establish a fair and performance-based sustainability compensation system to pay agents. This paper provides the analysis results of the numerical examples in the supporting literature and achieves practical feasibility.

Most studies on the principal–agent compensation system take wealth or real income as the goal of value consideration, but often ignore the utility differences in the value perception between the two sides. By introducing the HARA utility function and taking the negative exponential pattern of the principal’s utility and the agent’s different risk aversion perceptions as examples, this study constructs an optimal structure for the principal–agent sustainability compensation system for sustainability, with a subjective risk-aversion perception and objective sustainability value.

This study finds that the utility function of the principal declines sharply after introducing an increase in compensation, which is because agents will not ignore the fact that the effort cost scale is affected by the sustainability risk and risk coefficient dispersion, due to the high compensation paid by the principals. Obviously, there are differences between the principal and the agent in their sustainability risk perceptions. Although the principal increases the salary, the agent does not ignore the risks because of the salary; therefore, the agent’s utility does not increase. This shows that the principal–agent sustainability compensation system, as constructed under an objective sustainability value for sustainable development, has a certain reference value.

5.2. Practical Implications

When faced with the ever-changing financial market and the industry’s competition under sustainable development, how principals balance their industry sustainability rev-
erneue and sustainability compensation structure is a challenge, with regard to how the compensation measurement between principal and agent balances the principal–agent relation, the ultimate goal of which is to increase the value of sustainability. Previous research on compensation mainly conducted the measurement of wealth value, whereas this study expands the value of sustainability to consider the pattern of risk perception under the sustainable development value. Taking the negative exponential pattern of the HARA utility function as an example, this paper puts forward the main model design of an objective compensation system with a sustainability value, which is constructed under the different utility values generated by the subjective risk perception of insurers and professional managers, as well as the different risk perceptions of the principals and the agents. This study aims to offer an insight into the agency problems between the principal, the agent, and the sustainability compensation system, as designed from different perspectives of sustainability development, in order to construct a fair and motivating principal–agent sustainability compensation system, which has the purpose of enhancing the sustainability value of companies.

5.3. Research Limitations

In order to narrow down the research scope, this study only takes the risk aversion utility that combines operational efforts and risks in the agency problem as the issue of fair compensation systems for sustainability. However, the sustainability of insurance companies is not limited to the agent’s efforts, as the environmental and economic operating costs of companies are very extensive. Therefore, this study focused only on fair compensation systems that influence the social aspects of companies, which is one of its limitations.

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