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Characterization of Soft S-Open Sets in Bi-Soft Topological Structure Concerning Crisp Points

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Abstract: In this article, a soft s-open set in soft bitopological structures is introduced. With the help of this newly defined soft s-open set, soft separation axioms are regenerated in soft bitopological structures with respect to crisp points. Soft continuity at some certain points, soft bases, soft subbase, soft homeomorphism, soft first-countable and soft second-countable, soft connected, soft disconnected and soft locally connected spaces are defined with respect to crisp points under s-open sets in soft bitopological spaces. The product of two soft $\left(S^i : i = 1, 2\right)$ axioms with respect crisp points with almost all possibilities in soft bitopological spaces relative to semiopen sets are introduced. In addition to this, soft (countability, base, subbase, finite intersection property, continuity) are addressed with respect to semiopen sets in soft bitopological spaces. Product of soft first and second coordinate spaces are addressed with respect to semiopen sets in soft bitopological spaces. The characterization of soft separation axioms with soft connectedness is addressed with respect to semiopen sets in soft bitopological spaces. In addition to this, the product of two soft topological spaces is (S^1 and S^2) space if each coordinate space is soft (S^1 and S^2) space, product of two sot topological spaces is (S regular and C regular) space if each coordinate space is (S regular and C regular), the product of two soft topological spaces is connected if each coordinate space is soft connected and the product of two soft topological spaces is (first-countable, second-countable) if each coordinate space is (first countable, second-countable).

Keywords: soft sets; soft topological space; soft s-open set; soft bitopological spaces; soft s-separation axioms; soft product space; soft connectedness and soft coordinate spaces

1. Introduction

The soft set theory initiated by Molodtsov [1] has been demonstrated as an intelligent mathematical tool to deal with problems encompassing uncertainties or inexact data. Old-fashioned tools such as fuzzy sets [2], rough sets [3], vague sets [4], probability theory, etc. cannot be cast-off effectively because one of the root problems with these models is the absence of a sufficient number of expressive parameters to deal with uncertainty. In order to add a reasonable number of expressive parameters, Molodtsov [1] has shown that soft set philosophy has a rich potential to exercise in multifarious fields of mathematics. Maji et al. [5] familiarized comprehensive theoretical construction. Works on soft set philosophy are growing very speedily with all its potentiality and are being cast-off in different areas of mathematics [6–11]. In the case of the soft set, the parametrization is done with the assistance of words, sentences, functions, etc. For different characteristics of the decision variables present in soft set

theory, different hybridization viz. Fuzzy soft sets [12], rough soft sets [13], intuitionistic fuzzy soft sets [14], vague soft sets [15], neutrosophic soft sets [16], etc. have been introduced with the passage of pieces of time research was in progress in the field of soft set theory.

Pakistani mathematician M. Shabir and M. Naz [17] ushered in the novel concept of soft topological spaces, which are defined relative to an initial universe of discourse with a fixed set of decision variables. In his work, different basic recipes and fundamental results were discussed with respect to crisp points, and counter-examples were also planted to clear the doubts. Soft separation axioms with respect to crisp points were discussed. I. Zorlutune et al. [18] tried his hands to fill in the gap that exists in [17] and studied some new features of soft continuous mapping and gave some innovative categorizations of soft continuous, soft open, soft closed mappings and also soft homeomorphisms topological structures.

In 2012, E. Peyghan, B. Samadi and A. Tayebi [19] filled in the gap that exists in [18] and ushered in some new features of soft sets and manifestly explored the notions of soft connectedness in soft topological spaces with respect to crisp points. The same pieces of work were further scrutinized and modified in 2014, M. Al-Khafaj and M. Mahmood [20] with the introduction of some properties of soft connected spaces and soft locally connected spaces with respect to ordinary points. M. Akdag and A. Ozkan [21] investigated some basic characters of basic results for soft topology relative to soft weak open sets, namely soft semiopen (closed) sets and defined soft s (closure and interior) in STS relative to crisp points. S.A. El-sheikh et al. [22] introduced new soft separation structures rests on soft weak open sets, namely, soft b-open sets which are in true sense the generalization of soft open sets.

S. Hussain and B. Ahmad [23] introduced the concept of soft separation axiom in soft topological spaces in full detail for the first time with respect to soft points. They provided examples for almost all results with respect to soft points. Soft regular, soft T_3, soft normal and soft T_4 axioms using soft points are discussed. Khattak et al. [24] introduced the concept of α and soft β separation axioms in soft single point spaces and in soft ordinary spaces with respect to crisp points and soft points. M. Naz et al. [25] introduced the concept of soft bitopological spaces. First, the authors discussed the basic concepts of soft bitopology and then addressed different spaces in soft bitopology with respect to soft open sets. The results are supported by suitable examples.

M. Ittanagi [26] opened the door to pairwise notion of sbts and studied some types of soft separation axioms in a pairwise manner for sbts with respect to crisp points. Research on the same structures attracted the attention of researchers and resulted in T.Y., and C.G. Aras [27] giving birth to the concept of soft pairwise continuity, soft pairwise open (closed) mappings, pairwise soft homeomorphism and scrutinized their basic characters in sbts. A. Kandil [28] pointed out the notion of pairwise soft connectedness and disconnectedness under the restriction of the idea of pairwise separated soft sets in sbts. They tried their best to explain the newly defined concepts with the support of examples. All the papers that are related so far to soft bitopological spaces do not have any results about weak separation axioms related to soft points. This big gap was bridged by A.M. Khattak et al. [29] for the first time, characterized soft s-separation axioms in soft bitopological spaces with respect to soft points. Soft regularity and normality were also studied with respect to soft points in soft bitopological spaces. In continuation, A.M. Khattak did not stop his work and resulted in Khattak, and some other researchers [30] studied some basic results and hereditary properties in soft bitopological spaces with respect to soft points.

G. Senel [31] introduced the concept of soft bitopological Hausdorff space (SBT Hausdorff space) as an original study. First, the author introduced some new concepts in soft bitopological space such as SBT point, SBT continuous function and SBT homeomorphism. Second, the author defined SBT Hausdorff space. The authors analyzed whether an SBT space is Hausdorff or not by SBT homeomorphism defined from an SBT Hausdorff space to researched SBT space. Last, the author finished their study by defining SBT property and hereditary SBT by SBT homeomorphism and investigate the relations between SBT space and SBT subspace.

T.Y. Öztürk, S. Bayramov [32] introduced the concept of the pointwise topology of soft topological spaces. Finally, the authors investigated the properties of soft mapping spaces and the relationships between some soft mapping spaces.

S. Bayramov, G.[33] investigated some basic notions of soft topological spaces by using a new soft point concept. Later the authors addressed T_i-soft space and the relationships between them in detail. Finally, the authors define soft compactness and explore some of its important properties. The above references [31–33] became a source of motivation for my research work.

There is a big gap for the researchers to define and investigate new structures, namely soft compactness, soft connectedness, soft bases, soft subbases, soft countability, finite intersection properties, soft coordinate spaces etc. In our study, we introduce some new definitions, which are soft semiopen set, soft continuity at some certain points, soft bases, soft subbase, soft homeomorphism, soft first-countable, soft connected, soft disconnected and soft locally connected spaces with respect to crisp points under s-open sets in soft bitopological spaces.

In the present article, we will present the notion of soft bitopological structure relative to the soft semiopen set, which is a generalization of the soft open set. The rest of this work is organized as follows: In the next section, concepts, notations and basic properties of soft sets and soft topology are recalled. In Section 3, we introduced some new definitions, which are necessary for our future sections of this article. In Section 4, some important definitions are introduced in soft bitopological spaces with respect to crisp points under soft semiopen sets. In Section 5, the engagement of section two and section three with some important results with respect to the crisp points under soft semiopen sets are addressed. In Section 6, the characterization of soft separation axioms with soft connectedness is addressed with respect to semiopen sets in soft bitopological spaces. In the final section, some concluding comments are summarized, and future work is included.

2. Basic Concept

In this section, we present the vital definitions and results of soft set theory that are needed in this article.

Definition 1. ([1]) *Let X be the universal set, and E be the set of expressive parameters. Let $P(X)$ supposes the power set of X which contains all possible subsets of X and E be the super set of A. An ordered pair (f, A) is called a soft set over X, where f is a mapping given by $f : A \to P(X)$. This signifies that a soft set over X is a parametrized family of subsets of the universal set X. For $e \in A$, $f(e)$ is value at particular expressive parameter which engages a particular subset of the universal set and considered as the set of e-approximate element of the soft set (f, A) and if $e \notin A$, then $f(e) = \varphi$ implies meaningless that is $(f, A) = \left\{ f(e) : e \in A \widetilde{\subseteq} E \, , (f : A \to P(X) \right\}$. The family of all these soft sets is symbolized as $SS(X)^A$.*

Definition 2 ([1]). *For two soft sets (f, A) and (g, B) over the same universe of discourse X, we say $(f, A) \, \widetilde{\subseteq} (g, B)$ if $A \subseteq B$, $f(e) \subseteq g(e) \forall \in A$.*
And if $\subseteq A$, $f(e) \widetilde{\supseteq} g(e) \forall \in B$. Then the two soft sets are said to be soft equal. This is possible only if we are playing with the same expressive parameter set.

Definition 3 ([17]). *A soft set $(f, A) \in SS(X)^A$ is said to be an absolute soft set, denoted by $(\widetilde{X}, A)$ if $f(e) = X, \forall \, e \in A$. Moreover, soft set $(f, A) \in SS(X)^A$ is said to be null soft set denoted by $(\widetilde{\varphi}, A)$ if $f(e) = \varphi, \forall \, e \in A$.*

Definition 4 ([17]). *A soft set $(f, A) \in SS(X)^A$ is said to have complement denoted by $(f, A)^c$ where, $f^c : A \to P(X)$ is a mapping given by $f^c(e) = X - f(e), \forall \, e \in A$.*

Definition 5 ([17]). *The difference of two soft sets (f,A) and (g,A) over the same parameter set and universe of discourse X denoted by $(f,A)-(g,A)$ is the soft set namely, (H,A), where $\forall\, e \in A$, $H(e) = f(e) - g(e)$ or $(g,A)-(f,A)$.*

Definition 6 ([1]). *Let (f,A) and (g,B) be two soft sets over two different sets of parameters and the same universe of discourse X. The soft union of (f,A) and (g,B) is the soft set (H,C) such that $(f,A)\widetilde{\cup}(g,B) = (H,C)$, where $C = A \cup B$ and for all $e \in C$*

$$\mathrm{H(e)} = \left\{ \begin{array}{c} \mathrm{f(e)\ if\ B \subset A,\ i.e.,\ if\ e \in (A \backslash B)} \\ \mathrm{g(e)\ if\ A \subset B,\ i.e.,\ if\ e \in (B \backslash A)} \\ \mathrm{f(e) \cup g(e)\ if\ e \in (A \cap B)} \end{array} \right\}$$

Definition 7 ([1]). *Let (f,A) and (g,B) be two soft sets over two different sets of parameters and the same universe of discourse X. The soft intersection of (f,A) and (g,B) is the soft set (H,C) such that $(f,A)\widetilde{\cap}(g,B) = (H,C)$, where $C = A \cap B$ and $H(e) = f(e)\widetilde{\cap}g(e)$ for each $e \in C$.*

Definition 8 ([34]). *A soft set $(f,A) \in SS(X)^A$ is said to be a soft point in $(\widetilde{X},A)$ If there exists $x \in X$ and $e \in A$ in such a track that $f(e) = \{x\}$ subject to $f(e^c) = \varphi$ for each $e^c = A - \{e\}$. This soft point is signified by x^e where, $x^e : A \to P(X)$ following track*

$$(x^e, A) = \left(\begin{array}{cc} \{x\}, if & e = a \\ \varphi, if & e \neq a \ for\ all\ e \in A \end{array} \right)$$

A soft point (x^e, A) is said to be housed off in soft set(f,A) signifying $x^e \in (x^e, A)$, if $x^e(e)\widetilde{\subseteq}f(e)$, that is $\{x\}\widetilde{\subseteq}f(e)$. Obviously, $x^e \in (x^e, A)$ iff $(x^e, A)\widetilde{\subseteq}(f,A)$. In addition, two soft points x^{e_1}, y^{e_2} relative to the crisp set X are said to be equal if $x = y$ and $e_1 = e_2$ that is bi-equal. This means that if the equality symbol is disturbed in either case, then the equality between x^{e_1} and y^{e_2} will automatically be unbalanced.

Definition 9 ([17]). *Let $\widetilde{\tau}$ be a collection of soft sets over a universe of discourse X with a fixed set of expressive parameters E. Thence $\widetilde{\tau}\widetilde{\subseteq}SS(X)^E$ is called a soft topology on X if it qualifies the following axioms:*

(1) $(\widetilde{X},E)$, $(\widetilde{\varphi},E)\widetilde{\in}\widetilde{\tau}$, *Where $\widetilde{X}(e) = X, \forall\, e \in E$ and $\widetilde{\varphi}(e) = \varphi, \forall\, e \in E$*

(2) Union of any number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$

(3) The intersection of any two or finite number of soft sets in $\widetilde{\tau}$ belongs to $\widetilde{\tau}$.

The structure of ordered triple $(\widetilde{X},\widetilde{\tau},E)$ is called a soft topological structure. Any candidate of $\widetilde{\tau}$ is said to be a soft open set.

Definition 10 ([35]). *Suppose (f,E) be any soft set of soft topological space $(\widetilde{X},\widetilde{\tau},E)$. Then (f,E) is said to be a soft semiopen set of $(\widetilde{X},\widetilde{\tau},E)$ if $(f,E)\ \widetilde{\subseteq}cl(int((f,E)))$ and said to be soft semiclosed if $(f,E)\ \widetilde{\supseteq}int(cl((f,E)))$. The set of all soft semiopen sets is denoted by $SSO(X)^E$ and the set of all soft semiclosed sets is denoted by $SSC(X)^E$.*

3. Soft S-Open Sets in Soft Bi-Topological Space

In this section, we introduced some new definitions, which are necessary for our future sections of this article. Soft semiopen set, soft continuity at some certain points, soft bases, soft subbase, soft homeomorphism, soft first-countable and soft second-countable, soft connected, soft disconnected and soft locally connected spaces are defined with respect to crisp points under s-open sets in soft bitopological spaces. All most all the results are supported by examples.

Definition 11. *A quadrable system $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is SBT$\mathcal{S}pace$ relative to the crisp set X, where $\widetilde{\tau}_1$ and $\widetilde{\tau}_2$ are supposed to be arbitrary soft topologies on the crisp set X and E be the set of parameters.*

Example 1. *Suppose $X = \{x^1, x^2, x^3\}$ and $E = \{e^1, e^2\}$. We develop the following soft sets relative to crisp set X;*

$$\begin{cases} \left(\mathrm{f}^1, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^1, \mathrm{x}^2\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^2, \mathrm{x}^3\right\}\right)\right\}, \\ \left(\mathrm{f}^2, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^1\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^1, \mathrm{x}^2\right\}\right)\right\}, \\ \left(\mathrm{f}^3, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^2\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^2\right\}\right)\right\}, \\ \left(\mathrm{f}^4, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^1, \mathrm{x}^2\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^3\right\}\right)\right\}, \\ \left(\mathrm{g}^1, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^1\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^1, \mathrm{x}^2\right\}\right)\right\}, \\ \left(\mathrm{g}^2, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \left\{\mathrm{x}^2, \mathrm{x}^3\right\}\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^2, \mathrm{x}^3\right\}\right)\right\}, \\ \left(\mathrm{g}^3, \mathrm{E}\right) = \left\{\left(\mathrm{e}^1, \varnothing\right), \left(\mathrm{e}^2, \left\{\mathrm{x}^2\right\}\right)\right\}. \\ \mathrm{Then}(\mathrm{X}, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})\mathrm{is\ SBT}\mathcal{S}pace. \\ \widetilde{\tau}_1 = \left\{\widetilde{\varnothing}, \check{\mathrm{X}}, \left(\mathrm{f}^1, \mathrm{E}\right), \left(\mathrm{f}^2, \mathrm{E}\right), \left(\mathrm{f}^3, \mathrm{E}\right), \left(\mathrm{f}^4, \mathrm{E}\right)\right\}, \\ \widetilde{\tau}_2 = \left\{\widetilde{\varnothing}, \check{\mathrm{X}}, \left(\mathrm{g}^1, \mathrm{E}\right), \left(\mathrm{g}^2, \mathrm{E}\right), \left(\mathrm{g}^3, \mathrm{E}\right)\right\} \end{cases}$$

Definition 12. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X. A soft set (G, E) over X is said to be soft semiopen If there exists a soft semiopen set $\left(G^1, E\right) \in \widetilde{\tau}_1$ and soft semiopen set $\left(G^2, E\right) \in \widetilde{\tau}_2$ such that $(G, E) = \left(G^1, E\right)\widetilde{\bigcup}(\left(G^2, E\right)$. A soft set (G, E) over X is said to be close in $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ if its respective complement is soft semiopen in $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$.*

Definition 13. *Suppose (f, E) be any soft set of soft bitopological space $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$. Then (f, E) is said to be a soft semiopen set of $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ if $(f, E)\widetilde{\subseteq}\widetilde{\tau}_1 - cl(\widetilde{\tau}_2 - int((f, E)))$ and said to be soft semiclosed if $(f, E)\widetilde{\supseteq}\widetilde{\tau}_1 - int(\widetilde{\tau}_2 - cl((f, E)))$. The set of all soft semiopen sets is denoted by $SSOSOSBT(X)^E$ and the set of all soft semiclosed sets is denoted by $SSCSOSBT(X)^E$.*

Definition 14. *Let $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ and $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ be two SBT$\mathcal{S}paces$ over the crisp sets X^1 and X^2 respectively and $f : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) \to (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ be a soft mapping. Then f is said to be soft continuous at a soft point $x^e \widetilde{\in} SS(X)^E$ if f for each soft semiopen set (H, E) in $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ containing $f(x^e)$ there exists a soft semiopen (G, E) set in $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ containing x^e such that $f((G, E)\ )\widetilde{\subseteq}(H, E)$ that is $f(x^e) \widetilde{\in} f(G, E) \widetilde{\subseteq}(H, E)$ or $x^e \widetilde{\in}(G, E) \widetilde{\subseteq}(H, E)$.*

Example 2. *Suppose $X = \{x^1, x^2, x^3\}, Y = \{y^1, y^2, y^3\}$ and $E = \{e^1, e^2\}$. Then $\widetilde{\tau}_1 = \left\{\widetilde{\varnothing}, \check{X}, (F, E)\right\}$, $\widetilde{\tau}_2 = \left\{\widetilde{\varnothing}, \check{X}, (G, E)\right\}$ two soft topological spaces over X and $\widetilde{\eta_1} = \left\{\widetilde{\varnothing}, \check{Y}, (H, E)\right\}$, $\widetilde{\eta_2} = \left\{\widetilde{\varnothing}, \check{Y}, (K, E)\right\}$ are two soft topological spaces over Y. Here the soft sets over X and Y are defined as follows;*

$$\begin{cases} (F, E) = \left\{\left(e^1, \left\{x^1, x^3\right\}\right), \left(e^2, \left\{x^2, x^3\right\}\right)\right\} \\ (G, E) = \left\{\left(e^1, \left\{x^2, x^3\right\}\right), \left(e^2, \left\{x^1, x^3\right\}\right)\right\} \\ (H, E) = \left\{\left(e^1, \left\{y^1, y^2\right\}\right), \left(e^2, \left\{y^2, y^3\right\}\right)\right\} \\ (K, E) = \left\{\left(e^1, \left\{y^2, y^3\right\}\right), \left(e^2, \left\{y^1, y^2\right\}\right)\right\} \end{cases}$$

Then $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ and $\left(X^1, \widetilde{\eta_1}, \widetilde{\eta_2}, E\right)$ are two soft bitopological spaces.

$$\widetilde{\tau}_1 \cup \widetilde{\tau}_2 = \left\{\widetilde{\varnothing}, \check{X}, (F, E), (G, E)\right\}, \widetilde{\eta_1} \cup \widetilde{\eta_2} = \left\{\widetilde{\varnothing}, \check{Y}, (H, E), (K, E)\right\}.$$

If the mapping $f : X \to Y$ *defined as*

$$\begin{cases} f\left(X^1_{e^1}\right) = Y^1_{e^1} \\ f\left(X^1_{e^1}\right) = Y^1_{e^1} \\ f\left(X^1_{e^1}\right) = Y^1_{e^1} \\ f\left(X^1_{e^1}\right) = Y^1_{e^1} \end{cases}$$

Then f *soft semiopen and soft semiclosed.*

Definition 15. *Let* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *be a SBTSpace relative to the crisp set* X *and* $\subseteq (X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$*. If every element of* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *can be written as a soft union of elements of* $\mathfrak{W}$*, then* $\mathfrak{W}$ *is called a soft base for the soft bitopology* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *. Each element of* $\mathfrak{W}$ *is called a soft base element.*

Example 3. *Let* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *be a SBTSpace relative to the crisp set* X*. Where the crisp set is* $X = \left\{x^1, x^2, x^3\right\}$ *and* $\mathbb{E}^{paramater} = \left\{e^1, e^2, e^3, e^4, e^5, e^6\right\}$ *be the universal set of parameters and* $\mathscr{A}^{paramater} = \left\{e^1, e^2\right\}$ *be the subset of the universal set of parameters*

$$E = \left\{e^1, e^2, e^3, e^4, e^5, e^6\right\},\ (X, \widetilde{\tau}_1, E) = \left\{\widetilde{\varphi}, \widetilde{X}, f^1, f^2, f^3, f^4, f^5, \ldots\ldots\ldots f^{18}\right\}$$

then is a STSpace relative to the crisp set X *and* $(X, \widetilde{\tau}_2, E) = \{\widetilde{\varphi}, \widetilde{X}\}$ *is a STSpace relative to the crisp set* X *We define the soft functions as:*

$$\begin{cases} f^1\left(e^1\right) = \widetilde{\varphi}, f^1\left(e^2\right) = \left\{x^1\right\}, f^2\left(e^1\right) = \widetilde{\varphi}, \\ f^2\left(e^2\right) = \left\{x^2\right\}, f^3\left(e^1\right) = \widetilde{\varphi}, f^3\left(e^2\right) = \left\{x^1, x^2\right\} \\ f^4\left(e^1\right) = \left\{x^1\right\}, f^4\left(e^2\right) = \widetilde{\varphi}, f^5\left(e^1\right) = \left\{x^1\right\}, \\ f^5\left(e^2\right) = \widetilde{\varphi}, f^6\left(e^1\right) = \left\{x^1, x^2\right\}, f^6\left(e^2\right) = \widetilde{\varphi}, \\ f^7\left(e^1\right) = \left\{x^1\right\}, f^7\left(e^2\right) = \left\{x^2\right\}, f^8\left(e^1\right) = \left\{x^2\right\}, \\ f^8\left(e^2\right) = \left\{x^1\right\}, f^9\left(e^1\right) = \left\{x^2\right\}, f^9\left(e^2\right) = \left\{x^2\right\}, \\ f^{10}\left(e^1\right) = \left\{x^2\right\}, f^{10}\left(e^2\right) = \left\{x^1, x^3\right\}, f^{11}\left(e^1\right) = \left\{x^1, x^2\right\}, \\ f^{11}\left(e^2\right) = \left\{x^2\right\}, f^{12}\left(e^1\right) = \left\{x^1, x^3\right\}, f^{12}\left(e^2\right) = \left\{x^1, x^3\right\}, \\ f^{13}\left(e^1\right) = X, f^{13}\left(e^2\right) = \left\{x^1, x^3\right\}, f^{14}\left(e^1\right) = \left\{x^1, x^3\right\}, \\ f^{14}\left(e^2\right) = X, f^{15}\left(e^1\right) = \left\{x^1, x^2\right\}, f^{15}\left(e^2\right) = \left\{x^2\right\}, \\ f^{16}\left(e^1\right) = \left\{x^1\right\}, f^{16}\left(e^2\right) = \left\{x^2\right\}, f^{17}\left(e^1\right) = \left\{x^1\right\}, \\ f^{17}\left(e^2\right) = \left\{x^1, x^2\right\}, f^{18}\left(e^1\right) = \left\{x^1, x^2\right\}, f^{18}\left(e^2\right) = \left\{x^1, x^2\right\}. \\ Put\ \mathfrak{W} = \left\{\widetilde{\varphi}, \widetilde{X}, f^1, f^2, f^4, f^5, f^9, f^{14}, f^{15}, f^{17}\right\}. \\ Then\ \mathfrak{W}\ is\ soft\ base\ for\ (X, \widetilde{\tau}_1, \widetilde{\tau}_2, E). \end{cases}$$

Definition 16. *Let* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *be a SBTSpace relative to the crisp set* X *and* $\mathfrak{P} \subseteq \widetilde{(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)}$ *. Then this soft collection is said to be a soft subbase for* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *if the collection of all finite soft intersections of members of* $\mathfrak{P}$ *is a soft base for* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *.*

Definition 17. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces over the crisp sets* X^1 *and* X^2 *respectively. Let* $f : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) \to (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be soft mapping. This soft mapping is said to be a soft homeomorphism if this soft mapping is soft one-one, soft one-two and soft bi-continuous.*

Definition 18. *Let* $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *be a SBTSpace relative to the crisp set* X*. This soft space is said to be soft, first-countable if every point of* X *a soft-countable soft local base.*

Definition 19. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X. This soft space is said to be soft second if there exists a soft-countable soft base for $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$.*

Definition 20. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X and $\left(f^1, E\right)$, $\left(f^2, E\right)$ be soft subsets of the given space, then the soft sets $\left(f^1, E\right)$ and $\left(f^2, E\right)$, are said to be soft separated or soft disconnected if*

$$\begin{cases} (1): \left(f^1, E\right) \neq (\varphi, E), \left(f^2, E\right) \neq (\varphi, E) \\ (2): \left(f^1, E\right)\widetilde{\cap}\overline{(f^2, E)} = (\varphi, E), \\ \left(f^2, E\right)\widetilde{\cap}\overline{(f^1, E)} = (\varphi, E) \\ (3): (X, \widetilde{\tau}_1, \widetilde{\tau}_2, E) = \left(f^1, E\right)\widetilde{\cup}\left(f^2, E\right). \end{cases}$$

Definition 21. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X is said to soft connected if f is not soft disconnected.*

Definition 22. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X is said to be soft locally s-connected at $x \widetilde{\in} X$ if for every soft s-open set (f, E) with respect to $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ where, $(f, E) = \left(f^1, E\right)\widetilde{\cup}\left(f^2, E\right)$ there exists a soft connected s-open set (G, E) with respect to the same space $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ containing x contained in (f, E), where $(G, E) = \left(G^1, E\right)\widetilde{\cup}\left(G^2, E\right)$ with $\left(G^1, E\right) \in \widetilde{\tau}_1, \left(G^2, E\right) \in \widetilde{\tau}_2$. The space $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ said to be soft locally connected if f it is soft locally connected at each of its points.*

4. Separation Axioms in Soft Bi-Topological Spaces

In this section, some important definitions of soft structures are introduced in soft bitopological spaces with respect to crisp points under soft semiopen sets.

Definition 23. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X, $A \subseteq E$ and $x, y \in X$ such that $x > y$ or $x < y$ If there exists at least one soft $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ s-open set $\left(f^1, A\right)$ or $\left(f^2, A\right)$ such that $x \in \left(f^1, A\right)$, $y \notin \left(f^1, A\right)$ or $y \in \left(f^2, A\right)$, $x \notin \left(f^2, A\right)$ then $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said be a soft S^0 space.*

Definition 24. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X, $A \subseteq E$ and $x, y \in X$ such that $x > y$ or $x < y$ If there exists a soft $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ s-open sets $\left(f^1, A\right)$ and $\left(f^2, A\right)$ such that $x \in \left(f^1, A\right)$, $y \notin \left(f^1, A\right)$ and $y \in \left(f^2, A\right)$, $x \notin \left(f^2, A\right)$ then $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said be a soft S^1 space.*

Definition 25. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set X, $A \subseteq E$ and $x, y \in X$ such that $x > y$ or $x < y$ If there exists a soft $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ s-open sets $\left(f^1, A\right)$ and $\left(f^2, A\right)$ such that $x \in \left(f^1, A\right)$, $y \in \left(f^2, A\right)$ with $\left(f^1, A\right)\widetilde{\cap}\left(f^2, A\right) = \varnothing^A$. then $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said be a soft S^2 space.*

Definition 26. *Let $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ be a SBT$\mathcal{S}pace$ relative to the crisp set, $A \subseteq E$. Then $(X, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ is said to be regular if for every soft s-closed subset (f, A) and every point $x \notin (f, A)$, there exists a soft s-open sets (g, A) and (h, A) such that $(f, A) \subseteq (g, A)$, $x \in (h, A)$ and the possibility of (g, A) rules out the possibility of (h, A).*

5. Product of Soft Separation Axioms and Soft (First and Second) Coordinate Spaces

In this section, the engagement of section two and section three with some important results with respect to crisp points under soft semiopen sets is addressed. The product of two soft S^0 axioms with respect crisp points with almost all possibilities, the product of two soft S^1 axioms with respect crisp points with almost all possibilities and product of two soft S^2 axioms with respect to crisp points with almost all possibilities in soft bitopological spaces relative to semiopen sets are introduced. In addition

to this, soft (countability, base, subbase, finite intersection property, continuity) are addressed with respect to semiopen sets in soft bitopological spaces. Finally, the product of soft first and second coordinate spaces are addressed with respect to semiopen sets in soft bitopological spaces.

Theorem 1. *If* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBT*$\mathcal{S}paces$ *over the crisp sets* X^1 *and* X^2 *respectively such that these are two soft* S^0 $\mathcal{S}pace$*. Then their product* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *is soft* S^0 $\mathcal{S}pace$*.*

Proof. Let $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be the soft product space. We prove that it is soft S^0 $\mathcal{S}pace$. Suppose $(\mathrm{x}^1, \mathrm{y}^1)$ *and* $(\mathrm{x}^2, \mathrm{y}^2)$ are two distinct soft points in $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ then different cases are $\mathrm{x}^1 > \mathrm{x}^2$, *or* $\mathrm{y}^1 > \mathrm{y}^2, \mathrm{x}^1 \gg \mathrm{x}^2$ or $\mathrm{y}^1 \gg \mathrm{y}^2$.

Case (**I**) If $\mathrm{x}^1 > \mathrm{x}^2$, then $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^0 s$pace$, corresponding to this pair of distinct soft points there exists a $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ soft s-open set (G, E) such that $\mathrm{x}^1 \in (\mathrm{G}, \mathrm{E})$ and $\mathrm{x}^2 \notin (\mathrm{G}, \mathrm{E})$. Thus, $(\mathrm{G}, \mathrm{E}) * X^2$ is soft $((X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}))$ s-open set catching $(\mathrm{x}^1, \mathrm{y}^1)$ but bay passing $(\mathrm{x}^2, \mathrm{y}^2)$.

Case (**II**) If $\mathrm{x}^1 \gg \mathrm{x}^2$, then $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^0 s$pace$, corresponding to this pair of distinct soft points there exists a $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ soft s-open set (G, E) such that $\mathrm{x}^1 \in (\mathrm{G}, \mathrm{E})$ and $\mathrm{x}^2 \notin (\mathrm{G}, \mathrm{E})$. Thus, $(\mathrm{G}, \mathrm{E}) * X^2$ is soft $((X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}))$ s-open set catching $(\mathrm{x}^1, \mathrm{y}^1)$ but bay passing $(\mathrm{x}^2, \mathrm{y}^2)$.

Case (**III**) If $\mathrm{y}^1 > y^2$, then $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^0 s$pace$, corresponding to this pair of distinct points there exists a $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$–soft s-open set (H, A) such that $\mathrm{y}^1 \in (\mathrm{H}, \mathrm{A})$ and $y^2 \notin (\mathrm{H}, \mathrm{A})$. Thus, $X^1 * (\mathrm{H}, \mathrm{A})$ is soft $((X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}))$ s-open set arresting (x^1, y^1) and bypassing (x^2, y^2). Thus for any two distinct points of *in* $X^1 * X^2$ there exists a $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ soft s-open set arresting one of them and bypassing the other. Hence $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is soft S^0 $\mathcal{S}pace$.

Case (**IV**) If $\mathrm{y}^1 \gg y^2$, then $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^0 s$pace$, corresponding to this pair of distinct points there exists a $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ soft s-open set (H, A) such that $\mathrm{y}^1 \in (\mathrm{H}, \mathrm{A})$ and $y^2 \notin (\mathrm{H}, \mathrm{A})$. Thus, $X^1 * (\mathrm{H}, \mathrm{A})$ is soft $((X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}))$ s-open set arresting (x^1, y^1) and bypassing (x^2, y^2). Thus for any two distinct points of *in* $X^1 * X^2$ there exists a $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ soft s-open set arresting one of them and bypassing the other. Hence $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is soft S^0 space. □

Theorem 2. *If* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBT*$\mathcal{S}paces$ *over the crisp sets* X^1 *and* X^2 *respectively such that these are two soft* S^1 $\mathcal{S}paces$*. Then their product* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *is soft* S^1 $\mathcal{S}paces$*.*

Proof. Let $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ and $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be two *SBT*$\mathcal{S}paces$ such that they are soft S^1 $\mathcal{S}paces$, respectively. Then we have to show that their product that is $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is also soft S^1 $\mathcal{S}pace$. For this proof, it is sufficient to show that each soft subset of $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ consisting of exactly one soft point is a soft $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ s-closed set. Let $(x^1, y^1) \in X^1 * X^2$ so that $x^1 \in X^1$ and $y^1 \in X^2$.

Case (**I**) $x^1 > y^1$ Now, $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^1 $\mathcal{S}pace$, $(x^1, \mathrm{A})^c$ is soft $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ s−closed and therefore, $(y^1, \mathrm{A})^c$ is soft $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ s-open. Also, $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ being soft S^1 $\mathcal{S}pace$. So $((x^1, \mathrm{A})^c) * ((y^1, \mathrm{A})^c) \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ that is $\left\{((x^1, y^1)^c, \mathrm{A})\right\} \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ and hence, $\left\{(((x^1, y^1)^c, \mathrm{A})\right\}$ is soft $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ s−closed. Hence $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is soft S^1 $\mathcal{S}pace$.

Case (**II**) $x^1 \gg y^1$ Now, $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ being soft S^1 $\mathcal{S}pace$, $(\mathrm{x}^1, \mathrm{A})^c$ is soft $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ s−closed and therefore, $(y^1, \mathrm{A})^c$ is soft $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ s-open. Also, $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ being soft S^1 $\mathcal{S}pace$. So $((x^1, \mathrm{A})^c) * ((y^1, \mathrm{A})^c) \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ that is $\left\{((x^1, y^1)^c, \mathscr{A})\right\} \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ and hence, $\left\{(((x^1, y^1)^c, \mathscr{A})\right\}$ is soft $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ s-closed. Hence $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is soft S^1 $\mathcal{S}pace$. □

Theorem 3. *If* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces over the crisp sets* X^1 *and* X^2 *respectively such that these are two soft S-Hausdorff spaces. Then their product* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *is soft S-Hausdorff space.*

Proof. Let $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be the product space. We prove that it is a soft S−Hausdorff space. Suppose (x^1, x^2) and (y^1, y^2) are two distinct points in $X^1 * X^2$.

Case (**I**) When $x^1 = y^1$, then $x^2 \neq y^2$. let $x^2 > y^2$ therefore $(x^1, x^2) \neq (y^1, y^2)$ By the soft S−Hausdorff space property, given a pair of elements $x^2, y^2 \in X^2$ such that $x^2 > y^2$ there are disjoint soft s-open sets such that $(G^2, \mathrm{A}), (H^2, \mathrm{A}) \widetilde{\subset} (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ such that $x^2 \in (G^2, \mathrm{A})$, that $x^2 \in (H^2, \mathrm{A})$. Then $X^1 * (G^2, \mathrm{A})$ and $(X^1 * X^2) * (H^2, \mathrm{A})$ are disjoint soft s-open sets in $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. For, $x^1 \in (\mathscr{X}^{crisp^1})_{\mathscr{A}}$, $x^2 \in (G^2, \mathrm{A})$ implies that $(x^1, x^2) \in ((\mathscr{X}^{crisp^1})_{\mathscr{A}} * (G^2, \mathrm{A}))$, $y^1 \in X^1, y^2 \in (H^2, \mathrm{A})$ implies that $(y^1, y^2) \in X^1 * (H^2, \mathrm{A})$. this whole situation leads us to the conclusion that $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is a soft S−Hausdorff space.

Case (**II**) If $x^1 \neq y^1$ let $x^1 > y^1$ then too $(x^1, y^1) > (x^2, y^2)$. By the soft S−Hausdorff space property, given a pair of elements $(x^1, y^1) \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ such that $x^2 > y^2$, there are disjoint s-open sets in $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ such that $(G^1, \mathrm{A}), (H^1, \mathrm{A}) \widetilde{\subset} (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ such that $x^1 \in (G^1, \mathrm{A})$, that $y^1 \in (H^1, \mathrm{A})$. Then $(G^1, \mathrm{A}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ and $(H^1, \mathrm{A}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ are disjoint soft s-open sets in $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. For, $x^1 \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$, $x^2 \in (G^2, \mathrm{A})$ implies that $(x^1, x^2) \in (G^1, \mathrm{A}) * X^2$, $y^1 \in (H^1, \mathrm{A}) * X^2$, $y^2 \in (H^2, \mathrm{A})$ implies that $(\mathrm{y}^1, y^2) \in (H^1, \mathrm{A}) * X^2$. This whole situation leads us to the conclusion that $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is a soft S-Hausdorff space. □

Theorem 4. *Let* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two second-countable SBTSpaces then their product, that is* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *is also soft second-countable SBTSpace.*

Proof. To prove $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is second-countable *SBTSpaces*. Our assumption implies that there are countable soft s-bases $\mathscr{B}^1 = \left\{B^i : i \in N\right\}$ and $\mathscr{B}^2 = \left\{C^i = i \in N\right\}$ for $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ and $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ respectively. $\mathscr{B} = (G^1, \mathscr{A}) * (G^2, \mathscr{A})$; such that $(G^1, \mathscr{A})$ and $(G^2, \mathscr{A})$ are soft s-open such that $(G^1, \mathscr{A}) \in \left\{(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}), (G^2, \mathscr{A}) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})\right\}$ is a soft *base* for soft product topology $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. If we write $C = \left\{B^i * C^j; i, j \in N\right\} = B^1 * B^2$ is soft-countable, this implies that C is soft-countable. By definition of soft base $\mathscr{B}$, and $(x^1, y^1) \in N \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ implies that there exists a soft s-open sets $(\mathrm{G}, \mathrm{A}), (\mathrm{H}, \mathrm{A})$ such that $(\mathrm{G}, \mathrm{A}) * (\mathrm{H}, \mathrm{A}) \in \mathscr{B}$ such that $(x^1, y^1) \in (\mathrm{G}, \mathrm{A}) * (\mathrm{H}, \mathrm{A}) \widetilde{\subset} N$ implies $x^1 \in (\mathrm{G}, \mathrm{A}) \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}), y^1 \in (\mathrm{H}, \mathrm{A}) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ this implies that there exists $B^i \in B^1, C^j \in B^2$ such that $x^1 \in B^i \widetilde{\subset} (\mathrm{G}, \mathrm{A}), y^1 \in C^j \widetilde{\subset} (\mathrm{H}, \mathrm{A})$ this implies that $(x^1, y^1) \in B^i * C^j \widetilde{\subset} (\mathrm{G}, \mathrm{A}) * (\mathrm{H}, \mathrm{A}) \widetilde{\subset} N$. By definition this proves that C is a soft base for the soft product topology $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. In addition, C has been shown to be soft-countable. Hence, $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ is soft second-countable relative to soft s-open set. □

Theorem 5. *Let* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces on the crisp sets* X^1 *and* X^2 *respectively. the collection* $\mathscr{B} = \left\{(G^1, A) * (G^2, A) : (G^1, A) \in (\widetilde{\tau}_1 \cup \widetilde{\tau}_2), (G^2, A) \in (\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})\right\}$ *is a soft base for some product soft topologies* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$*, where* (G^1, A) *and* (G^2, A) *are soft s-open sets in their corresponding SBTSpaces.*

Proof. Let $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$ and $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be two *SBTSpaces* on the crisp sets X^1 and X^2 respectively. Suppose $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be the soft product topology.

$B = \left\{(\mathrm{u}^1, \mathscr{A}) * (\mathrm{u}^2, \mathrm{A}) : (\mathrm{u}, \mathrm{A}) \in (\widetilde{\tau}_1 \cup \widetilde{\tau}_2), (\mathrm{u}^2, \mathrm{A}) \in (\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})\right\}$ where $(\mathrm{u}^1, \mathrm{A})$ is soft s-open *in* $(\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})$ and $(\mathrm{u}^2, \mathrm{A})$ is soft-open *in* $(\widetilde{\tau}_1 \cup \widetilde{\tau}_2)$. We need to prove B is a *soft base* for some soft topology on $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. To prove that $\cup\{B : B \in \mathscr{B}\} = (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. Clearly, $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}) \in \mathscr{B}$ this implies that $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \mathfrak{F}_1, \widetilde{\mathfrak{F}_2}, \mathrm{E}) = \cup\{B : B \in \mathscr{B}\}$. Next, let $(\mathrm{u}^1, \mathrm{A}) * (\mathrm{u}^2, \mathrm{A}), (\mathscr{J}^1, \mathrm{A}) * (\mathscr{J}^2, \mathrm{A}) \in B$ where $(\mathscr{J}^1, \mathrm{A})$ is soft s-open *in* $(\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})$ *and* $(\mathscr{J}^2, \mathrm{A})$ *is soft* s-open *in* $(\widetilde{\tau}_1 \cup \widetilde{\tau}_2)$ and suppose $(x^1, x^2) \in$

$((u^1, A) * (u^2, A)) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A)) \in B$. To prove that there exists $(\mathcal{J}^1, A)$ *soft s*-open *in* $(\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})$ *and* $(\mathcal{J}^2, A)$ *is soft s*-open *in* $(\widetilde{\tau_1} \cup \widetilde{\tau_2})$ such that $(\mathcal{f}^1, A) * (\mathcal{f}^2, A) \in \mathcal{B}$ such that $(x^1, x^2) \in (\mathcal{f}^1, A) * (\mathcal{f}^2, A)\widetilde{\subseteq}((u^1, A) * (u^2, A)) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$ that is $(x^1, x^2) \in ((u^1, A) * (u^2, A)) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$ this implies $(x^1, x^2) \in ((u^1, A) * (u^2, A)), (x^1, x^2) \in ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$ This implies that $x^1 \in (u^1, A), x^2 \in (u^2, \mathcal{A})$, $x^1 \in (\mathcal{J}^1, A), x^2 \in (\mathcal{J}^2, A)$ implies that $x^1 \in ((u^1, A) \cap (\mathcal{J}^1, A))$, $x^2 \in ((u^2, A) \cap (\mathcal{J}^2, A))$ implies that $x^1 \in (\mathcal{f}^1, A)$, $x^2 \in (\mathcal{f}^2, A)$. On taking $(\mathcal{f}^1, A)$=$(u^1, A) \cap (\mathcal{J}^1, A)$, $(\mathcal{f}^2, A)$=$(u^2, A) \cap (\mathcal{J}^2, A)$ this implies $(x^1, x^2) \in (\mathcal{f}^1, A) * (\mathcal{f}^2, A)$. $(\mathfrak{U}^1, A) * (u^2, A) \in \mathcal{B}$, $(\mathcal{J}^1, A) * (\mathcal{J}^2, A) \in \mathcal{B}$ this implies that $(u^1, A) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), (u^2, A) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E), (\mathcal{J}^1, A) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), (\mathcal{J}^2, \mathcal{A}) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ this implies that $(u^1, A) \cap (\mathcal{J}^1, A) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), (u^2, A) \cap (\mathcal{J}^2, A) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$. This implies that $(\mathcal{f}^1, A) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), (\mathcal{f}^2, A) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ this implies that $(\mathcal{f}^1, A) * (\mathcal{f}^2, A) \in \mathcal{B}$. Thus we have shown that there exists $(\mathcal{f}^1, A) * (\mathcal{f}^2, A) \in \mathcal{B}$ such that $(x^1, x^2) \in (\mathcal{f}^1, A) * (\mathcal{f}^2, A)$. Remaining to prove that $(\mathcal{f}^1, A) * (\mathcal{f}^2, A)\widetilde{\subseteq}((u^1, A)*A) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$. Let $(y^1, y^2) \in (\mathcal{f}^1, A) * (\mathcal{f}^2, A)$ be arbitrary distinct points. $(y^1, y^2) \in ((\mathcal{f}^1, A) * (\mathcal{f}^2, A))$ implies that $y^1 \in (\mathcal{f}^1, A), y^2 \in (\mathcal{f}^2, A)$ implies that $y^1 \in ((u^1, \mathcal{A}) \cap (\mathcal{J}^1, A))$, $y^2 \in ((u^2, A) \cap (\mathcal{J}^2, A))$ implies that $y^1 \in (u^1, A), y^1 \in (\mathcal{J}^1, A)$; $y^2 \in (u^2, A), y^2 \in (\mathcal{J}^2, A)$ this implies that $(y^1, y^2) \in ((u^1, A) * (u^2, A)), (y^1, y^2) \in ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$ this implies that $(y^1, y^2) \in ((u^1, A) * (u^2, A)) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$. This implies $(\mathcal{f}^1, A) * (\mathcal{f}^2, A)((u^1, A) * (u^2, A))) \cap ((\mathcal{J}^1, A) * (\mathcal{J}^2, A))$. Thus $\mathcal{B}$ is a base. □

Theorem 6. *Let* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces on the crisp sets* X^1 *and* X^2 *respectively. Let* $\mathcal{B}^1$ *and* $\mathcal{B}^2$ *be soft bases for* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *respectively. Let* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be the soft product space.*

Then $B = \left\{B^1 * B^2 : B^1 \in \mathcal{B}^1, B^2 \in \mathcal{B}^2\right\}$ *is a soft base for the product soft topology* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *relative to soft s-open sets.*

Proof. $C = \left\{(G^1, \mathcal{A}) * (G^2, \mathcal{A}) : (G^1, \mathcal{A}) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), (G^2, \mathcal{A}) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)\right\}$

Then C is a soft base for the topology $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$. We have to prove that B is a soft base for $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$. According to the soft base, for any $(\widetilde{\cap}^1, \widetilde{\cap}^2) \in (G, \mathcal{A}) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ where (G, A) is soft s-open relative $(\widetilde{\tau_1}, \widetilde{\tau_2}) \cup ((\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2})$ implies there exists a soft $(\widetilde{\tau_1}, \widetilde{\tau_2})$ s-open sets $(G^1, \mathcal{A})$ and $(\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2})$ soft s-open set (G^2, A) such that $(G^1, A) * (G^2, A) \in C$ such that $(x^1, x^2) \in (G^1, A) * (G^2, A)\widetilde{\subseteq}(G, A)$. Again $(x^1, x^2) \in (G^1, A) * (G^2, A) \in C$ implies that $x^1 \in (G^1, \mathcal{A}) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E), x^2 \in (G^2, A) \in (\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2})$. Applying the definition of a soft base, $x^1 \in (G^1, A) \in (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ this implies that there exists $B^1 \in \mathcal{B}^1$ such that $x^1 \in B^1\widetilde{\subseteq}(G^1, A)$ .....(1)

$x^2 \in (G^2, \mathcal{A}) \in (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ implies that there exists $B^2 \in \mathcal{B}^2$ such that $x^2 \in B^2\widetilde{\subseteq}(G^2, A)$ .....(2). Now $B^1 \in \mathcal{B}^1, B^2 \in \mathcal{B}^2 \in B$ this implies that $\mathcal{B}^1 * B^2 \in B$. Now maxing (1) and (2) implies that there exists $\mathcal{B}^1 * B^2 \in B$ such that $(x^1, x^2) \in \mathcal{B}^1 * B^2\widetilde{\subseteq}(G^1, A)\widetilde{\subseteq}(G^2, A)\widetilde{\subseteq}(G, A)$ or $(x^1, x^2) \in \mathcal{B}^1 * B^2\widetilde{\subseteq}(G, A)$. By definition, this proves that B is a soft base for $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ relative to soft s -open sets. □

Theorem 7. *Let* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively. Suppose L and M be soft subbases for* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *respectively. Then the collection* $\mathcal{A}$ *of all soft subsets of the form* $L * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E)$ *and* $M * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$*, is a soft subbase for the product soft* $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *where* $L \in (\mathcal{L}, \mathcal{A}), M \in (\mathcal{M}, A)$

Proof. In order to prove that $\mathcal{A}$ is a soft subbase for $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$, we have to prove that the soft collection of s-open sets $(\mathcal{G}, A)$ of the finite intersection of soft members of A form a soft base for $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$. Since the intersection of empty soft subcollection of $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ and so $(X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E) \in (G, \mathcal{A})$. Next, we suppose that $\left\{(L^1\mathcal{A}) * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E), (L^1, A) * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E), (L^1, \mathcal{A}) * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)(L, {}^rA) * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E), \ldots\ldots (L^p, A) * (X^1, \widetilde{\tau_1}, \widetilde{\tau_2}, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)\right\}\uplus$

$\Big\{(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M^1,\mathrm{A}),(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M^2,\mathrm{A}),(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$
$*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M^3,\mathrm{A}),(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M^r,\mathrm{A}),(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$
$*(M^q,\mathrm{A})\Big\}$ be a non-empty infinite soft subcollection of A. This soft intersection of these elements belong to (G, A), by the construction of (G, A). This elements of (G,A) is $\Big\{((L,^1\mathrm{A})*(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}))\cap((L,^2\mathrm{A})*(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}))\cap((L,^3\mathrm{A})$
$*(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}))\cap((L,^r\mathscr{A})*(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^1,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})),\ldots\ldots(L,^p\mathscr{A})*(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$
$*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})\Big\}\cap\Big\{((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M,^1\mathrm{A}))\cap(((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M,^2\mathrm{A})$
$\cap((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M,^3\mathrm{A}))\cap((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M,^R\mathrm{A})),\ldots\ldots$
$((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*(M,^q\mathrm{A}))\Big\}$.

$[\cap\Big\{((X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})*((M,^1\mathrm{A}))\cap(M,^2\mathrm{A}))\cap((M,^3\mathrm{A}))\cap(M,^R\mathrm{A})),\ldots\ldots(M,^q\mathrm{A}))\Big\}$
$((L,^1\mathrm{A})\cap((L,^2\mathrm{A})\cap((L,^3\mathrm{A}))\cap((L,^r\mathrm{A}),\ldots\ldots(L,^p\mathrm{A}))*((M,^1\mathrm{A}))\cap(M,^2\mathrm{A}))\cap((M,^3\mathrm{A}))\cap(M,^R\mathrm{A})),$
$\ldots\ldots(M,^q\mathrm{A}))=\bigcap_{n=1}^{p}[(L,^r\mathrm{A})]*\bigcap_{r=1}^{q}[(M,^r\mathrm{A})]$. We supposed that B is a soft base for $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ generated by the elements of $(\mathscr{L},\ \mathrm{A})$ and C is a soft base for $(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ generated by the elements of $(\mathscr{M},\ \mathrm{A})$. Since the finite soft intersection of soft subbase form, the soft base for that soft topology. In view of the above statements, $\bigcap_{n=1}^{p}[(L,^r\mathrm{A})]\in\mathscr{B}$, $\bigcap_{r=1}^{q}[(M,^r\mathrm{A})]\in C$. Hence of (G,A) is expressible as $(G,\mathrm{A})=\Big\{B*C:B\in\mathscr{B},C\in\complement\Big\}$. Then (G,A) is soft s-open such that $(G,\mathrm{A})\in(\widetilde{\tau}_1,\widetilde{\tau}_2)\cup(\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2})$ is a soft base for $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. But (G,A) is obtained from the finite soft intersection of members of $\mathscr{A}$. It follows that $\mathscr{A}$ is a soft subbase for $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. □

Theorem 8. *Let $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E)$ and $(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E)$ be two SBTSpaces on the crisp set X^1 and X^2 respectively. Let $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E)*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E)$ be the product space. Then, the soft projection maps π^1 and π^2 are soft continuous, and soft s-open.*

Proof. Suppose $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ be a product space of *SBTSpaces* $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ and $(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. Then $\mathscr{X}=(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. Define soft maps $\pi^1:(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ $*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})\to(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ such that $\pi^1(x^1,x^2)=x^1$ *for all* $(x^1,x^2)\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ $*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}),\pi^2:(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})\to(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ such that $\pi^2(x^1,x^2)=x^2$ for all $(x^1,x^2)\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. Then π^1 and π^2 both are called soft projection maps on the soft first and second coordinate spaces, respectively.

Step (1) First, we show that π^1 is soft continuous. Let $(G,\mathrm{A})\widetilde{\subset}(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$ be arbitrary soft s-open set. $\pi^{-1}(G,\mathrm{A})=\Big\{(x^1,x^2)\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}):\pi^1(x^1,(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}))\in(G,\mathrm{A})\Big\}$
$=\Big\{(x^1,x^2)\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}):x^1\in(G,\mathrm{A})\Big\}=\Big\{(x^1,x^2)\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}):$
$x^1\in(G,\mathrm{A})\Big\}=(G,\mathrm{A})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$*is* soft s-open set in $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. This implies π^1 is soft continuous. The proof runs on similar lines to show that π^2 is soft continuous.

Step (2) To prove that the soft projection maps are soft s-open maps. Let $(G,\mathscr{A})\widetilde{\subset}(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*$ $(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ be an arbitrary soft s-open set. Let $x^1\in\pi^2\ [(G,\mathscr{A})]$ be arbitrary. This means that there exists $(\widetilde{u}^1,\widetilde{u}^2)\in(G,\mathrm{A})$ such that $\pi^2\ (\widetilde{u}^1,\widetilde{u}^2)x^2$ this implies that $\widetilde{u}^1=x^2$ because $\pi^2\ (\widetilde{u}^1,\widetilde{u}^2)=\widetilde{u}^2$. Now, $(\widetilde{u}^1,x^2\ )\in(G,\mathrm{A})$. Let B be the base for the soft topology $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$. Then by definition of a soft base, $(\widetilde{u}^1,x^2\ )\in(G,\mathrm{A})\in(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*$ $(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ implies that there exists a soft $(\widetilde{\tau}_1,\widetilde{\tau}_2)$ s-open set (U^1,A) and soft $(\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2})$ s-open set (U^2,A) such that $(U^1,\mathrm{A})*(U^2,\mathrm{A})\in\mathscr{B}$ such that $(\widetilde{u}^1,x^2\ )\in(U^1,\mathrm{A})*(U^2,\mathrm{A})\widetilde{\subset}(G,\mathrm{A})$ implies that $\pi^2((\widetilde{u}^1,x^2\ ),x^2\ )\in\pi^2\ ((U^1,\mathrm{A})*(U^2,\mathrm{A})\ )\widetilde{\subset}(G,\mathrm{A})\widetilde{\subset}\pi^2((G,\mathrm{A}))$. for $\pi^2((U^1,\mathrm{A})*(U^2,\mathrm{A})\ )=$ $\Big\{\pi^2\ (x^1,x^2):(x^1,x^2)\in((U^1,\mathrm{A})*(U^2,\mathrm{A})\ )\Big\}=\Big\{x^2\ :x^1\in(U^1,\mathrm{A}),x^2\ \in(U^2,\mathrm{A})\Big\}=(U^2,\mathrm{A})$. This proves that x^2 is a soft interior point of $\pi^2((G,\mathrm{A}))$. But x^2 is an arbitrary soft point of $\pi^2((G,\mathrm{A}))$. Therefore, every point of $\pi^2((G,\mathrm{A}))$ is a soft interior point. This proves that $\pi^2((G,\mathrm{A}))$ is soft s-open in $(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})$. This proves that the soft map $\pi^2:(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E})*(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})\to(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E})$ is the soft s-open map. A proof can be written on the same lines to show that π^2 is a soft s-open map. Consequently, projection maps are soft s-open maps. This finishes the proof. □

Theorem 9. *Let* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$ *and* $(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively.* $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be the product space,* $\pi^1 : (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E) \to (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E)$, $\pi^2 : (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E) \to (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$ *be the soft projections maps on the first and second coordinate spaces, respectively. Let* $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, E)$ *be another soft bitopological space such that* $\mathcal{F} : (\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, E) \to (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E)$. *Then* $\mathcal{F}$ *is soft continuous* $\Longleftrightarrow$ $\pi^1 o \mathcal{F}$ *and* $\pi^2 o \mathcal{F}$ *are soft continuous maps.*

Proof. Let $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be the soft product topological space. Let $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ be another soft bitopological space. Let $\mathcal{B}$ be the soft base for the soft product topology $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$. Now $\pi^1 o \mathcal{F} : (\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E}) \to (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E})$, $\pi^2 o \mathcal{F} : (\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E}) \to (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ are also soft maps. Let $\mathcal{F}$ be soft continuous. Now $\pi^1 o \mathcal{F}$ and $\pi^2 o \mathcal{F}$ are continuous soft maps. Conversely, suppose that $\pi^1 o \mathcal{F}$ and $\pi^2 o \mathcal{F}$ are continuous soft maps. To show that $\mathcal{F}$ is continuous. Let $(G, \mathrm{A}) \widetilde{\subset} (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ be arbitrary soft s-open set. If we prove that $\mathcal{F}^{-1}((G, \mathrm{A}))$ is soft s-$open$ in $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$, the result will follow. Let $\widetilde{y} \in \mathcal{F}^{-1}((G, \mathrm{A}))$ be an arbitrary, then $\mathcal{F}(\widetilde{y}) \in (G, \mathrm{A})$. therefore, $\mathcal{F}(\widetilde{y})$ is an element of $(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E})$ and hence it can be taken as $\mathcal{F}(\widetilde{y}) = (x^1, x^2) \in (G, \mathrm{A})$ by definition of the soft base, $(x^1, x^2) \in (G, \mathrm{A}) \in (X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}) * (\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ there exists a soft $(\widetilde{\tau}_1 \cup \widetilde{\tau}_2)$ s-open set (U^1, A) and soft $(\widetilde{\mathfrak{F}_1} \cup \widetilde{\mathfrak{F}_2})$ s-$open$ set (U^2, A) and such that $(U^1, \mathrm{A}) * (U^2, \mathrm{A}) \in \mathcal{B}$ such that $x^1 \in (U^1, \mathrm{A}) * (U^2, \mathscr{A}) \widetilde{\subset} (G, \mathrm{A})$ implies that $\pi^1((x^1, x^2)) \in \pi^1((U^1, \mathrm{A}) * (U^2, \mathrm{A})) \widetilde{\subset} \pi^1(G, \mathrm{A})$ and $\pi^2((x^1, x^2)) \in \pi^2((U^1, \mathrm{A}) * (U^2, \mathrm{A})) \widetilde{\subset} \pi^2(G, \mathrm{A})$ this implies that $x^1 \in (U^1, \mathscr{A}) \widetilde{\subset} \pi^1(G, \mathrm{A})$ and $x^2 \in (U^2, \mathscr{A}) \widetilde{\subset} \pi^2(G, \mathrm{A})$. For $\pi^1((U^1, \mathrm{A}) * (U^2, \mathrm{A})) = \left\{\pi^1((x^1, x^2)) : (x^1, x^2) \in ((U^1, \mathrm{A}) * (U^2, \mathrm{A}))\right\} = \left\{x^1 : x^1 \in (U^1, \mathrm{A}), x^2 \in (U^2, \mathrm{A}),\right\} = (U^1, \mathrm{A})$. Similarly $\pi^2((U^1, \mathrm{A}) * (U^2, \mathrm{A})) = (U^2, \mathrm{A})$. $(\pi^1 o \mathcal{F})(\widetilde{y}) = (\pi^1(\mathcal{F}(\widetilde{y})) = \pi^1((x^1, x^2)) = x^1$. Similarity, $(\pi^2 o \mathcal{F})(\widetilde{y}) = (\pi^2(\mathcal{F}(\widetilde{y})) = \pi^2((x^1, x^2)) = x^2$. Thus, $(\pi^1 o \mathcal{F})(\widetilde{y}) = x^1$, $(\pi^2 o \mathcal{F})(\mathrm{y}) = x^2$. From the above $(\pi^1 o \mathcal{F})(\widetilde{y}) \in (U^1, \mathrm{A}) \widetilde{\subset} \pi^1((G, \mathrm{A}))$, $(\pi^2 o \mathcal{F})(\widetilde{y}) \in (U^2, \mathrm{A}) \widetilde{\subset} \pi^2((G, \mathrm{A}))$ this implies that $(\widetilde{y}) \in (\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A})$ and $(\widetilde{y}) \in (\pi^2 o \mathcal{F})^{-1}(U^2, \mathrm{A})$ implies that $(\widetilde{y}) \in \left[(\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A}) \cap (\pi^2 o \mathcal{F})^{-1}(U^2, \mathscr{A})\right]$. Therefore, $(\pi^1 o \mathcal{F})$*and* $(\pi^2 o \mathcal{F})$ are given to be soft continuous and hence $(\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A})$ and $(\pi^2 o \mathcal{F})^{-1}(U^2, \mathrm{A})$ are soft s-open in $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ this implies that $\left[(\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A})\right] \cap [(\pi^2 o \mathcal{F})^{-1}(U^2, \mathrm{A})]$ is soft s-open in $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ On taking $(\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A}) = (V^2, \mathrm{A})$, $(\pi^1 o \mathcal{F})^{-1}(U^2, \mathrm{A}) = (V^2, \mathrm{A})$. We have $(V^1, \mathrm{A}) \cap (V^2, \mathrm{A})$ are soft s-open in $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ by the above result, $\widetilde{y} \in (V^1, \mathrm{A}) \cap (V^2, \mathrm{A}) = (V, \mathrm{A})$ (say). Any, $\widetilde{v} \in (V, \mathrm{A})$ implies that $\widetilde{v} \in (V^1, \mathrm{A})$ and $\widetilde{v} \in (V^2, \mathrm{A})$ implies that $\widetilde{v} \in (\pi^1 o \mathcal{F})^{-1}(U^1, \mathrm{A})$, $\widetilde{v} \in (\pi^2 o \mathcal{F})^{-1}(U^2, \mathrm{A})$ implies that $(\pi^1 o \mathcal{F})(\widetilde{v}) \in (U^1, \mathrm{A}), (\pi^2 o \mathcal{F})(\widetilde{v}) \in (U^2, \mathrm{A})$ implies that $(\pi^1 o \mathcal{F})(v) \in (U^1, \mathrm{A}) \widetilde{\subset} \pi^1((G, \mathrm{A}))$ and $(\pi^2 o \mathcal{F})(\widetilde{v}) \in (U^2, \mathrm{A}) \widetilde{\subset} \pi^2((G, \mathrm{A}))$ implies that $\widetilde{v} \in (\pi^1 o \mathcal{F})^{-1}[\pi^1((G, \mathrm{A}))]$ and $\widetilde{v} \in (\pi^2 o \mathcal{F})^{-1}[\pi^2((G, \mathrm{A}))]$ implies that $\widetilde{v} \in f^{-1}(G, \mathrm{A})$ and $\widetilde{v} \in f^{-1}(G, \mathrm{A})$. Therefore, any $\widetilde{v} \in (V, \mathrm{A})$ implies that $\widetilde{v} \in f^{-1}(G, \mathrm{A})$. This implies that $(V, \mathrm{A}) \widetilde{\subset} f^{-1}(G, \mathrm{A})$. Thus we have shown that any $\widetilde{y} \in f^{-1}(G, \mathrm{A})$ implies that there exists a soft s-open set $(V, \mathrm{A}) \widetilde{\subset} (\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$ such that $\widetilde{y} \in (V, \mathrm{A}) \widetilde{\subset} f^{-1}(G, \mathrm{A})$. This implies that y is a soft interior point of $f^{-1}(G, \mathrm{A})$ and hence every point of $f^{-1}(G, \mathrm{A})$ is a soft interior point, showing thereby $f^{-1}(G, \mathrm{A})$ is soft s-open in $(\mathcal{Y}^1, \mathcal{T}_1, \mathcal{T}_2, \mathrm{E})$. □

6. Attachment of Separation Axioms with Soft Connectedness and Soft Coordinate Spaces

In this section, the characterization of soft separation axioms with soft connectedness is addressed with respect to semiopen sets in soft bitopological spaces. In addition to this, the product of two soft topological spaces is (S^1 *and* S^2) space if each coordinate space is soft (S^1 *and* S^2) space, a product of two sot topological spaces is (S-regular and S-C regular) space if each coordinate space is (S-regular and S-C regular), a product of two soft topological spaces is connected if each coordinate space is soft

connected and the product of two soft topological +spaces is (first-countable, second-countable) if each coordinate space is (first-countable, second-countable).

Theorem 10. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be two SBTSpaces on the crisp sets* X^1 *and* X^2 *respectively.* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be the soft product space, then the product space* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *is soft S-connected if both* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *are soft S-connected.*

Proof. Suppose $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ be a product space of soft bitopological spaces $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ and $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$. If the soft product $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ is soft *S*-connected then we have to prove that $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ and $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ are soft *S*-connected. Define soft maps $\pi^1 : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right) \to \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ such that $\pi^1\left(x^1, x^2\right) = x^1$ *for all* $\left(x^1, x^2\right) \in \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$, $\pi^2 : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right) \to \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ such that $\pi^2\left(x^1, x^2\right) = x^2$ *for all* $\left(x^1, x^2\right) \in \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ then π^1 and π^2 are called soft projection maps. Also π^1 *and* π^1 Now $\pi^1 : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right) \to \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ is soft continuous and $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ is soft *S*-connected.

This implies that $\pi^1\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)$ is soft *S*-connected that is $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ is soft S-connected. Again $\pi^2\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)$ is continuous and $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ is soft *S*-connected. This implies $\pi^2\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right) = \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$. For what we have done, it follows that $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ is soft *S*-connected sets. Conversely, let $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ be the product space of soft *S*-connected spaces $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$. We have to prove that $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ is soft *S*-connected. Pick any point $\left(x^1, x^2\right) \in \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ and consider the soft sets $\left(x^1, \mathrm{A}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ and $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(x^2, \mathrm{A}\right)$. Define the soft maps $\mathscr{F} : \left(x^1, \mathrm{A}\right) * \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) \to \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ by requiring that $\mathscr{F} : \left(\left(x^1, x^2\right)\right) = x^2$ for all $x^2 \in \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ and $\mathscr{g} : \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(x^2, \mathrm{A}\right) \to \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$ by writing $\mathscr{g} : \left(\left(x^1, x^2\right)\right) = \widetilde{\curvearrowright}^1$ for all $x^1 \in \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)$. Now $\mathscr{F}^{-1}$ and $\mathscr{g}^{-1}$ are continuous soft maps so that $\mathscr{F}^{-1}[\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ and $\mathscr{g}^{-1}\left[\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right)\right]$ are soft *S*-connected sets that are $\left(x^1, \mathrm{A}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ and $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(x^2, \mathrm{A}\right)$ are soft *S*-connected sets. Write $\mathscr{X}^{crisp\ x^1} = \left(\widetilde{\curvearrowright}^1, \mathscr{A}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right), X^{crisp\ x^2} = \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(\widetilde{\curvearrowright}^2, \mathscr{A}\right)$, $C^{x^1x^2} = X^{crisp\ x^1} \cup X^{crisp\ x^2}$. Then $X^{crisp\ x^1}$ and $X^{crisp\ x^2}$ are soft *S*-connected sets. $\left(x^1, x^2\right) \in X^2 \cup X^2$. So that $X^{crisp\ x^1} \cup X^{\ x^2} \neq \varphi^{\mathscr{A}}$. being a finite soft union of soft *S*-connected sets having non-empty intersection. $\mathscr{X}^{crisp\ x^1} \cup \mathscr{X}^{crisp\ x^2}$ is soft *S*-connected that is, $C^{x^1x^2}$ is soft *S*-connected. Consider the family of soft *S*-connected sets $\left\{C^{x^1x^2} : \left(x^1, x^2\right) \in \mathscr{X}\right\}$ that is $\cup\left\{C^{x^1x^2} : \left(x^1, x^2\right) \in \mathscr{X}^{crisp}\right\}$ this implies that $\bigcup_{(x^1,x^2)\in\mathscr{X}}[X^{crisp\ x^1} \cup X^{crisp\ x^2}] = \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$. Finally, $\bigcup_{(x^1,x^2)\in\mathscr{X}}[X^{crisp\ x^1} \cup X^{crisp^{x^2}}] = \left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$. If we select fixed members of the above family, then their soft intersection is not soft null. Hence, $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)$ is soft S-connected. □

Theorem 11. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be the soft product space, then the product space* $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)\right)^{\partial : \partial \in \Delta}\right\}$ *is soft* S^1 *Space iff each soft coordinate space* $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)\right)^{\partial}\right\}$ *is soft* S^1 *Space.*

Proof. Suppose each coordinate space $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial}\right\}$ is soft S^1 $\mathcal{S}pace$ and let $\langle(\varkappa, \mathrm{A})^{\partial:\partial\in\Delta}\rangle$ be an element of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. Then, $(\varkappa, \mathrm{A})^{\partial} \in \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial}\right\}$ for each $\partial \in \Delta$. Since $\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial}$ is soft S^1 space, it follows that $\left\{(\varkappa, \mathrm{A})^{\partial}\right\}$ is soft $(\widetilde{\tau}_1, \widetilde{\tau}_2)\left(\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}\right) - \mathrm{s}$ closed for each $\partial \in \Delta$. Now, each soft projection mapping π^{α} being soft continuous, it follows that $(\pi^{\partial})^{-1}\left\{(\varkappa, \mathrm{A})^{\partial}\right\}$ is soft $(\widetilde{\tau}_1, \widetilde{\tau}_2)\left(\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}\right) - \mathrm{s}$ closed is $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ for every $\partial \in \Delta$. Consequently, $\bigcap_{\partial}(\pi^{\partial})^{-1}\left\{(\varkappa, \mathrm{A})^{\partial}\right\} = (\varkappa, \mathrm{A})^{\partial}$. So, every singleton soft subspace of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft $(\widetilde{\tau}_1, \widetilde{\tau}_2)\left(\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}\right) - s$ closed. Hence, $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S^1 $\mathcal{S}pace$. Conversely, let $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S^1 $\mathcal{S}pace$ and let $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ be an arbitrary soft coordinate space of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. Let $\left(\left\{(\varkappa, \mathrm{A})^{\partial}\right\}\right)^{\beta}$ and $\left(\left\{(\widetilde{y}, \mathscr{A})^{\partial}\right\}\right)^{\beta}$ be any two soft distinct points of $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$. Choose $(\varkappa, \mathrm{A})$ and (y, A) in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ whose β-*th* coordinate is $\left(\left\{(\varkappa, \mathrm{A})^{\partial}\right\}\right)^{\beta}$ and $\left(\left\{(y, \mathrm{A})^{\partial}\right\}\right)^{\beta}$ respectively. Since $\left(\left\{(\varkappa, \mathrm{A})^{\partial}\right\}\right)^{\beta} \neq \left(\left\{(y, \mathrm{A})^{\partial}\right\}\right)^{\beta}$ we have $(\varkappa, \mathrm{A}) > (y, \mathrm{A})$ or $(\varkappa, \mathrm{A}) < (y, \mathrm{A})$ or $(\varkappa, \mathrm{A}) \gg (y, \mathrm{A})$ or $(\varkappa, \mathrm{A}) \leq (y, \mathrm{A})$ But $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ being soft S^1 $\mathcal{S}pace$, corresponding to the soft distinct points $(\varkappa, \mathrm{A})$ and (y, A) of $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ there exists soft s-open sets (G, A) and (H, A) in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such that $(\varkappa, \mathrm{A}) \in (G, \mathrm{A})$ but $(y, \mathrm{A}) \notin (G, \mathrm{A})$ and $(y, \mathrm{A}) \in (H, \mathrm{A})$ but $(\varkappa, \mathrm{A}) \notin (H, \mathrm{A})$. So, there exists basic soft s-open sets $(I, \mathrm{A}) = \times\left\{(G, \mathrm{A})^{\alpha} : \alpha \in \Delta\right\}$ and $(J, \mathrm{A}) = \times\left\{(H, \mathrm{A})^{\alpha} : \alpha \in \Delta\right\}$ such that $(\varkappa, \mathrm{A}) \in (I, \mathrm{A}) \subseteq (G, \mathrm{A})$ and $(\widetilde{y}, \mathrm{A}) \in (J, \mathrm{A}) \subseteq (H, \mathrm{A})$. Clearly, $(y, \mathrm{A}) \notin (I, \mathrm{A})$ and $(\varkappa, \mathrm{A}) \notin (J, \mathrm{A})$. Thus, $(G, \mathrm{A})^{\beta}$ is soft s-open set containing $(\varkappa, \mathrm{A})^{\beta}$ but not $(y, \mathrm{A})^{\beta}$; and, $(H, \mathrm{A})^{\beta}$ is soft s-open set containing $(y, \mathrm{A})^{\beta}$ but not $(\varkappa, \mathrm{A})^{\beta}$. This shows that $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is soft S^1 $\mathcal{S}pace$. □

Theorem 12. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be two SBT*$\mathcal{S}paces$ *such that they are soft* S^2 $paces$ *on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be the soft product space, then the product space* $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ *is soft* S^2 $\mathcal{S}pace$ iff *each soft coordinate space* $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)\right)^{\partial}\right\}$ *is soft* S^2 $\mathcal{S}pace$.

Proof. Suppose each soft coordinate space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial}\right\}$ is soft S^2 $\mathcal{S}pace$ and let $\langle(\varkappa, \mathrm{A})^{\partial:\partial\in\Delta}\rangle$ and $\langle(y, \mathrm{A})^{\partial:\partial\in\Delta}\rangle$ be two points of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such that $\langle(\varkappa, \mathrm{A})^{\partial:\partial\in\Delta}\rangle > \langle(y, \mathrm{A})^{\partial:\partial\in\Delta}\rangle$ Then, $(y, \mathrm{A})^{\beta} > (y, \mathrm{A})^{\beta}$ for some for each $\beta \in \Delta$ where $(\varkappa, \mathrm{A})^{\beta}, (y, \mathrm{A})^{\beta} \in \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beta}\right\}$. Now, $\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beta}$ is soft S^2 $\mathcal{S}pace$ and $(\varkappa, \mathrm{A})^{\beta} > (y, \mathrm{A})^{\beta}$ are points of $\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beta}$, So there exists a soft $(\widetilde{\tau}_1, \widetilde{\tau}_2)\left(\widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}\right) - s$ open sets $(G, \mathrm{A})^{\beta}$ and $(H, \mathrm{A})^{\beta}$ such that $(\varkappa, \mathrm{A})^{\beta} \in (G, \mathrm{A})^{\beta}$ and $(y, \mathrm{A})^{\beta} \in (H, \mathrm{A})^{\beta}$ and $(G, \mathrm{A})^{\beta} \cap (H, \mathrm{A})^{\beta} = \varphi$. Since $\pi^{\beta}\left((\varkappa, \mathrm{A})^{\beta}\right) = (\varkappa, \mathrm{A})^{\beta} \in (G, \mathrm{A})^{\beta}$ and $\pi^{\beta}\left((y, \mathrm{A})^{\beta}\right) = (y, \mathrm{A})^{\beta} \in (H, \mathrm{A})^{\beta}$ each soft projection mapping π^{β} being soft continuous, it follows that $(\varkappa, \mathrm{A}) \in \left(\pi^{\beta}\right)^{-1}((G, \mathrm{A})^{\beta})$

and $(y,\mathrm{A}) \in \left(\pi^{\beta}\right)^{-1}((H,\mathrm{A})^{\beta})$ and $\left(\pi^{\beta}\right)^{-1}\left[(G,\mathrm{A})^{\beta} \cap (H,\mathrm{A})^{\beta}\right] = \left(\pi^{\beta}\right)^{-1}[\varphi] = \varnothing$. Moreover by soft continuity of π^{β} , $\left(\pi^{\beta}\right)^{-1}\left((G,\mathrm{A})^{\beta}\right)$, $\left(\pi^{\beta}\right)^{-1}\left((H,\mathrm{A})^{\beta}\right)$ are soft $(\widetilde{\tau}_1,\widetilde{\tau}_2)\left(\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2\right)$ s-open in $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. Hence, $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S^2 $\mathcal{S}pace$. Conversely, let $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S^2 $\mathcal{S}pace$ and let $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ be an arbitrary soft coordinate space of $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. We must show that $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is soft S^2 $\mathcal{S}pace$. Let $(\widetilde{z},\mathrm{A}) = \langle(\widetilde{z},\mathrm{A})^{\alpha:\alpha\in\partial}\rangle$ be soft fixed point of $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial}\right\}$. Let (K,A) be soft subset of $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial}\right\}$ consisting of all points of the form $(\varkappa,\mathrm{A}) = \langle(\varkappa,\mathrm{A})^{\alpha:\alpha\in\Delta}\rangle$ such that $(\varkappa,\mathrm{A})^{\alpha} = (z,\mathrm{A})^{\alpha}$ if $\alpha > \beta$ and $(\varkappa,\mathrm{A})^{\beta}$ may be any point of $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$. Let $\pi^{(\widetilde{z},\mathscr{A})}$ be the restriction of the soft projection mapping $\pi^{(z,\mathscr{A})}: \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial}\right\} \to \left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ *to* (z,A) such that $\pi^{(z,\mathscr{A})}: (z,\mathrm{A}) \to \left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta} : \pi^{(z,\mathscr{A})}(((\varkappa,\mathrm{A})) = \pi^{\beta}(((\varkappa,\mathrm{A}))$ $= (z,\mathrm{A})^{\beta}\ \forall(\varkappa,\mathrm{A}) \in (z,\mathrm{A})$. Then, $\pi^{(z,\mathscr{A})}$ is clearly soft one-one and soft onto. Also, the projection mapping π^{β} being soft continuous, is a restriction $\pi^{(z,\mathscr{A})}$ is therefore soft continuous. Now, let (G,A) be any soft basic s-open set in the soft subspace (z,A). Then, $(G,\mathscr{A}) = (z,\mathrm{A}) \cap (\mathscr{L},\mathrm{A})$ for some soft basic s-open set $(\mathscr{L},\mathrm{A})$ in $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial}\right\}$. Let $(\mathscr{L},\mathrm{A}) = \times\left\{(G,\mathrm{A})^{\alpha} : \alpha \in \wedge\right\}$, where, $(G,\mathscr{A})^{\alpha}$ is soft s-open in $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\alpha}$. Let $(\mathscr{L},\mathscr{A}) = \times\left\{\left\{(G,\mathscr{A})^{\alpha} : \alpha \in \wedge\right\}\right\}$, where $(G,\mathscr{A})^{\alpha}$ is soft s-$\mathfrak{open}$ in $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)$. Consequently, $(G,\mathrm{A}) = (z,\mathrm{A}) \cap \left(\times\left\{\left\{(G,\mathrm{A})^{\alpha} : \alpha \in \wedge\right\}\right\}\right)$, Where $(G,\mathrm{A})^{\alpha}$ is soft s-open in $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\alpha}$ for each $\alpha \in \wedge$. Thus, either $(G,\mathscr{A}) = \varnothing$ *or* $(G,\mathscr{A}) =$ {(ϰ,A)∈(z,A): βth coordinate of (x,A) in (G,A)^β}. Therefore $\pi^{(z,\mathscr{A})}(((G,\mathscr{A})) = \varnothing$ *or* $(G,\mathrm{A})^{\beta}$, each one of which is soft s-open. This shows that the soft image under $\pi^{(\widetilde{z},\mathscr{A})}$ of every soft basic s-open set in (z,A) is soft s-open and therefore, $\pi^{(z,\mathscr{A})}$ is soft s-open. Thus, $\pi^{(\varkappa,\mathrm{A})}$, is homeomorphism and therefore, $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is the soft homeomorphic image of (z,A). Now, every soft subspace of a S^2 $\mathcal{S}pace$ being S^2 $\mathcal{S}pace$, (z,A) is soft s-open and therefore, $\pi^{(z,\mathrm{A})}$ is soft S^2 $\mathcal{S}pace$ and so its soft homeomorphic image $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)$ is soft S^2 $\mathcal{S}pace$. Hence, each coordinate space is soft S^2 $\mathcal{S}pace$. □

Theorem 13. *Let* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)$ *and* $\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,E\right)$ *be two SBT*$\mathcal{S}paces$ *on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,E\right)$ *be the soft product space, then the product space* $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,E\right)\right)^{\partial:\partial\in\Delta}\right\}$ *is soft S-regular space iff each soft coordinate space* $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,E\right)\right)^{\partial}\right\}$ *is soft S-regular space.*

Proof. Suppose each coordinate space $\times\left\{= \left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial}\right\}$ is soft S-regular space. Let $(\varkappa,\mathrm{A}) = \langle(\varkappa,\mathrm{A})^{\partial:\partial\in\Delta}\rangle$ be any point of the soft product space $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and (G,A) be any soft s-open set in $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such that $(\varkappa,\mathrm{A}) \in (G,\mathrm{A})$. Then there exists a soft basic s-open set (H,A) in $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}}_1,\widetilde{\mathfrak{F}}_2,\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such

that $(\varkappa, \mathrm{A}) \in (H, \mathrm{A})\widetilde{\subseteq}(G, \mathrm{A})$. Let, $\times\left\{((G, \mathscr{A}))^{\partial:\partial\in\Delta}\right\}$ is the soft product space such that $((G, \mathrm{A}))^{\partial}$ is soft s-open in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial}\right\}$. Since each $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial}\right\}$ is soft S-regular and $((G, \mathrm{A}))^{\partial}$ is soft s-open in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial}\right\}$ containing $(\varkappa, \mathrm{A})^{\partial}$ there exists a soft s-open set $((K, \mathscr{A}))^{\partial}$ in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial}\right\}$ such that $(\varkappa, \mathrm{A})^{\partial} \in ((K, \mathrm{A}))^{\partial}$ and $\overline{((K, \mathrm{A}))^{\partial}}\,\widetilde{\subseteq}(G, \mathrm{A})^{\partial}$. Let $\times\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$, then $\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$ is soft s-open set in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and contains $(\varkappa, \mathrm{A})$. Also, $\overline{\times((K, \mathrm{A}))^{\partial::\partial\in\Delta}} = \times\left\{\overline{((K, \mathrm{A}))^{\partial:\partial\in\Delta}} : \partial \in \Delta\right\}$. Further, since $\overline{((K, \mathrm{A}))^{\partial}}\,\widetilde{\subseteq}(G, \mathrm{A})^{\partial}$ for each ∂, we have $\times\left\{\overline{((K, \mathrm{A}))^{\partial:\partial\in\Delta}} : \partial \in \Delta\right\}\widetilde{\subseteq} \times \left\{((G, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$. this shows that for every soft point $(\varkappa, \mathrm{A}) \in \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and every soft s-open set (G, A) containing $(\varkappa, \mathrm{A})$ there exists a soft s-open set $\times\left\{\overline{((K, \mathscr{A}))^{\partial:\partial\in\Delta}} : \partial \in \Delta\right\}$ in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such that $(\varkappa, \mathrm{A}) \in \times\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$ and $\overline{\times((K, \mathrm{A}))^{\partial::\partial\in\Delta}}\,\widetilde{\subseteq} \times \left\{((G, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$. Hence, $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S-regular. Conversely, let the non-empty soft product space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ be soft S-regular and let $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ be an arbitrary soft coordinate space. Then, we must show that it is a soft S-regular. Let $(\widetilde{\varkappa}, \mathscr{A})^{\beta}$ be any soft point of $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ and let $(G, \mathrm{A})^{\beta}$ be any soft s-open in $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ such that $(\varkappa, \mathrm{A})^{\beta} \in (G, \mathrm{A})^{\beta}$. now, choose, soft element $(\varkappa, \mathrm{A})$ in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ whose βth coordinate in $(\varkappa, \mathrm{A})^{\beta}$. Let $(G, \mathrm{A}) = \pi^{\beta-1}((\varkappa, \mathrm{A})^{i}$. Then, $(\widetilde{\varkappa}, \mathscr{A}) \in (G, \mathrm{A})$ and by soft continuity of π^{β}, (G, A) is soft p-$open$ in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. Since, $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S-regular space so there exists a soft basic s-open set $\times\left\{((K, \mathscr{A}))^{\partial:\partial\in\Delta}\right\}$, Where each $((K, \mathrm{A}))^{\partial}$ is soft s-open in $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ such that $(\varkappa, \mathrm{A}) \in \times\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$ and $\overline{\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}}\widetilde{\subseteq}(G, \mathrm{A})$. Now $(\varkappa, \mathrm{A}) \in \overline{\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}}\widetilde{\subseteq}(G, \mathrm{A})$ this implies that $(\varkappa, \mathrm{A}) \in (G, \mathrm{A}) = \pi^{\beta-1}\left((G, \mathscr{A})^{\beta}\right)$ implies that $(\varkappa, \mathrm{A})^{\beta} \in (G, \mathrm{A})$. Moreover, $\overline{\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}} = \left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}$ and $\overline{\left\{((K, \mathrm{A}))^{\partial:\partial\in\Delta}\right\}}\widetilde{\subseteq}(G, \mathrm{A})^{\beta}$ This shows that $\left(\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is soft S-regular and hence, each coordinate space is soft S-regular. □

Theorem 14. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)$ *be the soft product space, then the product space* $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ *is soft S-completely regular space if each soft coordinate space* $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\}$ *is soft completely S-regular space.*

Proof. Let each soft coordinate space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial}\right\}$ is soft S-completely regular. Then, we must show that the soft product space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, \mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. Let $(\aleph, \mathrm{A})$ be any member of the usual soft subbase for the soft product topology and let $(\varkappa, \mathrm{A}) = \langle(\varkappa, \mathrm{A})^{\partial:\partial\in\Delta}\rangle$ be any

soft point in $(\aleph, A)$. Then $(\aleph, A) = \pi^{\beta-1}\left((G, A)^{\beta}\right)$ is soft s-open in $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\}$ and contains $(\varkappa, A)^{\beta})$. Since, $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\}$ is soft completely S-regular there exists a soft mapping $f : \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\} \to \left[\widetilde{0}, \widetilde{1}\right]$ such that $f\left((\varkappa, A)^{\beta}\right) = \widetilde{0}$ and $f(y, A) = \widetilde{1}\ \forall\ y \in \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\} - (G, \mathscr{A})^{\beta}$. Since π^{β} is soft continuous and f is soft continuous, so the soft composite mapping $\left(fo\pi^{\beta}\right) : \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\} \to \left[\widetilde{0}, \widetilde{1}\right]$ is soft continuous. Now, if $(\varkappa, A) \in (\aleph, \mathscr{A})$, then $(\varkappa, A) \in \pi^{\beta-1}\left((G, \mathscr{A})^{\beta}\right)$ implies that $\pi^{\beta}((\varkappa, A)) \in (G, A)^{\beta}$ implies that $f\left[\pi^{\beta}((\varkappa, A))\right] = f\left((\varkappa, A)^{\beta}\right) = \widetilde{0}$ implies that $\left(fo\pi^{\beta}\right)((\varkappa, A)) = \widetilde{0}$. Again, if $(\varkappa, A) \in \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\} - (\aleph, A)$, then, $(\varkappa, A) \in \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\} - (\aleph, A)$ implies that $(\varkappa, A) \in \pi^{\beta-1}(\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\} - \pi^{\beta-1}\left((G, A)^{\beta}\right)$ implies that $(\varkappa, A) \in \pi^{\beta-1}\left(\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\} - (G, \mathscr{A})^{\beta}\right)$ implies that $\pi^{\beta}(\ (\varkappa, A)) \in \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\} - (G, \mathscr{A})^{\beta})$ implies that $f\left[\pi^{\beta}(\ (\varkappa, A))\right] = \widetilde{1}$ implies that $\left(fo\pi^{\beta}\right)((\widetilde{\varkappa}, \mathscr{A})) = \widetilde{1}$. Thus, $f\left[\pi^{\beta}(\ (\varkappa, A))\right] = \begin{cases} \widetilde{0} = if\ (\varkappa, A) \in (G, \mathscr{A})^{\beta}) \\ \widetilde{0} = if\ (\varkappa, A) \in \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\} - (\aleph, A). \end{cases}$

Hence, $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft, completely S-regular. Conversely, let the soft product space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ be soft completely S-regular and let $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\}$ be an arbitrary soft coordinate space. Then, continuing on the same lines, we can show that $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\}$ is the soft homeomorphic image of a soft subspace of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$. Now, every soft subspace of a soft completely S-regular space being a soft completely regular and soft homeomorphic image of a soft completely S-regular space being soft S-completely regular, it follows that $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\beta}\right\}$ is soft completely S-regular. Hence, each coordinate space of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S-completely regular. □

Theorem 15. *Let $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ and $\left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)$ be two SBTSpaces on the crisp sets X^1 and X^2 respectively. $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)$ be the soft product., then the product space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft S-connected if each soft coordinate space $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\}$ is S -soft connected space.*

Proof. Suppose $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\}$ is soft S-connected for each $\partial \in \Delta$. Fix a soft point $(\widetilde{\varkappa}, \mathscr{A}) = \langle(\widetilde{\varkappa}, \mathscr{A})^{\partial:\partial\in\Delta}\rangle$ in soft product space $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial:\partial\in\Delta}\right\}$ and let $\widetilde{C}$ be the soft component to which $(\widetilde{\varkappa}, \mathscr{A})$ belongs. We shall show that every soft point of $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\}$ is in $\widetilde{C}$. Let $(G, A)^{\partial:\partial\in\Delta}$ be any soft basic s-open in $\times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\} = \times\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}}_1, \widetilde{\mathfrak{F}}_2, E\right)\right)^{\partial}\right\}$ for all but a

finite soft number of ∂. Let $(G,\mathscr{A})^{\partial:\partial\in\Delta} = \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ when $\partial > \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots\ldots\ldots\ldots,\partial^m,\partial < \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots\ldots\ldots\ldots,\partial^m$ or when $\partial \gg \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots\ldots\ldots\ldots,\partial^m,\partial \ll \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots\ldots\ldots\ldots,$ ∂^m. Then, we can construct

$$(\beth,\mathscr{A})^{\partial^1} = \left\{\left\{(\beth,\mathscr{A})^{\partial^1} = (\widetilde{\varkappa},\mathscr{A}) = \langle(\widetilde{\varkappa},\mathscr{A})^{\partial:\partial\in\Delta}\rangle : (\widetilde{\varkappa},\mathscr{A})^{\partial} = (\widetilde{z},\mathscr{A})^{\partial} if\ \partial > \partial^1 or\ \partial < \partial^1 or\ \partial \gg \partial^1 or\ \partial \ll \partial^1\ and\ (\widetilde{\varkappa},\mathscr{A})^{\partial^1} \in \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^1}\right\}\right\}$$

and show that $(\beth,\mathscr{A})^{\partial^1}$ is soft homeomorphic to $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^1}\right\}$. Similarly, we can construct $(\beth,\mathscr{A})^{\partial^2},(\beth,\mathscr{A})^{\partial^3},(\beth,\mathscr{A})^{\partial^4},(\beth,\mathscr{A})^{\partial^5},(\beth,\mathscr{A})^{\partial^6},(\beth,\mathscr{A})^{\partial^7},(\beth,\mathscr{A})^{\partial^8},(\beth,\mathscr{A})^{\partial^9},(\beth,\mathscr{A})^{\partial^{10}}$ $\ldots\ldots\ldots(\beth,\mathscr{A})^{\partial^m}$ and as argued before, they are homeomorphic to

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^2}\right\}$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^3}\right\}\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^4}\right\}$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^5}\right\}\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^6}\right\}$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^7}\right\}\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^8}\right\}$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^9}\right\},\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots,\ \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^m}\right\},$

respectively. So $(\beth,\mathscr{A})^{\partial^2}*(\beth,\mathscr{A})^{\partial^3}\times(\beth,\mathscr{A})^{\partial^4}*(\beth,\mathscr{A})^{\partial^5}*(\beth,\mathscr{A})^{\partial^6}*(\beth,\mathscr{A})^{\partial^7}*(\beth,\mathscr{A})^{\partial^8}*(\beth,\mathscr{A})^{\partial^9}$ $*(\beth,\mathscr{A})^{\partial^{10}}*\ldots\ldots\ldots*(\beth,\mathscr{A})^{\partial^m}$ is soft homeomorphic to $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^2}\right\}*$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^3}\right\}\times*\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^4}\right\}*$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^5}\right\}\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^6}\right\}*$

$\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^7}\right\}\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^8}\right\}*$

$\times\ \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^9}\right\},\quad \ldots.\quad \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^m}\right\}.$ Now each $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^i}\right\}$ being soft S-connected and finite soft product of soft S-connected spaces being soft S-connected, it follows that $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^2}\right\}*\ \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^3}\right\}\ \times$

$*\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^4}\right\}*\ \times\ \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^5}\right\}\ \times$

$\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^6}\right\}*\ \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^7}\right\}\ \times$

$\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^8}\right\}*\ \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^9}\right\},\ \ldots$

$*\ \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^m}\right\}.$ Now each $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial^m}\right\}$ is soft S-connected and therefore it is soft homeomorphic to image $(\beth,\mathscr{A})^{\partial^2}*(\beth,\mathscr{A})^{\partial^3}\times(\beth,\mathscr{A})^{\partial^4}*(\beth,\mathscr{A})^{\partial^5}*(\beth,\mathscr{A})^{\partial^6}*(\beth,\mathscr{A})^{\partial^7}*(\beth,\mathscr{A})^{\partial^8}*(\beth,\mathscr{A})^{\partial^9}*(\beth,\mathscr{A})^{\partial^{10}}*\ldots\ldots\ldots*(\beth,\mathscr{A})^{\partial m} = (\beth,\mathscr{A})$ is soft S -connected. Now, $(\beth,\mathscr{A})$ is $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and C is a soft maximal S-connected and therefore, its soft homeomorphic image $(\beth,\mathscr{A})^{\partial^2}*(\beth,\mathscr{A})^{\partial^3}\times(\beth,\mathscr{A})^{\partial^4}*(\beth,\mathscr{A})^{\partial^5}*(\beth,\mathscr{A})^{\partial^6}*(\beth,\mathscr{A})^{\partial^7}*(\beth,\mathscr{A})^{\partial^8}*(\beth,\mathscr{A})^{\partial^9}*(\beth,\mathscr{A})^{\partial^{10}}*\ldots\ldots\ldots*(\beth,\mathscr{A})^{\partial^m} = ((\beth,\mathscr{A}))\widetilde{\subset C}$. But $(\beth,\mathscr{A})$ contains the soft points $(t,\mathscr{A}) = \langle\left(\widetilde{t},\mathscr{A}\right)^{\partial:\partial\in\Delta}\rangle$ for which $\left(\widetilde{t},\mathscr{A}\right)^{\partial} = (z,\mathscr{A})^{\partial}$ if

$\partial > \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots,\partial^m, \partial < \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots,\partial^m$ or when $\partial \gg \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots,\partial^m, \partial \ll \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots,\partial^m$ and $\left(\widetilde{t},\mathscr{A}\right)^{\partial} = (\widetilde{\varkappa},\mathscr{A})^{\partial}$ for which $\partial = \partial^1,\partial^2,\partial^1,\partial^3,\partial^4,\partial^5,\partial^6,\partial^7,\partial^8,\ldots,\partial^m$. This soft point $\left(\widetilde{t},\mathscr{A}\right)$ lies in $(G,\mathscr{A})^{\partial:\partial\in\Delta}$ which was an arbitrary soft basic *s*-open set containing $(\widetilde{\varkappa},\mathscr{A})$. This shows that $(\widetilde{\varkappa},\mathscr{A})$ is a soft adherent point of *C* that is $t(\widetilde{\varkappa},\mathscr{A}) \in \overline{C}$. but *C* being a soft component, it is soft s-closed and therefore, $\overline{C} = C$. Thus, $(\widetilde{\varkappa},\mathscr{A}) \in C$. So, it follows that every soft point of $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is in *C*, that is $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\subseteq\widetilde{C}$. Hence, is $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}=\widetilde{C}$. But, $\overline{C}$ being soft *S*-connected, so is, therefore, $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$.

Conversely, let the product space $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ be soft *S*-connected. Since each soft projection mapping $\pi^{(z,\mathscr{A})}: \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ $\rightarrow \left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is soft continuous and the soft continuous image of each soft *S*-connected set is soft *S*-connected, so it follows that $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^{\beta}$ is soft *S*-connected $\beta \in \Delta$. That is, each soft coordinate space is soft *S*-connected. □

Theorem 16. *Let* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)$ *and* $\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)$ *be two SBTSpaces on the crisp sets* X^1 *and* X^2 *respectively.* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)$ *be the soft product, then the product space* $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)\right)^{\partial:\partial\in\Delta}\right\}$ *is soft locally S-connected if each soft coordinate space* $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)\right)^{\partial}\right\}$ *is soft S-locally connected, and all but a finite soft number are soft S-connected.*

Proof. Suppose $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ is soft *S*-connected for each $\partial \in \Delta$ and soft connected for $\partial > \beth^1,\beth^2,\beth^3,\beth^4,\beth^5,\beth^6,\beth^7,\beth^8,\beth^9,\ldots,\beth^m, \partial < \beth^1,\beth^2,\beth^3,\beth^4,\beth^5,\beth^6,\beth^7,\beth^8,\beth^9,\ldots,\beth^m$ when $\partial \gg \beth^1,\beth^2,\beth^3,\beth^4,\beth^5,\beth^6,\beth^7,\beth^8,\beth^9,\ldots,\beth^m, \partial \ll \beth^1,\beth^2,\beth^3,\beth^4,\beth^5,\beth^6,\beth^7,\beth^8,\beth^9,\ldots,\beth^m$. Then we must show that $\prod_{\partial}\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ is soft *S*-locally connected. Let $(\widetilde{\varkappa},\mathscr{A}) = \langle(\widetilde{\varkappa},\mathscr{A})^{\partial:\partial\in\Delta}\rangle$ be any soft point of $\prod_{\partial}\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ contained in an arbitrary soft basic *s*-open set $\prod_{\partial}\left\{(G,\mathscr{A})^{\partial}\right\}$ where $(G,\mathscr{A})$ soft *s*-open in is $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ and $(G,\mathscr{A})^{\partial} = \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ for $\partial > \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}, \partial < \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}$, when $\partial \gg \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}, ,\partial \ll \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}$. Since each $\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ is soft *S*-locally connected and $(\widetilde{\varkappa},\mathscr{A})^{\partial} \in (G,\mathscr{A})^{\partial}$ there exists a soft connected *s*-open set $(\mathfrak{H},\mathscr{A})^{\partial}$ in $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ such that $(x,\mathscr{A})^{\partial} \in (\mathfrak{H},\mathscr{A})^{\partial}\widetilde{\subseteq}(G,\mathscr{A})^{\partial}$. Now consider the soft subset $((H,\mathscr{A})) = \sqcap^{\partial}(H,\mathscr{A})^{\partial}$ of $\prod_{\partial}\left\{(G,\mathscr{A})^{\partial}\right\}$ where $(G,\mathscr{A})$ soft *s*-open in is $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ and $(H,\mathscr{A})^{\partial} = (\mathfrak{H},\mathscr{A})^{\partial}$ if $\partial > \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots\ldots\ldots\ldots.\beth^{m'}, \partial < \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}$, when $\partial \gg \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}, \partial \ll \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},\beth^{7'},\beth^{8'},\beth^{9'}\beth^{10'},\ldots.\beth^{m'}$ and $(H,\mathscr{A})^{\partial} = \left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ if $\partial = \beth^{1'},\beth^{2'},\beth^{3'},\beth^{4'},\beth^{5'},\beth^{6'},,\ldots.\beth^{m'}$. Clearly, each $(H,\mathscr{A})^{\partial}$ is soft *S*-connected and so is, therefore, their soft product $(H,\mathscr{A})$. Also, $(H,\mathscr{A})$ is clearly a soft basic *s*-open set containing $(\widetilde{\varkappa},\mathscr{A})$ and contained in $\prod_{\partial}\left\{(G,\mathscr{A})^{\partial}\right\}$. Thus, to each

$(\widetilde{\varkappa},\mathscr{A}) \in \times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and each soft basic *s*-open set $\prod_\partial\left\{(G,\mathscr{A})^\partial\right\}$ containing $(\widetilde{\varkappa},\mathscr{A})$ there exists a soft basic *s*-open connected set $(H,\mathscr{A})$ such that $(\widetilde{\varkappa},\mathscr{A}) \in (H,\mathscr{A})\widetilde{\subseteq}\prod_\partial\left\{(G,\mathscr{A})^\partial\right\}$. This shows that the soft product space $\prod_\partial\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\partial\right\}$ is soft *S*-locally connected. Conversely, let the soft product space $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ be soft *S*-locally connected. Then, we must show that each soft coordinate space $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\partial$ is soft *S*-locally connected and all but a finite soft number are soft connected. Let $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\beth$ be soft and arbitrary soft coordinate space. Let $(\widetilde{\varkappa},\mathscr{A})^\beth$ be any soft point of $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\beth$ contained in some *s*-open set $(G,\mathscr{A})^\beth$ in $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\beth$. Consider the soft point $(\widetilde{y},\mathscr{A}) = \langle(\widetilde{y},\mathscr{A})^{\partial:\partial\in\Delta}\rangle$ such that $(\widetilde{y},\mathscr{A})^\beth = (\widetilde{\varkappa},\mathscr{A})^\beth$. Then, clearly $(\widetilde{y},\mathscr{A}) \in \pi^{\beth^{-1}}\left((G,\mathscr{A})^\beth\right)$. Since $(G,\mathscr{A})^\beth)$ is soft *s*-open in $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\beth$ and $\pi^\beth$ is soft continuous, it follows that $\pi^{\beth^{-1}}\left((G,\mathscr{A})^\beth\right)$ is soft *s*-open in $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. So, by soft *S*-locally connectedness of $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ there exists soft *S*-connected open set *C* such that $(y,\mathscr{A}) \in C\widetilde{\subseteq}\pi^{\beth^{-1}}\left((G,\mathscr{A})^\beth\right)$. Therefore $\pi^\beth((\widetilde{y},\mathscr{A})) \in \pi^\beth(C)\widetilde{\subseteq}\left((G,\mathscr{A})^\beth\right)$ or $\pi^\beth((\widetilde{x},\mathscr{A})) \in \pi^\beth(C)\widetilde{\subseteq}\left((G,\mathscr{A})^\beth\right)$. Since *C* is soft *s*-open and $\pi^\beth$ is soft *s*-$\mathfrak{open}$, so $\pi^\beth(C)$ is soft *s*-open. Also, *C* is soft *S*-connected and $\pi^\beth$ is soft continuous, so, $\pi^\beth(C)$ is soft *S*-connected. Thus, every soft *s*-open set $(G,\mathscr{A})^\beth)$ in $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^\beth$ containing an arbitrary soft point $(x,\mathscr{A})^\beth \in \left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^\beth$ contains a soft connected s-open set which contains $(\widetilde{\varkappa},\mathscr{A})^\beth$. this shows that $\left(\times\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}\right)^\beth$ is soft locally S-connected. Thus, nothing is lost to say straight-forwardly that every soft coordinate space is soft *S*-locally connected. Further let $(\widetilde{\varkappa},\mathscr{A})$ be any point of the soft product space $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$. then, by soft *S*-locally connectedness of $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ there exists a soft connected *s*-open set *C* such that $(\widetilde{y},\mathscr{A})) \in C$. So there exists a soft basic *s*-open set $\prod_\partial\left\{(G,\mathscr{A})^{\partial:\partial\in\Delta}\right\}$ such that $(\widetilde{\varkappa},\mathscr{A}) \in \prod_\partial\left\{(G,\mathscr{A})^\partial\right\}\widetilde{\subseteq}C$. Now in $\prod_\partial\left\{(G,\mathscr{A})^{\partial:\partial\in\Delta}\right\}$ $(G,\mathscr{A})^\partial$ is soft *s*-open in $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}$ for every $(G,\mathscr{A})^\partial = \left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial \text{ for all but finite soft number of}\partial,s}$. Clearly, $\pi^\beta\left(\prod_\partial\left\{(G,\mathscr{A})^\partial\right\}\right) = in\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial \text{ for all but finite soft number of}\partial,s}$. But $\prod_\partial\left\{(G,\mathscr{A})^{\partial:\partial\in\Delta}\right\}\widetilde{\subseteq}C$ implies that $\pi^\beth\left(\prod_\partial\left\{(G,\mathscr{A})^{\partial:\partial\in\Delta}\right\}\right)\widetilde{\subseteq}\pi^v(C)$ this implies that $\pi^\beth(C) = \left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial \text{ for all but finite soft number of}\partial,s}$. Moreover, *C* is soft *S*-connected and $\pi^\beth$ is soft continuous; it follows that $\pi^\beth(C)$ is soft *S*-connected, that is $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial \text{ for all but finite soft number of}\partial,s}$. □

Theorem 17. *Let* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)$ *and* $\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)$ *be the soft product, then the product space* $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,E\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},E\right)\right)^{\partial:\partial\in\Delta}\right\}$ *is soft first-countable iff each of the soft coordinate spaces is soft first-countable and all but a soft-countable number are soft indiscrete.*

Proof. Let $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial\ is\ countable\ for\ each\ \partial\in\Delta}\right\}$. Let all but a soft-countable number of them be indiscrete. Let $\Delta^{/}=\left\{\partial\in\Delta:\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial\ is\ not\ soft\ indiscrete}\right\}\right\}$. Then the bu hypothesis $\Delta^{/}$ is soft-countable. Let $(\widetilde{\varkappa},\mathscr{A})=\langle(\widetilde{\varkappa},\mathscr{A})^{\partial:\partial\in\Delta}\rangle$ be soft arbitrary point of $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ so that $(\widetilde{\varkappa},\mathscr{A})^{\partial}\in\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ for each $\partial\in\Delta$. Since each $\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial\ is\ soft\ first\ countable}$ so there exists a soft-countable local base $\delta^{(\widetilde{\varkappa},\mathscr{A})}$ at $(\widetilde{\varkappa},\mathscr{A})$ for every $\partial\in\Delta$. Since for each $\partial\in\left(\Delta-\Delta^{/}\right)$, $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ is soft indiscrete we have $\delta^{(\widetilde{\varkappa},\mathscr{A})}=\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial}\right\}$ for each ∂. For every $\partial\in\Delta^{/}$, Let $C^{\partial}=\left\{\pi^{\beta-1}(B):B\in\ \delta^{(\widetilde{\varkappa},\mathscr{A})}\right\}$. Then, C^{∂} is clearly soft-countable collection of soft s-open sets, each containing $(\widetilde{\varkappa},\mathscr{A})$. Let $C=\cup\left\{C^{\partial:\partial\in\Delta^{/}}\right\}$. Then, C being the soft union of soft-countable collection of soft-countable sets, is soft-countable. So, there are only a soft-countable number of finite subcollections of C. Let $C^{/}$ be the soft collection of soft intersections of all finite soft subcollections of C. Then, $C^{/}$ is clearly soft-countable and is a soft local base at $(\widetilde{\varkappa},\mathscr{A})$. Hence $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft first-countable. Conversely, let $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)\right)^{\partial:\partial\in\Delta}\right\}$ be soft first-countable space. Let $(\widetilde{\varkappa},\mathscr{A})^{\beth}\in\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\beth}\right\}$ and let $(\widetilde{\varkappa},\mathscr{A})=\langle(\widetilde{\varkappa},\mathscr{A})^{\partial:\partial\in\Delta}\rangle\in\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)\right)^{\partial:\partial\in\Delta}\right\}$. Since $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft first-countable there exists a soft-countable s-open base at $(\widetilde{\varkappa},\mathscr{A})$. Let it be $\wp=\left\{B^{n:n\in N}\right\}$. Clearly, each B^n is soft s-open in $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ and soft continuous $(\widetilde{\varkappa},\mathscr{A})$. Let $\wp^*=\left\{\pi^3(B^n):n\in N\right\}$, it follows that each member of $\wp^*$ is soft s-open subset of $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\beth}\right\}$. Also $\wp^*$ is clearly soft-countable. More-over, $(\widetilde{\varkappa},\mathscr{A})\in B^n$ for each $n\in N$ implies that $\pi^{\beth}((\widetilde{\varkappa},\mathscr{A})\in\pi^{\beta}(B^n)$ implies that $(\widetilde{\varkappa},\mathscr{A})^{\beth}\in\pi^{\beth}(B^n)$. Now, let $(N,\mathscr{A})^{\beth}$ be soft arbitrary nbd of $(\widetilde{\varkappa},\mathscr{A})^{\beta}$. Then, there exists soft s-$\mathfrak{open}$ set $(G,\mathscr{A})^{\beta}$ in $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\beth}\right\}$ such that $(\widetilde{\varkappa},\mathscr{A})^{\beth}\in(G,\mathscr{A})^{\beth}\widetilde{\subseteq}(N,\mathscr{A})^{\beth}$. Consequently, $(\widetilde{\varkappa},\mathscr{A})=\pi^{\beth^{-1}}\left((\widetilde{\varkappa},\mathscr{A})^{\beth}\right)\in\pi^{\beth^{-1}}\left((G,\mathscr{A})^{\beth}\right)\widetilde{\subseteq}\pi^{\beth^{-1}}\left((N,\mathscr{A})^{\beth}\right)$. Since $\pi^{\beth}$ is soft continuous, $\pi^{-\beth}\left((G,\mathscr{A})^{\beth}\right)$ is soft s-open neighbourhood (nbhd). So, there exists some member $(G,\mathscr{A})^{\beth^0}\in\wp$ such that $(\widetilde{\varkappa},\mathscr{A})\in(G,\mathscr{A})^{\beth^0}\widetilde{\subseteq}\pi^{\beta^{-1}}\left((G,\mathscr{A})^{\beth}\right)\widetilde{\subseteq}\pi^{\beth^{-1}}\left((N,\mathscr{A})^{\beth}\right)$. Therefore, $\pi^{\beth}\left((\widetilde{\varkappa},\mathscr{A})\right)\in\pi^{\beth}\left(B^{n\ 0}\right)\widetilde{\subseteq}\left((N,\mathscr{A})^{\beth}\right)$. This shows that $\wp^*$ is soft-countable local base at $(\widetilde{\varkappa},\mathscr{A})^{\beth}$ and therefore $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\beth}\right\}$ is soft first-countable. Hence, each coordinate space is soft, first-countable. Further, let Let $\Delta^{/}=\left\{\partial\in\Delta:\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial\ is\ not\ soft\ indiscrete}\right\}\right\}$. Then, we must show that $\Delta^{/}$ is soft-countable. We prove this result by contradiction that is we cotra-positively suppose that $\Delta^{/}$ is uncountable. Now, for each $\alpha\in\Delta^{/}$, $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\alpha}\right\}$ is not soft indiscrete. So, there exists a soft non-empty s-open proper soft subset $(G,\mathscr{A})^{\beth}$ of $\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\alpha}\right\}$ for each $\alpha\in\Delta^{/}$. Let $(\widetilde{y},\mathscr{A})^{\alpha}\in(G,\mathscr{A})^{\alpha}$ and $(\widetilde{z},\mathscr{A})$ be soft element of $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ such that $(\widetilde{z},\mathscr{A})^{\alpha}=(\widetilde{y},\mathscr{A})^{\alpha}$ *for all* $\alpha\in\Delta^{/}$. Since of $\prod\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\partial:\partial\in\Delta}\right\}$ is soft first-countable so there exists a soft-countable local base $\wp^*=(B^n):n\in N$ at $(\widetilde{z},\mathscr{A})$. Clearly, $\pi^{\alpha}(B^n)=\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\alpha}\right\}$ for all except at finite soft number of α,s. Let $(Y,\mathscr{A})^n$ bea finite soft subset of Δ for which $\pi^{\beth}(B^n)\lneqq\left\{\left(\left(X^1,\widetilde{\tau}_1,\widetilde{\tau}_2,\mathrm{E}\right)*\left(X^2,\widetilde{\mathfrak{F}_1},\widetilde{\mathfrak{F}_2},\mathrm{E}\right)\right)^{\alpha}\right\}$ where the equality sign is ruled out because if it is there then we can not proceed further more. Then, $(Y,\mathscr{A})=\bigcup\left\{(Y,\mathscr{A})^{n:n\in N}\right\}$ being soft-countable

cannot contain the soft uncountable set $\Delta^{/}$. so, there exists some $\beth \in \Delta^{/}$ such that $\beth \notin (Y, \mathscr{A})^{n \text{ for all } n \in N}$ and hence $\pi^{\beth}(B^n) = \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beth}\right\}$ for every n $\in N$. Now, there exists a non-empty soft s-open proper subset $(G, \mathscr{A})^{\beth}$ of $\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beth}\right\}$ such that $(\widetilde{z}, \mathscr{A})^{\beth} = (\widetilde{y}, \mathscr{A})^{\beth} \in (G, \mathscr{A})^{\beth}$ and therefore $(\widetilde{z}, \mathscr{A}) = \pi^{\beth^{-1}}\left((\widetilde{z}, \mathscr{A})^{\beth}\right) \in \pi^{\beth^{-1}}\left((G, \mathscr{A})^{\beth}\right)$. this shows that $\pi^{\beth^{-1}}\left((G, \mathscr{A})^{\beth}\right)$ is soft s-open nhd of $(\widetilde{z}, \mathscr{A})$. But no soft member of $\wp$ is soft subset of $\pi^{\beth^{-1}}\left((G, \mathscr{A})^{\beth}\right)$, since $\pi^{\beth}(B^n) = \left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, \mathrm{E}\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, \mathrm{E}\right)\right)^{\beth}\right\}$ for every $B^n \in \wp$. This is purely contradiction. This contradiction is taking birth due to our wrong supposition, which we made at the start. Hence, we are obliged that all but the soft-countable numbers of the coordinate spaces are soft indiscrete. □

Theorem 18. *Let* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right)$ *and* $\left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be two SBTSpaces on the crisp set* X^1 *and* X^2 *respectively.* $\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)$ *be the soft product, then the product space* $\prod\left\{\left(\left(X^1, \widetilde{\tau}_1, \widetilde{\tau}_2, E\right) * \left(X^2, \widetilde{\mathfrak{F}_1}, \widetilde{\mathfrak{F}_2}, E\right)\right)^{\partial : \partial \in \Delta}\right\}$ *is second-countable if each of the soft coordinate spaces is soft second-countable and all but a soft-countable soft number are soft indiscrete.*

Proof. Similar to that of Theorem 17. □

7. Conclusions

In this article, the product of soft bitopological spaces in connection with different particular structures with respect to crisp points under most soft generalized open sets that are soft s-open sets is discussed. With the help of soft s-open sets, soft separation axioms are regenerated in soft bitopological spaces with respect to crisp points and a little bit with soft points with all kinds of possibilities. With the help of soft s-open sets, the soft products of different structures with respect to crisp points are addressed. The soft base is connected with the soft product of soft bitopologies with respect to soft s-open sets. In a similar fashion, soft subbases are engaged with soft product spaces with respect to soft s-open sets. Soft projection over the soft product spaces of soft bitopologies is defined, and their soft continuity is addressed. Soft product spaces relative to soft first and soft second coordinate spaces are studied in soft bitopologies. The characterization of soft separation axioms with soft connectedness is addressed with respect to semiopen sets in soft bitopological spaces. In addition to this, the product of two soft topological spaces is ($S^1$and S^2) space if each coordinate space is soft (S^1 and S^2) space, product of two sot topological spaces is (S regular and C regular) space if each coordinate space is (S regular and C regular), the product of two soft topological spaces is connected if each coordinate space is soft connected and the product of two soft topological spaces is (first countable, second-countable) if each coordinate space is (first countable, second-countable). All the above results are developed with respect to crisp (ordinary) points of the space. In the future, we will try to generate the above structures with respect to the soft points of the space. In this study, the product was limited to two structures. We will try to study the product of finite structures and infinite structures. In the development of the structures, we will use the nearly soft open sets, soft generalized open sets and soft most generalized open sets. After finishing this, we will try our hands to extend the soft bitopological structures to soft tri-topological structures with respect to crisp points and soft points of the spaces under nearly soft open sets, soft generalized open sets and soft most generalized open sets. When studying something in soft topology, we will be very careful because our domain of soft topology is not so strong. By extending the domain will extend the number of soft topologies. The more the topologies, the more accurate the structures will be because soft topology is actually giving information about soft structures. This article is just the beginning of the investigation of a new kind of structure. Hence, it will be necessary to continue the study and carry out more theoretical research in order to build a general framework for practical applications.

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References

1. Molodtsov, D. Soft set theory-first results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
2. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
3. Pawlak, Z. Rough sets. *Int. J. Inf. Comput. Sci.* **1982**, *11*, 341–356. [CrossRef]
4. Gau, W.L.; Buehrer, D.J. Vague sets. *IEEE Trans. Sys. Man Cybernet.* **1993**, *23*, 610–614. [CrossRef]
5. Maji, P.K.; Biswas, R.; Roy, A.R. Soft set theory. *Comput. Math. Appl.* **2003**, *45*, 555–562. [CrossRef]
6. Herawan, T.; Deris, M.M. A soft set approach for association rules mining. *Knowl.-Based Syst.* **2011**, *24*, 186–195. [CrossRef]
7. Kalayathankal, S.J.; Singh, G.S. A fuzzy Soft flood alarm model. *Math. Comput. Simulat.* **2010**, *80*, 887–893. [CrossRef]
8. Xiao, Z.; Gong, K.; Zou, Y. A Combined forecasting approach based on fuzzy soft sets. *J. Comput. Appl. Maths.* **2009**, *228*, 326–333. [CrossRef]
9. Chen, D.; Tsang, E.C.C.; Yeung, D.S.; Wang, X. The parametrization reduction of soft sets, its applications. *Comput. Math. Appl.* **2005**, *49*, 757–763. [CrossRef]
10. Maji, P.K.; Biswas, R.; Roy, A.R. An application of soft set in a decision making problem. *Comput. Math. Appl.* **2002**, *44*, 1077–1083. [CrossRef]
11. Roy, R.; Maji, P.K. A fuzzy soft set theoretic approach to decision making problems. *Comput. Appl. Math.* **2007**, *203*, 412–418. [CrossRef]
12. Maji, P.K.; Biswas, R.; Roy, A.R. Fuzzy soft sets. *J. Fuzzy Math.* **2001**, *9*, 589–602.
13. Feng, F.; Liu, X.; Leoreanu-Foeta, V.; Jun, Y.B. Soft sets, soft rough sets. *Inf. Sci.* **2011**, *181*, 1125–1137. [CrossRef]
14. Maji, P.K.; Biswas, R.; Roy, A.R. On intuitionistic fuzzy soft sets. *J. Fuzzy Math.* **2001**, *9*, 669–684.
15. Xu, W.; Ma, J.; Wang, S.; Hao, G. Vague soft sets, their Properties. *Comput. Math. Appl.* **2010**, *59*, 787–794. [CrossRef]
16. Maji, P.K. Neutrosophic soft sets. *Ann. Fuzzy Maths. Inf.* **2013**, *5*, 157–168.
17. Shabir, M.; Naz, M. On soft toplogical spaces. *Comput. Math. Appl.* **2011**, *61*, 1786–1799. [CrossRef]
18. Zorlutuna, I.; Çakir, H. On continuity of soft mapping. *Appl. Math. Inf. Sci.* **2015**, *1*, 403–409. [CrossRef]
19. Peyghan, E.; Samadi, B.; Tayebi, A. On soft connectedness. *arXiv* **2012**, arXiv:math/12021668.
20. Al-Khafaj, M.; Mahmood, M. Some properties of soft connected spaces, soft locally connected spaces. *IOSR J. Math.* **2014**, *10*, 102–107. [CrossRef]
21. Adag, M.; Ozkan, A. On soft pre open sets, soft pre separation axioms. *Gazi Univ. J. Sci.* **2014**, *27*, 1077–1083.
22. El-Sheikh, S.A.; El-latif, R.A.H.A.M.A. Characterizations of b-soft separation axioms in soft topological spaces. *Inf. Sci. Lett.* **2015**, *4*, 125–133.
23. Hussainand, S.; Ahmad, B. Soft separation axioms in soft topological spaces. *Hacet. J. Math. Stat.* **2015**, *44*, 559–568.
24. Khattak, M.; Khan, G.A.; Ishfaq, M.; Jamal, F. Characterization of soft α-separation axioms, soft β-separation axioms in soft single point spaces, in soft ordinary spaces. *J. New Theory* **2017**, *1*, 63–81.
25. Naz, M.; Shabir, M.; Ali, M.I. Separation axioms in bi-soft topological spaces. *arXiv* **2015**, arXiv:math/150900866.
26. Ittanagi, B.M. Soft bitopological spaces. *Int. J. Comput. Appl.* **2011**, *107*, 1–4.
27. Ozturk, T.Y.; Aras, C.G. Soft pair-wise sontinuity in soft bitopological spaces. *CBU J. Sci.* **2017**, *13*, 413–422.
28. Kandil, A.; El-Tantawy, O.; El-Sheikh, S.; Hazza, S.A. Pairwise soft connected in soft bi topological spaces. *Int. J. Comput. Appl.* **2017**, *169*, 0975–8887.
29. Khattak, A.M.; Khan, G.A.; Khan, Y.; Khattak, M.I.; Jamal, F. Characterization of Soft b-separation axioms in soft bi-topological spaces. *Matrix Sci. Math.* **2018**, *2*, 11–17. [CrossRef]

30. Khattak, M.; Zamir, M.; Alam, M.; Nadeem, F.; Jabeen, S.; Zaighum, A. Weak soft open sets in soft bi topological spaces. *J. Math. Ext.* **2020**, *14*, 85–116.
31. Senel, G. A New Approach to Hausdorff space theory via the soft sets. *Math. Probl. Eng.* **2016**, 1–6. [CrossRef]
32. Öztürk, T.Y.; Bayramov, S. Topology on soft continuous function spaces. *Math. Comput. Appl.* **2017**, *22*, 32. [CrossRef]
33. Bayramov, S.; Aras, C.G. A New approach to separibility and compactness in soft topological spaces. *TWMS J. Pure Appl. Math.* **2018**, *1*, 82–93.
34. Yu, G. Soft points, s-relations and soft rough approximate operations. *Int. J. Comput. Intell. Syst.* **2017**, *10*, 90–103. [CrossRef]
35. Kandil, A.; Tantawy, O.A.E.; El-Sheikh, S.A.; Abd El-latif, A.M. γ-operation, decompositions of some for forms of soft continuity in soft topological spaces. *Ann. Fuzzy Math. Inf.* **2014**, *7*, 181–196.