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An Innovative Approach towards Possibility Fuzzy Soft Ordered Semigroups for Ideals and Its Application

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Abstract: The objective of this paper is put forward the novel concept of possibility fuzzy soft ideals and the possibility of fuzzy soft interior ideals. The various results in the form of the theorems with these notions are presented and further validated by suitable examples. In modern life decision-making problems, there is a wide applicability of the possibility fuzzy soft ordered semigroup which has also been constructed in the paper to solve the decision-making process. Elementary and fundamental concepts including regular, intra-regular and simple ordered semigroups in terms of possibility fuzzy soft ordered semigroup are presented. Later, the concept of left (resp. right) regular and left (resp. right) simple in terms of possibility fuzzy soft ordered semigroups are delivered. Finally, the notion of possibility fuzzy soft semiprime ideals in an ordered semigroup is defined and illustrated by theorems and example.

Keywords: soft ordered semigroup; fuzzy soft ideals; possibility fuzzy soft interior ideals; simple possibility fuzzy soft ordered semigroup; possibility fuzzy soft semiprime ideals

1. Introduction

As classical mathematical tools are unable to solve the modern uncertain problems, scientists are paying much attention to deriving tools for solving these uncertainties. Fuzzy mathematics is an approach put forwarded by Zadeh that opens up a new direction for scientists in solving such hard problems [1]. Fuzzy mathematics is a key tool to solve many problems from the field of control engineering, robotics, and artificial intelligence, medical diagnose, operational research, and neural networking. The algebraic structures act as a main tool in solving formal coding languages, machine learning, and in providing logical and mathematical basis for many engineering problems. The theory of semigroups has been used in various fields of applied sciences and engineering. Kuroki in [2] introduced fuzzy sets in terms of semigroups in 1991. He also studied fuzzy ideals in semigroups [3,4]. Several classes including simple, regular, intra-regular, weakly regular, and many more were introduced.

Later, Kehayopulu and Tsingelis in [5] studied some elementary properties of fuzzy sets in ordered groupoids, relating the concept of fuzzy ideals in terms of ordered semigroups for the first time. They further put forward the idea of fuzzy bi-ideals and fuzzy interior ideals in ordered semigroups [6,7]. Shabir and khan [8,9] studied some of the properties of fuzzy ideals in ordered semigroups and initiated the theory of fuzzy generalized bi-ideals. Khan et al. [10] presented generalized fuzzy interior ideals in ordered semigroups. Khan et al. [11] induced some new forms of fuzzy interior ideals of ordered semigroups.

An extension to fuzzy set theory, named as the soft set approach, was introduced by Moldstov [12,13] and Maji [14,15], which provided a parametrization tool. Alkalzaleh et al. [16] following Moldstov's approach and gave a new conception of the soft multi-set. An extension to fuzzy set theory, named as multi-fuzzy sets, is initiated by Sebastian in [17]. The fuzzy soft set approach provides a parameterization tool that reduces the uncertainties for theoretical problems and yielded more precise and better results. Well ahead, Yang et al. in [18] broached a new concept of interval-valued fuzzy set theory coupled with the soft set approach. Several other concepts, including intuitionistic fuzzy soft sets, expert sets, neutrosopic sets, vague sets, complex neutrosophic sets, and so on, were introduced recently for handling decision-making and other physical problems [19–31]. A possibility fuzzy soft set is also included in one of the aforementioned concepts introduced by Alkhalzaleh et al. in [32]. Authors have mentioned some applications of possibility fuzzy soft set theory in medical diagnoses and decision-making. Zhang and Shu [33] proposed a concept of possibility multi-fuzzy soft set theory and its application in decision sciences. Jun et al. [34] put forward a new side by coupling soft sets with ordered semigroups. Later, the ideas of fuzzy soft sets with ordered semigroups and different types of fuzzy soft ideals were further investigated in [35,36]. On basis of the aforementioned research, the concept of possibility fuzzy soft sets in ordered semigroup has been introduced by Habib et al. [37].

In the modern decision-making process, many researchers are studying different approaches of fuzzy soft sets coupling with semigroup theory and relating their possible applications in the field of decision sciences. This progress in the field of fuzzy mathematics motivates the Authors to develop a new direction to solve decision-making problems. The aim of this research is to initiate the concept possibility fuzzy soft ordered semigroups and its extension for interior ideals and left (resp. right) ideals. This new research had a massive scope in solving decision-making problems in the field of applied sciences. Thus, based on these observations, the primary objectives of the present work are summarized as follows:

(1) To propose a novel approach of possibility fuzzy soft sets in terms of ordered semigroups and to present the characterizations of various classes of ordered semigroups coupling with possibility fuzzy soft sets.
(2) To derive the application of possibility fuzzy soft ordered semigroup in decision sciences.
(3) To introduce a concept of possibility fuzzy soft left (resp. right) ideals and possibility fuzzy soft interior ideals and establish a relationship between these types of ideals. This relationship is well explained via suitable examples.
(4) To investigate various classes of ordered semigroups including regular and intra-regular in terms of possibility fuzzy soft ideals and possibility fuzzy soft interior ideals.
(5) To present a new concept of possibility fuzzy soft simple ordered semigroups and characterize this class of ordered semigroups in terms of possibility fuzzy soft ideals and possibility fuzzy soft interior ideals.
(6) To introduce a novel concept of possibility fuzzy soft ordered semigroup in terms of semiprime ideals.

The rest of the manuscript is organized as follows: In Section 2, the authors reiterate some basic concepts and definitions that will help the readers to understand the concepts well. In Section 3, fundamental concepts of possibility fuzzy soft ordered semigroup along with suitable examples are introduced. Some basic definitions and theorems are included. Section 4 includes a new notion of possibility fuzzy soft interior ideals in ordered semigroups. Further, possibility fuzzy soft left (resp. right) ideals are also defined. Several examples, properties of possibility fuzzy soft left (resp. right) ideals and possibility fuzzy soft interior ideals are also investigated. In Section 5, the concept of possibility fuzzy soft simple ordered semigroup is introduced. Furthermore, important relations between regular and intra-regular ordered semigroups with possibility fuzzy soft interior ideals and possibility fuzzy soft left (resp. right) ideals are drawn out. Lastly, the notion of semi-prime possibility fuzzy soft ideals in ordered semigroups has been defined.

2. Preliminaries

In this section, we recapitulate some basic definitions and lemmas that will further be used to derive some new and fundamental concepts.

For an ordered semigroup Z, represents a structure $(Z,.,\leq)$ that satisfies the following conditions: $(Z,.)$ defines a semigroup, $(Z,\leq)$ is an ordered set and for all, $z_1 \leq z_2$ implies $z_1 a \leq z_2 a$ or $az_1 \leq az_2$ [38]. A non-empty subset X of $(Z,.,\leq)$ is called a subsemigroup of Z if $X^2 \subseteq X$. For any non-empty subset L of Z is called a left (resp. right) ideal of Z if $ZL \subseteq L$ (resp. $LZ \subseteq L$) and for all $z \in Z, x \in L$ if$z \leq x \Rightarrow z \in L$. Left (resp. right) ideals can be represented as $L \lhd_l Z$ (resp. $L \lhd_r Z$) [39]. If L is both left and right ideal of Z then L is known as an ideal of Z and is represented as $L \lhd Z$.

For an ordered semigroup Z, X is a non-empty subset of Z then X is known as an interior ideal of Z if

1. X is a subsemigroup of Z
2. $(\forall z \in Z, x \in X)(z \leq x \Rightarrow z \in X)$
3. $ZXZ \subseteq X$

For an ordered semigroup Z, P is a non-empty subset of Z then P is known as semiprime [39] if $x^2 \in P \Rightarrow x \in P$ $(\forall x \in Z)$ or $X^2 \subseteq P$ $(\forall X \subseteq Z)$. An ordered semigroup Z is regular [40] if $\forall x \in Z$ there exists $z \in Z$ such that $x \leq xzx$ or in other words $\forall x \in Z$ we have $x \in (xZx]$ and $\forall X \subseteq Z$, $X \subseteq (XZX]$. Let $\forall x \in Z$ there exists $z \in Z$ then Z is called left (resp. right) regular if $x \leq zx^2$ (resp. $x \leq x^2 z$) or in other words $\forall x \in Z$ we have $x \in (Zx^2]$ (resp. $x \in (x^2 Z]$) and $(\forall X \subseteq Z,\ X \subseteq (ZX^2]$ (resp. $X \subseteq (X^2 Z]$). If an ordered semigroup Z is both left and right regular then it is known as completely regular.

If for all $L \lhd_l Z$ (resp. $L \lhd_r Z$, $L = Z$ then Z is called left (resp. right) simple [41]. An ordered semigroup is simple if it is both right and left simple.

Definition 1. *[12] Let, for a universal set U, there exist a set of parameters represented as N. If $M \subseteq N$, then (f, M) is called a soft set, where f is a mapping from M to power set of U, i.e., $f: M \to P(U)$.*

Definition 2. *[34] Let (f, N) be a soft set over Z, then (f, N) is called a soft ordered semigroup if $\forall n \in M$ $f(n) \neq \phi$ $(f(n)$ is a subsemigroup of Z.*

Definition 3. *[34] If (f, N) and (g, M) are two soft sets over U then their union is represented by $(f, N) \cup (g, M) = (h, O)$ with the following conditions:*

(1) $0 = N \cup M$

(2) $h(\mathrm{n}) = \begin{cases} f(\mathrm{n}) & \text{if} \quad n \in M - N \\ g(\mathrm{n}) & \text{if} \quad n \in N - M \\ f(\mathrm{n}) \cup g(\mathrm{n}) & \text{if} \quad n \in M \cap N \end{cases} \quad \forall n \in O$

Definition 4. *[34] If (f, N) and (g, M) are two soft sets over U then their intersection is represented as $(f, N) \cap (g, M) = (h, O)$ with the following conditions:*

(1) $0 = N \cap M$

(2) $\big(h(n) = f(n)\big) Or\ (h(n) = g(n))\ \ \forall n \in O$

Definition 5. *[42] Let a pair of sets (U, N), where U and N represent a universal set and a set of parameters, respectively. If $\forall M \subseteq N$, $f: M \to F(U)$ where $F(U)$ consists of all fuzzy subsets of U, then (f, M) is a fuzzy soft set.*

Definition 6. *[43] A soft set (f, N) over Z is a soft left (resp. right) ideal over Z if and only if $f(n)$ is a fuzzy soft left (resp. right) ideal over Z.*

Definition 7. *[43] A soft set (f, N) over Z is a soft left (resp. right) ideal over Z if and only if (f, N) is a fuzzy soft left (resp. right) ideal over Z.*

Definition 8. [43] *A soft set (f, N) over Z is a semiprime soft left (resp. right) ideal over Z if and only if $f(n)$ is a semiprime ideal over Z for all $n \in N$. Additionally, for each $z \in Z$ and $n \in N$, $z^2 \in f(n)$.*

Definition 9. *[32] Let $U = \{u_1, u_2, u_3, \ldots\ldots, u_r\}$ be a universal set and $N = \{n_1, 2, \ldots, n_q\}$ be the set of parameters, then the possibility fuzzy soft set $(\bar{f}_{\bar{\upsilon}}, N)$ is defined by a mapping $\bar{f}_{\bar{\upsilon}}: N \to F(U) \times I(U)$ ($\bar{f}_{\bar{\upsilon}}$ Is the fuzzy subset of N) for all $u \in U$, where $\bar{f}_{\bar{\upsilon}}(n_i) = (\bar{f}(n_i)(u), \bar{\upsilon}(n_i)(u))$. Here, $\bar{f}(n_i)$ and $\bar{\upsilon}(n_i)$ define the degree of the membership value and the possibility of the degree of the membership value, respectively. In generalized form the possibility fuzzy soft is represented by:*

$$\bar{f}_{\bar{\upsilon}}(n_i) = \begin{matrix} \{(u_1/\bar{f}(n_i)(u_1), \bar{\upsilon}(n_i)(u_1)), (u_2/\bar{f}(n_i)(u_2), \bar{\upsilon}(n_i)(u_2)), \\ \ldots\ldots, (u_n/\bar{f}(n_i)(u_n), \bar{\upsilon}(n_i)(u_r))\} \end{matrix} \quad i = 1, 2, \ldots., q \tag{1}$$

The possibility fuzzy soft set is among the generalized techniques used to resolve decision-making problems.

Definition 10. *[32] The union of any two possibility fuzzy soft sets $\bar{f}_{\bar{\upsilon}}$ and $\bar{g}_{\bar{\mu}}$ over* U *is represented as $\bar{h}_{\bar{\delta}}$ where $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$ $\bar{h}_{\bar{\delta}}: N \to F(U) \times I(U)$ and is defined as $\bar{h}_{\bar{\delta}}(n) = \bar{h}(n)(u), \bar{\delta}(n)(u)$. $\bar{h}_{\bar{\delta}}$ must satisfy the following relation:*

(1) $\bar{h}(n) = \bar{f}(n) \cup \bar{g}(n) \forall n \in N$

(2) $\bar{\delta}(n) = \bar{\upsilon}(n) \cup \bar{\mu}(n)$

Definition 11. *[32] The intersection of any two possibility fuzzy soft sets $\bar{f}_{\bar{\upsilon}}$ and $\bar{g}_{\bar{\mu}}$ over U is represented as $\bar{h}_{\bar{\delta}}$ where $\bar{h}_{\bar{\delta}}: N \to F(U) \times I(U)$ and is defined as $\bar{h}_{\bar{\delta}}(n) = \bar{h}(n)(u), \bar{\delta}(n)(u)$, $\bar{h}_{\bar{\delta}}$ must satisfy the following relation:*

(1) $\bar{h}(n) = \bar{f}(n) \cap \bar{g}(n) \forall n \in N$

(2) $\bar{\delta}(n) = \bar{\upsilon}(n) \cap \bar{\mu}(n)$

Definition 12. *Let $(\bar{f}_{\bar{\upsilon}}, N)$ and $(\bar{g}_{\bar{\mu}}, M)$ be the possibility fuzzy soft sets over U, then the AND operation is denoted as $(\bar{f}_{\bar{\upsilon}}, N) \wedge (\bar{g}_{\bar{\mu}}, M)$ and is defined as, $(\bar{f}_{\bar{\upsilon}}, N) \wedge (\bar{g}_{\bar{\mu}}, M) = (\bar{h}_{\bar{\delta}}, O), \forall O = M \times N$, where $\bar{h}_{\bar{\delta}}(m, n) = \bar{h}(m, n)(u), \bar{\delta}(m, n)(u)$, $\forall (m, n) \in M \times N$ such that:*

(1) $\bar{h}(m, n) = \bar{f}(m) \cap \bar{g}(n)$

(2) $\bar{\delta}(m, n) = \bar{\upsilon}(m) \cap \bar{\mu}(n)$

3. Possibility Fuzzy Soft Ordered Semigroup

Definition 13. *[37] Let $(\boldsymbol{Z}, ., \leq)$ be an ordered semigroup and N is a set of parameters of Z defined by a mapping $\bar{\boldsymbol{f}}_{\bar{\upsilon}}: \boldsymbol{N} \to \boldsymbol{F}(\boldsymbol{Z}) \times \boldsymbol{I}(\boldsymbol{Z})$ then $(\bar{\boldsymbol{f}}_{\bar{\upsilon}}, \boldsymbol{N})$ is called a possibility fuzzy soft ordered semigroup of Z, if it satisfies the following: $\bar{\boldsymbol{f}}_{\bar{\upsilon}}(\boldsymbol{n}) \neq \boldsymbol{\phi}$ and $\bar{\boldsymbol{f}}_{\bar{\upsilon}}(\boldsymbol{n})$ are fuzzy subsemigroups of Z. The possibility fuzzy soft ordered semigroup is represented as PFSS until and unless explained.*

Example 1. *Let $Z = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ be an ordered semigroup as defined in Table 1 and ordered relation is represented in Figure 1:*

$$\begin{gathered} \leq := \{(w_1, w_1), (w_1, w_6), (w_2, w_2), (w_2, w_3), (w_2, w_6), (w_3, w_3), (w_3, w_6), (w_4, w_1), (w_4, w_3), (w_4, w_4) \\ (w_4, w_6), (w_5, w_1), (w_5, w_2), (w_5, w_3), (w_5, w_4), (w_5, w_5), (w_5, w_6), (w_6, w_6)\} \end{gathered}$$

Let $N = \{n_1, n_2, n_3\}$ *be a parameterized set* Z *then possibility fuzzy soft set* $(\bar{f}_{\bar{\upsilon}}, N)$ *is defined by mapping:* $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$.

Thus:

$$\bar{f}_{\bar{\upsilon}}(n) = \begin{bmatrix} 0.2, 0.3 & 0.1, 0.1 & 0.2, 0.2 & 0.6, 0.6 & 0.6, 0.6 & 0.1, 0.2 \\ 0.1, 0.2 & 0.2, 0.3 & 0.3, 0.4 & 0.5, 0.6 & 0.5, 0.6 & 0.1, 0.1 \\ 0.2, 0.3 & 0.3, 0.4 & 0.1, 0.2 & 0.5, 0.7 & 0.5, 0.7 & 0.2, 0.2 \end{bmatrix}, \tag{2}$$

By using Definition 13, $\bar{f}_{\bar{\upsilon}}(n) \neq \emptyset$ *also* $\bar{f}(n)(w), \bar{\upsilon}(n)(w)$ *are all fuzzy subsemigroups of Z. Thus,* $(\bar{f}_{\bar{\upsilon}}, N)$ *is called PFSS over Z.*

Table 1. Multiplication table.

$\cdot$	$\boldsymbol{w_1}$	$\boldsymbol{w_2}$	$\boldsymbol{w_3}$	$\boldsymbol{w_4}$	$\boldsymbol{w_5}$	$\boldsymbol{w_6}$
w_1	w_4	w_4	w_4	w_4	w_4	w_1
w_2	w_5	w_5	w_5	w_5	w_5	w_2
w_3	w_4	w_4	w_4	w_4	w_5	w_3
w_4	w_4	w_4	w_4	w_4	w_4	w_4
w_5	w_5	w_5	w_5	w_5	w_5	w_5
w_6	w_4	w_4	w_4	w_4	w_4	w_6

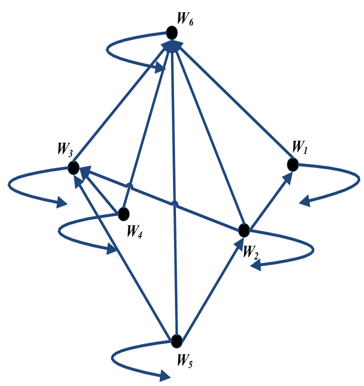

Figure 1. Hesse Diagram.

Definition 14. *[35] Let for an ordered semigroup* $(Z, ., \leq)$, $(\bar{f}_{\bar{\upsilon}}, N)$ *is a PFSS over Z. Then, the possibility fuzzy soft level set of* $\bar{f}_{\bar{\upsilon}}$ $(\forall t \in [0,1])$ *is defined as:*

$$U(\bar{f}_{\bar{\upsilon}}; t) = \{z \in Z | \bar{f}(z) \geq t, \bar{\upsilon}(z) \geq t\} \tag{3}$$

where $\bar{f}$ *and* $\bar{\upsilon}$ *be two fuzzy subsets of Z.*

Based on possibility fuzzy soft ordered semigroups, the following theorems are stated and since the proofs are straightforward the proofs are omitted.

Theorem 1. *[35] Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *be a possibility fuzzy soft ordered semigroup of Z. Then* $(\bar{g}_{\bar{\mu}}, M)$ *is also a possibility fuzzy soft ordered semigroup over Z for all M and is a subset of N.*

Theorem 2. *[35] Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be the two possibility fuzzy soft ordered semigroups over Z. Then their union* $(\bar{f}_{\bar{\upsilon}}, N) \cup (\bar{g}_{\bar{\mu}}, M), \forall M \cap N = \emptyset$ *is also a possibility fuzzy soft ordered semigroup over Z.*

Theorem 3. *[35] Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be the two possibility fuzzy soft ordered semigroups over Z. If* $M \cap N \neq \emptyset$*, then their intersection* $((\bar{f}_{\bar{\upsilon}}, N) \cap (\bar{g}_{\bar{\mu}}, M))$ *is also a possibility fuzzy soft ordered semigroup over Z.*

Theorem 4. [35] *Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be the two possibility fuzzy soft ordered semigroups over Z, then* $((\bar{f}_{\bar{\upsilon}}, N) \wedge (\bar{g}_{\bar{\mu}}, M))$ *is also a possibility fuzzy soft ordered semigroup over Z.*

Theorem 5. *[35] Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be the two possibility fuzzy soft ordered semigroups over Z. If* $M \cap N = \emptyset$ *then* $((\bar{f}_{\bar{\upsilon}}, N) \vee (\bar{g}_{\bar{\mu}}, M))$ *is also a possibility fuzzy soft ordered semigroup over Z.*

Application of Possibility Fuzzy Soft Ordered Semigroups

Let $Z = \{P_1, P_2, P_3\}$ be a set of three players of cricket and we have defined the following multiplication table in Table 2 and ordered relation in Figure 2, on the basis of their average performance in last ten matches.

Table 2. Multiplication table of players.

$\cdot$	P_1	P_2	P_3
P_1	P_3	P_2	P_1
P_2	P_2	P_2	P_3
P_3	P_3	P_3	P_3

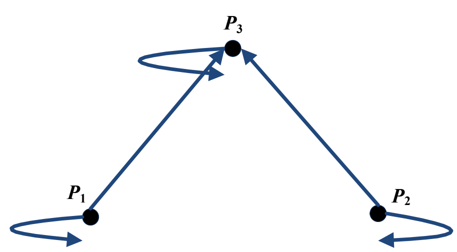

Figure 2. Ordered relation of Z.

$$\leq := \{(P_1, P_1), (P_2, P_2), (P_3, P_3), (P_1, P_3), (P_2, P_3)\}$$

Let us define a mapping $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$, here $N = \{n_1, n_2\}$ as a set of parameters where n_1 is batting and n_2 is bowling. Then $\bar{f}_{\bar{\upsilon}}(n_j) = \bar{f}(n_j)(P_i), \bar{\upsilon}(n_j)(P_i)$ for all $P_i \in Z$ and $n_i \in N$ where $i = 1,2$ and $j = 1,2,3$. Here we will define a possibility fuzzy soft set $(\bar{f}_{\bar{\upsilon}}, N)$ for the first observer or the committee member to choose the best player for the team:

$$\bar{f}_{\bar{\upsilon}} = \begin{bmatrix} 0.4,0.6 & 0.6,0.7 & 0.8,0.9 \\ 0.4,0.1 & 0.5,0.4 & 0.7,0.8 \end{bmatrix} \quad (4)$$

Hence, from the above matrix we found that $\bar{f}_{\bar{\upsilon}}(n) \neq \emptyset$ and $\bar{f}_{\bar{\upsilon}}(n)$ are fuzzy subsemigroup of Z. Thus, we concluded that, $(\bar{f}_{\bar{\upsilon}}, N)$ is a possibility fuzzy soft ordered semigroup.

Similarly, let us define a mapping $\bar{g}_{\bar{\mu}}: N \to F(Z) \times I(Z)$, where $(\bar{g}_{\bar{\mu}}, N)$ is the second observer or the committee member:

$$\bar{g}_{\bar{\mu}} = \begin{bmatrix} 0.2,0.6 & 0.5,0.4 & 0.6,0.7 \\ 0.3,0.2 & 0.6,0.5 & 0.8,0.9 \end{bmatrix} \quad (5)$$

After satisfying the above condition, $(\bar{g}_{\bar{\mu}}, N)$ is a possibility fuzzy soft ordered semigroup. Now we will evaluate the combine results of the committee members, an AND operation must be applied. Let us define $(\bar{h}_{\bar{\delta}}, N)$ by using Theorem 4, keeping in mind that $(\bar{h}_{\bar{\delta}}, N) = (\bar{f}_{\bar{v}}, N) \wedge (\bar{g}_{\bar{\mu}}, N)$, we obtain:

$$\bar{h}_{\bar{\delta}} = \begin{bmatrix} 0.2, 0.6 & 0.5, 0.4 & 0.6, 0.7 \\ 0.3, 0.2 & 0.6, 0.5 & 0.8, 0.9 \\ 0.3, 0.1 & 0.5, 0.4 & 0.7, 0.8 \\ 0.2, 0.1 & 0.5, 0.4 & 0.6, 0.7 \end{bmatrix} \quad (6)$$

The above matrix clearly shows that the membership value as well as the possible membership value for the third player is the highest. Hence, committee choose third player. Additionally, $(\bar{h}_{\bar{\delta}}, N)$ is a possibility fuzzy soft ordered semigroup of Z.

4. Possibility Fuzzy Soft Interior Ideals

This section elaborates some innovative concepts of possibility fuzzy soft left (resp. right) ideals and possibility fuzzy soft interior ideals. Some relatable classes of ordered semigroup including regular, intra-regular, and simple ordered relations are defined along with their suitable examples.

Definition 15. *Let $(\bar{f}_{\bar{v}}, N)$ be a PFSS over Z then a possibility fuzzy soft set $(\bar{g}_{\bar{\mu}}, L)$ over Z is called a possibility fuzzy soft l-ideal (resp. r-ideal) of $(\bar{f}_{\bar{v}}, N)$ represented as $(\bar{g}_{\bar{\mu}}, L) \lhd_r (\bar{f}_{\bar{v}}, N)$ (resp. $((\bar{g}_{\bar{\mu}}, L) \lhd_r (\bar{f}_{\bar{v}}, N))$), if it follows:*

(1) $L \subseteq N$
(2) $\forall n \in L$, $\bar{g}_{\bar{\mu}}(n)$ is a fuzzy soft left ideal (resp. right ideal) of $\bar{f}_{\bar{v}}(n)$ implies$(\bar{g}_{\bar{\mu}}, L) \lhd_r (\bar{f}_{\bar{v}}, N)$ (resp. $((\bar{g}_{\bar{\mu}}, L) \lhd_r (\bar{f}_{\bar{v}}, N))$)

If $(\bar{g}_{\bar{\mu}}, L)$ is l-ideal and r-ideal of $(\bar{f}_{\bar{v}}, N)$, then $(\bar{g}_{\bar{\mu}}, L)$ is a possibility fuzzy soft ideal of $(\bar{f}_{\bar{v}}, N)$, can be further denoted as $(\bar{g}_{\bar{\mu}}, L) \lhd (\bar{f}_{\bar{v}}, N)$.

Example 2. *Let $Z = \{w_1, w_2, w_3, w_4, w_5\}$ be an ordered semigroup as defined in Table 3 and ordered relation as represented by Figure 3:*

$$\leq := \{(w_1, w_1), (w_1, w_3), (w_1, w_4), (w_1, w_5), (w_2, w_2), (w_2, w_3), (w_2, w_4),$$
$$(w_2, w_5), (w_3, w_3), (w_3, w_4), (w_3, w_5), (w_4, w_4), (w_5, w_5), \}$$

Let $(\bar{f}_{\bar{v}}, N)$ be a possibility fuzzy soft set defined under $(Z, ., \leq)$, where $N = \{n_1, n_2, n_3\}$ and $\bar{f}_{\bar{v}}: N \to F(Z) \times I(Z)$. Let L be a set of parameters such that $L \subseteq N$ and then define $\bar{g}_{\bar{\mu}}: L \to F(Z) \times I(Z)$ where $\bar{g}_{\bar{\mu}}(n) = \bar{g}(n)(w_i), \bar{\mu}(n)(w_i) \forall w_i \in Z$

$$\bar{g}_{\bar{\mu}}(\mathrm{n}) = \begin{bmatrix} 0.8, 0.9 & 0.3, 0.4 & 0.5, 0.6 & 0.4, 0.5 & 0.5, 0.6 \\ 0.7, 0.8 & 0.5, 0.5 & 0.6, 0.7 & 0.5, 0.6 & 0.6, 0.7 \\ 0.6, 0.7 & 0.4, 0.4 & 0.5, 0.6 & 0.4, 0.4 & 0.5, 0.6 \end{bmatrix} \quad (7)$$

Table 3. Multiplication table.

$\cdot$	w_1	w_2	w_3	w_4	w_5
w_1	w_1	w_1	w_1	w_1	w_1
w_2	w_1	w_1	w_1	w_1	w_1
w_3	w_1	w_1	w_3	w_3	w_5
w_4	w_1	w_1	w_3	w_4	w_5

w_5	w_1	w_1	w_3	w_3	w_5

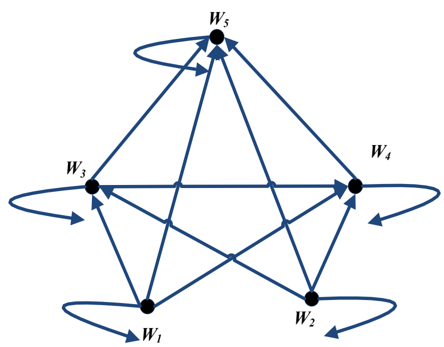

Figure 3. Hesse diagram.

As in Equation (7) $\bar{g}_{\bar{\mu}}(n)(w_i \cdot w_j) \geq \bar{g}_{\bar{\mu}}(n)(w_i)$*,* $\forall w_i, w_j \in Z$ *hence* $\bar{g}_{\bar{\mu}}(n)$ *is a fuzzy soft right ideal of* $\bar{f}_{\bar{\upsilon}}(n)$*. Similarly,* $\bar{g}_{\bar{\mu}}(n)(w_i \cdot w_j) \geq \bar{g}_{\bar{\mu}}(n)(w_j)$ *implies* $\bar{g}_{\bar{\mu}}(n)$ *is a fuzzy soft left ideal of* $\bar{f}_{\bar{\upsilon}}(n)$*. Thus,* $(\bar{g}_{\bar{\mu}}, L) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, L) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$*, hence* $\bar{g}_{\bar{\mu}}(n)$ *is a Possibility fuzzy soft ideal of* $\bar{f}_{\bar{\upsilon}}(n)$*.*

Theorem 6. *Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *be a PFSS over Z, then for any two possibility fuzzy soft sets* $(\bar{g}_{\bar{\mu}}|_1, L_1)$ *and* $(\bar{g}_{\bar{\mu}}|_2, L_2)$ *over Z, where* $L_1 \cap L_2 \neq \emptyset$ *then following relations can be proved:*

1. *If* $(\bar{g}_{\bar{\mu}}|_1, L_1) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}|_2, L_2) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ *then* $(\bar{g}_{\bar{\mu}}|_1, L_1) \cap (\bar{g}_{\bar{\mu}}|_2, L_2) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$
2. *If* $(\bar{g}_{\bar{\mu}}|_1, L_1) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}|_2, L_2) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$ *then* $(\bar{g}_{\bar{\mu}}|_1, L_1) \cap (\bar{g}_{\bar{\mu}}|_2, L_2) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$

Proof. Intersection of any two possibility fuzzy soft sets is defined as $(\bar{g}_{\bar{\mu}}|_1, L_1) \cap (\bar{g}_{\bar{\mu}}|_2, L_2) = (\bar{g}_{\bar{\mu}}, L)$, where $L_1 \cap L_2 = L$ then $\forall n \in L$ implies $n \in L_1$ and $n \in L_2$ so either $\bar{g}_{\bar{\mu}}(n) = \bar{g}_{\bar{\mu}}|_1(n)$ or $\bar{g}_{\bar{\mu}}(n) = \bar{g}_{\bar{\mu}}|_2(n)$ also $L \sqsubseteq N$ hence $(\bar{g}_{\bar{\mu}}, L)$ is a PFSS over *Z* using Theorem 3 Since $(\bar{g}_{\bar{\mu}}|_1, L_1) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ implies $(\bar{g}_{\bar{\mu}}, L) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ as $(\bar{g}_{\bar{\mu}}|_1, L_1) \cap (\bar{g}_{\bar{\mu}}|_2, L_2) = (\bar{g}_{\bar{\mu}}, L) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$.

Similarly, we can prove the 2nd relation. □

Theorem 7. *Let* $(\bar{f}_{\bar{\upsilon}}, N)$ *be a PFSS over S, then for any two possibility fuzzy soft sets* $(\bar{g}_{\bar{\mu}}, M)$ *and* $(\bar{h}_{\bar{\delta}}, O)$*over S, where* $M \cap O = \emptyset$*, then following relations can be proved:*

1. *If* $(\bar{g}_{\bar{\mu}}, M) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{h}_{\bar{\delta}}, O) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$ *then* $(\bar{g}_{\bar{\mu}}, M) \cup (\bar{h}_{\bar{\delta}}, O) \lhd_l (\bar{f}_{\bar{\upsilon}}, N)$.
2. *If* $(\bar{g}_{\bar{\mu}}, M) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{h}_{\bar{\delta}}, O) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$ *then* $(\bar{g}_{\bar{\mu}}, M) \cup (\bar{h}_{\bar{\delta}}, O) \lhd_r (\bar{f}_{\bar{\upsilon}}, N)$

Proof. The union of any two possibility fuzzy soft sets is defined $\bar{g}_{\bar{\mu}}(n) \cup \bar{h}_{\bar{\delta}}(n) = \bar{k}_{\bar{\gamma}}(n)$ where $M \cup O = K$:

$$\bar{k}_{\bar{\gamma}}(\mathrm{n}) = \begin{cases} \bar{g}_{\bar{\mu}}(\mathrm{n}) & if \quad n \in M - O, \\ \bar{h}_{\bar{\delta}}(\mathrm{n}) & if \quad n \in O - M, \\ \bar{g}_{\bar{\mu}}(\mathrm{n}) \bigcup \bar{h}_{\bar{\delta}}(\mathrm{n}) & if \quad n \in M \bigcap O. \end{cases} \tag{8}$$

Since here $M \cap O = \emptyset$ so either $n \in M - O$ or $n \in O - M$. If $n \in M - O$ then $\bar{k}_{\bar{\gamma}}(n) = \bar{g}_{\bar{\mu}}(n)$ where $\bar{g}_{\bar{\mu}}(n)$ is a left ideal of $\bar{f}_{\bar{v}}(n)$. Thus, $\bar{k}_{\bar{\gamma}}(n)$ is also a left ideal of $\bar{f}_{\bar{v}}(n)$. Thus, $(\bar{k}_{\bar{\gamma}}, K) \lhd_l (\bar{f}_{\bar{v}}, N)$. If $n \in O - M$ then $\bar{k}_{\bar{\gamma}}(n) = \bar{h}_{\bar{\delta}}(n)$ where $\bar{h}_{\bar{\delta}}(n)$ is a left ideal of $\bar{f}_{\bar{v}}(n)$. Thus, $\bar{k}_{\bar{\gamma}}(n)$ is also a left ideal of $\bar{f}_{\bar{v}}(n)$. Thus, $(\bar{k}_{\bar{\gamma}}, K) \lhd_l (\bar{f}_{\bar{v}}, N)$. Hence, we have $(\bar{g}_{\bar{\mu}}, M) \cup (\bar{h}_{\bar{\delta}}, O) \lhd_l (\bar{f}_{\bar{v}}, N)$.

Similarly, we can prove $(\bar{g}_{\bar{\mu}}, M) \cup (\bar{h}_{\bar{\delta}}, O) \lhd_r (\bar{f}_{\bar{v}}, N)$. □

Definition 16. *Let $(\bar{f}_{\bar{v}}, N)$ be a possibility fuzzy soft ordered semigroup of Z. Then $(\bar{f}_{\bar{v}}, N)$ is said to be possibility fuzzy soft interior ideal of Z if:*

(1) $\bar{f}_{\bar{v}}(n)$ *is fuzzy subsemigroup of Z*

(2) $\bar{f}(n)(z_1 a z_2) \geq \bar{f}(n)(a) \forall a, z_1, z_2 \in Z$

(3) $\bar{v}(n)(z_1 a z_2) \geq \bar{v}(n)(a)$

Example 3. *Let $Z = \{v_1, v_2, v_3, v_4\}$ is an ordered semigroup of Z as defined in Table 4 and ordered relation.*

Table 4. Multiplication table.

.	v_1	v_2	v_3	v_4
v_1	v_1	v_1	v_1	v_1
v_2	v_1	v_1	v_1	v_1
v_3	v_1	v_1	v_1	v_2
v_4	v_1	v_1	v_2	v_3

$$\leq := \{(v_1, v_1), (v_2, v_2), (v_3, v_3), (v_4, v_4), (v_1, v_2)\}$$

Let $(\bar{f}_{\bar{v}}, N)$ be a possibility fuzzy soft set of $(Z, ., \leq)$, where $N = \{n_1, n_2, n_3\}$ then we can define a mapping $(\bar{f}_{\bar{v}}: N \to F(Z) \times I(Z))$ as $\bar{f}_{\bar{v}}(n) = \left(\bar{f}(n)(v_i), \bar{v}(n)(v_i)\right), \forall v_i \in Z$:

$$\bar{f}_{\bar{v}}(n) = \begin{bmatrix} 0.7, 0.6 & 0.6, 0.5 & 0.5, 0.4 & 0.5, 0.4 \\ 0.6, 0.5 & 0.5, 0.4 & 0.2, 0.3 & 0.1, 0.2 \\ 0.8, 0.7 & 0.5, 0.5 & 0.3, 0.2 & 0.3, 0.2 \end{bmatrix} \tag{9}$$

As $\bar{f}_{\bar{v}}(n)$ are fuzzy subsemigroup of Z:

$$\bar{f}(n)(v_1 a v_2) \geq \bar{f}(n)(a) \ \forall a, v_i, v_j \in Z \tag{10}$$

$$\bar{v}(n)(v_1 a v_2) \geq \bar{v}(n)(a) \ \forall a, v_i, v_j \in Z. \tag{11}$$

Thus, $(\bar{f}_{\bar{v}}, N)$ is possibility fuzzy soft interior ideal of Z.

Definition 17. *A possibility fuzzy soft set $(\bar{f}_{\bar{v}}, N)$ is called possibility fuzzy soft interior ideal over Z if and only if $\bar{f}_{\bar{v}}(n)$ is a possibility fuzzy soft interior ideal over Z $\forall n \in N$.*

Lemma 1. *Every possibility fuzzy soft ideal of Z is also a possibility fuzzy soft interior ideal of Z.*

Proof. The proof for Lemma 1 is straightforward and can easily be understand by Example 4. □

Example 4. *Let $Z = \{w_1, w_2, w_3, w_4\}$ is an ordered semigroup of Z as represented in Table 5 and ordered relation.*

Table 5. Multiplication table.

.	w_1	w_2	w_3	w_4
w_1	w_1	w_1	w_1	w_1
w_2	w_1	w_1	w_1	w_1
w_3	w_1	w_1	w_2	w_1
w_4	w_1	w_1	w_2	w_2

$$\leq := \{(w_1, w_1), (w_2, w_2), (w_3, w_3), (w_4, w_4), (w_1, w_2)\}$$

Let $(\bar{f}_{\bar{\upsilon}}, N)$ *be a possibility fuzzy soft set defined under* $(Z, ., \leq)$, *where* $N = \{n_1, n_2\}$ *and* $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$. *Let N be a set of parameters and then define* $\bar{f}_{\bar{\upsilon}}(n) = \left(\bar{f}(n)(w_i), \bar{\upsilon}(n)(w_i)\right), \forall w_i \in Z$

$$\bar{f}_{\bar{\upsilon}}(n) = \begin{bmatrix} 0.8, 0.9 & 0.6, 0.7 & 0.5, 0.6 & 0.2, 0.3 \\ 0.7, 0.8 & 0.5, 0.5 & 0.2, 0.3 & 0.3, 0.4 \end{bmatrix} \tag{12}$$

As $\bar{f}_{\bar{\upsilon}}(n)(w_i \cdot w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(w_i)$ $\forall w_i, w_j \in Z$ *hence* $\bar{f}_{\bar{\upsilon}}(n)$ *is a fuzzy soft right ideal of Z. Similarly,* $\bar{f}_{\bar{\upsilon}}(n)(w_i \cdot w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(w_j)$ *implies* $\bar{f}_{\bar{\upsilon}}(n)$ *is a fuzzy soft left ideal of Z. Hence,* $(\bar{f}_{\bar{\upsilon}}, N)$ *is possibility fuzzy soft ideal of Z. Here* $\bar{f}_{\bar{\upsilon}}(n)(w_i a w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(a)$ $\forall a, w_i, w_j \in Z$. *Thus,* $(\bar{f}_{\bar{\upsilon}}, N)$ *is also possibility fuzzy soft interior ideal of Z.*

Hence, it is obvious that every possibility fuzzy soft ideal of Z is also a possibility fuzzy soft interior ideal of Z. But the converse is not true for all as elaborated in the following example.

Example 5. *Let* $Z = \{w_1, w_2, w_3, w_4\}$ *is an ordered semigroup of Z as represented in Example 4.*

$$\leq := \{(w_1, w_1), (w_2, w_2), (w_3, w_3), (w_4, w_4), (w_1, w_2)\}$$

Let $(\bar{f}_{\bar{\upsilon}}, N)$ *be a possibility fuzzy soft set defined under* $(Z, ., \leq)$, *where* $N = \{n_1, n_2\}$ *and* $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$. *Let N be a set of parameters and then define* $\bar{f}_{\bar{\upsilon}}(n) = \left(\bar{f}(n)(w_i), \bar{\upsilon}(n)(w_i)\right), \forall w_i \in Z$:

$$\bar{f}_{\bar{\upsilon}}(n) = \begin{bmatrix} 0.8, 0.9 & 0.5, 0.6 & 0.6, 0.7 & 0.2, 0.3 \\ 0.7, 0.8 & 0.2, 0.3 & 0.5, 0.5 & 0.3, 0.4 \end{bmatrix} \tag{13}$$

Thus, from Definition 16 $(\bar{f}_{\bar{\upsilon}}, N)$ *is possibility fuzzy soft interior ideal of Z. But the inequality* $\bar{f}_{\bar{\upsilon}}(n)(w_i a w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(w_j)$ *doesn't satisfy here for all the value of Z implies* $\bar{f}_{\bar{\upsilon}}(n)$ *is not a fuzzy soft left ideal of Z. Hence,* $(\bar{f}_{\bar{\upsilon}}, N)$ *is not possibility fuzzy soft ideal of Z.*

Proposition 1. *If Z is a regular ordered semigroup then for all possibility fuzzy soft interior ideal is a possibility fuzzy soft ideal of Z.*

Proof. Let $(\bar{f}_{\bar{\upsilon}}, N)$ be a possibility fuzzy soft interior ideal of Z and for all $z_1, z_2 \in Z$, $\exists a \in Z$ such that $z_1 \leq z_1 a\, z_1$ then by using the Lemma 1:

$$\bar{f}_{\bar{\upsilon}}(n)(z_1 z_2) \geq \bar{f}_{\bar{\upsilon}}(n)((z_1 a z_1) z_2) = \bar{f}_{\bar{\upsilon}}(n)((z_1 a) z_1 z_2) \geq \bar{f}_{\bar{\upsilon}}(n)(z_1) \tag{14}$$

Similarly:

$$\bar{f}_{\bar{\upsilon}}(n)(z_1 z_2) \geq \bar{f}_{\bar{\upsilon}}(n)(z_1(z_2 a z_2)) = \bar{f}_{\bar{\upsilon}}(n)(z_1 z_2 (a z_2)) \geq \bar{f}_{\bar{\upsilon}}(n)(z_2) \tag{15}$$

Thus, from the above two relations it can be proved that $\bar{f}_{\bar{\upsilon}}(n)(z_1 z_2) \geq \bar{f}_{\bar{\upsilon}}(n)(z_1)$ also for all $n \in N$ and $z_1, z_2 \in Z$. Hence, $(\bar{f}_{\bar{\upsilon}}, N)$ is a possibility fuzzy soft ideal of Z. □

Corollary 1. *For regular ordered semigroups, both the notion of possibility fuzzy soft ideal and possibility fuzzy soft interior ideals over Z must satisfy.*

Theorem 8. *Let for an ordered semigroup Z,* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be two possibility fuzzy soft left (resp. right) ideals of Z then for all* $M \cap N \neq \phi$, $(\bar{h}_{\bar{\delta}}, O) = (\bar{f}_{\bar{\upsilon}}, N) \wedge (\bar{g}_{\bar{\mu}}, M)$ *is also a possibility fuzzy soft left (resp. right) ideal of Z.*

Proof. Let $(\bar{f}_{\bar{\upsilon}}, N)$ and $(\bar{g}_{\bar{\mu}}, M)$ be two possibility fuzzy soft left (resp. right) ideals of Z. By using Theorem 4, $\forall n \in O = M \cap N$ there exist $\bar{f}_{\bar{\upsilon}}(n) \wedge (\bar{g}_{\bar{\mu}}(n) = \bar{h}_{\bar{\delta}}(n)$ and since $\bar{f}_{\bar{\upsilon}}(n)$ and $\bar{g}_{\bar{\mu}}(n)$ are both possibility fuzzy soft left (resp. right) ideal of Z their intersection is also a possibility fuzzy soft left (resp. right) ideal of Z. Thus, $\bar{h}_{\bar{\delta}}(n)$ is a possibility fuzzy soft left (resp. right) ideal of Z. Accordingly, $(\bar{h}_{\bar{\delta}}, O) = (\bar{f}_{\bar{\upsilon}}, N) \wedge (\bar{g}_{\bar{\mu}}, M)$ is a possibility fuzzy soft left (resp. right) ideal of Z. □

Theorem 9. *Let for an ordered semigroup Z,* $(\bar{f}_{\bar{\upsilon}}, N)$ *and* $(\bar{g}_{\bar{\mu}}, M)$ *be two possibility fuzzy soft left (resp. right) ideals of Z then for all* $M \cap N = \emptyset$, $(\bar{h}_{\bar{\delta}}, O) = (\bar{f}_{\bar{\upsilon}}, N) \vee (\bar{g}_{\bar{\mu}}, M)$ *is also a possibility fuzzy soft left(resp. right) ideal of Z.*

Proof. The proof directly follows Theorem 5. □

Example 6. *Suppose a company is hiring an employer and the CEO will choose between three internees working there for past six months.*

Let $Z = \{W_1, W_2, W_3\}$ *be a set of three internees and we have defined in Table 6 and ordered relation on the basis of their regularity in last six months.*

Table 6. Multiplication table defined on internee's regularity.

.	W_1	W_2	W_3
W_1	W_3	W_2	W_1
W_2	W_2	W_2	W_3
W_3	W_3	W_3	W_3

$$\leq := \{(W_1, W_1), (W_2, W_2), (W_3, W_3), (W_1, W_3), (W_2, W_3)\}$$

Let the set of parameters N includes the qualities list that CEO is looking for.

Here $N = \{n_1, n_2, n_3\}$ *is a set of parameters, where* n_1 *is qualification;* n_2 *is behavior; and* n_3 *is achievements.*

Then we can define $(\bar{f}_{\bar{\upsilon}}, N)$ *as* $\bar{f}_{\bar{\upsilon}}: N \to F(Z) \times I(Z)$:

$$\bar{f}_{\bar{\upsilon}} = \begin{bmatrix} 0.5, 0.4 & 0.6, 0.5 & 0.9, 0.8 \\ 0.4, 0.2 & 0.6, 0.4 & 0.7, 0.6 \\ 0.6, 0.4 & 0.7, 0.5 & 0.8, 0.5 \end{bmatrix} \quad (16)$$

Here $\bar{f}_{\bar{\upsilon}}(n)$ *represents the membership values and possible membership values attain by the internees and given by the CEO.*

Hence, $\bar{f}_{\bar{\upsilon}}(n) \neq \emptyset$, *also* $\bar{f}_{\bar{\upsilon}}(n)$ *is a fuzzy subsemigroup of Z. Thus,* $(\bar{f}_{\bar{\upsilon}}, N)$ *is a possibility fuzzy soft ordered semigroup of Z. Additionally, noticing* $\bar{f}_{\bar{\upsilon}}$, *we clearly concluded that the third internee attains maximum scores for all the parameters with respect to ordered semigroup relation defined on the base of their regularity. Also* $\bar{f}_{\bar{\upsilon}}(n)(w_i . w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(w_i)$ *implies* $\bar{f}_{\bar{\upsilon}}(n)$ *is a possibility fuzzy soft right ideal of Z. However,* $\bar{f}_{\bar{\upsilon}}(n)(w_i . w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(w_j)$ *does not satisfy here for all the value of Z. Thus, this implies* $\bar{f}_{\bar{\upsilon}}(n)$ *is not a fuzzy soft left ideal of Z. Similarly,* $\bar{f}_{\bar{\upsilon}}(n)(w_i a w_j) \geq \bar{f}_{\bar{\upsilon}}(n)(a)$, $\forall a, w_i, w_j \in Z$ *does not satisfy here for all values of Z Thus, this implies* $(\bar{f}_{\bar{\upsilon}}, N)$ *is not a possibility fuzzy soft interior ideal of Z. Possibility fuzzy soft ordered semigroups help in providing more precise and accurate results that help in solving decision sciences problems.*

5. Possibility Fuzzy Soft Simple Ordered Semigroup

Let $(Z, ., \leq)$ be an ordered semigroup and $(\bar{f}_{\bar{\upsilon}}, N)$ be a possibility fuzzy soft set over Z. Then for all $n \in N$ and $z \in Z$, a mapping $\bar{f}_{\bar{\upsilon}}: N \to I_z$ such that $(I_z(n), M)$ is a PFSS over Z and is defined as, $I_z(n) := \{a \in Z | \bar{f}_{\bar{\upsilon}}(n)(a) \geq \bar{f}_{\bar{\upsilon}}(n)(z)\} \forall n \in M$.

Proposition 2. *Let Z be an ordered semigroup and* $(\bar{f}_{\bar{\upsilon}}, N)$ *be a possibility fuzzy soft right ideal over Z, then* (I_z, M) *is a soft right ideal of Z.*

Proof. Let $(\bar{f_{\bar{\upsilon}}}, N)$ be a possibility fuzzy soft right ideal over Z and if $z \in Z$ there exists $I_z(n) \neq \emptyset$ $n \in M$ then we have $\bar{f_{\bar{\upsilon}}}(n)(z) \geq \bar{f_{\bar{\upsilon}}}(n)(z)$ so $z \in I_z(n)$ and $I_z(n) \neq \emptyset$. Let $a \in I_z(n)$ and for any $t \in Z$, $t \leq a$ then $t \in I_z(n)\ \forall n \in M$. Since $(\bar{f_{\bar{\upsilon}}}, N)$ is a possibility fuzzy soft right ideal over Z and $t \leq a$ then for every $n \in M$ we have $\bar{f_{\bar{\upsilon}}}(n)(t) \geq \bar{f_{\bar{\upsilon}}}(n)(a)$ as $a \in I_z(n)$ implies, $\bar{f_{\bar{\upsilon}}}(n)(a) \geq \bar{f_{\bar{\upsilon}}}(n)(t)$ and so, $\bar{f_{\bar{\upsilon}}}(n)(t) \geq \bar{f_{\bar{\upsilon}}}(n)(z)$ Hence, $t \in I_z(n)$. Now let $a \in I_z(n)$ for every $n \in M$, $t \in Z$ then here we prove that $at \in I_z(n)$. As $(\bar{f_{\bar{\upsilon}}}, N)$ is a possibility fuzzy soft right ideal over Z here $n \in M$:

$$\bar{f_{\bar{\upsilon}}}(n)(at) \geq \bar{f_{\bar{\upsilon}}}(n)(a) \text{ as } a \in I_z(n), \tag{17}$$

$$\bar{f_{\bar{\upsilon}}}(n)(a) \geq \bar{f_{\bar{\upsilon}}}(n)(z) \tag{18}$$

$$\bar{f_{\bar{\upsilon}}}(n)(at) \geq \bar{f_{\bar{\upsilon}}}(n)(z) \tag{19}$$

Hence, $at \in I_z(n)$ such that $I_z(n)Z \subseteq I_z(n)$, so $I_z(n)$ is a soft right ideal of Z.

Similarly, the proposition can easily be proved for left ideals following the above proof. □

Theorem 10. *Let Z be an ordered semigroup and $(\bar{f_{\bar{\upsilon}}}, N)$ be a possibility fuzzy soft ideal over Z, then (I_z, M) is a soft ideal of Z, for all $z \in Z$.*

Definition 18. *Let Z be an ordered semigroup and $(\bar{f_{\bar{\upsilon}}}, N)$ be a possibility fuzzy soft ordered semigroup over Z then Z is called possibility fuzzy left (resp. right) simple if and only if it is possibility fuzzy soft left (resp. right) ideal $(\bar{f_{\bar{\upsilon}}}, N)$ over Z with $\bar{f_{\bar{\upsilon}}}(n)(z_1) = \bar{f_{\bar{\upsilon}}}(n)(z_2)$ for all $n \in N$ and $z_1, z_2 \in Z$.*

An ordered semigroup Z is called possibility fuzzy soft simple if it is both possibility fuzzy soft left and right simple.

Example 7. *Let $Z = \{z_1, z_2, z_3\}$ is an ordered semigroup of Z as represented in Table 7 and ordered relation.*

Table 7. Multiplication table.

.	z_1	z_2	z_3
z_1	z_3	z_2	z_1
z_2	z_2	z_2	z_3
z_3	z_3	z_3	z_3

$$\leq := \{(z_1, z_1), (z_2, z_2), (z_3, z_3), (z_1, z_3), (z_2, z_3)\}$$

Let a possibility fuzzy soft set $(\bar{f_{\bar{\upsilon}}}, N)$ is defined by a mapping $\bar{f_{\bar{\upsilon}}}: N \to F(Z) \times I(Z)$. Where N is a set of parameters and then $\bar{f_{\bar{\upsilon}}}(n) = \bar{f}\ (n)\ (z_i)$, $\bar{\upsilon}(n)(z_i) \forall z_i \in Z$ where:

$$\bar{f}_{\bar{\upsilon}}(n) = \begin{bmatrix} (0.9, 0.8) & (0.9, 0.8) & (0.9, 0.8) \\ (0.2, 0.5) & (0.2, 0.5) & (0.2, 0.5) \end{bmatrix} \tag{20}$$

As $\bar{f_{\bar{\upsilon}}}(n)$ are fuzzy subsemigroups of Z, thus, $(\bar{f_{\bar{\upsilon}}}, N)$ is a possibility fuzzy soft left (resp. right) ideal of Z. Additionally, $\bar{f_{\bar{\upsilon}}}(n)(z_1) = \bar{f_{\bar{\upsilon}}}(n)(z_2)$ for all $n \in N$ and $z_1, z_2 \in Z$. Hence, $(\bar{f_{\bar{\upsilon}}}, N)$ is a possibility fuzzy soft simple ideal of Z.

Theorem 11. *An ordered semigroup Z is called soft simple if it is possibility fuzzy soft simple.*

Proof. Let $(\bar{f_{\bar{\upsilon}}}, N)$ be a possibility fuzzy soft ideal over Z and let $z_1, z_2 \in Z$ then by Theorem 10 (I_{z_1}, M) is a soft ideal of Z. As Z is a soft simple, it means that $I_{z_1}(n) = Z, \forall\, n \in M$ and $\forall z_2 \in I_{z_1}(n)$. We have $\bar{f_{\bar{\upsilon}}}(n)(z_1) \geq \bar{f_{\bar{\upsilon}}}(n)(z_2) \forall\, n \in M$ similarly, $\bar{f_{\bar{\upsilon}}}(n)(z_2) \geq \bar{f_{\bar{\upsilon}}}(n)(z_1)$, hence $\bar{f_{\bar{\upsilon}}}(n)(z_1) = \bar{f_{\bar{\upsilon}}}(n)(z_2)$. Thus, Z is a possibility fuzzy soft simple.

Conversely, let (I, M) be a soft ideal over Z such that $I(\mathrm{n}) \neq Z, n \in M$. Since (I, M) is a soft ideal over Z, then it is obvious that $I(\mathrm{n})$ is a soft ideal over Z. By Definition 6, every soft ideal is also a fuzzy soft ideal hence $\bar{g}_{I(m)}(n)$ is a fuzzy ideal of Z also $\bar{\mu}_{I(m)}(n)$ is fuzzy ideal of Z, $\forall\, n \in N$ implies $\bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}$ is possibility fuzzy ideal of Z. Since Z is fuzzy simple Thus, $\bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}(n)$ must be a constant function that is $\bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}(n)(z_1) = \bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}(n)(z_2)$ for all $z_1, z_2 \in Z$, Thus, for any $a \in I(m)$ we can write $\bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}(n)(z_1) = \bar{\mathrm{g}}_{\bar{\mu}}|_{I(m)}(n)(z_2) = 1$ and so $a \in I(m)$ thus, therefore, $Z = I(m)$, which is a contradiction, so Z is a soft simple. □

Theorem 12. *An ordered semigroup Z is soft simple if and only if for every possibility fuzzy soft interior ideal* $(\bar{f_{\bar{v}}}, N)$ *over Z,* $\bar{f_{\bar{v}}}(n)(z_1) = \bar{f_{\bar{v}}}(n)(z_2)$ *for all* $n \in N$ *and* $z_1, z_2 \in Z$.

Proof. Let an ordered semigroup Z is soft simple Assume that $(\bar{f_{\bar{v}}}, N)$ be a possibility fuzzy soft interior ideal over Z and $z_1, z_2 \in Z$. As Z is a simple and $\forall z_2 \in Z$, $Z = (Zz_2Z]$, then $z_1 \le az_2b$ for any $a, b \in Z$. As $(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft interior ideal over Z, Thus, we can write, $\bar{f_{\bar{v}}}(n)(z_1) \ge \bar{f_{\bar{v}}}(n)(az_2b$ as $\bar{f_{\bar{v}}}(n)(az_2b) = \bar{f_{\bar{v}}}(n)(a(z_2b))$ implies $\bar{f_{\bar{v}}}(n)(a(z_2b)) \ge \bar{f_{\bar{v}}}(n)(z_2)$, Hence, $\bar{f_{\bar{v}}}(n)(z_1) \ge \bar{f_{\bar{v}}}(n)(z_2)$. Following the same steps we can prove $\bar{f_{\bar{v}}}(n)(z_1) \le \bar{f_{\bar{v}}}(n)(z_2)$. Thus, $\bar{f_{\bar{v}}}(n)(z_1) = \bar{f_{\bar{v}}}(n)(z_2)$.

Conversely, $(\bar{f_{\bar{v}}}, N)$ be a possibility fuzzy soft interior ideal over Z with $\bar{f_{\bar{v}}}(n)(z_1) = \bar{f_{\bar{v}}}(n)(z_2)$ $n \in N$, then by Proposition 1 $(\bar{f_{\bar{v}}}, N)$ is also a possibility fuzzy soft ideal over Z. Thus, by Theorem 11, Z is a possibility fuzzy soft simple with $\bar{f_{\bar{v}}}(n)(z_1) = \bar{f_{\bar{v}}}(n)(z_2)$. Hence, Z is a soft simple. □

Theorem 13. *An ordered semigroup Z is intra-regular if for every possibility fuzzy soft ideal* $(\bar{f_{\bar{v}}}, N)$ *over Z,* $\bar{f_{\bar{v}}}(n)(z) = \bar{f_{\bar{v}}}(n)(z^2)$ *for all* $n \in N$ *and* $z \in Z$.

Proof. Let for an intra-regular ordered semigroup Z, suppose $(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft ideal then for any $z \in Z$ there exist $a, b \in Z$ such that $a \le az^2b$. As $(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft ideal Thus, we can write, $\bar{f_{\bar{v}}}(n)(z) \ge \bar{f_{\bar{v}}}(n)(az^2b)$ as $\bar{f_{\bar{v}}}(n)(az^2b) = \bar{f_{\bar{v}}}(n)(a(z^2b))$ implies $\bar{f_{\bar{v}}}(n)(a(z^2b)) \ge \bar{f_{\bar{v}}}(n)(z^2b)$ ($(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft left ideal). Also $\bar{f_{\bar{v}}}(n)(z^2b) \ge \bar{f_{\bar{v}}}(n)(z^2)$, $(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft right ideal. Thus, $\{\bar{f_{\bar{v}}}(n)(z) \ge \bar{f_{\bar{v}}}(n)(z^2) = \bar{f_{\bar{v}}}(n)(z.z) \ge \bar{f_{\bar{v}}}(n)(z)\}$ and, hence, $\bar{f_{\bar{v}}}(n)(z) = \bar{f_{\bar{v}}}(n)(z^2)$. □

6. Semiprime Possibility Fuzzy Soft Ideals

Let Z be an ordered semigroup, a possibility fuzzy soft ideal $(\bar{f_{\bar{v}}}, N)$ is called a semiprime possibility fuzzy soft ideal over Z if and only if $(\bar{f_{\bar{v}}}, N)$ is a semiprime possibility fuzzy ideal of Z. That is for all $n \in N$ and $z \in Z$, $\bar{f_{\bar{v}}}(n)(z) \ge \bar{f_{\bar{v}}}(n)(z^2)$.

Lemma 2. *Let Z be an ordered semigroup then a soft set* (f, M) *over Z is a semiprime soft ideal over Z if and only if* $(\bar{f_{\bar{v}}}, N)$ *is a semiprime possibility fuzzy soft ideal over Z.*

Proof. Let (f, M) is a semiprime soft ideal over Z. By Definition 8 $f(n)$ is also a semiprime soft ideal over Z then suppose $z \in Z$ such that $z^2 \epsilon \bar{f}(n)$ also $z^2 \epsilon \bar{v}(n)$ for possibility membership value concluding $\bar{f_{\bar{v}}}(n)(z^2) = 1$, $n \in N$. Since $f(n)$ is a semiprime ideal so $z \epsilon \bar{f_{\bar{v}}}(n)$ then $\bar{f_{\bar{v}}}(n)(z) = 1 = \bar{f_{\bar{v}}}(n)(z^2)$. Contrary, suppose $z^2 \notin \bar{f_{\bar{v}}}(n)$, then $\bar{f_{\bar{v}}}(n)(z) \ge 0 = \bar{f_{\bar{v}}}(n)(z^2)$. Hence, $\bar{f_{\bar{v}}}(n)$ is a semiprime possibility fuzzy soft ideal over Z. Thus, $(\bar{f_{\bar{v}}}, N)$ is a semiprime possibility fuzzy soft ideal over Z.

Conversely, let $(\bar{f_{\bar{v}}}(n), N)$ is a semiprime possibility fuzzy soft ideal over Z. Let $z^2 \epsilon \bar{f_{\bar{v}}}(n)$ for all $n \in M$ then $\bar{f_{\bar{v}}}(n)(z^2) = 1$. As $\bar{f_{\bar{v}}}(n)(z) \ge \bar{f_{\bar{v}}}(n)(z^2) = 1$ concluded that $\bar{f_{\bar{v}}}(n)(z) = 1$, thus, $z \epsilon \bar{f_{\bar{v}}}(n)$. Hence, $\bar{f_{\bar{v}}}(n)$ is a semiprime soft ideal over Z implies $f(\mathrm{n})$ is also a semiprime soft ideal over Z and so does (f, M), following Definition 8. □

Theorem 14. *An ordered semigroup Z is left (resp. right) regular if every possibility fuzzy soft left (resp. right) ideal over Z is semiprime.*

Proof. Let Z be a left regular ordered semigroup and $(\bar{f_{\bar{v}}}, N)$ is a possibility fuzzy soft left ideal over Z. As Z is left regular, by definition $\forall z \in Z$ $\exists$ $x \in Z$ such that $z \le xz^2$ Then we have $\bar{f_{\bar{v}}}(n)(z) \ge \bar{f_{\bar{v}}}(n)(xz^2) \ge \bar{f_{\bar{v}}}(n)(z^2)$. Additionally, $\bar{f_{\bar{v}}}(n)$ is a possibility fuzzy soft left ideal, hence, $(\bar{f_{\bar{v}}}, N)$ is a semiprime possibility fuzzy soft left ideal over Z. Similarly, we can prove for right regular ordered semigroups as well. □

Theorem 15. *An ordered semigroup Z is intra-regular if every possibility fuzzy soft left (resp. right) ideal over Z is semiprime.*

Proof. The proof directly follows the Theorem 14. □

Example 8. *Let $Z = \{z_1, z_2, z_3, z_4\}$ be an ordered semigroup defined in Table 8 and ordered relation respectively.*

Table 8. Multiplication table.

$\cdot$	z_1	z_2	z_3	z_4
z_1	z_1	z_1	z_1	z_1
z_2	z_1	z_1	z_1	z_1
z_3	z_1	z_1	z_2	z_1
z_4	z_1	z_1	z_2	z_2

$$\leq := \{(z_1, z_1), (z_2, z_2), (z_3, z_3), (z_4, z_4), (z_1, z_2)\}$$

Then possibility fuzzy soft set $(\bar{f_{\bar{v}}}, N)$ for parameters $N = \{n_1, n_2\}$ is defined by mapping,

$$\bar{f_{\bar{v}}}: N \to F(Z) \times I(Z)$$

Here we obtain:

$$\bar{f}_{\bar{v}}(n) = \begin{bmatrix} 0.7, 0.9 & 0.6, 0.7 & 0.5, 0.6 & 0.2, 0.3 \\ 0.6, 0.8 & 0.5, 0.4 & 0.2, 0.2 & 0.3, 0.3 \end{bmatrix} \tag{21}$$

As $\bar{f_{\bar{v}}}(n)$ is non-empty also $\bar{f_{\bar{v}}}(n)(z) \forall z \in Z$ are all fuzzy subsemigroup of Z then as per the Definition 13, $(\bar{f_{\bar{v}}}, N)$ is possibility fuzzy soft ordered semigroup over Z. As $\bar{f_{\bar{v}}}(n)(z_i \cdot z_j) \geq \bar{f_{\bar{v}}}(n)(z_j)$ implies $\bar{f_{\bar{v}}}(n)$ is a fuzzy soft left ideal of Z. Hence, $(\bar{f_{\bar{v}}}, N)$ is possibility fuzzy soft left ideal of Z. $(\bar{f_{\bar{v}}}, N)$ is also semiprime possibility fuzzy ideal of Z as per the Definition 8, along with the condition $\bar{f_{\bar{v}}}(n)(z) \geq \bar{f_{\bar{v}}}(n)(z^2)$ for all $n \in N$ and $z \in Z$.

Hence, $(\bar{f_{\bar{v}}}, N)$ is a semiprime possibility fuzzy soft ideal over Z.

7. Conclusions

This article was established to investigate a new subsystem named as possibility fuzzy soft ordered semigroups. An application to possibility fuzzy soft ordered semigroups in decision-making has been explained. As this theory relates two types of membership values that precisely characterize all the elements along with the suitable ordered semigroup relations, this leads to the correct and more appropriate choice by experts in decision-making. The presented work has introduced the novel concept of possibility fuzzy soft interior ideals and their relation with possibility fuzzy soft left (resp. right) ideals. Various algebraic relations related with the proposed theory have been investigated. Different applications of algebraic structures in applied sciences is a new direction of entrust for the researcher these days. This research can also be applied for solving such problems of applied sciences. This research will also lead:

(1) To initiate new notions including possibility fuzzy soft bi-ideals, possibility fuzzy soft generalized bi-ideals, possibility fuzzy soft quasi ideals, and so on.
(2) To derive the same approach for rough sets introducing possibility fuzzy rough ordered semigroups.
(3) To derive various application using possibility fuzzy soft ordered semigroups in decision-making.

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