A Rectified Reiterative Sieved-Pollaczek Polynomials Neural Network Backstepping Control with Improved Fish School Search for Motor Drive System

Chih-Hong Lin

Department of Electrical Engineering, National United University, Miaoli 360, Taiwan; jhlin@nuu.edu.tw; Tel.: +886-3-7382464

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Abstract: As the six-phase squirrel cage copper rotor induction motor has some nonlinear characteristics, such as nonlinear friction, nonsymmetric torque, wind stray torque, external load torque, and time-varying uncertainties, better control performances cannot be achieved by utilizing general linear controllers. The snug backstepping control with sliding switching function for controlling the motion of a six-phase squirrel cage copper rotor induction motor drive system is proposed to reduce nonlinear uncertainty effects. However, the previously proposed control results in high chattering on nonlinear system effects and overtorque on matched uncertainties. So as to reduce the immense chattering situation, we then put forward the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method to estimate the external bundled torque uncertainties and to recoup the smallest reorganized error of the evaluated rule. In the light of Lyapunov stability, the online parametric training method of the rectified reiterative sieved-Pollaczek polynomials neural network can be derived by utilizing an adaptive rule. Moreover, to improve convergence and obtain beneficial learning manifestation, the improved fish school search algorithm is made use of to readjust two fickle learning rates of the weights in the rectified reiterative sieved-Pollaczek polynomials neural network. Lastly, the effectuality of the proposed control system is validated by examination results.

Keywords: sieved-Pollaczek polynomials neural network; fish school search; Lyapunov stability theorem; backstepping control; six-phase squirrel cage copper rotor induction motor

1. Introduction

A six-phase induction motor [1–3] has been broadly applied in various kinds of industrial applications [4,5] because it has higher efficiency, higher reliability, and lower torque ripple in comparison with a three-phase squirrel cage aluminum rotor induction motor. Moreover, nonlinear dynamics continuously variable transmission systems [6,7] driven by the six-phase squirrel cage copper rotor induction motor, which adopted the orthogonal polynomials neural network controls, were proposed by Lin [8–10]. An indirect field-oriented control, which is a popular control technique, is used in the six-phase squirrel cage copper rotor induction motor. As a result, the torque ripple of the six-phase squirrel cage copper rotor induction motor can be extremely low in comparison with that of a switched reluctance motor and a synchronous reluctance motor. On the other hand, the six-phase squirrel cage copper rotor induction motor, which is controlled by an indirect field-oriented control that can achieve fast four-quadrant operation, is much less sensitive to the parameter variations of the motor [1–3]. Therefore, robotics and other mechatronics [1–3] that are driven by the six-phase squirrel cage copper rotor induction motor drive systems have appeared lately.
The backstepping designs [11–13] befit a large type of linearizable nonlinear system. Each backstepping phase can produce a novel fictitious-control design that is denoted by previous design phases. When the procedure ends, a feedback design can achieve the primitive design aim by utilizing the last Lyapunov function that is made up by adding the Lyapunov functions of all individual design phases [11–14]. Moreover, an adaptive backstepping control is also applied to a power switcher and a synchronous generator [15,16].

Artificial intelligent systems have been widely used in many commercial and industrial applications. Artificial neural networks [17–20] were one of the popular methods in modeling, control, estimation, and prediction of nonlinear systems with better learning ability. However, these artificial neural networks need a longer time to process training and learning procedures. Hence, a lot of orthogonal polynomials neural networks [21–24] were proposed for application to various kinds of modeling, identification, approximation, and control of nonlinear systems as a result of less computational complexity. However, the adjustment mechanics of weights were not discussed in these control methods that are combined with neural networks. It leads to a larger difference in control and identification for nonlinear systems. Moreover, the sieved-Pollaczek polynomials [25] belong to the sieved orthogonal polynomials that were enlightened by Ismail [26]. However, the sieved-Pollaczek polynomials, combined with a neural network, have never been presented in any control of nonlinear systems. Although the feedforward sieved-Pollaczek polynomials neural network can approximate nonlinear function, it may not be able to approximate the dynamic act of nonlinear uncertainties as a result of lacking a reflect loop. As it has more benefits than the feedforward sieved-Pollaczek polynomials neural network, the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control has not yet been presented for controlling the motion of a six-phase squirrel cage copper rotor induction motor drive system to raise the certification property of the nonlinear system and reduce calculation complexity. The rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with error recouped agency to decrease uncertainty influences is, thus, the main motivation in this research. However, these learning rates, by utilizing some acceleration factors, are absent so that the convergent speed of weight regulation is slower.

Fish school search was first proposed by Carmelo et al. [27] and it is a unimodal optimization algorithm inspired by the collective behavior of fish schools. It is a population-based on search algorithm inspired by the behavior of swimming fish schools, which expand and contract while looking for food. Each fish-dimensional location represents a possible solution for the optimization problem. Salomao et al. [28] proposed the density-based fish school search method that includes modifications to the previous operators (feeding and swimming), as well as new operators (memory and partition). Fernando and Marcelo [29] proposed the weight-based fish school search method that is a weight-based niching version of the fish school search intended to produce multiple solutions. However, these fish school search methods have absented the updated mechanics of weights, so this has resulted in the slower convergence. Thus, the improved fish school search method is proposed as a novel method to regulate two fickle rates with two optimal values and to raise the convergent speed of weights in this paper.

However, those neural networks need longer calculation time to carry out the nonlinear system. Therefore, dynamics control of the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method for the six-phase squirrel cage copper rotor induction motor drive system is a novel method, and its main aim is to develop the assessed law, to reduce calculation time, and to speed-up the convergence of weight regulation. The main motivation of the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method is a novel structure in order to reduce calculation complexity that raises the certification property of the nonlinear system because this control construct has faster learning ability and better generalization. The rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method can carry out the backstepping control, the rectified reiterative sieved-Pollaczek polynomials neural network control with a cozy law, and the indemnified control
with an appraised rule and a snug law. In the light of Lyapunov stability, the cozy law for training parameters in the rectified reiterative sieved-Pollaczek polynomials neural network is derived online. Therefore, the rectified reiterative sieved-Pollaczek polynomials neural network with online learning ability can respond to nonlinear uncertainty. Moreover, the improved fish school search is proposed to search for two optimal fickle rates in the connecting weights and recurrent weights of the rectified reiterative sieved-Pollaczek polynomials neural network and to speed-up the convergent speed of weight regulation. Finally, tested results are demonstrated to confirm that the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method can reach fine control performance.

Better control performance of the six-phase squirrel cage copper rotor induction motor drive system cannot be reached by utilizing the linear controller due to the influences of these uncertainties. So as to heighten robustness, the snug backstepping control with a sliding switching function is put forward to control the movement of the six-phase squirrel cage copper rotor induction motor drive system for tracking cyclic references. With the snug backstepping control with a sliding switching function, the rotor place of the six-phase squirrel cage copper rotor induction motor drive system preserves the merits of fine transient control performance and robustness to uncertainties for the tracking of cyclic reference loci. Moreover, so as to ameliorate larger chattering influence for the aforementioned control system under uncertainty effects, we then put forward the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method to estimate the internal bundled and external bundled force uncertainties and to recoup the smallest reorganized error of the evaluated rule.

The main issue in this paper is as below: Segment 2 presents the models and conformation of the six-phase squirrel cage copper rotor induction motor drive system. Segment 3 presents the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method. Segment 4 presents the examination consequences for the six-phase squirrel cage copper rotor induction motor drive system by utilizing three control methods at five tested fettles. Segment 5 presents the conclusions.

2. Models and Conformation of the Six-Phase Squirrel Cage Copper Rotor Induction Motor Drive

This section consists of two subsections that are the models of the six-phase squirrel cage copper rotor induction motor and the conformation of the six-phase squirrel cage copper rotor induction motor drive system.

2.1. Models of the Six-Phase Squirrel Cage Copper Rotor Induction Motor

The mathematical models of the six-phase squirrel cage copper rotor induction motor with six sets of voltage equations in the six-phase \(a_1 - b_1 - c_1 - a_2 - b_2 - c_2\) axes are described by Munoz and Lipo [1–3]. For simplicity in the six-phase \(a_1 - b_1 - c_1 - a_2 - b_2 - c_2\) axes coordinate frames via the Clarke and Park transformations, the voltage equations in the coordinate frames transformation from the six-phase \(a_1 - b_1 - c_1 - a_2 - b_2 - c_2\) axes to the \(d_1 - q_1 - d_2 - q_2\) axes in the six-phase squirrel cage copper rotor induction motor can be represented by

\[
v_{q1} = \eta i_{q1} + \omega_l \lambda_{d1} + \frac{d\lambda_{q1}}{dt} \tag{1}\n\]

\[
v_{d1} = \eta i_{d1} - \omega_l \lambda_{q1} + \frac{d\lambda_{d1}}{dt} \tag{2}\n\]

\[
v_{q2} = \eta i_{q2} + \frac{d\lambda_{q2}}{dt} \tag{3}\n\]
\[ v_{d2} = n_2 i_{d2} + \frac{d\lambda_{d2}}{dt} \]  

(4)

\[ 0 = r_2 i_{qr} + (\omega_c - \omega_b) \lambda_{dr} + \frac{d\lambda_{qr}}{dt} \]  

(5)

\[ 0 = r_1 i_{dr} + (\omega_c - \omega_b) \lambda_{qr} + \frac{d\lambda_{dr}}{dt} \]  

(6)

where \( \lambda_{q1} = L_{i1} i_{q1} + L_{i2} i_{q2} \), \( \lambda_{d1} = L_{i1} i_{d1} + L_{i2} i_{d2} \), and \( \lambda_{q2} = L_{i1} i_{q1} + L_{i2} i_{q2} \) are the \( d_1 - q_1 \) and \( d_2 - q_2 \) axes flux linkages, respectively; \( \lambda_{qr} = L_{ii} i_{qr} + L_{i2} i_{q2} \), \( \lambda_{dr} = L_{ii} i_{dr} + L_{i2} i_{d2} \) are the \( d_r - q_r \) axes flux linkages; \( v_{d1} \), \( v_{q1} \), \( v_{d2} \), \( v_{q2} \) are the \( d_1 - q_1 - d_2 - q_2 \) axes voltages; \( i_{d1} \), \( i_{q1} \), \( i_{d2} \), \( i_{q2} \) are the \( d_1 - q_1 - d_2 - q_2 \) axes currents; \( i_{dr} \), \( i_{qr} \) are the \( d_r - q_r \) axes currents; \( r_1 \) and \( r_2 \) are the stator resistance and rotor resistance; \( L_1 = L_{ls} + 3L_{ms} \), \( L_2 = L_{ir} + 3L_{ms} \), and \( L_2 = 3L_{ms} \) are the self-inductance of the stator winding, the self-inductance of rotor winding, and the mutual inductance between stator winding and the rotor winding, respectively. \( \omega_c \), \( \omega_b = \frac{P_1 \omega_{rs}}{2} \), and \( \omega_{rs} \) are the electrical angular speed of synchronous flux, the electrical angular speed of the rotor, and the mechanical angular speed of the rotor, respectively. The electromagnetic torque \( \tau_{es} \) of the six-phase squirrel cage copper rotor induction motor can be represented by

\[ \tau_{es} = 3R_2 L_2 (\lambda_{dr} i_{q1} - \lambda_{qr} i_{d1})/(4L_3) \]  

(7)

where \( \lambda_{dr} \), \( \lambda_{qr} \) are the \( d_r - q_r \) axes flux linkages; \( P_1 \) is the number of poles.

The dynamic equation can be typified by

\[ X_s \frac{d\omega_{rs}}{dt} + Y_s \omega_{rs} = \tau_{es} - \tau_{ls} - \tau_{ws} - \tau_{fs} - \tau_{cs} \]  

(8)

where \( \omega_{es} \) and \( \omega_{rs} \) stand for the electrical angular velocity and the angular velocity of the rotor; \( \theta_{es} \) and \( \theta_{rs} \) stand for the electrical angular position and the mechanical angular position of the rotor; \( \tau_{es} \), \( \tau_{ls} \), \( \tau_{ws} \), \( \tau_{fs} \), and \( \tau_{cs} \) stand for the electromagnetic torque, the external load torque, the wind stray torque, the flux saturation torque, and the cogging torque; \( X_s \) and \( Y_s \) stand for the total moment of inertia and the total viscous frictional coefficient.

### 2.2. Conformation of the Six-Phase Squirrel Cage Copper Rotor Induction Motor Drive System

The common control skill of the six-phase squirrel cage copper rotor induction motor drive system is to adopt the indirect field-oriented control [1–3]. When \( \frac{\lambda_{qr} i_{d1}}{\lambda_{dr} i_{q1}} \) is the linear value between 0.1 and 0.793, and \( \lambda_{dr} \) is equal to a constant value, as 2.7495 Wb in Equation (7) by some experimental tests [3] via the open circuit test as well as the short circuit test, then the electromagnetic torque \( \tau_{es} \) will be in direct ratio to \( i_{q1} \) for the six-phase squirrel cage copper rotor induction motor drive system in the closed-loop control. The expected ranges of two parameters are obtained by some experimental tests [3] via the open circuit test, the short circuit test, and the loading test. When the produced torque is in a linearly direct ratio to \( \frac{\lambda_{qr} i_{d1}}{\lambda_{dr} i_{q1}} \), the larger torque per ampere can be reached [3]. The mandate electromagnetic torque equation from Equation (7) can be typified by.
where \( \tau_{es}^* \) stands for the mandate electromagnetic torque between 1.177 and 5.118 Nm; 
\( k_{es} = 3R_2L_2\lambda_{d}\lambda_{q1}/(4L_3) \) stands for the torque coefficient between 0.214 and 0.9306 Nm/A; 
\( i_{q1}^* \) stands for the mandate propulsion current, i.e., the mandated control propulsion between 0 and 5.5 A. The expected ranges of the three parameters are obtained by some experimental tests [3] via the open circuit test, the short circuit test, and the loading test.

Figure 1 is the conformation of indirect field-oriented control over the six-phase squirrel cage copper rotor induction motor drive system with a digital signal processor control system, which is made up of an encoder, a set of analog–digital current sensing circuit, a six-phase squirrel cage copper rotor induction motor, a space-vector pulse-width-modulation modulator control, a coordinate transformer, a sin/cos generator, a speed control loop, and a place control loop. The rotor of the six-phase squirrel cage copper rotor induction motor is equipped with different sizes of iron disks to change the values in the moment of inertia of the rotor and the viscous frictional torque.

The six-phase squirrel cage copper rotor induction motor drive system with indirect field-oriented control was realized by a digital signal processor control system. The rated format of the six-phase squirrel cage copper rotor induction motor is six-phase two poles 220 V, 2 kW, and 3452 rpm. The electrical and mechanical parameters of the six-phase squirrel cage copper rotor induction motor are \( r_1 = 1.58 \Omega \), \( r_2 = 1.21 \Omega \), \( L_1 = 19.28 \text{ mH} \), \( L_2 = 4.8 \text{ mH} \), \( L_3 = 19.12 \text{ mH} \), \( X_s = 18.24 \times 10^{-3} \text{ Nm} \), and \( Y_s = 2.16 \times 10^{-3} \text{ Nms/rad} \) by means of the open circuit test, the short circuit test, the rotor block test, and the loading test [3]. With the fulfillment of indirect field-oriented control, the six-phase squirrel cage copper rotor induction motor and drive system can be predigested as the control block diagram portrayed in Figure 2. The six-phase squirrel cage copper rotor induction motor drive system can be predigested as

\[
H_t(s) = \frac{1}{s X_s + Y_s} = \frac{1}{(0.01824s + 0.00216)},
\]

which is the transfer function; the \( s \) is Laplace’s operation.

The perfect electromagnetic property for the drive system is hence realized by controlling the propulsion current distributions, where \( \lambda_{qr}i_d/\lambda_{d}i_q \) is the linear value and \( \lambda_{dr} \) is equal to a constant value. Two gains of the proportional-integral current controller are given as \( k_{cc} = 17.5 \) and \( k_{cd} = k_{cc}/T_{cc} = 8.1 \) via certain heuristic comprehension [30–32] so as to get a fine dynamic response by using the Kronecker method to construct a stability boundary in the \( k_{cc} \) and \( k_{cd} \) planes [30–32].
Figure 1. Conformation of the indirect field-oriented control of the six-phase squirrel cage copper rotor induction motor drive system with a digital signal processor control system.

Figure 2. Predigested control block diagram.
3. Design of Rectified Reiterative Sieved-Pollaczek Polynomials Neural Network Backstepping Control with Improved Fish School Search Method

When the six-phase squirrel cage copper rotor induction motor drive system with the electromagnetic torque, the wind stray torque, the flux saturation torque, the cogging torque, the parametric variations, and the external load torque is applied, a disturbance is acted upon; then, Equation (7) is typified by

\[
\frac{dz_s}{dt} = (c_s + \Delta c_s)z_s + (d_s + \Delta d_s)v_s + h_s(\tau_{ls} + \tau_{ws} + \tau_{fs} + \tau_{cs})
\]

(10)

\[
\frac{d\theta_{rs}}{dt} = \omega_{rs} = z_s
\]

(11)

where \( \theta_{rs} \) and \( z_s \) stand for the rotor place and rotor velocity of the six-phase squirrel cage copper rotor induction motor drive system to be presumed as bound; \( \Delta c_s \) and \( \Delta d_s \) stand for two parametric uncertainties from \( X_s \) and \( Y_s \) to be presumed as bound; \( c_s = -\frac{Y_s}{X_s} \); \( d_s = k_{cs}/X_s > 0 \); \( h_s = -\frac{1}{l_{s}X_s} \) stand for three real numbers to be presumed as bound; \( v_s = i_{s1} \) is the control propulsion to the six-phase squirrel cage copper rotor induction motor drive system, i.e., the propulsion current. Equation (9) can be typified by

\[
\frac{dz_s}{dt} = c_s z_s + d_s v_s + \tau_{s1} + \tau_{s2} + \tau_{s3} + \tau_{s4}
\]

(12)

where \( \tau_{s1} = \Delta c_s z_s \) and \( \tau_{s2} = \Delta d_s v_s \) stand for two parametric variations that are to be presumed as bound; \( \tau_{s3} = h_s (\tau_{fs} + \tau_{cs}) \) and \( \tau_{s4} = h_s (\tau_{ls} + \tau_{ws}) \) stand for the internal bundled uncertainty and external bundled torque uncertainty to be presumed as bound.

The control goal is to track the reference locus \( \theta_{rm} \). The snug backstepping control with a sliding switching function is designed so as to reach a good tracking performance. The design procedure is as below.

The tracking error is typified by

\[
e_{s1} = \theta_{rm} - \theta_{rs}
\]

(13)

Take the differential of (13) by

\[
\frac{de_{s1}}{dt} = \frac{d\theta_{rm}}{dt} - \frac{d\theta_{rs}}{dt} = \frac{d\theta_{rm}}{dt} - z_s
\]

(14)

The stabilizing function is typified by

\[
\rho_{s1} = c_{s1} e_{s1} + \frac{d\theta_{rm}}{dt} + c_{s2} \beta_x
\]

(15)

where \( c_{s1} \) and \( c_{s2} \) stand for two positive constants; \( \beta_x = e_{s1}(t)dt \) stands for the integral factor.

The fictitious tracking error is typified by

\[
e_{s2} = z_s - \rho_{s1}
\]

(16)

Take the differential of (16) by

\[
\frac{de_{s2}}{dt} = \frac{dz_s}{dt} - \frac{d\rho_{s1}}{dt} = (c_s z_s + d_s v_s + \tau_{s1} + \tau_{s2} + \tau_{s3} + \tau_{s4}) - \frac{d\rho_{s1}}{dt}
\]

(17)

where \( \tau_{s1}, \tau_{s2}, \) and \( \tau_{s3} \) stand for three unknown parameters. The assessed errors are typified by

\[
e_{s1} = \tilde{e}_{s1} - \tau_{s1}
\]

(18)
\[ e_{s2} = \hat{\tau}_{s2} - \tau_{s2} \]  
\[ e_{s3} = \hat{\tau}_{s3} - \tau_{s3} \]  

where \( e_{s1}, e_{s2}, \) and \( e_{s3} \) are the assessed errors; \( \hat{\tau}_{s1}, \hat{\tau}_{s2}, \) and \( \hat{\tau}_{s3} \) are the assessed values of \( \tau_{s1}, \tau_{s2}, \) and \( \tau_{s3} \). The external bundled torque uncertainty \( \tau_{s4} \) satisfies the condition \( |\tau_{s4}| \leq \gamma_1 \) and is to be presumed as bound. So as to design the snug backstepping control with a sliding switching function, the Lyapunov function can be typified by

\[ A_{s1} = e_{s1}^2/2 + e_{s2}^2/2 + e_{s3}^2/(2\eta_1) + e_{s2}^2/(2\eta_2) + e_{s3}^2/(2\eta_3) + c_{x2}\beta_x^2/2 \]  

By taking the differential of \( A_{s1} \) and by utilizing Equations (14)–(20) and the integral factor \( \beta_x = [e_{s1}(v)]dv \), Equation (21) is typified by

\[ \frac{dA_{s1}}{dt} = e_{s1}\frac{d\theta_m}{dt} - z_s + e_{s2}((e_{s1}z_s + d_1v_s + \tau_{s1} + \tau_{s2} + \tau_{s3} + \tau_{s4}) - \frac{d\rho_{s1}}{dt} + e_{s1}\frac{d\rho_{s2}}{dt} + e_{s2}\frac{d\rho_{s3}}{dt} + c_{x2}\beta_x \frac{d\beta_x}{dt} 
\]
\[ + e_{s3}\frac{d\beta_x}{dt} + c_{x2}\beta_x \frac{d\beta_x}{dt} 
\]
\[ + (\hat{\tau}_{s2} - \tau_{s2}) \frac{d\hat{\tau}_{s2}}{dt} + (\hat{\tau}_{s3} - \tau_{s3}) \frac{d\hat{\tau}_{s3}}{dt} 
\]
\[ \frac{d\hat{\tau}_{s2}}{dt} - e_{s2}((e_{s1}z_s + d_1v_s + \tau_{s1} + \tau_{s2} + \tau_{s3} + \tau_{s4}) - \frac{d\rho_{s1}}{dt} + \hat{\tau}_{s1} \frac{d\hat{\tau}_{s1}}{dt} 
\]
\[ + e_{s2}\frac{d\hat{\tau}_{s2}}{dt} + e_{s3}\frac{d\hat{\tau}_{s3}}{dt} + (e_{s2}\tau_{s2} - \tau_{s2}) \frac{d\hat{\tau}_{s2}}{dt} + (e_{s2}\tau_{s3} - \tau_{s3}) \frac{d\hat{\tau}_{s3}}{dt} \]  

In accordance with Equation (22), the control propulsion \( v_s \) of the snug backstepping control with a sliding switching function can be typified by

\[ v_s = \dot{u}_q = d_s^{-1}[e_{s1} - c_{x3}e_{s2} - c_{x4}z_s + \frac{d\rho_{s1}}{dt} - (\hat{\tau}_{s1} + \hat{\tau}_{s2} + \hat{\tau}_{s3}) - \gamma_1 \text{sgn}(e_{s2})] \]  

where \( C_{x3} \) stands for a positive constant; \( \gamma_1 \) stands for upper boundary that is a constant; \( \gamma_1 \text{sgn}(e_{s2}) \) stands for the sliding switching function. By utilizing Equation (23), Equation (22) can be typified by

\[ \frac{dA_{s1}}{dt} = -c_{x1}e_{s1}^2 - c_{x3}e_{s2}^2 - c_{x4}e_{s3}^2 - e_{s2}\left[\gamma_1 \text{sgn}(e_{s2}) - \tau_{s4}\right] - e_{s2}\left[\hat{\tau}_{s1} + \hat{\tau}_{s2} + \hat{\tau}_{s3}\right] + \hat{\tau}_{s1} \frac{d\hat{\tau}_{s1}}{dt} + \tau_{s1}(e_{s2} - \frac{d\hat{\tau}_{s1}}{dt}) 
\]
\[ + \hat{\tau}_{s2} \frac{d\hat{\tau}_{s2}}{dt} + \tau_{s2}(e_{s2} - \frac{d\hat{\tau}_{s2}}{dt}) + \hat{\tau}_{s3} \frac{d\hat{\tau}_{s3}}{dt} + \tau_{s3}(e_{s2} - \frac{d\hat{\tau}_{s3}}{dt}) \]  

\[ = -c_{x1}e_{s1}^2 - c_{x3}e_{s2}^2 - c_{x4}e_{s3}^2 - e_{s2}\left[\gamma_1 \text{sgn}(e_{s2}) - \tau_{s4}\right] - \hat{\tau}_{s1}(e_{s2} - \frac{d\hat{\tau}_{s1}}{dt}) + \tau_{s1}(e_{s2} - \frac{d\hat{\tau}_{s1}}{dt}) \]
\[
-\dot{\tau}_s(e_{x_2} - \frac{dt_x}{dt_2}) + \tau_2(e_{x_2} - \frac{dt_x}{dt_2}) - \dot{\tau}_s(e_{x_2} - \frac{dt_x}{dt_3}) + \tau_3(e_{x_2} - \frac{dt_x}{dt_3})
\]

where three snug laws \( \frac{dt_x}{dt_1}, \frac{dt_x}{dt_2}, \) and \( \frac{dt_x}{dt_3} \) can be typified by
\[
\frac{dt_x}{dt_1} = \eta_1 e_{x_2} \\
\frac{dt_x}{dt_2} = \eta_2 e_{x_2} \\
\frac{dt_x}{dt_3} = \eta_3 e_{x_2}
\]

(25)
(26)
(27)

So as to reach \( \frac{dA_1(t)}{dt} \leq 0 \) by utilizing Equations (25)–(27) and \( |\tau_s| \leq \gamma_1 \), Equation (24) can be typified by
\[
\frac{dA_1}{dt} = -c_{x_1} e_{x_1}^2 - c_{x_3} e_{x_2}^2 - e_{x_2} [\gamma \text{sgn}(e_{x_2}) - \tau_s]
\]
\[
\leq -c_{x_1} e_{x_1}^2 - c_{x_3} e_{x_2}^2 - |e_{x_2}| |\gamma_1 - |\tau_s|
\]
\[
\leq -c_{x_1} e_{x_1}^2 - c_{x_3} e_{x_2}^2 \\
\leq 0
\]

Equation (28) shows \( \frac{dA_1(t)}{dt} \) to be negative and semidefinite (i.e., \( A_1(t) \leq A_1(0) \)), meaning that \( e_{x_1} \) and \( e_{x_2} \) are bound. The following term is typified by
\[
B_{x_1}(t) = c_{x_1} e_{x_1}^2 + c_{x_3} e_{x_2}^2 = -\frac{dA_1}{dt}
\]

(29)

By taking the integration of, Equation (29) is typified by
\[
\int_{0}^{t} B_{x_1}(v) \, dv = u_{x_1}(e_{x_1}(0), e_{x_2}(0)) - u_{x_1}(e_{x_1}(t), e_{x_2}(t))
\]

(30)

As \( u_{x_1}(e_{x_1}(0), e_{x_2}(0)) \) is bound, and \( u_{x_2}(e_{x_1}(t), e_{x_2}(t)) \) is nonincreasing and presumed to be bound, then \( \lim_{t \to \infty} \int_{0}^{t} B_{x_1}(v) \, dv < \infty \). Moreover, \( \frac{dB_{x_1}(t)}{dt} \) is presumed to be bound, hence \( B_{x_1}(t) \) is a uniformly continuous function. By utilizing Barbalat’s lemma [33,34], it can be portrayed that
\[
\lim_{t \to \infty} B_{x_1}(t) = 0
\]
That is, \( e_{x_1} \) and \( e_{x_2} \) will converge to zero when \( t \to \infty \). Furthermore, \( \lim_{t \to \infty} B_{x_1}(t) = 0 \)

\[
\lim_{t \to \infty} z_s = d\theta_m/\eta_3
\]
The stability of the snug backstepping control with a sliding switching function can be guaranteed, and, consequently, the control block diagram is portrayed in Figure 3.

As the external bundled torque uncertainty \( \tau_s \) is unknown, and its upper boundary is troublesome to decide, the appraised value \( \hat{\tau}_s \) of the external bundled torque uncertainty \( \tau_s \) is not easy to estimate. Consequently, the rectified reiterative sieved-Pollaczek polynomials neural network is proposed to adapt the real value of the external bundled torque uncertainty \( \tau_s \). The rectified reiterative sieved-Pollaczek polynomials neural network with a three-layer constitution, which is made up of the primary layer, the central layer, and the tertiary layer, is portrayed in Figure 4. The information intentions in each node for each layer are explained in the following expression.

In the primary layer, input information and output information are typified by
\[
d_{c, a}^1 = \prod_{e} k^1_a(H) \phi^1_{\omega e}(H) \phi^1_c(H-1) \phi^1_a(H-1), \quad d_{c, a}^1(H) = g^1_{c, a}(d_{c, a}^1) = v^1_{c, a}, \quad a = 1, 2
\]

(31)
where \( h^1 = \theta_{m} - \theta_{r} = e_{x1} \) and \( h^2 = e_{x1}(1 - z^{-1}) = \Delta e_{x1} \) denote the position discrepancy and position discrepancy alteration, respectively. \( H \) denotes the iteration count. \( \phi^{1}_{ac} \) denotes the reiterative weight between the tertiary layer and the primary layer. \( q^{3}_{a} \) denotes the output information of the node in the tertiary layer. The symbol \( \Pi \) denotes a multiplication factor.

In the central layer, input information and output information are typified by

\[
dx_{b}^{2} = \sum_{i=1}^{2} l_{i}^{2}(H) + \epsilon l_{b}^{2}(H-1), \quad l_{b}^{2}(H) = q_{b}^{2}(dx_{b}^{2}) = SP_{b}^{a}(dx_{b}^{2}; q), \quad b = 0, 1, \ldots, n-1
\]

where \( \epsilon \) denotes the reiterative gain in the central layer. \( n \) is the number of nodes in the central layer. Sieved-Pellaczk polynomials function [25,26] is adopted as the activation function \( q_{b}^{2} \). \( SP_{b}^{a}(x; q) \) denotes the sieved-Pellaczk polynomials in the interval \([-1, 1]\). \( SP_{b}^{a}(x; q) = 1 \), \( SP_{1}^{a}(x; q) = 2x \), and \( SP_{2}^{a}(x; q) = 4x^2 - 1 \) denote the 0-, 1-, and 2-order sieved-Pellaczk polynomials, respectively. The sieved-Pellaczk polynomials may be generated by the recurrence relation [25,26]

\[
SP_{n+1}(x; q) = 2xSP_{n}^{a}(x; q) - SP_{n-1}(x; q)
\]

The symbol \( \sum \) denotes a summation factor.

In the tertiary layer, input information and output information are typified by

\[
dx_{c}^{3} = \sum_{b=0}^{n-1} \phi_{cb}^{2}(H) l_{b}^{2}(H), \quad l_{c}^{2}(H) = q_{c}^{2}(dx_{c}^{3}) = dx_{c}^{3}, \quad c = 1
\]

where \( \phi_{cb}^{2} \) denotes the connecting weight between the central layer and the tertiary layer. \( q_{c}^{3} \) denotes the linear activation function. The output \( l_{c}^{2}(H) \) in the tertiary layer of the rectified reiterative sieved-Pellaczk polynomials neural network can be typified by
\[ \hat{\tau}_{s4}(A) = l_{s1}^3(H) = A^T B \]

where \( A = [h_{0}^2 \ldots h_{m-1}^2]^T \) and \( B = [l_{0}^2 \ldots l_{m-1}^2]^T \) denote the weight vector in the tertiary layer and the input vector in the tertiary layer, respectively.

The minimum reorganized error \( e_{s4} \) is typified by

\[ e_{s4} = \tau_{s4} - \tau_{s4}(A^*) = \tau_{s4} - (A^*)^T B \]

where \( A^* \) stands for an ideal weight vector that reaches the smallest reorganized error. So as to make up the smallest reorganized error \( e_{s4} \), the indemnified controller \( v_{fr} \) with an appraised rule is proposed. It is presumed that the small positive number \( \sigma_{s4} \) stands for more than the absolute value of \( e_{s4} \), i.e., \( \sigma_{s4} \geq |e_{s4}| \). The Lyapunov function is typified by

\[ \Delta x_2 = \frac{1}{2} \eta_5 (e_{s5} + (A - A^*)^T (A - A^*)/(2\beta_1)) \]

where \( \eta_5 \) stands for an adaptive gain. \( \beta_1 \) stands for the fickle rate of the connecting weight. \( e_{s5} = \hat{e}_{s4} - e_{s4} \) stands for the appraised error to be presumed as bound. \( \hat{e}_{s4} \) stands for the appraised value of smallest reorganized error \( e_{s4} \); By taking the derivative of \( \Delta x_2 \) by utilizing Equation (14) to Equation (20) and the integral factor \( \frac{1}{\eta_5} \), Equation (36) is typified by

\[ \frac{dA_{x2}}{dt} = e_{s1}(-c_{x1}e_{x1} - e_{s2}) + e_{s2}((c_{x2}z_{2} + d_{v}s + \tau_{s4}) - \frac{dp_{x1}}{dt}) + \hat{e}_{s1} \frac{d\hat{e}_{s1}}{\eta_{dt}} + (c_{x1} \tau_{s1} - \tau_{s1} \tau_{s1} \frac{d\hat{e}_{s1}}{\eta_{dt}}) \]

\[ + \hat{e}_{s2} \frac{d\hat{e}_{s2}}{\eta_{dt}} + (e_{x2}z_{2} - e_{s3} \frac{d\hat{e}_{s2}}{\eta_{dt}}) + (e_{x2}z_{3} - e_{s3} \frac{d\hat{e}_{s2}}{\eta_{dt}}) + (e_{s5} + (A - A^*)^T \frac{dA}{\beta_0 dt}) \]
In accordance with Equation (37), the control propulsion \( v_s = \dot{v}_s \) of the RRSPPNNB control with the IFSS (improved fish school search) method can be typified by

\[
v_s = \dot{v}_s = i_d^* + d_{\text{st}}^{-1}\left[ e_s - c_{s3} e_s + c_{s4} z_s + \frac{dP_\text{st}}{dt} - (\dot{r}_{s1} + \dot{r}_{s2} + \dot{r}_{s3} + \dot{r}_{s4}(A) + v_r) \right]
\]  

(38)

By utilizing Equation (38), Equation (37) can be typified by

\[
\frac{dA_s}{dt} = \begin{pmatrix} -c_{s1} e_{s1}^2 - c_{s3} e_{s2}^2 - e_{s2}[\dot{r}_{s4} + v_r - \dot{r}_{s4}] - e_{s2}[\dot{r}_{s4}(A) - \dot{r}_{s4}(A^*)] - e_{s2}^2 \dot{r}_{s4} + e_{s5} \frac{d\dot{r}_{s4}}{dt} + \dot{r}_{s4} \frac{d\dot{r}_{s4}}{dt} \end{pmatrix} + \dot{r}_{s2} \frac{d\dot{r}_{s2}}{dt} + \dot{r}_{s3} e_{s2} - \frac{d\dot{r}_{s3}}{dt} \frac{d\dot{r}_{s3}}{dt} + e_{s5} \frac{d\dot{r}_{s5}}{dt} + (A - A^*)^T \frac{dA}{\beta_d dt}
\]

(39)

By utilizing Equations (25)–(27) and \( e_{s3} = \dot{r}_{s4} - \dot{r}_{s4} \), Equation (39) can be typified by

\[
\frac{dA_s}{dt} = \begin{pmatrix} -c_{s1} e_{s1}^2 - c_{s3} e_{s2}^2 + e_{s2} e_{s4} - e_{s2} (A - A^*)^T B - e_{s2} v_r + (\dot{r}_{s4} - \dot{r}_{s4}) \frac{d\dot{r}_{s5}}{dt} + (A - A^*)^T \frac{dA}{\beta_d dt} \end{pmatrix}
\]

(40)

So as to reach \( A_{s2} \leq 0 \), the cozy law \( \frac{dA}{dt} \) and the indemnified controller \( v_r \), with an appraised rule \( \dot{r}_{s4} \) and a snug law \( \frac{d\dot{r}_{s5}}{dt} \) to reduce uncertainty influences, can be typified by

\[
\frac{dA}{dt} = \beta_1 e_{s2} B
\]

(41)

\[
v_r = \dot{r}_{s4} \text{sgn}(e_{s2})
\]

(42)

\[
\frac{d\dot{r}_{s5}}{dt} = \eta_5 |e_{s2}|
\]

(43)

By substituting Equations (41)–(43) into Equation (40) and by utilizing \( \dot{r}_{s4} \geq |e_{s4}| \), Equation (40) can be typified by

\[
\frac{dA_s}{dt} = \begin{pmatrix} -c_{s1} e_{s1}^2 - c_{s3} e_{s2}^2 + e_{s2} e_{s4} - e_{s2} (A - A^*)^T B - e_{s2} \dot{r}_{s4} \text{sgn}(e_{s2}) + (\dot{r}_{s4} - \dot{r}_{s4}) \beta_1 e_{s2} \frac{dA}{\beta_1 dt} \end{pmatrix}
\]

\[
\leq \begin{pmatrix} -c_{s1} e_{s1}^2 - c_{s3} e_{s2}^2 + e_{s2} e_{s4} - (\dot{r}_{s4} - \dot{r}_{s4}) |e_{s2}| \end{pmatrix}
\]

\[
\leq \begin{pmatrix} -c_{s1} e_{s1}^2 - c_{s3} e_{s2}^2 \end{pmatrix}
\]

(44)

Equation (44) portrays \( A_{s2}(t) \) to be negative and semidefinite, i.e., \( A_{s2}(t) \leq A_{s2}(0) \), meaning that \( e_{s1} \) and \( e_{s2} \) are bound. By utilizing Barbalat’s lemma, it can be represented that \( -B_{s1}(t) \to 0 \) at \( t \to \infty \) by way of Equation (29), Equation (30), and Equation (44), i.e., \( e_{s1} \) and \( e_{s2} \) will converge to zero at \( t \to \infty \). The stability of the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with the improved fish school search method can be ensured, and, consequently, the control block diagram is portrayed in Figure 5.

A training skillfulness of parameters in the rectified reiterative sieved-Pollaczek polynomials neural network can be unearthed by utilizing Lyapunov stability and the gradient descent skill. The
improved fish school search method with two adjusted factors is applied to look for two better fickle rates in the rectified reiterative sieved-Pollaczek polynomials neural network to acquire faster convergence. The connecting weight parametric presented in Equation (41) can be typified by

\[
\frac{d\varphi_{cb}^2}{dt} = \beta_1 e_{x2}l_b^2
\]  

(45)

So as to describe the online training process of the rectified reiterative sieved-Pollaczek polynomials neural network, a goal function that explains the online training procedure of the rectified reiterative sieved-Pollaczek polynomials neural network is typified by

\[
L_2 = e_{x2}^2 / 2
\]  

(46)

The cozy learning rule of the connecting weight, by utilizing the gradient descent skill with the chain rule, is typified by

\[
\frac{d\varphi_{cb}^2}{dt} = -\beta_1 \frac{\partial L_2}{\partial \varphi_{cb}^2} = -\beta_1 \frac{\partial L_2}{\partial l_c^3} \frac{\partial l_c^3}{\partial x_c^3} \frac{\partial x_c}{\partial \varphi_{cb}^2} = -\beta_1 \frac{\partial L_2}{\partial l_c^3} \frac{\partial l_c^3}{l_b^2}
\]  

(47)

It is well-known that \( \partial L_2 / \partial l_c^3 = -e_{x2} \) by way of Equations (45)–(47). The adaptive fickle rule of recurring weight \( \varphi_{ac}^1 \), by utilizing the gradient descent technology with the chain rule, is hence typified by

\[
\frac{d\varphi_{ac}^1}{dt} = -\alpha_2 \frac{\partial L_2 \partial l_c^3 \partial l_b^2 \partial dx_c^2 \partial l_a^1 \partial dx_a^1}{\partial l_c^3 \partial l_b^2 \partial dx_c^2 \partial l_a^1 \partial dx_a^1} \frac{\partial l_a}{\partial \varphi_{ac}^1} = \beta_2 e_{x2} \varphi_{cb}^2 SL_b^4 (\cdot)h_a(H)l_k^1(H - 1)^1(H)
\]  

(48)

where \( \beta_2 \) stands for the fickle rate of the reiterative weight. So as to obtain better convergence, the improved fish school search method is used for looking for two adjusted fickle rates of the two weights in the rectified reiterative sieved-Pollaczek polynomials neural network. The proposed improved fish school search method not only improved convergent speed but also searched for two optimal fickle rates in this study.
Figure 5. Control block diagram of the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with improved fish school search method.

In the improved fish school search method, every fish in the school realizes a local search that looks for promising regions in the search space. Individual components of the movement algorithm, with regard to the learning rate, is expressed below.

$$x_{n_1,i}(t_1 + 1) = x_{n_1,i}(t_1) + rand(-1,1) \times S p_{n_1,i}, n_1 = 1, 2, i = 1, ..., M_1, t_1 = 1, 2, ..., N_1$$  (49)

where $x_{n_1,i}(t_1 + 1)$ and $x_{n_1,i}(t_1)$ represent the position of fish $i$ before and after the individual movement operator, respectively. $rand(-1,1)$ is a uniformly distributed random number varying between -1 and 1. $S p_{n_1,i}$ is the parameter that defines the maximum displacement for this movement. The new position $x_{n_1,i}(t_1 + 1)$ is only accepted if the fitness of fish $i$ improves with the position change. If it is not the case and fish $i$ remains in the same position, then $x_{n_1,i}(t_1 + 1) = x_{n_1,i}(t_1)$.

An average of the individual movements in the collective–instinctive component of the movement is also expressed below.
The vector \( F_{n,av}(t_1) \) represents the weighted average of the displacements of each fish. It means that the fish that experienced a greater improvement will attract more fish into its position. After the vector \( F_{n,av}(t_1) \) is computed, every fish can be expressed below as

\[
x_{n,i}(t_1 + 1) = x_{n,i}(t_1) + F_{n,av}(t_1), \quad n_1 = 1, 2, \ldots, M_1, \quad t_1 = 1, 2, \ldots, N_1
\]

This operator in the collective–volitive component of the movement is used so as to regulate the exploration/exploitation ability of the school during the search process. First of all, the barycenter \( G_{n_1}(t_1) \) of the school that is calculated based on the position \( x_{n,i}(t_1) \) and the weight \( F_{n,i}(t_1) \) of each fish \( i \) can be expressed by

\[
G_{n_1}(t_1) = \frac{\sum_{i=1}^{M_1} x_{n,i}(t_1) F_{n,i}(t_1)}{\sum_{i=1}^{M_1} F_{n,i}(t_1)}, \quad n_1 = 1, 2, \ldots, M_1, \quad t_1 = 1, 2, \ldots, N_1
\]

Moreover, the movement operators also define a feeding operator so as to update the weights of every fish by

\[
F_{n,i}(t_1 + 1) = F_{n,i}(t_1) - \frac{\Delta y_{n,i}}{\max(\{\Delta y_{n,i}\})}, \quad n_1 = 1, 2, \ldots, M_1, \quad t_1 = 1, 2, \ldots, N_1
\]

where \( F_{n,i}(t_1) \) is the weight parameter for fish \( i \). \( \Delta y_{n,i} \) is the fitness variation between the last and the new positions. \( \max(\{\Delta y_{n,i}\}) \) represents the maximum absolute value of the fitness variation among all the fish in the school. \( F_{n,i}(t_1) \) is only allowed to vary from 1 up to \( F_{n,\text{scale}} / 2 \), which is a user-defined attribute. The weights of all fish are initialized with the value \( F_{n,\text{scale}} / 2 \). Finally, if the total school weight \( \sum_{i=1}^{M_1} F_{n,i}(t_1) \) has increased from the last to the current iteration, the fish are attracted to the barycenter \( G_{n_1}(t_1) \) by using Equation (54).

\[
x_{n,i}(t_1 + 1) = x_{n,i}(t_1) - \frac{\Delta y_{n,i}}{\text{distance}(x_{n,i}(t_1) - G_{n_1}(t_1))}, \quad n_1 = 1, 2, \ldots, M_1
\]

If the total school weight has not improved, the fish are spread away from the barycenter \( G_{n_1}(t_1) \) by using Equation (55).

\[
x_{n,i}(t_1 + 1) = x_{n,i}(t_1) + \frac{\Delta y_{n,i}}{\text{distance}(x_{n,i}(t_1) - G_{n_1}(t_1))}, \quad n_1 = 1, 2, \ldots, M_1
\]

where \( \text{distance}(x_{n,i}(t_1) - G_{n_1}(t_1)) \) is the Euclidean distance between the fish in position \( i \) and the school barycenter \( G_{n_1}(t_1) \). \( \text{rand}(0,1) \) is a uniformly distributed random number varying between 0 and 1. Lastly, \( x_{n,i}(t_1 + 1), \quad n_1 = 1, 2 \) is the best solution in regard to the learning rates \( \beta_{n,i}(t_1 + 1), \quad n_1 = 1, 2 \) of the two weights in the rectified reiterative sieved-Pollaczek polynomials neural network. Hence, the better numbers can be optimized by using the improved fish school search method for adjusting...
the two fickle rates of two weights so as to find two optimal values and speed-up the convergence of two weights.

**Remark 1.** The key point of the proposed design is to utilize the Lyapunov function for constructing the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method expressed by Equation (38), which reduces the input dimensions of the rectified reiterative sieved-Pollaczek polynomials neural network.

**Remark 2.** A block diagram of the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method is shown in Figure 5, which reduces the input dimensions of the rectified reiterative sieved-Pollaczek polynomials neural network.

**Remark 3.** A rectified reiterative sieved-Pollaczek polynomials neural network approximation holds only in a compact set. Thus, the obtained result is semiglobal in the sense that they hold for the compact sets; there exists the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method with a sufficiently large number of rectified reiterative sieved-Pollaczek polynomials neural network nodes, such that all the closed-loop signals are bound.

**Remark 4.** Owing to the inherent uncertainty in the six-phase squirrel cage copper rotor induction motor drive system in Equation (8), it will be shown that we cannot conclude the convergence of the tracking error to zero. Therefore, it is only reasonable to expect that all errors defined on the proposed rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method with the indemnified control, the assessed law, and the cozy learning law will converge into a neighborhood with reachable radius and remains within it thereafter, which is so-called uniformly ultimate boundedness.

4. Experimental Results

A block diagram of indirect field-oriented control of the six-phase squirrel cage copper rotor induction motor drive system by utilizing a TMS 320F28335 digital signal processor control system is portrayed in Figure 1. A photo picture of the examination structure is portrayed in Figure 6. A TMS 320F28335 digital signal processor control system involves 4 channels of digital–analog converters and 2 channels of encoder connection ports. The coordinate transformation in the indirect field-oriented control is realized by the digit signal processor control system. The used control technologies in the real-time realization by utilizing the TMS 320F28335 digital signal processor control system are composed of the core program and the subcore interrupt service routine in the digital signal processor control system, as portrayed in Figure 7.
In the core program, parameters and input/output initialization are processed. The interrupt time for the subcore interrupt service routine is set. After permitting the interruption, the core program is used to monitor control data. The subcore interrupt service routine, with 2 msec sampling time, is used for reading the rotor place of the six-phase squirrel cage copper rotor induction motor drive system from the encoder and three-phase currents by way of the analog–digital converter, calculating reference model and position error, executing lookup tables and coordinate transformation, executing the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with the improved fish school search method, and outputting six-phase current mandates to swap the pulse-width-modulation voltage source inverter with six sets of insulated-gate bipolar transistor power modules by way of the interlocked circuit and isolated circuit. Additionally, the tested bandwidth of the place control loop and the tested bandwidth of the current control loop are about 100 and 1000 Hz for the six-phase squirrel cage copper rotor induction motor drive system under the nominal fettle. The proposed controllers are realized by the TMS 320F28335 digital signal processor control system. The coordinate transformation in the indirect field-oriented controller is realized by the TMS 320F28335 digital signal processor control system. The control goal is to control the rotor to rotate 6.26 rad cyclically. Then, when the mandate is a sinusoidal reference locus, the reference model is set to unit value.

For comparison of control performance with the four control systems, five cases were provided in the experiment. The four control systems are the popular proportional–integral controller as control system W1, the snug backstepping control with sliding switching function as controller W2, the backstepping control using reformed recurrent Hermite polynomial neural networks with adaptive law and error-estimated law [35] as controller W3, and the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with the improved fish school search method as controller W4. The five tested cases are as follows. Case R1 is the nominal case at the periodic step command from 0 to 6.28 rad. Case R2 is the cogging torque, the column friction torque, and the Stribeck effect torque and the parameter variations case with 4 times the nominal value at the periodic step command from 0 to 6.28 rad. Case R3 is the nominal case due to periodic sinusoidal command from $-6.28$ to 6.28 rad. Case R4 is the cogging torque, the column friction torque, and the Stribeck effect torque and the parameter variations case with 4 times the nominal value due to the periodic sinusoidal command from $-6.28$ to 6.28 rad. Case R5 is the load torque disturbance $\tau_{le} = 5 \, Nm$ while adding load cases.
To achieve good transient and steady-state control performance, two gains of the popular proportional–integral controller as control system W1 are $k_{pp} = 4.5$ and $k_{ip} = k_{pp}/T_{ip} = 1.8$ by using the Kronecker method to construct a stability boundary in the $k_{pp}$ and $k_{ip}$ planes [30–32] on the tuning of the PI controller in Case R1 for position tracking.

The parameters of the snug backstepping control with a sliding switching function as controller W2 are given as $c_{x1} = 2.2$, $c_{x2} = 2.6$, $c_{x3} = 2.1$, $\eta_1 = 0.1$, $\eta_2 = 0.1$, $\eta_3 = 0.1$, $\gamma_1 = 9.1$ according to

The parameters of the backstepping control using a reformed recurrent Hermite polynomial neural network with adaptive law and error-estimated law [35] as controller W3 are given as $c_1 = 2.2$, $c_2 = 2.6$, $c_3 = 2.1$, $\eta_1 = 0.1$, $\eta_2 = 0.1$, $\eta_3 = 0.1$, $\tau = 0.3$, $\gamma = 0.2$ according to heuristic knowledge in Case R1 for position tracking to achieve good transient and steady-state control performance. Furthermore, to show the effectiveness of the control system with a small number of neurons, the reformed recurrent Hermite polynomial neural network has 2, 4, and 1 neurons in the input layer, the hidden layer, and the output layer, respectively. The parameter adjustment process remains continually active for the duration of the experimentation. The reformed recurrent Hermite polynomial neural network in Lewis et al. [36] is adopted to initialize the parameters in this paper. The parameter adjustment process remains continually active for the duration of the experimentation.

The parameters of the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with the improved fish school search method as controller W4 are given as $c_1 = 2.2$, $c_2 = 2.6$, $c_3 = 2.1$, $\eta_1 = 0.1$, $\eta_2 = 0.1$, $\eta_3 = 0.1$, $\varepsilon = 0.3$, $\tau_5 = 0.2$ according to heuristic knowledge in Case R1 for position tracking to achieve good transient and steady-state control performance. Furthermore, to show the effectiveness of the control system with a small number of neurons, the rectified reiterative sieved-Pollaczek polynomials neural network has 2, 4, and 1 neurons in the first layer, the second layer, and the third layer, respectively. The parameter adjustment process remains continually active for the duration of the experimentation. The rectified reiterative sieved-Pollaczek polynomials neural network in Lewis et al. [36] is adopted to initialize the parameters in this paper. The parameter adjustment process remains continually active for the duration of the experimentation.

Experimental results of the rotor position response by utilizing the four control systems for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R1 are shown in Figure 8. The position responses of the rotor by utilizing the control systems W1, W2, W3, and W4 in Case R1 are shown in Figure 8a–d, respectively. Additionally, experimental results of the associated control propulsions by utilizing the control systems W1, W2, W3, and W4 for controlling the SPSCCRIM drive system in Case R1 are shown in Figure 9. The associated control propulsions by utilizing the control systems W1, W2, W3, and W4 in Case R1 are shown in Figure 9a–d, respectively. The maximum errors of $e_{x_1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R1 are 0.21, 0.19, 0.15, and 0.10 rad, respectively. The root-mean-square errors of $e_{x_1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R1 are 0.11, 0.09, 0.07, and 0.05 rad, respectively.

Experimental results of the rotor position response by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R2 are shown in Figure 10. The position responses of the rotor by utilizing the control systems W1, W2, W3, and W4 in Case R2 are shown in Figure 10a–d, respectively. Additionally, experimental results of the associated control propulsions by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R2 are shown in Figure 11. The associated control propulsions by utilizing the control systems W1, W2, W3, and W4 in Case R2 are shown in Figure 11a–d, respectively. The maximum errors of $e_{x_1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R2 are 0.56, 0.37, 0.28, and 0.19 rad, respectively. The root-mean-square errors of $e_{x_1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R2 are 0.27, 0.18, 0.13, and 0.09 rad, respectively.

Experimental results of the rotor position response by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R3 are shown in Figure 12. The position responses of the rotor by utilizing the control systems W1, W2, W3, and W4 in Case R3 are shown in Figure 12a–d, respectively. Additionally, experimental results of the associated control propulsions by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R3 are
shown in Figure 13. The associated control propulsions by utilizing the control systems W1, W2, W3, and W4 in Case R3 are shown in Figure 13a–d, respectively. The maximum errors of $e_{x1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R3 are 0.21, 0.18, 0.14, and 0.10 rad, respectively. The root-mean-square of $e_{x1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R3 are 0.10, 0.09, 0.07, and 0.05 rad, respectively.

Experimental results of the rotor position response by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R4 are shown in Figure 14. The position responses of the rotor by utilizing the control systems W1, W2, W3, and W4 in Case R4 are shown in Figure 14a–d, respectively. Additionally, experimental results of the associated control propulsions by utilizing the control systems W1, W2, W3, and W4 for controlling the six-phase squirrel cage copper rotor induction motor drive system in Case R4 are shown in Figure 15. The associated control propulsions by utilizing the control systems W1, W2, W3, and W4 in Case R4 are shown in Figure 15a–d, respectively. The maximum errors of $e_{x1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R4 are 0.52, 0.36, 0.27, and 0.18 rad, respectively. The root-mean-square errors of $e_{x1}$ by utilizing the control systems W1, W2, W3, and W4 in Case R4 are 0.25, 0.18, 0.13, and 0.09 rad, respectively.

From the experimental results, the fine tracking responses of the position in Case R1 and Case R3, shown in Figures 8a and 12a, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W1. Additionally, the sluggish tracking responses of the position in Case R2 and Case R4, shown in Figures 10a and 14a, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W1 due to greater nonlinear disturbance. The linear controller has weak robustness under greater nonlinear disturbance because of inappropriate gains tuning or degenerate nonlinear effects. From the experimental results, good tracking responses of the position in Cases R1, R2, R3, and R4, shown in Figures 8b, 10b, 12b, and 14b, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W2. However, the beating function with a higher upper boundary results in very serious vibration in the control propulsions in Cases R1, R2, R3, and R4, shown in Figures 9b, 11b, 13b, and 15b. It is a well-known fact that the control propulsions with serious vibration will wear the bearing mechanism and might excite unstable system dynamics. From the experimental results, better tracking responses of the position in Cases R1, R2, R3, and R4, shown in Figures 8c, 10c, 12c, and 14c, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W3 due to adaptive mechanism action. Moreover, an adaptive mechanism action of the reformed recurrent Hermite polynomial neural network [35] results in smaller vibrations in the control propulsions in Cases R1, R2, R3, and R4, shown in Figures 9c, 11c, 13c, and 15c. Furthermore, due to the online adaptive mechanism action of the reformed recurrent Hermite polynomial neural network [35], vibrations of the control propulsions in Cases R1, R2, R3, and R4, shown in Figures 9c, 11c, 13c, and 15c, have been improved. From the experimental results, the best tracking responses of the position in Cases R1, R2, R3, and R4, shown in Figures 8d, 10d, 12d, and 14d, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W4 due to adaptive mechanism action. Moreover, an adaptive mechanism action of the rectified reiterative sieved-Pollaczek polynomials neural network results in smaller vibrations in the control propulsions in Cases R1, R2, R3, and R4, shown in Figures 9d, 11d, 13d, and 15d. From the experimental results, excellent tracking responses of the position in Cases R1, R2, R3, and R4, shown in Figures 9d, 11d, 13d, and 15d, are obtained for controlling the six-phase squirrel cage copper rotor induction motor drive system by utilizing control system W4 due, in large part, to the online adaptive adjustment of under greater nonlinear disturbance. Furthermore, due to the online adaptive mechanism action of the rectified reiterative sieved-Pollaczek polynomials neural network, vibrations of the control propulsions at Cases R1, R2, R3, and R4, shown in Figures 9d, 11d, 13d, and 15d, have obviously been improved.

Finally, the experimental results of the measured rotor position response caused in Case R5 are shown in Figure 16. Experimental results of the measured rotor position responses by utilizing
control system W1, W2, W3, and W4 are shown in Figure 16a–d, respectively. The maximum errors of $e_{x1}$ by utilizing control systems W1, W2, W3, and W4 in Case R5 are 3.00, 1.55, 1.12, and 0.72 rad, respectively. The root-mean-square errors of $e_{x1}$ by utilizing control systems W1, W2, W3, and W4 in Case R5 are 1.49, 0.77, 0.55, and 0.31 rad, respectively.

From these experimental results, the transient response of control system W4 is better than control systems W1, W2, and W3 at load regulation. However, the robust control performance of control system A4 was outstanding for controlling the six-phase squirrel cage copper rotor induction motor drive system in the tracking of periodic steps and sinusoidal commands under the occurrence of parameter disturbance and load regulation due, in large part, to the online adaptive adjustment of the rectified reiterative sieved-Pollaczek polynomials neural network. Control system W4 results in the smallest tracking error in comparison with control systems W1, W2, and W3, as shown in Table 1. According to the tabulated measurements, control system W4 indeed yields superior control performance.

Additionally, some comparisons of the control performances by using control systems W1, W2, W3, and W4 are summarized in Table 2 with respect to some experimental results. Moreover, the characteristic performances in comparison with control systems W1, W2, W3, and W4, on the basis of the above experimental results, are recapitulated below. The chatterings in control intensity obtained by using control systems W1, W2, W3, and W4 are 5%, 12%, 4%, and 3% of the rated value in Case R2, respectively. The dynamic responses obtained by using control systems W1, W2, W3, and W4 are the rising time of 0.09 sec in Case R2, 0.08 sec in Case R2, 0.08 sec in Case R2, and 0.07 sec in Case R2, respectively. The ability of load regulation, obtained by using control systems W1, W2, W3, and W4, is the maximum error, with 3.01 rad with added load in Case R5, 1.57 rad in Case R5, 1.12 rad in Case R5, and 0.75 rad in Case R5. The convergent speeds obtained by using control systems W1, W2, W3, and W4 are the tracking error response at 0.2 rad within 0.05 sec in Case R2, 0.1 rad within 0.04 sec in Case R2, 0.1 rad within 0.04 sec in Case R2, and 0.1 rad within 0.01 sec in Case R2, respectively. The position tracking errors obtained by using control systems W1, W2, W3, and W4 are the maximum error as 0.56 rad in Case R2, 0.37 rad in Case R2, 0.28 rad in Case R2, and 0.19 rad in Case R2, respectively. The rejection ability for parameter disturbances obtained by using control systems W1, W2, W3, and W4 are the maximum error as 0.56 rad in Case R2, 0.37 rad in Case R2, 0.28 rad in Case R2, and 0.19 rad in Case R2, respectively. From these performances, the vibration in the control intensity, the dynamic response, the convergence speed, the position tracking error, and the ability of load regulation are displayed in Table 2. Control system W4 is more successful than control systems W1, W2, W3, and W4, from Table 2.
Figure 8. Experimental results of the rotor position response in Case R1 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 9. Experimental results of the response of control propulsion in Case R1 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 10. Experimental results of the rotor position response in Case R2 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 11. Experimental results of the response of control propulsion in Case R2 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 12. Experimental results of the rotor position response in Case R3 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 13. Experimental results of the response of control propulsion in Case R3 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 14. Experimental results of the rotor position response in Case R4 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 15. Experimental results of the response of control propulsion in Case R4 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Figure 16. Experimental results of the rotor position response in Case R5 (a) by utilizing control system W1, (b) W2, (c) W3, and (d) W4.
Table 1. Performance comparison of control systems.

<table>
<thead>
<tr>
<th>Performance</th>
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<tbody>
<tr>
<td></td>
<td>Case R1</td>
<td>Case R2</td>
<td>Case R3</td>
<td>Case R4</td>
<td>Case R5</td>
</tr>
<tr>
<td>Maximum error of $e_{x1}$</td>
<td>0.21 rad</td>
<td>0.56 rad</td>
<td>0.20 rad</td>
<td>0.54 rad</td>
<td>3.00 rad</td>
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<tr>
<td>Root-mean-square error of $e_{x1}$</td>
<td>0.11 rad</td>
<td>0.27 rad</td>
<td>0.10 rad</td>
<td>0.25 rad</td>
<td>1.49 rad</td>
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<table>
<thead>
<tr>
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<tr>
<td></td>
<td>Case R1</td>
<td>Case R2</td>
<td>Case R3</td>
<td>Case R4</td>
<td>Case R5</td>
</tr>
<tr>
<td>Maximum error of $e_{x1}$</td>
<td>0.19 rad</td>
<td>0.37 rad</td>
<td>0.18 rad</td>
<td>0.36 rad</td>
<td>1.55 rad</td>
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<tr>
<td>Root-mean-square error of $e_{x1}$</td>
<td>0.09 rad</td>
<td>0.18 rad</td>
<td>0.09 rad</td>
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<thead>
<tr>
<th>Performance</th>
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<tbody>
<tr>
<td></td>
<td>Case R1</td>
<td>Case R2</td>
<td>Case R3</td>
<td>Case R4</td>
<td>Case R5</td>
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<tr>
<td>Maximum error of $e_{x1}$</td>
<td>0.15 rad</td>
<td>0.28 rad</td>
<td>0.14 rad</td>
<td>0.27 rad</td>
<td>1.12 rad</td>
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<td>Root-mean-square error of $e_{x1}$</td>
<td>0.07 rad</td>
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<table>
<thead>
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<tbody>
<tr>
<td></td>
<td>Case R1</td>
<td>Case R2</td>
<td>Case R3</td>
<td>Case R4</td>
<td>Case R5</td>
</tr>
<tr>
<td>Maximum error of $e_{x1}$</td>
<td>0.10 rad</td>
<td>0.19 rad</td>
<td>0.10 rad</td>
<td>0.18 rad</td>
<td>0.72 rad</td>
</tr>
<tr>
<td>Root-mean-square error of $e_{x1}$</td>
<td>0.05 rad</td>
<td>0.09 rad</td>
<td>0.05 rad</td>
<td>0.09 rad</td>
<td>0.31 rad</td>
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Table 2. Control characteristic performance comparisons of control systems.

<table>
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<tr>
<th>Characteristic Performance</th>
<th>Control System W1</th>
<th>Control System W2</th>
<th>Control System W3</th>
<th>Control System W4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vibration in control intensity</td>
<td>Small (5% of rated value in Case R2)</td>
<td>Middle (12% of rated value in Case R2)</td>
<td>Smaller (4% of rated value in Case R2)</td>
<td>Smallest (3% of rated value in Case R2)</td>
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<tr>
<td>Dynamic response</td>
<td>Slow (rising time as 0.09 sec in Case R2)</td>
<td>Fast (rising time as 0.08 sec in Case R2)</td>
<td>Fast (rising time as 0.08 sec in Case R2)</td>
<td>Faster (rising time as 0.07 sec in Case R2)</td>
</tr>
<tr>
<td>Ability of load regulation</td>
<td>Poor (maximum error as 3.00 rad with added load in Case R5)</td>
<td>Good (maximum error as 1.56 rad with added load in Case R5)</td>
<td>Better (maximum error as 1.12 rad with added load in Case R5)</td>
<td>Best (maximum error as 0.72 rad with added load in Case R5)</td>
</tr>
<tr>
<td>Convergence speed</td>
<td>Middle (tracking error response at 0.2 rad within 0.05 sec in Case R2)</td>
<td>Fast (tracking error response at 0.1 rad within 0.04 sec in Case R2)</td>
<td>Faster (tracking error response at 0.1 rad within 0.03 sec in Case R2)</td>
<td>Fastest (tracking error response at 0.1 rad within 0.01 sec in Case R2)</td>
</tr>
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</table>
Position tracking error

<table>
<thead>
<tr>
<th></th>
<th>Middle (maximum error as 0.56 rad in Case R2)</th>
<th>Small (maximum error as 0.37 rad in Case R2)</th>
<th>Smaller (maximum error as 0.28 rad in Case R2)</th>
<th>Smallest (maximum error as 0.19 rad in Case R2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejection ability for parameter disturbance</td>
<td>Poor (maximum error as 0.56 rad in Case R2)</td>
<td>Good (maximum error as 0.37 rad in Case R2)</td>
<td>Better (maximum error as 0.28 rad in Case R2)</td>
<td>Best (maximum error as 0.19 rad in Case R2)</td>
</tr>
<tr>
<td>Fickle rate</td>
<td>None</td>
<td>None</td>
<td>Vary (optimal fickle rate)</td>
<td>Vary (optimal fickle rate)</td>
</tr>
</tbody>
</table>

5. Conclusions

The snug backstepping control with a sliding switching function for controlling the motion of a six-phase squirrel cage copper rotor induction motor drive system is proposed to reduce nonlinear uncertainty effects. However, the previously proposed control results in higher chattering on nonlinear system effects and overtorque on matched uncertainties. So as to reduce the immense chattering situation, we then put forward the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method to estimate the external bundled torque uncertainties and to recoup the smallest reorganized error of the evaluated rule. The rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method is proposed to control the six-phase squirrel cage copper rotor induction motor drive system for the tracking of periodic reference inputs.

The main contribution of this study is as follows. (1) The indirect field-oriented control has been smoothly applied to control the six-phase squirrel cage copper rotor induction motor drive system. (2) The snug backstepping control with a sliding switching function has been smoothly derived in order to overcome influences under the external bundled torque uncertainty disturbances in the light of the Lyapunov function. (3) The rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method to estimate external bundled torque uncertainties has been smoothly derived in the light of the Lyapunov function in order to diminish the external bundled torque uncertainties effect. (4) The indemnified controller, with an appraised rule and a snug law to recoup the minimum reorganized error, has been smoothly derived in the light of the Lyapunov function. (5) Two optimal fickle rates of the rectified reiterative sieved-Pollaczek polynomials neural network have been smoothly calculated by utilizing the improved fish school search method to speed-up the parameter’s convergence.

Lastly, some control performances with regard to the vibration in the control intensity, the dynamic response, the convergence speed, the position tracking error, and the ability of load regulation by using the rectified reiterative sieved-Pollaczek polynomials neural network backstepping control with an improved fish school search method are more exceptional than the popular proportional–integral controller, the snug backstepping control with a sliding switching function, and the backstepping control using reformed recurrent Hermite polynomial neural networks with adaptive law and error-estimated law [35].

As the hardware and the software of the system have many limitations in the realizations, some control performances obtained by the used control systems resulted in error values within an unexpected range. Therefore, the study’s directions in future works are as below. (1) The advancement of more accurate modeling in the six-phase squirrel cage copper rotor induction motor drive system will be derived so as to obtain more exact experimental results. (2) High performance of the digital signal processor control systems will be adopted to reduce execution time and enhance computation efficiency. (3) The more advanced control methods combined in this study will be adopted so as to heighten robustness in the six-phase squirrel cage copper rotor induction motor drive system. (4) The advanced control structures, with tracking for some different reference trajectories, will be developed so as to enhance the feasibility of the control systems.
Author Contributions: C.-H.L. conceived and designed the system and experiments; C.-H.L. performed the experimental tests and used the software programs; C.-H.L. analyzed the data for writing the paper; C.-H.L. revised the paper. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest: The author declares no conflict of interest.

References


