On the MHD Casson Axisymmetric Marangoni Forced Convective Flow of Nanofluids

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Abstract: The proposed investigation concerns the impact of inclined magnetohydrodynamics (MHD) in a Casson axisymmetric Marangoni forced convective flow of nanofluids. Axisymmetric Marangoni convective flow has been driven by concentration and temperature gradients due to an infinite disk. Brownian motion appears due to concentration of the nanosize metallic particles in a typical base fluid. Thermophoretic attribute and heat source are considered. The analysis of flow pattern is perceived in the presence of certain distinct fluid parameters. Using appropriate transformations, the system of Partial Differential Equations (PDEs) is reduced into non-linear Ordinary Differential Equations (ODEs). Numerical solution of this problem is achieved invoking Runge–Kutta fourth-order algorithm. To observe the effect of inclined MHD in axisymmetric Marangoni convective flow, some suitable boundary conditions are incorporated. To figure out the impact of heat/mass phenomena on flow behavior, different physical and flow parameters are addressed for velocity, concentration and temperature profiles with the aid of tables and graphs. The results indicate that Casson fluid parameter and angle of inclination of MHD are reducing factors for fluid movement; however, stronger Marangoni effect is sufficient to improve the velocity profile.

Keywords: Casson nanoliquid; Marangoni convection; inclined MHD; Joule heating; heat source

1. Introduction

The theory of magnetohydrodynamics (MHD) is highly appreciated for the industrial purposes. It is based on magnetic properties of electrically conducting liquids. The characteristic of MHD field is to generate currents in moving liquid and produce forces that act upon the liquid flow and reconstruct the magnetic field itself. To modify flow features of heat and mass analysis, the applied magnetic field impacts the deferred nanoparticles and reforms their absorption inside the liquid. This efficient phenomenon was first utilized for astrophysical and geophysical related problems. Recently, heat transportation and MHD flows have played significant roles in agricultural engineering, petroleum industries and medical treatment such as MHD strategy used for reduction of blood during surgeries, magnetic cell separation and treatment of certain arterial diseases. Basically, the MHD parameter is not only working as a significant parameter to control the cooling/heating rate but also to achieve desired quality of product for different flows. Further, MHD can be used in continuous casting of metal.
processing to suppress instabilities and control flow field. In this context, Hayat et al. [1,2] explored the MHD flow through moving surfaces and concluded that enhancement in magnetic parameter shows increase in nanoparticles concentration and temperature profiles. Hayat et al. [3,4] numerically studied heat transfer impact on MHD axisymmetric third grade liquid flow. Shafiq et al. [5] presented the study of bioconvective MHD tangent hyperbolic nanoliquid flow with Newtonian heating. Shateyi and Makinde [6] prepared MHD stagnant point flow through a radially stretching convectively heated disk. Hayat et al. [7] investigated the third grade axisymmetric MHD flow over a stretched cylinder and showed that momentum layer thickness and velocity profile are increasing when the curvature parameter increases. Moreover, Shafiq et al. [8] discussed magnetohydrodynamics axisymmetric third grade liquid flow between two porous disks.

The novelty of Marangoni convection is generally the edge dissipative layer between two phase fluid flows such as gas–liquid and liquid–liquid interfaces. It depends upon the variation of surface tension driven by temperature, chemical concentration and applied magnetic field. These gradients can occur only when fluid interfaces contain different fluid properties from each other. Due to the viscosity of interacting liquids, external forces such as gravitational and shear forces come into action. Most researchers have focused their interest on simulating these external forces by utilizing governing equations due to its widespread application in the fields of space processing, industrial manufacturing processes and microgravity science. The significance of Marangoni convective flows in the transportation process of heat and mass into different systems have been thoroughly scrutinized in [9–11]. Kumar et al. [12] discussed Marangoni convective Casson nanoliquid flow in the presence of chemical reaction and uniform heat source/sink and observed that Marangoni parameter showed dominant behavior in terms of velocity as well as temperature fields. Din et al. [13] examined the effect of Marangoni convection on based nanoliquid with thermal radiation and demonstrated that decreasing behavior of velocity profile depends on suction parameter, whereas the temperature distribution and boundary layer thickness increased with an increase in nanofluid volume fraction. Sheikholeslami and Ganji [14] studied the impact of magnetic field on nanoliquid flow by Marangoni convection by Runge–Kutta technique and observed that an increment in heat transfer depended on an increment in solid volume fraction of nanofluid. Hayat et al. [15] investigated the impact of radiation and Joule heating on Marangoni mixed convective flow.

For the last few decades, survey of non-Newtonian fluid flows has been the center of attraction for researchers, engineers and scientists. This is due to the application of non-Newtonian liquid flows in the real world, e.g., in bio-engineering, drilling operations, plastic polymers, paint, optical fibers, coated sheets, cosmetics, salt solutions, food item, etc. The existing problems in nature related with larger diameter and higher shear rates can be solved easily; however, when these flows are related to small diameter with low shear rates, the importance of non-Newtonian fluids (see [16,17]) are non-negligible. The deviation from classical Newton’s law of viscosity and flow behavior under shear stress to the non-Newtonian fluids become complex. These flows are challenging task for researchers due to their non-linear rheological behavior. Casson liquid model is one of simplest models of non-Newtonian fluids. The idea of Casson fluid administrated by Casson (see [18]) is to build up the blood flow problems. Due to its rheological properties, Casson liquid behaves as a soft solid when yield stress is higher than shear stress, whereas, if shear stress approaches to infinity, then it starts to deform (see [19]). This structure is widely used for different materials, such as jelly, chocolate, honey, blood, tomato sauce and condensed fruit juices. Charm and Kurland [20] used Casson fluid model and investigated the viscosity of human blood. Bhattacharyya and Hayat [21] analyzed the Casson fluid on MHD boundary layer flow through shrinking sheet. Kumar et al. [22] investigated the viscous dissipation phenomenon in Casson nanoliquid over a moving radiative surface. Moreover, Casson fluid flow model [23–25] has been considered for different geometries and various effects in the literature.

The introduction of nanoparticles in different systems is most favorable to intensify thermal conductivity of classical liquid flows, convection heat transfer coefficient and to control loss in energy.
The advantages of nanosize particles in fluid systems is to increase surface area, capacity of heat transfer, intensify the flow interface after collision and interact fluid particles with each other. Thus, this phenomenon is a backbone of the industrial processes and is also beneficial for solar energy resources and bio-medical treatment (see [26–34]). The proficiency of the solar systems [35] can be improved by incorporating the nanoparticles as working fluid into the systems. The iron based nanoparticles may be utilized as drug and radiation transportation for the treatment of cancer patient (see [36,37]). Using magnets the particles can be enter through blood stream to tumor. This type of cancer treatment permits high local doses of drugs into the body without any significant side effect. Further, micelles nanoparticles have been recently introduced to target the kidney cells diseases. These particles can pass into the kidney and remain there. Similarly, magnetic based nanoparticles are also used for cell separation, hyperthermia therapies and for the increment in Magnetic Resonance Imaging (MRI) with contrast behavior. Hayat et al. [38] judged that nanofluid enhanced the temperature and associated boundary layer width of Casson flow. Naseem et al. [39] numerically investigated third grade nanoliquid flow using the Cattaneo–Christov model over a Riga plate and observed that, with an increment in thermal and concentration relaxation parameters, a reduction occurred in concentration and temperature distribution, respectively. Rasool et al. [40] examined the MHD Darcy–Forchheimer nanoliquid flow under the nonlinear stretched surface. Rashid et al. [41] investigated the entropy generation in Darcy–Forchheimer flow of nanofluid with five nanomaterials due to stretching cylinder. Naseem et al. [42] considered the MHD biconvective flow of a Powell–Eyring nanoliquid over a stretching plate. Rasool et al. [43–48] reported some interesting results involving the role of nanoparticles in typical base fluids flowing over different surfaces.

In the studies mentioned above, one can see that an utmost attention is given to natural convection and heat and mass transfer analysis but less importance has been given to the convection through Marangoni phenomena especially in nanofluid flows. The thermo-capillary and soluto-capillary affects are the main factors in Marangoni convection of fluids and nanofluids. Furthermore, flat surfaces with linear stretching are assumed frequently but axisymmetric analyses are less reported. The main contribution of this research is to examine the process of heat and mass transportation for axisymmetric Marangoni convective flow with an inclined MHD by taking Casson nanofluid flowing towards an infinite disk. Brownian motion and thermophoresis are deliberated on account of nanoparticles structure. Finally, the problem is solved by an accurate numerical technique known as Runge–Kutta fourth-order algorithm, whereas previous studies are given mostly by HAM.

2. Problem Formulation and Coordinate System

The geometry of the problem (see Figure 1) is based on the MHD effect for axisymmetric Marangoni convective, incompressible, steady and laminar flow utilizing the electrically conducting Casson nanoliquid model. Marangoni convective flow is caused due to concentration and temperature gradients on surfaces generated by surface tension. A uniform magnetic field is applied in such a way that it makes an angle $a_1$ in non-vertical direction. The cylindrical coordinates system is considered along and normal to the interface of flow problem. Concentration and temperature interfaces of the flow structure are altered at the surface of the disk. The analysis of heat transfer is examined through Joule heating and viscous dissipation. The formulated governing equations for the MHD effect on Marangoni convective flow structure are given as (see, for example, [4–15]):

$$\frac{\partial \tilde{u}}{\partial \tilde{r}} + \frac{\partial \tilde{w}}{\partial \tilde{z}} + \frac{\partial \tilde{w}}{\partial \tilde{z}} = 0, \quad (1)$$

$$\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{r}} + \tilde{w} \frac{\partial \tilde{u}}{\partial \tilde{z}} = \frac{\mu}{\rho} \left( 1 + \frac{1}{\beta_1} \right) \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} - \frac{\sigma_b^2}{\rho} \sin^2 a_1 \tilde{u}, \quad (2)$$
In Equations (1)–(4), \( \rho \) characterizes fluid density; \( \mu \) signifies dynamic viscosity; \( \beta \) indicates parameter of Casson fluid; \( \sigma \) symbolizes surface tension; \( \tilde{C} \) and \( \tilde{T} \) represent fluid concentration and temperature, respectively; \( \tilde{C}_\infty \) and \( \tilde{T}_\infty \) characterize fluid ambient concentration and temperature far away from the surface, respectively; \( \tau \) shows shear stress; \( D_B \) is the coefficient of Brownian diffusion; \( k \) indicates coefficient of absorption; \( c_\omega \) denotes specific heat; \( D_\tilde{T} \) characterizes coefficient of thermophoretic diffusion; \( Q_1 \) represents heat source sink coefficient; and \( \alpha_1 \) signifies angle of inclination.

**Figure 1.** Physical diagram of the flow model.

The subjected boundary conditions are (see, for example, [10]):

\[
\begin{align*}
\mu \left( 1 + \frac{1}{\beta_1} \right) \left. \frac{\partial \tilde{u}}{\partial \tilde{z}} \right|_{\tilde{z}=0} &= \left. \frac{\partial \sigma}{\partial \tilde{T}} \right|_{\tilde{z}=0} + \left. \frac{\partial \sigma}{\partial \tilde{C}} \right|_{\tilde{z}=0} \left. \frac{\partial \tilde{T}}{\partial \tilde{r}} \right|_{\tilde{z}=0} \left. \frac{\partial \tilde{C}}{\partial \tilde{r}} \right|_{\tilde{z}=0} = 0, \\
\left. \tilde{T} \right|_{\tilde{z}=0} &= \tilde{T}_\infty + A r^2 \phi, \quad \left. \tilde{C} \right|_{\tilde{z}=0} = \tilde{C}_\infty + B r^2 \zeta, \\
\left. \tilde{u} \right|_{\tilde{z} \to \infty} &\to 0, \quad \left. \tilde{T} \right|_{\tilde{z} \to \infty} \to \tilde{T}_\infty, \quad \left. \tilde{C} \right|_{\tilde{z} \to \infty} \to \tilde{C}_\infty.
\end{align*}
\]
The suitable transformations incorporated in the proposed flow structure are (see, for example, [6]):

\[
\eta = \sqrt{\frac{b}{v}} z, \quad \tilde{u} = b \tilde{v} g'(\eta), \quad \tilde{v} = -2 \sqrt{bu} g(\eta), \quad \tilde{\zeta} = \frac{C - C_\infty}{C_K - C_\infty}, \quad \varphi = \frac{\tilde{T} - \tilde{T}_\infty}{T_K - T_\infty}.
\]  

Moreover, assumptions indicate that surface tension is a linear function of concentration and temperature, which may be represented as (see, for example, [10]):

\[
\sigma = \sigma_0 - \gamma_T (\tilde{T} - \tilde{T}_\infty) - \gamma_C (\tilde{C} - \tilde{C}_\infty),
\]

where \(\sigma_0, \gamma_T\) and \(\gamma_C\) represent the positive constants. After incorporating the above-mentioned transformations into Equations (1)–(4), we obtain

\[
\left(1 + \frac{1}{\beta_1}\right) g'' + 2g'g'' - (g')^2 - M_1^2 \sin^2 a_1 g' = 0,
\]  

\[
g(0) = 0, \quad \left(1 + \frac{1}{\beta_1}\right)g''(0) = -2 M_1 (1 + R_a \tilde{\zeta}(0)), \quad g'(') = 0,
\]

\[
\phi'' + 2 \Pr g \phi' + \Pr N_1 \phi' \tilde{\zeta} + \Pr N_2 (\phi')^2 + \Pr Ec \left(1 + \frac{1}{\beta_1}\right) (g'')^2 + \Pr Ec M_1^2 \sin^2 a_1 (g')^2 + \Pr B_1 \phi = 0,
\]

\[
\phi(0) = 1, \quad \phi(\infty) = 0, \quad N_1 \tilde{\zeta}(0) + N_2 \phi'(0) = 0,
\]

\[
\zeta'' + 2Le g\zeta' + N_2 \tilde{\zeta} = 0,
\]

\[
\zeta(0) = 1, \quad \zeta(\infty) \to 0.
\]

In Equations (8)–(13), \(N_1 = \frac{\tau D_0 (C_K - C_\infty)}{v} \) indicates Brownian motion parameter, \(N_2 = \frac{\tau D_1 (T_\infty - T_w)}{vT_\infty} \)
characterizes thermophoresis parameter, \(M_1 = \frac{e R_0^2}{v \sigma_0^2} \) shows magnetic number, \(R_a = \frac{\gamma_T B}{\tau T} \) shows Marangoni ratio parameter, \(B_1 = \frac{\Omega c_b}{\eta \rho \omega} \) represents heat source
sink, \(Pr = \frac{k \nu \rho}{\kappa} \) signifies Prandtl number, \(Ec = \frac{\eta \rho c_b}{\kappa (T_w - T_\infty)} \) indicates Eckert number, and \(Le = \frac{v}{D_a} \)
denotes Lewis number. Additionally, \(Nu\) the local Nusselt number is given as

\[
Nu = -\frac{f}{k(T_\infty - T_w)} \bigg| \xi = 0,
\]

and in dimensionless form becomes

\[
R_d^{-1/2} Nu = \frac{Nu}{\sqrt{R_d}} = -\phi'(0),
\]

where \(R_d = \frac{\rho a f}{k} \) is local Reynold’s parameter.

3. Computational Scheme

We now solve the governing Equations (8)–(13), numerically by employing Runge–Kutta fourth-order technique. For different sundry parameters, we perform numerical computation.

4. Physical Interpretation and Analysis

The main objective of this segment is to communicate the physical importance of heat and mass transportation phenomenon in axisymmetric Marangoni convective flow with the impact of inclined MHD on Casson nanoliquid over an infinite disk. To clearly check the insight of proposed model, the
impact of different parameters (Casson fluid parameter $\beta_1$, magnetic number $M_1$, angle of inclination $\alpha_1$, Marangoni number $M_a$, Brownian motion parameter $N_1$, thermophoresis parameter $N_2$, Prandtl number $Pr$, heat source sink $B_1$ and Lewis number $Le$) are considered on velocity field $g(\eta)$, temperature profile $\phi(\eta)$, concentration distribution $\zeta(\eta)$ and local Nusselt number $Nu$.

4.1. Assessment of Velocity Distribution

The performance of Casson fluid parameter $\beta_1$ on velocity field $g'(\eta)$ is demonstrated in Figure 2. In this figure, one can see that, for enhancement in $\beta_1$, velocity profile increases near the wall but decreases when $\eta > 1.4$ and vanishes far away from the surface. This is because an increment in Casson fluid parameter produces a decrease in yield stress and the fluid adopts rheological behavior and associated boundary layer width reduces. In Figure 3, it is analyzed that a rise in magnetic parameter $M_1$ drops the fluid velocity. This logic is dependent on the fact that an increment in magnetic field $M_1$, which causes an increase in the resistive nature of Lorentz force, and consequently decreases the velocity field. Figure 4 demonstrates the influence of inclination angle $\alpha_1$ on $g'(\eta)$. It is apparent from the sketch that velocity profile $g'(\eta)$ reduces when the angle of inclination $\alpha_1$ increases. This is because, when angle of inclination increases, the impact of magnetic field rises on liquid and as a result Lorentz force increases, which in turn decreases the velocity profile. In addition, for $\alpha_1 = 0$, there is no effect of magnetic field on velocity profile, whereas, for $\alpha_1 = \pi/2$, maximum resistance is noted. In Figure 5, graphical representation signifies that velocity field is mounting function of Marangoni number $M_a$. This behavior is because of Marangoni number, as it is the ratio between tangential stress and viscosity. Therefore, the fluid with higher surface tension acts more strongly on the surrounding liquid and consequently it enhances velocity of the fluid.

$M_1 = 0.5,\ \alpha_1 = \pi/3,\ R_a = 0.5,\ B_1 = 0.2,\ M_a = 0.3,\ Q_1 = 0.5,\ Pr = 1.0,\ Ec = 0.2$

$N_1 = 0.5,\ N_2 = 1.5,\ Le = 1.5,$

$\beta_1 = 1.3, 1.5, 2.0, 2.5$

![Figure 2. Influence of $\beta_1$ on velocity field.](image-url)
$\beta_1 = 1.3, \ \alpha_1 = \pi/3, \ Ra = 0.5, \ B_1 = 0.2, \ Ma = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.3,$
$N_1 = 0.5, \ N_2 = 1.5, \ Le = 1.5,$

Figure 3. Influence of $M_1$ on velocity field.

$\beta_1 = 1.3, \ R_a = 0.5, \ B_1 = 0.2, \ M_1 = Ma = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.5,$
$N_1 = 0.5, \ N_2 = 1.5, \ Le = 1.5,$

$M_1= 0.0, 0.2, 0.4, 0.6$

Figure 4. Influence of $\alpha_1$ on velocity field.
\[ \beta_1 = 1.3, \quad R_a = 0.5, \quad B_1 = 0.2, \quad M_1 = M_a = 0.3, \quad Q_1 = 0.5, \quad Pr = 1.0, \quad Ec = 0.5, \quad N_1 = 0.5, \quad N_2 = 1.5, \quad Le = 1.5, \]

\[ \alpha_1 = \frac{\pi}{3}, \quad \alpha_1 = \frac{\pi}{2}, \quad N_2 = 0.5, \quad Le = 1.0, \quad N_1 = 0.01, \ 0.2, \ 0.4, \ 0.6 \]

**Figure 5.** Influence of \( M_a \) on velocity field.

**4.2. Assessment of Temperature Distribution**

In this subsection, the temperature field \( \phi(\eta) \) corresponds to various sundry parameter such as thermophoresis parameter \( N_2 \), Brownian motion parameter \( N_1 \) and heat source parameter \( B_1 \) are plotted in Figures 6–8. The behavior of \( N_1 \) on \( \phi(\eta) \) is sketched in Figure 6. The temperature field \( \phi(\eta) \) is increased with a rise in Brownian motion parameter \( N_1 \). With the increment in Brownian motion parameter, the fluid molecules becomes more energetic. As a result, the temperature field is enhanced. Figure 7 shows the significance of Thermophoresis parameter \( N_2 \) on \( \phi(\eta) \). It is noted that the temperature field is a mounting function of \( N_2 \). Figure 8 shows the variation of heat source \( B_1 \) via temperature field \( \phi(\eta) \). It is examined that temperature as well as the associated boundary layer is increased by increment in heat source parameter \( B_1 \). Physically, the rise in rate of heat source parameter \( B_1 \) leads to the thermal boundary layer thickness becoming greater, as does the temperature field.

**Figure 6.** Influence of \( N_1 \) on temperature field.
$\beta_1 = 1.3, \ R_a = 0.2, \ B_1 = 0.2, \ M_1 = M_a = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.8,$
$\alpha_1 = \pi/3, \ N_1 = 0.5, \ Le = 1.0,$

**Figure 7.** Influence of $N_2$ on temperature field.

$\beta_1 = 1.3, \ R_a = 0.2, \ B_1 = 0.2, \ M_1 = M_a = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.8,$
$\alpha_1 = \pi/3, \ N_2 = 0.0, 0.5, 1.0, 1.5$

**Figure 8.** Influence of $B_1$ on temperature field.

### 4.3. Assessment of Concentration Distribution

The significance of Brownian motion $N_1$ on concentration profile $\zeta(\eta)$ is displayed in Figure 9. It is observed from the sketch that larger values of $N_1$ fluid concentration reduce far away from the surface and vanish after $\eta \geq 5$. On the other side, it increases near the surface. This is due to the existence of slip mechanisms of fluid particles, which influence the hydrodynamic and thermal bounce. Hence, the presence of this terminology does not have significant impact on flow concentration. Further, both thermophoresis parameter $N_2$ and Lewis number $Le$ show increasing impact for concentration profiles (see Figures 10 and 11). The improvement in fluid concentration profile via Lewis number $Le$
is due to the fact that it is characterized by fluid flows where simultaneously mass and heat transport are involved. Therefore, an improvement is found in fluid concentration.

\[ \beta_1 = 1.3, \ R_a = 0.2, \ B_1 = 0.2, \ M_1 = M_a = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.8, \]
\[ \alpha_1 = \pi/3, \ N_2 = 0.5, \ Le = 1.0, \]

Figure 9. Influence of \( N_1 \) on concentration field.

\[ \beta_1 = 1.3, \ R_a = 0.2, \ B_1 = 0.2, \ M_1 = M_a = 0.3, \ Q_1 = 0.5, \ Pr = 1.0, \ Ec = 0.8, \]
\[ \alpha_1 = \pi/3, \ N_1 = 0.5, \ Le = 1.0, \]

Figure 10. Influence of \( N_2 \) on concentration field.
4.4. Assessment of local Nusselt Number

In Table 1, one can see that a good agreement is found between the present results and previous literature. A very good agreement is found in RK-45 results; however, the results of HAM are a little different but the variation trend is similar in all the cases. Table 2 presents the significance of physical parameters through local Nusselt number $Nu$ for axisymmetric Marangoni convective flow of Casson liquid over an infinite disk with the impact of an inclined MHD. Near the wall or boundary, Nusselt number has a dominant role, for the computation of thermal profile variations. The numerical quantities of Nusselt number is supportive to convey the cumulative tendency of temperature gradient in flow domain. It is observed in the table that rises in the Marangoni convective fluid parameter, $M_a$, Marangoni ratio, $R_a$, and $N_2$ monotonically decrease the Nusselt number $Nu$ by keeping other fluid parameters fixed. On the other hand, the parameters $N_1$, $Pr$, $B_1$ and $Le$ manifest rises in heat flux behavior $Nu$. The small increase on average Nusselt number indicates that greater heat exchange rate occurs near boundary of the disk due to these parameters.

Table 1. Comparison table of current results with previously published literature setting the additional parameters equals to zero.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$N_1 = N_b$</th>
<th>$N_2 = N_t$</th>
<th>$Pr$</th>
<th>$Nu_x$ (Present)</th>
<th>$Nu_x$ ([12])</th>
<th>$Nu_x$ ([15])</th>
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<td>1.488646</td>
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<td></td>
<td></td>
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Table 2. Numeric values of local Nusselt parameter $Nu$ for distinct values of sundry parameter where $\beta_1 = 1.5$, $M_1 = 1.0$, $Ec = 0.8$ and $a_1 = \pi/5$.

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$Ra_a$</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$Pr$</th>
<th>$B_1$</th>
<th>$Le$</th>
<th>$-R_d^{-1/2}Nu$</th>
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5. Conclusions

In the present research work, we used RK45 scheme to simulate the two-dimensional Marangoni convective flow along with MHD effect and related heat and mass transfer problem over an infinite disk. The efficiency of proposed model was observed numerically and graphically, and found in good agreement for heat transportation process. The influence of distinct parameters on proposed flow problem are discussed in detail above. Further, the main findings of the present study are highlighted below:

- Increase in Brownian motion parameter enhances the flow temperature field, however the same goes for a declination of concentration field.
- Rise in thermophases parameter improves the fluid temperature as well as concentration field.
- Larger values of Lewis number corresponds to the high concentration profile.
- Casson fluid parameter is found to be a reducing factor for fluid movement; therefore, admitting the higher quantity of Casson fluid parameter causes a reduction in fluid velocity.
- Increment in magnetic parameter and angle of inclination are reducing factors for the motion of fluid; however, the opposite performance in terms of heat transfer rate via Nusselt number is noted for the two parameters.
- The higher amount of Marangoni number condenses the active connectivity, which leads to improve the velocity profile.
- Temperature distribution rises up for the larger values of heat source sink.
- Increase in the Marangoni and Prandtl numbers show high increment on average Nusselt number, which leads to the conclusion that less heat exchange happens near the disk, while small values of fractional and physical parameters $\beta_1$, $M_1$, $a_1$, $Ra_a$, $N_1$, $N_2$, $Ec$, $B_1$, and $Le$ manifest the high heat exchange rate near the boundary of the disk.
Author Contributions: Conceptualization, I.Z.; methodology, I.Z.; software, A.S.; validation, G.R.; formal analysis, T.S.K.; investigation, I.Z.; resources, I.T.; data curation, G.R.; writing—original draft preparation, I.Z.; writing—review and editing, A.S., G.R. and I.T.; visualization, A.S.; supervision, T.S.K.; project administration, A.S.; and funding acquisition, I.T.

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Conflicts of Interest: The authors declare no conflict of interest.

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