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A Multi-Criteria Decision-Making Method Based on the Improved Single-Valued Neutrosophic Weighted Geometric Operator

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Abstract: The aggregation operator is one of the most common techniques to solve multi-criteria decision-making (MCDM) problems. The aim of this paper is to propose an MCDM method based on the improved single-valued neutrosophic weighted geometric (ISVNWG) operator. First, the defects of several existing single-valued neutrosophic weighted geometric aggregation operators in terms of producing uncertain results in some special cases are analyzed. Second, an ISVNWG operator is proposed to avoid the defects of existing operators. Further, the properties of the proposed ISVNWG operator, including idempotency, boundedness, monotonicity, and commutativity, are discussed. Finally, a single-valued neutrosophic MCDM method based on the developed ISVNWG operator is proposed to overcome the defects of existing MCDM methods based on existing operators. Application examples demonstrate that our proposed operator and corresponding MCDM method are effective and rational for avoiding uncertain results in some special cases.

Keywords: single-valued; neutrosophic numbers (SVNNs); improved single-valued neutrosophic geometric aggregation operator; multi-criteria decision-making (MCDM)

1. Introduction

Since Zadeh put forward the concept of fuzzy sets (FSs) [1], the multi-criteria decision-making (MCDM) methods based on FSs and their extensions, including intuitionistic fuzzy sets (IFSs) [2] and picture fuzzy sets (PFSs) [3], have been extensively used to deal with different decision-making problems [4–12]. However, because of the complexity of the real decision-making environment and the limitations of decision-makers' (DMs) knowledge, FSs and their extensions cannot accurately describe the uncertain preferences of DMs. On account of this, the definition of neutrosophic sets (NSs), including truth-membership, indeterminacy-membership, and falsity-membership degrees, is developed [13,14]. Notably, these three memberships in NSs are all subsets of non-standard unit interval $[0^-, 1^+]$, limiting the wide applications of NSs [15]. In order to further promote the applications of NSs, Wang et al. [16] and Ye [17] extended the scope of membership degrees from non-standard unit interval $[0^-, 1^+]$ to standard unit interval [0,1], and proposed the concept of single-valued neutrosophic sets (SVNSs) or simplified neutrosophic sets (SNSs). Since then, the single-valued neutrosophic aggregation operators [17–25], measures [26–33], and outranking relations [34,35], and the corresponding MCDM methods have been widely concerned by many scholars [36–40]. In particular, single-valued neutrosophic aggregation operators, as one of the main techniques of MCDM methods, have been successfully applied in different fields [17–25]. For example, Ye [17] proposed an MCDM method based on the single-valued neutrosophic weighted geometric (SVNWG) operator; Peng et al. [18] defined an MCDM method based on simplified neutrosophic weighted algebraic geometric (SNNWG) and simplified neutrosophic weighted Einstein geometric ($SNNWG^{\varepsilon}$) operators; Garg [19] proposed an MCDM method based on the single-valued neutrosophic weighted Frank geometric (SVNFWG) operator. With comprehensive consideration of those above single-valued geometric aggregation operators, Wu et al. [22] developed the simplified neutrosophic prioritized aggregation operator, Liu and Wang [23] and Liu et al. [24] proposed the single-valued neutrosophic weighted Bonferroni mean and Schweizer-Sklar prioritized aggregation operators, respectively.

However, these existing single-valued geometric aggregation operators and their corresponding MCDM methods have some considerable defects as follows. (1) Existing aggregation operators may produce uncertain results in the aggregation process, that is, the aggregated terms are uncertain in some special cases. (2) Those MCDM methods based on existing aggregation operators can also produce unreasonable decision-making results, which are contradictory to the real decision-making. On account of this, an improved single-valued neutrosophic weighted geometric (ISVNWG) operator is developed to avoid the defects of existing aggregation operators presented above. Moreover, a novel MCDM method based on the proposed aggregation operator is proposed to overcome the defects of existing MCDM methods based on corresponding aggregation operators.

The structure of this article is organized as follows. Section 2 reviews the related concepts, including SVNSs, single-valued neutrosophic number (SVNNs), and the comparison method of SVNNs. Section 3 analyzes the defects of existing aggregation operators. Then, Section 4 proposes the ISVNWG operator, and discusses its related properties. Further, Section 5 puts forward the corresponding MCDM method based on the proposed aggregation operator. In Section 6, the rationality of the proposed MCDM method is investigated in combination with three different application examples. Finally, some conclusions are drawn in Section 7.

2. Preliminaries

In this section, some related concepts, including SVNSs, SVNNs, and the comparison method of SVNNs, are reviewed.

Definition 1 [16,17]**.** *Let X be a space of points (objects), with a generic element in X, denoted by x. An SVNS, χ, in X is characterized by*

$$\chi = \left\{ \left\langle x, \vartheta_{\chi}(x), \tau_{\chi}(x), \xi_{\chi}(x) \right\rangle \middle| x \in X \right\}$$

where $\vartheta_{\chi}(x)$, $\tau_{\chi}(x)$, and $\xi_{\chi}(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively, satisfying $0 \le \vartheta_{\chi}(x) \le 1$, $0 \le \tau_{\chi}(x) \le 1$, and $0 \le \xi_{\chi}(x) \le 1$. Specially, if X has only one element, then $\chi = \left\langle \vartheta_{\chi}(x), \tau_{\chi}(x), \xi_{\chi}(x) \right\rangle$ is called an SVNN, which is denoted by $\chi = \left\langle \vartheta, \tau, \xi \right\rangle$.

Definition 2 [18]**.** *Let χ_1 and χ_2 be any two SVNNs. Then the comparison method of the two SVNNs is defined as:*

(1) If $s(\chi_1) > s(\chi_2)$, then $\chi_1 \succ \chi_2$;

(2) If $s(\chi_1) = s(\chi_2)$ and $a(\chi_1) > a(\chi_2)$, then $\chi_1 \succ \chi_2$;

(3) If $s(\chi_1) = s(\chi_2)$, $a(\chi_1) = a(\chi_2)$, and $c(\chi_1) > c(\chi_2)$, then $\chi_1 \succ \chi_2$;

(4) If $s(\chi_1)=s(\chi_2)$, $a(\chi_1)=a(\chi_2)$, *and* $c(\chi_1)=c(\chi_2)$, *then* $\chi_1=\chi_2$.

Here $s(\chi_i)=\frac{(\vartheta_i+1-\tau_i+1-\xi_i)}{3}$, $a(\chi_i)=\vartheta_i-\xi_i$, *and* $c(\chi_i)=\vartheta_i$ $(i=1,2)$ *denote the score function, the accuracy function, and the certainty function, respectively.*

3. Limitations of Existing Aggregation Operators

The defects of some existing aggregation operators are outlined as follows.

Definition 3 [17]. *Let* $\chi_i=(\vartheta_i,\tau_i,\xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs, and* $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ *be a weight vector for* $\chi_i(i=1,2,\ldots,n)$ *with* $0\le\omega_i\le1$ *and* $\sum_{i=1}^{n}\omega_i=1$.

Then, the SVNWG operator is a function as $\chi^n\to\chi$, *which is given by:*

$$SVNWG_\omega(\chi_1,\chi_2,\ldots,\chi_n)=\left\langle\prod_{i=1}^{n}\vartheta_i^{\omega_i},\prod_{i=1}^{n}\tau_i^{\omega_i},\prod_{i=1}^{n}\xi_i^{\omega_i}\right\rangle \quad (1)$$

The SVNWG aggregation operator, i.e., Equation (1) in Definition 3, is unreasonable, since the truth-membership, indeterminacy-membership, and falsity-membership values cannot be equal to 0. In other words, the SVNWG operator is invalid for some special cases. For example, if $\chi_1=\langle1,0,0\rangle$, $\chi_2=\langle0,0,1\rangle$, *and* $\chi_3=\langle0.3,0.4,0.5\rangle$ *are three SVNNs, and* $\omega=(0,0,1)^T$ *is the weight vector of three SVNNs, then according to our intuition, the aggregated result should be* $\langle0.3,0.4,0.5\rangle$. *However, based on the SVNWG operator, it is impossible to calculate the terms* $0^1\times0^0\times0.3^1$, $0^0\times0^0\times0.4^1$, *and* $0^0\times1^0\times0.5^1$, *as* 0^0 *is an indeterminate value. Hence, the expression of the SVNWG operator in Equation (1) is unreasonable.*

Definition 4 [18]. *Let* $\chi_i=(\vartheta_i,\tau_i,\xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs, and* $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ *be a weight vector for* $\chi_i(i=1,2,\ldots,n)$ *with* $0\le\omega_i\le1$ *and* $\sum_{i=1}^{n}\omega_i=1$.

Then, the SNNWG operator is a function as $\chi^n\to\chi$, *which is given by:*

$$SNNWG_\omega(\chi_1,\chi_2,\ldots,\chi_n)=\left\langle\prod_{i=1}^{n}\vartheta_i^{\omega_i},1-\prod_{i=1}^{n}(1-\tau_i)^{\omega_i},1-\prod_{i=1}^{n}(1-\xi_i)^{\omega_i}\right\rangle \quad (2)$$

The SNNWG operator, i.e., Equation (2) in Definition 4, is also unreasonable and has the same defects as discussed in Definition 3. Specially, the truth-membership value cannot be equal to 0, and the indeterminacy-membership and falsity-membership values cannot be equal to 1 in Equation (2). For example, if $\chi_1=\langle0,1,1\rangle$, $\chi_2=\langle0,0,1\rangle$, *and* $\chi_3=\langle0,1,0\rangle$ *are three SVNNs, and* $\omega=(0,0,1)^T$ *is the weight vector of three SVNNs, then the aggregated result should be* $\langle0,1,0\rangle$. *However, we cannot calculate the terms* $0^0\times0^0\times0^1$, $1-0^0\times1^0\times0^1$, *and* $1-0^0\times0^0\times1^1$, *which are indeterminate values. In other words, the SVNWG operator is invalid for such special cases.*

Definition 5 [18]. *Let* $\chi_i=(\vartheta_i,\tau_i,\xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs, and* $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ *be a weight vector for* $\chi_i(i=1,2,\ldots,n)$ *with* $0\le\omega_i\le1$ *and* $\sum_{i=1}^{n}\omega_i=1$.

Then, the $SNNWG^{\varepsilon}$ *operator is a function as* $\chi^{n}\to\chi$, *which is given by:*

$$SNNWG_{\omega}^{\varepsilon}(\chi_1,\chi_2,\ldots,\chi_n)=\left\langle \frac{2\prod_{i=1}^{n}\vartheta_i^{\omega_i}}{\prod_{i=1}^{n}(2-\vartheta_i)^{\omega_i}+\prod_{i=1}^{n}\vartheta_i^{\omega_i}}, \frac{\prod_{i=1}^{n}(1+\tau_i)^{\omega_i}-\prod_{i=1}^{n}(1-\tau_i)^{\omega_i}}{\prod_{i=1}^{n}(1+\tau_i)^{\omega_i}+\prod_{i=1}^{n}(1-\tau_i)^{\omega_i}}, \frac{\prod_{i=1}^{n}(1+\xi_i)^{\omega_i}-\prod_{i=1}^{n}(1-\xi_i)^{\omega_i}}{\prod_{i=1}^{n}(1+\xi_i)^{\omega_i}+\prod_{i=1}^{n}(1-\xi_i)^{\omega_i}} \right\rangle \quad (3)$$

However, the $SNNWG^{\varepsilon}$ operator, i.e., Equation (3) in Definition 5, is unreasonable. It has the same defects as the SNNWG operator, and cannot deal with some special cases where the truth-membership value is equal to 0, and the indeterminacy-membership and falsity-membership values are equal to 1. Similarly, assume $\chi_1=\langle 0,1,1\rangle$, $\chi_2=\langle 0,0,1\rangle$, and $\chi_3=\langle 0,1,0\rangle$ are three SVNNs, and that $\omega=(0,0,1)^T$ is a weight vector of three SVNNs, then the aggregated result should be $\langle 0,1,0\rangle$. However, if the $SNNWG^{\varepsilon}$ operator is used, then truth-membership, indeterminacy-membership, and falsity-membership of aggregated values all include the terms 0^0, which are indeterminate values. Thus, the $SNNWG^{\varepsilon}$ operator also has defects in the aggregation process.

Definition 6 [19]. *Let* $\chi_i=(\vartheta_i,\tau_i,\xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs, and* $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ *be a weight vector for* $\chi_i(i=1,2,\ldots,n)$ *with* $0\le\omega_i\le 1$ *and* $\sum_{i=1}^{n}\omega_i=1$, *and* $\lambda>1$. *Then, the SVNFWG operator is a function as* $\chi^{n}\to\chi$, *which is given by:*

$$SVNWG_{\omega}(\chi_1,\chi_2,\ldots,\chi_n)=\left\langle \log_{\lambda}\left(1+\prod_{i=1}^{n}\left(\lambda^{\vartheta_i}-1\right)^{\omega_i}\right), 1-\log_{\lambda}\left(1+\prod_{i=1}^{n}\left(\lambda^{1-\tau_i}-1\right)^{\omega_i}\right), 1-\log_{\lambda}\left(1+\prod_{i=1}^{n}\left(\lambda^{1-\xi_i}-1\right)^{\omega_i}\right)\right\rangle \quad (4)$$

Similarly, the SVNFWG operator, i.e., Equation (4) in Definition 6, is unreasonable. Apparently, the truth-membership value cannot be equal to 0, and the indeterminacy-membership and falsity-membership values cannot be equal to 1. Assume $\chi_1=\langle 0,1,1\rangle$, $\chi_2=\langle 0,1,0\rangle$, and $\chi_3=\langle 0,0,1\rangle$ are three SVNNs, and that $\omega=(0,0,1)^T$ is a weight vector of three SVNNs, then the aggregated result should be $\langle 0,0,1\rangle$. However, if the SVNFWG operator is used, and $\lambda=2$, then the aggregated result also includes the indeterminate term 0^0. Hence, the SVNFWG aggregation operator has the same limitations as previously discussed in Definitions 3–5.

4. The Improved Single-Valued Neutrosophic Weighted Geometric Operator

In the following, the ISVNWG operator is defined, and the corresponding properties are discussed as well.

Definition 7. *Let* $\chi_i=(\vartheta_i,\tau_i,\xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs, and* $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ *be a weight vector for* $\chi_i(i=1,2,\ldots,n)$ *with* $0\le\omega_i\le 1$, $\sum_{i=1}^{n}\omega_i=1$ *and* $0<t<1$. *Then, the ISVNWG operator is a function as* $\chi^{n}\to\chi$, *which is given by:*

$$ISVNWG_{\omega}(\chi_1,\chi_2,\ldots,\chi_n)=\left\langle 1-\frac{1}{t}\left(1-\prod_{i=1}^{n}\left(1-t(1-\vartheta_i)\right)^{\omega_i}\right), \frac{1}{t}\left(1-\prod_{i=1}^{n}(1-t\tau_i)^{\omega_i}\right), \frac{1}{t}\left(1-\prod_{i=1}^{n}(1-t\xi_i)^{\omega_i}\right)\right\rangle \quad (5)$$

Specially, if $t = 1$, then the ISVNWG operator, i.e., Equation (5), can be reduced to the SNNWG operator presented in Equation (2). The larger the parameter value of t, the higher the similarity between the ISVNWG operator and the SNNWG operator. Generally speaking, DMs can choose different parameter values of t based on their preferences.

Moreover, if the proposed ISVNWG operator is used to deal with the problems discussed in Definitions 3–5, and assuming $t = 0.95$, then we can get the following results.

(1) If the same case in Definition 3 is considered, that is, $\chi_1 = \langle 1,0,0\rangle$, $\chi_2 = \langle 0,0,1\rangle$, and $\chi_3 = \langle 0.3,0.4,0.5\rangle$ are three SVNNs, and $\omega = (0,0,1)^T$ is a weight vector of the three SVNNs, then

$$\begin{aligned}&ISVNWG_\omega(\chi_1,\chi_2,\ldots,\chi_n)\\&=\left\langle 1-\frac{1}{0.95}\left(1-1^0\times 0.05^0\times 0.335^1\right),\frac{1}{0.95}\left(1-1^0\times 1^0\times 0.62^1\right),\frac{1}{0.95}\left(1-1^0\times 0.05^0\times 0.525^1\right)\right\rangle\\&=\left\langle 1-\frac{1}{0.95}\times 0.665,\frac{1}{0.95}\times 0.38,\frac{1}{0.95}\times 0.475\right\rangle=\langle 0.3,0.4,0.5\rangle.\end{aligned}$$

Apparently, the result by using the proposed ISVNWG operator is consistent with our intuition. Hence, the proposed ISVNWG operator can avoid the defects of the SVNWG operator.

(2) If the same case in Definitions 4 and 5 is considered, that is $\chi_1 = \langle 0,1,1\rangle$, $\chi_2 = \langle 0,0,1\rangle$, and $\chi_3 = \langle 0,1,0\rangle$ are three SVNNs, and $\omega = (0,0,1)^T$ is a weight vector of the three SVNNs, then

$$\begin{aligned}&ISVNWG_\omega(\chi_1,\chi_2,\ldots,\chi_n)\\&=\left\langle 1-\frac{1}{0.95}\left(1-0.05^0\times 0.05^0\times 0.05^1\right),\frac{1}{0.95}\left(1-0.05^0\times 0.05^0\times 0.05^1\right),\frac{1}{0.95}\left(1-0.05^0\times 0.05^0\times 1^1\right)\right\rangle\\&=\left\langle 1-\frac{1}{0.95}\times 0.95,\frac{1}{0.95}\times 0.95,\frac{1}{0.95}\times 0\right\rangle=\langle 0,1,0\rangle.\end{aligned}$$

The result by using the proposed ISVNWG operator is also consistent with our intuition. Hence, the proposed operator can avoid the defects of the SNNWG and the $SNNWG^{\varepsilon}$ operators simultaneously.

(3) If the same case in Definition 6 is considered, that is, $\chi_1 = \langle 0,1,1\rangle$, $\chi_2 = \langle 0,1,0\rangle$, and $\chi_3 = \langle 0,0,1\rangle$ are three SVNNs, and $\omega = (0,0,1)^T$ is a weight vector of the three SVNNs, then

$$\begin{aligned}&ISVNWG_\omega(\chi_1,\chi_2,\ldots,\chi_n)\\&=\left\langle 1-\frac{1}{0.95}\left(1-0.05^0\times 0.05^0\times 0.05^1\right),\frac{1}{0.95}\left(1-0.05^0\times 0.05^0\times 1^1\right),\frac{1}{0.95}\left(1-0.05^0\times 1^0\times 0.05^1\right)\right\rangle\\&=\left\langle 1-\frac{1}{0.95}\times 0.95,\frac{1}{0.95}\times 0,\frac{1}{0.95}\times 0.95\right\rangle=\langle 0,0,1\rangle.\end{aligned}$$

The result by using the proposed ISVNWG operator is also consistent with our intuition. Hence, the proposed ISVNWG operator can avoid the defects of the SVNFWG operator.

Thus, the proposed ISVNWG operator can overcome the defects of existing aggregation operators. Moreover, the proposed ISVNWG operator also satisfies some properties, including idempotency, boundedness, monotonicity, and commutativity, which are discussed in the following.

Theorem 1. *(Idempotency) Let* $\chi_i = (\vartheta_i, \tau_i, \xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs. If* χ_i *are equal, i.e.,* $\chi_i = \chi = (\vartheta, \tau, \xi)(i=1,2,\ldots,n)$, *then* $ISVNWG_\omega(\chi_1, \chi_2, \ldots, \chi_n) = \chi$.

Proof. Since $\chi_i = \chi = (\vartheta, \tau, \xi)(i=1,2,\ldots,n)$, then

$$ISVNWG_\omega(\chi_1, \chi_2, \ldots, \chi_n) = \left\langle 1-\frac{1}{t}\left(1-(1-t(1-\vartheta))^{\sum_{i=1}^{n}\omega_i}\right), \frac{1}{t}\left(1-(1-t\tau)^{\sum_{i=1}^{n}\omega_i}\right), \frac{1}{t}\left(1-(1-t\xi)^{\sum_{i=1}^{n}\omega_i}\right)\right\rangle$$
$$= \left\langle 1-\frac{1}{t}(1-(1-t(1-\vartheta))), \frac{1}{t}(1-(1-t\tau)), \frac{1}{t}(1-(1-t\xi))\right\rangle$$
$$= \left\langle 1-\frac{1}{t}t(1-\vartheta), \frac{1}{t}t\tau, \frac{1}{t}t\xi\right\rangle = \langle \vartheta, \tau, \xi\rangle.$$

Theorem 2. *(Boundedness) Let* $\chi_i = (\vartheta_i, \tau_i, \xi_i)(i=1,2,\ldots,n)$ *be a class of SVNNs. If* $\chi^- = \langle \min \vartheta_i, \max \tau_i, \max \xi_i\rangle(i=1,2,\ldots,n)$ *and* $\chi^+ = \langle \max \vartheta_i, \min \tau_i, \min \xi_i\rangle(i=1,2,\ldots,n)$ *, then* $\chi^- \le ISVNWG_\omega(\chi_1, \chi_2, \ldots, \chi_n) \le \chi^+$.

Proof. Since $\chi^- = \langle \min \vartheta_i, \max \tau_i, \max \xi_i\rangle(i=1,2,\ldots,n)$ and $\chi^+ = \langle \max \vartheta_i, \min \tau_i, \min \xi_i\rangle(i=1,2,\ldots,n)$, then for any i, we have

$$\min \vartheta_i \le \vartheta_i \le \max \vartheta_i \Leftrightarrow 1-\max \vartheta_i \le 1-\vartheta_i \le 1-\min \vartheta_i$$

$$\Leftrightarrow t(1-\max \vartheta_i) \le t(1-\vartheta)_i \le t(1-\min \vartheta_i) \Leftrightarrow 1-t(1-\min \vartheta_i) \le 1-t(1-\vartheta_i) \le 1-t(1-\max \vartheta_i)$$

$$\Leftrightarrow \prod_{i=1}^{n}(1-t(1-\min \vartheta_i))^{\omega_i} \le \prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i} \le \prod_{i=1}^{n}(1-t(1-\max \vartheta_i))^{\omega_i}$$
$$\Leftrightarrow (1-t(1-\min \vartheta_i))^{\sum_{i=1}^{n}\omega_i} \le \prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i} \le (1-t(1-\max \vartheta_i))^{\sum_{i=1}^{n}\omega_i}$$
$$\Leftrightarrow (1-t(1-\min \vartheta_i)) \le \prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i} \le (1-t(1-\max \vartheta_i))$$

$$\Leftrightarrow 1-(1-t(1-\max \vartheta_i)) \le 1-\prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i} \le 1-(1-t(1-\min \vartheta_i))$$
$$\Leftrightarrow t(1-\max \vartheta_i) \le 1-\prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i} \le t(1-\min \vartheta_i)$$
$$\Leftrightarrow 1-\max \vartheta_i = \frac{1}{t}t(1-\max \vartheta_i) \le \frac{1}{t}\left(1-\prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i}\right) \le \frac{1}{t}t(1-\min \vartheta_i) = 1-\min \vartheta_i$$
$$\Leftrightarrow \min \vartheta_i = 1-(1-\min \vartheta_i) \le 1-\frac{1}{t}\left(1-\prod_{i=1}^{n}(1-t(1-\vartheta_i))^{\omega_i}\right) \le 1-(1-\max \vartheta_i) = \max \vartheta_i$$

Moreover,

$$\min\tau_i \le \tau_i \le \max\tau_i \Leftrightarrow t\min\tau_i \le t\tau_i \le t\max\tau_i$$

$$\Leftrightarrow 1-t\max\tau_i \le 1-t\tau_i \le 1-t\min\tau_i$$
$$\Leftrightarrow \left(1-t\max\tau_i\right)^{\sum_{i=1}^{n}\omega_i} = \prod_{i=1}^{n}\left(1-t\max\tau_i\right)^{\omega_i} \le \prod_{i=1}^{n}\left(1-t\tau_i\right)^{\omega_i} \le \prod_{i=1}^{i=n}\left(1-t\min\tau_i\right)^{\omega_i} = \left(1-t\min\tau_i\right)^{\sum_{i=1}^{n}\omega_i}$$
$$\Leftrightarrow \left(1-t\max\tau_i\right) \le \prod_{i=1}^{n}\left(1-t\tau_i\right)^{\omega_i} \le \left(1-t\min\tau_i\right)$$
$$\Leftrightarrow t\min\tau_i = 1-\left(1-t\min\tau_i\right) \le 1-\prod_{i=1}^{n}\left(1-t\tau_i\right)^{\omega_i} \le 1-\left(1-t\max\tau_i\right) = t\max\tau_i$$
$$\Leftrightarrow \min\tau_i = \frac{1}{t}t\min\tau_i \le \frac{1}{t}\left(1-\prod_{i=1}^{n}\left(1-t\tau_i\right)^{\omega_i}\right) \le \frac{1}{t}t\max\tau_i = \max\tau_i$$
$$\Leftrightarrow 1-\max\tau_i \le 1-\frac{1}{t}\left(1-\prod_{i=1}^{n}\left(1-t\tau_i\right)^{\omega_i}\right) \le 1-\min\tau_i$$

Similarly, $1-\max\xi_i \le 1-\frac{1}{t}\left(1-\prod_{i=1}^{n}\left(1-t\xi_i\right)^{\omega_i}\right) \le 1-\min\xi_i$ can be obtained.

Thus, $\min\vartheta_i + \left(1-\max\tau_i\right) + \left(1-\max\xi_i\right) \le s\left(ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right)\right) \le \max\vartheta_i + \left(1-\min\tau_i\right) + \left(1-\min\xi_i\right)$, i.e., $s\left(\chi^-\right) \le s\left(ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right)\right) \le s\left(\chi^+\right)$. From the comparison method in Definition 2, we can get $\chi^- \le ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right) \le \chi^+$.

Theorem 3. *(Monotonicity) Let* $\chi_i = \left(\vartheta_i,\tau_i,\xi_i\right)\left(i=1,2,\ldots,n\right)$ *and* $\chi_i^* = \left(\vartheta_i^*,\tau_i^*,\xi_i^*\right)\left(i=1,2,\ldots,n\right)$ *be two classes of SVNNs. If* $\chi_i \le \chi_i^*\left(i=1,2,\ldots,n\right)$ *and* $\vartheta_i \le \vartheta_i^*$, $\tau_i \le \tau_i^*$, *and* $\xi_i \le \xi_i^*\left(i=1,2,\ldots,n\right)$, *then* $ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right) \le ISVNWG_\omega\left(\chi_1^*,\chi_2^*,\ldots,\chi_n^*\right)$.

Proof. Since $\vartheta_i \le \vartheta_i^*$, $\tau_i \le \tau_i^*$, and $\xi_i \le \xi_i^*\left(i=1,2,\ldots,n\right)$, then based on Theorem 2, we can get $s\left(ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right)\right) \le s\left(ISVNWG_\omega\left(\chi_1^*,\chi_2^*,\ldots,\chi_n^*\right)\right)$. From the comparison method in Definition 2, $ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right) \le ISVNWG_\omega\left(\chi_1^*,\chi_2^*,\ldots,\chi_n^*\right)$ is true.

Theorem 4. *(Commutativity) Let* $\chi_i = \left(\vartheta_i,\tau_i,\xi_i\right)\left(i=1,2,\ldots,n\right)$ *be a class of SVNNs. If* $\tilde{\chi}_i = \left(\tilde{\vartheta}_i,\tilde{\tau}_i,\tilde{\xi}_i\right)\left(i=1,2,\ldots,n\right)$ *is an arbitrary permutation of* $\chi_i = \left(\vartheta_i,\tau_i,\xi_i\right)\left(i=1,2,\ldots,n\right)$, *then* $ISVNWG_\omega\left(\chi_1,\chi_2,\ldots,\chi_n\right) = ISVNWG_\omega\left(\tilde{\chi}_1,\tilde{\chi}_2,\ldots,\tilde{\chi}_n\right)$.

Proof. Since $\tilde{\chi}_i = (\tilde{\vartheta}_i, \tilde{\tau}_i, \tilde{\xi}_i)(i = 1, 2, \ldots, n)$ is an arbitrary permutation of $\chi_i = (\vartheta_i, \tau_i, \xi_i)(i = 1, 2, \ldots, n)$, we can get

$$\begin{aligned} ISVNWG_\omega(\chi_1, \chi_2, \ldots, \chi_n) &= \left\langle 1 - \frac{1}{t}\left(1 - \prod_{i=1}^{n}\left(1 - t(1 - \vartheta_i)\right)^{\omega_i}\right), \frac{1}{t}\left(1 - \prod_{i=1}^{n}(1 - t\tau_i)^{\omega_i}\right), \frac{1}{t}\left(1 - \prod_{i=1}^{n}(1 - t\xi_i)^{\omega_i}\right)\right\rangle \\ &= \left\langle 1 - \frac{1}{t}\left(1 - \prod_{i=1}^{n}\left(1 - t(1 - \tilde{\vartheta}_i)\right)^{\omega_i}\right), \frac{1}{t}\left(1 - \prod_{i=1}^{n}(1 - t\tilde{\tau}_i)^{\omega_i}\right), \frac{1}{t}\left(1 - \prod_{i=1}^{n}(1 - \tilde{t}\xi_i)^{\omega_i}\right)\right\rangle \\ &= ISVNWG_\omega(\tilde{\chi}_1, \tilde{\chi}_2, \ldots, \tilde{\chi}_n). \end{aligned}$$

5. The Single-Valued Neutrosophic MCDM Method Based on the Improved Aggregation Operator

Assume $\Phi = \{\Phi_1, \Phi_2, \ldots, \Phi_n\}$ is a set of alternatives and that $C = \{c_1, c_2, \ldots, c_m\}$ is the corresponding criteria. Assume $\chi = (\chi_{ij})_{n \times m}$ is the single-valued neutrosophic decision matrix, where $\chi_{ij} = (\vartheta_{ij}, \tau_{ij}, \xi_{ij})(i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)$ denotes the evaluation value provided by DMs about Φ_i under criterion c_j. Then the steps to obtain the optimal alternative(s) are presented as follows:

Step 1. Normalize the Decision Matrix.

Since the criteria may belong to the cost type or benefit type, the normalized method of the criteria can be determined as:

$$\overline{\chi}_{ij} = \begin{cases} \chi_{ij}, & \text{for benefit criteria } c_j \\ (\chi_{ij})^c, & \text{for cost criteria } c_j \end{cases}, \quad (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \tag{6}$$

where $(\chi_{ij})^c = (\xi_{ij}, \tau_{ij}, \vartheta_{ij})$ denotes the complement of χ_{ij}.

Step 2. Determine the General Value.

Based on the ISVNWG operator, i.e., Equation (5), the general value of alternative $\Phi_i(i = 1, 2, \ldots, n)$ can be obtained as:

$$\Phi_i = ISVNWG_\omega(\chi_{i1}, \chi_{i2}, \ldots, \chi_{im}) = \left\langle 1 - \frac{1}{t}\left(1 - \prod_{j=1}^{m}\left(1 - t(1 - \vartheta_{ij})\right)^{\omega_j}\right), \frac{1}{t}\left(1 - \prod_{j=1}^{m}(1 - t\tau_{ij})^{\omega_j}\right), \frac{1}{t}\left(1 - \prod_{j=1}^{m}(1 - t\xi_{ij})^{\omega_j}\right)\right\rangle \tag{7}$$

where $t = 0.95$ is assumed in the following calculation process.

Step 3. Calculate the Score, Accuracy, and Certainty Values.

From Definition 2, the score, accuracy, and certainty values, i.e., $s(\Phi_i)$, $a(\Phi_i)$, and $c(\Phi_i)$ of Φ_i, can be obtained, respectively.

Step 4. Rank the Alternatives.

Based on Step 3 and Definition 2, the candidate alternatives can be ranked.

In order to demonstrate that the proposed MCDM method based on the developed ISVNWG operator can avoid the defects of MCDM methods based on existing aggregation operators, three different examples are illustrated in the following.

Example 1. *Assume* $\Phi=(\Phi_1,\Phi_2)$ *is a group of candidate alternatives,* $c=(c_1,c_2)$ *is a group of benefit criteria, and that* $\omega=(0.4,0.6)^T$ *is the corresponding weight vector of the two criteria. Assume the decision matrix* $\chi=(\chi_{ij})_{2\times 2}$ *provided by DMs is presented as:*

$$\chi=\begin{matrix} & c_1 & c_2 \\ \Phi_1 & \langle 0,0.2,0.3\rangle & \langle 0.7,0.1,0.1\rangle \\ \Phi_2 & \langle 0,0.2,0.3\rangle & \langle 0.3,0.1,0.1\rangle \end{matrix}.$$

(1) If the MCDM method based on the proposed ISVNWG operator is used, and $t=0.95$, we can get $ISVNWG(\chi_{11},\chi_{12})=\langle 0.207,0.141,0.186\rangle$ and $ISVNWG(\chi_{21},\chi_{22})=\langle 0.112,0.141,0.186\rangle$. From the comparison method of SVNNs presented in Definition 2, $s(ISVNWG(\chi_{11},\chi_{12}))=0.627$ and $s(ISVNWG(\chi_{21},\chi_{22}))=0.595$ can be obtained. Since $s(ISVNWG(\chi_{11},\chi_{12}))>s(ISVNWG(\chi_{21},\chi_{22}))$, we can get $\Phi_1\succ\Phi_2$. The result is consistent with the actual decision-making environment.

(2) If the MCDM method based on the SVNWG operator is used, we can get $SVNWG(\chi_{11},\chi_{12})=SVNWG(\chi_{21},\chi_{22})=\langle 0,0.132,0.155\rangle$. Thus, we can get $\Phi_1=\Phi_2$, which contradicts the actual decision-making environment.

(3) If the MCDM method based on the SNNWG operator is used, we can get $SNNWG(\chi_{11},\chi_{12})=SNNWG(\chi_{21},\chi_{22})=\langle 0,0.141,0.186\rangle$. Apparently, we can get $\Phi_1=\Phi_2$, which contradicts the actual decision-making environment.

(4) If the MCDM method based on the $SNNWG^{\varepsilon}$ operator is used, we can get $SNNWG^{\varepsilon}(\chi_{11},\chi_{12})=SNNWG^{\varepsilon}(\chi_{21},\chi_{22})=\langle 0,0.14,0.182\rangle$. Thus, we can get $\Phi_1=\Phi_2$, which contradicts the actual decision-making environment.

(5) If the MCDM method based on the SVNFWG operator is used, and $\lambda=2$, we can get $SVNFWG(\chi_{11},\chi_{12})=SVNFWG(\chi_{21},\chi_{22})=\langle 0,0.141,0.185\rangle$. Thus, we can get $\Phi_1=\Phi_2$, which contradicts the actual decision-making environment.

Apparently, if the MCDM methods based on existing aggregation operators, such as the SVNNWG, SNNWG, $SNNWG^{\varepsilon}$, or SVNFWG operator, are used, we can get the same unreasonable results, i.e., $\Phi_1=\Phi_2$. However, the MCDM method based on the proposed ISVNWG operator can produce the reasonable result, i.e., $\Phi_1\succ\Phi_2$. Thus, the proposed MCDM method based on the developed ISVNWG operator can avoid the defects of the MCDM methods based on existing aggregation operators.

Example 2. *Assume* $\Phi=(\Phi_1,\Phi_2)$ *is a group of candidate alternatives,* $c=(c_1,c_2)$ *is a group of benefit criteria, and that* $\omega=(0.5,0.5)^T$ *is the corresponding weight vector of the two criteria. Assume the decision matrix* $\chi=(\chi_{ij})_{2\times 2}$ *provided by DMs is presented as:*

$$\chi=\begin{matrix} & c_1 & c_2 \\ \Phi_1 & \langle 0.3,1,1\rangle & \langle 0.5,0.1,0.1\rangle \\ \Phi_2 & \langle 0.3,1,1\rangle & \langle 0.5,0.1,0.4\rangle \end{matrix}.$$

(1) If the MCDM method based on the proposed ISVNWG operator is used, and $t=0.95$, we can get $ISVNWG(\chi_{11},\chi_{12})=\langle 0.389,0.829,0.829\rangle$ and $ISVNWG(\chi_{21},\chi_{22})=\langle 0.389,0.829,0.867\rangle$.

From the comparison method of SVNNs presented in Definition 2, $s\left(ISVNWG\left(\chi_{11},\chi_{12}\right)\right)=0.244$ and $s\left(ISVNWG\left(\chi_{21},\chi_{22}\right)\right)=0.231$ can be obtained. Since $s\left(ISVNWG\left(\chi_{11},\chi_{12}\right)\right)>s\left(ISVNWG\left(\chi_{21},\chi_{22}\right)\right)$, we can get $\Phi_1 \succ \Phi_2$.

(2) If the MCDM method based on the SVNWG operator is used, we can get $SVNWG\left(\chi_{11},\chi_{12}\right)=\left\langle 0.387,0.316,0.316\right\rangle$ and $SVNWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.387,0.316,0.632\right\rangle$. From the comparison method of SVNNs presented in Definition 2, $s\left(SVNWG\left(\chi_{11},\chi_{12}\right)\right)=0.585$ and $s\left(SVNWG\left(\chi_{21},\chi_{22}\right)\right)=0.48$. Since $s\left(SVNWG\left(\chi_{11},\chi_{12}\right)\right)>s\left(SVNWG\left(\chi_{21},\chi_{22}\right)\right)$, we can get $\Phi_1 \succ \Phi_2$.

(3) If the MCDM method based on the SNNWG operator is used, we can get $SNNWG\left(\chi_{11},\chi_{12}\right)=SNNWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.387,1,1\right\rangle$. Apparently, $\Phi_1 = \Phi_2$ can be obtained.

(4) If the MCDM method based on the $SNNWG^{\varepsilon}$ operator is used, we can get $SNNWG^{\varepsilon}\left(\chi_{11},\chi_{12}\right)=SNNWG^{\varepsilon}\left(\chi_{21},\chi_{22}\right)=\left\langle 0.39,1,1\right\rangle$. From the comparison method of SVNNs presented in Definition 2, we can get $\Phi_1 = \Phi_2$.

(5) If the MCDM method based on the SVNFWG operator is used, and $\lambda = 2$, we can get $SVNFWG\left(\chi_{11},\chi_{12}\right)=SVNFWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.611,1,1\right\rangle$. From the comparison method of SVNNs presented in Definition 2, we can get $\Phi_1 = \Phi_2$.

Apparently, the result by using the proposed MCDM method based on the developed ISVNWG operator is the same as that by using the MCDM method based on the SVNWG operator, i.e., $\Phi_1 \succ \Phi_2$, which is more reasonable in the real decision-making environment. By contrast, the results by using the MCDM methods based on the SNNWG, $SNNWG^{\varepsilon}$, and SVNFWG operators are identical to each other, i.e., $\Phi_1 = \Phi_2$, which contradicts the actual decision-making results. Thus, the proposed MCDM method based on the ISVNWG operator can avoid the defects of that based on the SNNWG, $SNNWG^{\varepsilon}$, or SVNFWG operator.

Example 3. *Assume* $\Phi=\left(\Phi_1,\Phi_2\right)$ *is a group of candidate alternatives,* $c=\left(c_1,c_2\right)$ *is a group of benefit criteria, and that* $\omega=\left(0.5,0.5\right)^T$ *is the corresponding weight vector of two criteria. Assume the decision matrix* $\chi=\left(\chi_{ij}\right)_{2\times 2}$ *provided by DMs is presented as:*

$$\chi=\begin{matrix} & c_1 & c_2 \\ \Phi_1 & \left\langle 0.4,0,0\right\rangle & \left\langle 0.5,0.1,0.2\right\rangle \\ \Phi_2 & \left\langle 0.4,0,0\right\rangle & \left\langle 0.5,0.2,0.3\right\rangle \end{matrix}.$$

(1) If the MCDM method based on the proposed ISVNWG operator is used, and $t = 0.95$, we can get $ISVNWG\left(\chi_{11},\chi_{12}\right)=\left\langle 0.448,0.05,0.105\right\rangle$ and $ISVNWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.448,0.105,0.163\right\rangle$. From the comparison method of SVNNs presented in Definition 2, $s\left(ISVNWG\left(\chi_{11},\chi_{12}\right)\right)=0.764$ and $s\left(ISVNWG\left(\chi_{21},\chi_{22}\right)\right)=0.727$ can be obtained. Since $s\left(ISVNWG\left(\chi_{11},\chi_{12}\right)\right)>s\left(ISVNWG\left(\chi_{21},\chi_{22}\right)\right)$, we can get $\Phi_1 \succ \Phi_2$.

(2) If the MCDM method based on the SVNWG operator is used, we can get $SVNWG\left(\chi_{11},\chi_{12}\right)=SVNWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.447,0,0\right\rangle$, i.e., $\Phi_1 = \Phi_2$.

(3) If the MCDM method based on the SNNWG operator is used, we can get $SNNWG\left(\chi_{11},\chi_{12}\right)=\left\langle 0.447,0.05,0.106\right\rangle$ and $SNNWG\left(\chi_{21},\chi_{22}\right)=\left\langle 0.447,0.106,0.163\right\rangle$. From the comparison method of SVNNs presented in Definition 2, $s\left(SNNWG\left(\chi_{11},\chi_{12}\right)\right)=0.764$ and $s\left(SNNWG\left(\chi_{21},\chi_{22}\right)\right)=0.726$. Since $s\left(SNNWG\left(\chi_{11},\chi_{12}\right)\right)>s\left(SNNWG\left(\chi_{21},\chi_{22}\right)\right)$, we

can get $\Phi_1 \succ \Phi_2$.

(4) If the MCDM method based on the $SNNWG^{\varepsilon}$ operator is used, we can get $SNNWG^{\varepsilon}(\chi_{11},\chi_{12})=\langle 0.448,0.05,0.101\rangle$ and $SNNWG^{\varepsilon}(\chi_{21},\chi_{22})=\langle 0.448,0.101,0.149\rangle$. From the comparison method of SVNNs presented in Definition 2, $s(SNNWG^{\varepsilon}(\chi_{11},\chi_{12}))=0.766$ and $s(SNNWG^{\varepsilon}(\chi_{21},\chi_{22}))=0.733$. Since $s(SNNWG^{\varepsilon}(\chi_{11},\chi_{12}))>s(SNNWG^{\varepsilon}(\chi_{21},\chi_{22}))$, we can get $\Phi_1 \succ \Phi_2$.

(5) If the MCDM method based on the SVNFWG operator is used, and $\lambda=2$, we can get $SVNFWG(\chi_{11},\chi_{12})=\langle 0.448,0.051,0.104\rangle$ and $SVNFWG(\chi_{21},\chi_{22})=\langle 0.448,0.104,0.16\rangle$. From the comparison method of SVNNs presented in Definition 2, $s(SVNFWG(\chi_{11},\chi_{12}))=0.764$ and $s(SVNFWG(\chi_{21},\chi_{22}))=0.728$. Since $s(SNNWG(\chi_{11},\chi_{12}))>s(SNNWG(\chi_{21},\chi_{22}))$, we can get $\Phi_1 \succ \Phi_2$.

From the results presented above, the result by using the proposed MCDM methods based on the developed ISVNWG operator is the same as that by using the MCDM method based on the SNNWG, $SNNWG^{\varepsilon}$, or SVNFWG operator, and the final ranking is $\Phi_1 \succ \Phi_2$. By contrast, if the MCDM method based on the SVNWG operator is used, the final ranking is $\Phi_1 = \Phi_2$, which is different from the results by using other MCDM methods based on existing aggregation operators. In other words, the proposed MCDM method based on the ISVNWG operator can avoid the defects of that based on the SVNWG operator.

6. Application Examples

Example 4 [18]. *An investment company wants to choose one from four candidate companies to invest. The four companies are automobile, food, computer, and arm enterprises, which are denoted as Φ_1, Φ_2, Φ_3, and Φ_4, respectively. In the decision-making process, the decision-making department of the investment company needs to evaluate the candidate companies according to three criteria, including risk, growth, and environmental impact, which are denoted as c_1, c_2, and c_3, respectively. The weight vector of the criteria is $w=(0.35,0.25,0.4)^T$. The decision-making department includes three DMs, and the importance of different DMs should be considered. The corresponding weight vector of these three DMs is $d=(0.5,0.3,0.2)^T$. Assume the DMs evaluate each criterion of candidate companies in the form of SVNNs, and then the decision matrix can be obtained as follows:*

$$\chi^1=\begin{pmatrix} \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.2,0.2,0.5\rangle \\ \langle 0.3,0.1,0.2\rangle & \langle 0,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle \\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0,0.3,0.2\rangle \\ \langle 0.7,0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.4,0.3,0.2\rangle \end{pmatrix}$$

$$\chi^2=\begin{pmatrix} \langle 0.3,0.2,0.2\rangle & \langle 0,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle \\ \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.5,0.1,1\rangle & \langle 0.6,0.3,0.2\rangle & \langle 0.5,0.1,0.2\rangle \end{pmatrix}$$

$$\chi^3=\begin{pmatrix} \langle 0.5,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.3,0.1,0.3\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle \\ \langle 0,0.2,1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.2\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.5,0.2,1\rangle & \langle 0.6,0.2,0.2\rangle \end{pmatrix}$$

Since criterion c_3 is belongs to the cost type, it should be normalized. For example, $\overline{\chi}_{13}=(\chi_{13})^c=\langle 0.2,0.2,0.5\rangle$. From the ISVNWG operator, i.e., Equation (7), let $t=0.95$, and then the aggregated values of the three DMs can be obtained as:

$$\chi=\begin{pmatrix} \langle 0.3844,0.1809,0.2516\rangle & \langle 0.1944,0.2,0.281\rangle & \langle 0.4231,0.1809,0.2516\rangle \\ \langle 0.4114,0.1439,0.2\rangle & \langle 0.1017,0.1312,0.2516\rangle & \langle 0.2458,0.221,0.5\rangle \\ \langle 0.2072,0.1712,0.6105\rangle & \langle 0.5,0.2129,0.3\rangle & \langle 0.2,0.2336,0.3343\rangle \\ \langle 0.5923,0.0973,0.6618\rangle & \langle 0.5786,0.1845,0.5641\rangle & \langle 0.2,0.2245,0.4756\rangle \end{pmatrix}$$

Thus, the general value of alternatives can be determined by using the proposed ISVNWG operator as:

$$\Phi=\begin{pmatrix} \langle 0.3394,0.1857,0.259\rangle \\ \langle 0.2427,0.1725,0.3469\rangle \\ \langle 0.2576,0.207,0.4394\rangle \\ \langle 0.3883,0.1717,0.5697\rangle \end{pmatrix}$$

From the comparison method in Definition 2, the score values can be calculated, i.e., $s(\Phi_1)=0.6316, s(\Phi_2)=0.5745, s(\Phi_3)=0.5371$, and $s(\Phi_4)=0.549$. Since the score values of the four alternatives are different, there is no need to calculate the accuracy and certainty values. Apparently, $s(\Phi_1)>s(\Phi_2)>s(\Phi_4)>s(\Phi_3)$. Thus, the final ranking is $\Phi_1\succ\Phi_2\succ\Phi_4\succ\Phi_3$, and the optimal alternative is Φ_1.

In the following, the different parameter values of t in the proposed ISVNWG operator are considered respectively to investigate the influence on the final decision-making, and the corresponding results of sensitivity analysis are shown in Table 1. Apparently, if $t=0.95,0.75$, then the final ranking is always $\Phi_1\succ\Phi_2\succ\Phi_4\succ\Phi_3$, and the best alternative is Φ_1; while if $t=0.55,0.35,0.15$, then the positions of Φ_2 and Φ_4 have been changed. However, the best one is always Φ_1. Thus, different parameter values of t do not affect the final decision-making results. Generally speaking, DMs can choose the parameter values of t based on their preferences. The larger the parameter value of t, the higher the similarity between the ISVNWG operator and the SNNWG operator. Moreover, since the values of the degree of three memberships of SVNNs belongs to [0,1], the bigger parameter value of t can be determined to improve the accuracy of the calculation process.

Table 1. Sensitivity analysis.

Parameter	$s(\Phi_i)$				Rank	Optimal one
	Φ_1	Φ_2	Φ_3	Φ_4		
$t=0.95$	0.6316	0.5745	0.5371	0.549	$\Phi_1\succ\Phi_2\succ\Phi_4\succ\Phi_3$	Φ_1
$t=0.75$	0.6411	0.5877	0.5632	0.5852	$\Phi_1\succ\Phi_2\succ\Phi_4\succ\Phi_3$	Φ_1
$t=0.55$	0.6443	0.5935	0.5741	0.6021	$\Phi_1\succ\Phi_4\succ\Phi_2\succ\Phi_3$	Φ_1
$t=0.35$	0.6454	0.5958	0.5782	0.6084	$\Phi_1\succ\Phi_4\succ\Phi_2\succ\Phi_3$	Φ_1
$t=0.15$	0.6464	0.5981	0.5824	0.6148	$\Phi_1\succ\Phi_4\succ\Phi_2\succ\Phi_3$	Φ_1

In order to further illustrate the advantages of the proposed ISVNWG operator and the corresponding MCDM method, comparison analysis is conducted in the following by using the MCDM methods based on different aggregation operators to deal with the same MCDM problem, respectively, and $\lambda=2$, then the comparison results are shown in Table 2.

Table 2. Comparison results.

Operator	$s(\Phi_i)$				Rank	Optimal one
	Φ_1	Φ_2	Φ_3	Φ_4		
SVNNWG	0.5224	0.5138	0.6033	0.6877	$\Phi_4 \succ \Phi_3 \succ \Phi_1 \succ \Phi_2$	Φ_4
SNNWG	0.5184	0.4931	0.2643	0.4030	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
$\text{SNNWG}^{\varepsilon}$	0.5189	0.4963	0.265	0.4092	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
SVNFWG	0.5186	0.4944	0.2645	0.406	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
The proposed operator	0.6316	0.5745	0.5371	0.549	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1

From Table 2, the result by using the proposed MCDM method based on the ISVNWG operator is the same is that of by using the MCDM methods based on the SNNWG, $\text{SNNWG}^{\varepsilon}$, or SVNFWG operator, and the final ranking is $\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$, with Φ_1 as the best investment one. By contrast, if the MCDM method based on the SVNWG operator is used, the final ranking is $\Phi_4 \succ \Phi_3 \succ \Phi_1 \succ \Phi_2$, and the best investment one is Φ_4, which is different from the results by using the MCDM method based on other aggregation operators.

Example 5. *Based on Example 4, if the weight vector of the criteria is* $w=(0.3,0.4,0.3)^T$ *and the importance of the three DMs is* $d=(0.5,0.3,0.2)^T$*, then the decision matrix can be provided as:*

$$\chi^1 = \begin{pmatrix} \langle 0.35,0.2,0.3\rangle & \langle 0,0.2,0.3\rangle & \langle 0.2,0.2,0.25\rangle \\ \langle 0.3,0.1,0.2\rangle & \langle 0,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle \\ \langle 0.3,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0,0.3,0.2\rangle \\ \langle 0,0,0.1\rangle & \langle 0.6,0.1,0.2\rangle & \langle 0.4,0.3,0.2\rangle \end{pmatrix}$$

$$\chi^2 = \begin{pmatrix} \langle 0.3,0.2,0.2\rangle & \langle 0,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle \\ \langle 0.45,0.1,0.2\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle \\ \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.5,0.1,1\rangle & \langle 0.6,0.3,0.2\rangle & \langle 0.5,0.1,0.2\rangle \end{pmatrix}$$

$$\chi^3 = \begin{pmatrix} \langle 0.35,0.1,0.2\rangle & \langle 0.25,0.2,0.2\rangle & \langle 0.3,0.1,0.3\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle \\ \langle 0,0.2,1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.2\rangle \\ \langle 0.5,0.3,0.2\rangle & \langle 0.5,0.2,1\rangle & \langle 0.6,0.2,0.2\rangle \end{pmatrix}$$

Similar to Example 4, the MCDM methods based on different aggregation operators are used to deal with the same MCDM problem, respectively, and the comparison results are shown in Table 3.

From Table 3, the result by using the MCDM method based on the proposed ISVNWG operator is different from that by using the MCDM method based on the SVNNWG, SNNWG, $\text{SNNWG}^{\varepsilon}$, or SVNFWG operator, and the final ranking is $\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$, with Φ_2 as the best investment one. By contrast, if the MCDM method based on the SVNWG operator is used, the final ranking is $\Phi_3 \succ \Phi_2 \succ \Phi_1 \succ \Phi_4$, and the best investment one is Φ_3; if the MCDM method based on the SNNWG, $\text{SNNWG}^{\varepsilon}$, or SVNFWG operator is used, the final ranking is $\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$, and the best investment one is Φ_1.

Table 3. Comparison results.

Operator	$s(\Phi_i)$				Rank	Optimal one
	Φ_1	Φ_2	Φ_3	Φ_4		
SVNNWG	0.5196	0.5227	0.6032	0.5064	$\Phi_3 \succ \Phi_2 \succ \Phi_1 \succ \Phi_4$	Φ_3
SNNWG	0.5159	0.5045	0.2643	0.2669	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
SNNWG$^{\varepsilon}$	0.5164	0.5074	0.265	0.2678	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
SVNFWG	0.5161	0.5057	0.2645	0.2672	$\Phi_1 \succ \Phi_2 \succ \Phi_4 \succ \Phi_3$	Φ_1
The proposed operator	0.5634	0.5726	0.541	0.4999	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$	Φ_2

Example 6. *Assume all the assumption conditions in Example 4 are unchanged, and then the decision matrix can be provided as:*

$$\chi^1 = \begin{pmatrix} \langle 0.35,0.2,1\rangle & \langle 0,0.2,0.3\rangle & \langle 0.2,0.2,0.25\rangle \\ \langle 0.3,0.1,0.2\rangle & \langle 0,0.1,0.2\rangle & \langle 0.5,0.2,0.2\rangle \\ \langle 0.3,0.2,0.3\rangle & \langle 0.45,0.2,0.3\rangle & \langle 0,0.3,1\rangle \\ \langle 0.2,0,0.1\rangle & \langle 0.3,0.1,0.2\rangle & \langle 0.4,0.3,0.2\rangle \end{pmatrix}$$

$$\chi^2 = \begin{pmatrix} \langle 0.3,0.2,0.2\rangle & \langle 0,0.2,1\rangle & \langle 0.3,0.2,0.4\rangle \\ \langle 0.45,0.1,0.2\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.5,0.2,0.3\rangle \\ \langle 0.6,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle & \langle 1,0.2,0\rangle \\ \langle 0,0.1,1\rangle & \langle 0,0.3,1\rangle & \langle 0.5,0.1,0.2\rangle \end{pmatrix}$$

$$\chi^3 = \begin{pmatrix} \langle 0.35,0.1,0.2\rangle & \langle 0.25,0.2,1\rangle & \langle 0.3,0.1,0.3\rangle \\ \langle 0,0.3,0.2\rangle & \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.3,0.3\rangle \\ \langle 0,0.2,1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.2\rangle \\ \langle 0.5,1,0.2\rangle & \langle 0,0.2,1\rangle & \langle 1,0.2,0\rangle \end{pmatrix}$$

Similarly, the MCDM methods based on different aggregation operators are used to deal with the same MCDM problem, respectively, and the comparison results are shown in Table 4.

Table 4. Comparison results.

Operator	$s(\Phi_i)$				$a(\Phi_i)$		$c(\Phi_i)$		Rank
	Φ_1	Φ_2	Φ_3	Φ_4	Φ_3	Φ_4	Φ_3	Φ_4	
SVNNWG	0.4704	0.5227	0.5907	0.4766					$\Phi_3 \succ \Phi_2 \succ \Phi_4 \succ \Phi_1$
SNNWG	0.2705	0.5045	0	0	-1	-1	0	0	$\Phi_2 \succ \Phi_1 \succ \Phi_3 = \Phi_4$
SNNWG$^{\varepsilon}$	0.2706	0.5074	0	0	-1	-1	0	0	$\Phi_2 \succ \Phi_1 \succ \Phi_3 = \Phi_4$
SVNFWG	0.2705	0.5057	0	0	-1	-1	0	0	$\Phi_2 \succ \Phi_1 \succ \Phi_3 = \Phi_4$
The proposed operator	0.403	0.5614	0.3563	0.3451					$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$

From Table 4, the result by using the proposed MCDM method based on the ISVNWG operator is different from that by using the MCDM method based on the SVNNWG, SNNWG, SNNWG$^{\varepsilon}$, or SVNFWG operator, and the final ranking is $\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$, with Φ_2 as the best investment one. By contrast, if the MCDM method based on the SVNNWG operator is used, the final ranking is

$\Phi_3 \succ \Phi_2 \succ \Phi_4 \succ \Phi_1$, and the best investment one is Φ_3; if the MCDM method based on the SNNWG, $\mathrm{SNNWG}^{\varepsilon}$, or SVNFWG operator is used, the final ranking is $\Phi_2 \succ \Phi_1 \succ \Phi_3 = \Phi_4$, and the best investment one is Φ_2. Moreover, the MCDM method based on the SNNWG, $\mathrm{SNNWG}^{\varepsilon}$, or SVNFWG operator cannot distinguish between Φ_3 and Φ_4.

7. Conclusions

In summary, a novel single-valued neutrosophic MCDM method based on the developed ISVNWG operator was proposed in this paper. The proposed ISVNWG operator can avoid the defects of existing aggregation operators, such as the SVNNWG, SNNWG, $\mathrm{SNNWG}^{\varepsilon}$, and SVNFWG operators, and the corresponding MCDM method can overcome the defects of MCDM methods based on those existing aggregation operators. The provided application examples verify that the proposed ISVNWG operator and the corresponding MCDM method are valid and feasible for solving MCDM problems. In the future, we will further study other improved aggregation operators, including Muirhead mean operator, Bonferroni mean operator, and Heronian mean operators, and improve the corresponding MCDM methods based on these aggregation operators.

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