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Article

Classroom Methodologies for Teaching and Learning Ordinary Differential Equations: A Systemic Literature Review and Bibliometric Analysis

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Abstract: In this paper, we develop a review of the research focused on the teaching and learning of ordinary differential equations with the following three purposes: to get an overview of the existing literature of the topic, to contribute to the integration of the actual knowledge, and to define some possible challenges and perspectives for the further research in the topic. The methodology we followed is a combination of a systematic literature review and a bibliometric analysis. The contributions of the paper are given by the following: shed light on the latest research in this area, present a characterization of the actual research lines regarding the teaching and learning of ordinary differential equations, present some topics to be addressed in the next years and define a starting point for researchers who are interested in developing research in this field.

Keywords: teaching differential equations; teaching mathematics; mathematical modeling; solving problem

1. Introduction

The teaching and learning of ordinary differential equations has experienced a dramatic change in the last two decades [1–19]. The motivation for innovation in the traditional teaching obey different reasons, at least three of those are given below. First, from the second half of the 20th century until now, the ordinary differential equations have been recognized as useful tools for teaching and learning mathematical models arising in different areas of science like physics, biomathematics, engineering, and chemistry [20–26]. Second, in our current era, the development of information technologies has strongly influenced and modified the traditional ways of inquiring in science. In particular, the information technologies have increased the innovation and application of numerical methods which are essential to solve a wide class of differential equations and are also useful to understand some qualitative properties [10,11,19,27–30]. Third, as a consequence of the above, in the last years, more attention has been given to the transformation of teaching and learning mathematical concepts by incorporating didactic methodologies to encourage students to be actively engaged in their learning process [21,31–34]. Thus, in a brief sense, the changes in the teaching of differential equations has been mainly influenced by the incorporation of active learning didactic methodologies and technology enriched learning environments.

Traditionally the curricula in many careers, like engineering, physics, mathematics, or statistics, begin with three courses of calculus (differential, integral, and several variables) and they are followed by an ordinary differential equations course. From the last decade of the twentieth century, several efforts to change the calculus curriculum have been proposed and conducted by numerous authors worldwide [15,35,36]. Specifically, in the teaching of differential equations, the changes consider new contents, new pedagogical methods, and the incorporation of the exploration of dynamical systems concepts with graphical (or qualitative) and numerical approximations by using technological resources. Nowadays, the study of curricular modifications that must be undertaken in order to adequately overcome the diverse deficiencies and difficulties in the teaching and learning of differential equations is a very active topic of research with different subjects and perspectives.

Despite the interest in the curricular innovation for the teaching and learning of ordinary differential equations, we have found that the research lines for the next years are still diffusely stated. Some advances for integration of the findings from the research can be seen in the articles [21,37]. In [21], the authors developed an extensive bibliographic survey of 16 works published between 2000 and 2011. Meanwhile in [37], the authors focused on the factors that influence the problem solving abilities for undergraduate students in differential equations. However, to the best of our knowledge, there is not a literature review with an open period of time and the specific didactic methodologies are missing in those works. In other words, a literature review to find, critically evaluate, and synthesize the relevant research topics related to the teaching and learning of ordinary differential equations remains open.

Consequently, in the light of the increasing development of research related to teaching methods and, in order to lead emerging trends and challenges of teaching and learning ordinary differential equations, it is evident the lack of a systematic study of the existing literature. To shed light on this gap, in this paper we present an analysis of the literature related to teaching and learning differential equations, based on a systematic review and a bibliometric investigation. We propose a systematic arrangement of the main existing literature. Thus, we follow the methodology of five steps introduced in [38]: (i) framing questions for a review, (ii) identifying relevant work, (iii) assessing the quality of studies, (iv) summarizing the evidence, and (v) interpreting the findings. For step (i), we considered three questions. In the case of (ii), we retain 120 articles that come from the following databases: Web of Science, Scopus, Qualis, Zbemath, and Scielo. In step (iii), we provide some statistical properties of the retained literature. In steps (iv) and (v), we expose explicitly the subjects of ordinary differential equations covered by the research, the teaching methodologies used in the classroom, and also we present the answers to the questions of step (i). Then, in step (v), we summarize our findings.

We survey a set of 120 articles from 1970 to 2020, where initially two standard classifications for teaching differential equations were identified. The first considers the traditional and contemporary teaching methodologies pointed, for example, by [1]. The second classification includes the separation in analytic, graphical, and numerical approaches advised by some authors [39,40]. However, these classifications are currently imprecise, since the actual state-of-the-art in the research is extensive and there are works which are out of those classes, for instance the teaching under the mathematical modeling as a cyclic process can gather the analytic, graphical, and numerical approaches as a particular phase of the cycle.

The main improvements for the research field which are established in this article are described below. From our analysis of the set of 120 articles we mainly obtained the following contributions:

1. We propose a classification for the research in teaching and learning ordinary differential equations according to the didactic methodologies: traditional teaching and learning methodology, graphical and numerical approaches to the teaching, active learning methods, mathematical modeling-based methodology, information and communication technology-based methodologies, and project-based learning.
2. We introduce five groups given a categorization of the mathematical topics addressed in the papers: basic concepts of ordinary differential equation, biomathematical models, scalar-based models, systems-based on physic models, and other concepts.
3. We found that results about effectiveness of innovation were reported only in a few articles.

Moreover, the review of the literature shows an increasing trend since the first research around 1990. The best ranked journal regarding to the h-Index in the area of Mathematical

Education is "The journal for research in mathematics education" and the most prolific author is Chris Rasmussen with 13 articles in the collected list, and the article with the largest number of cites in Google scholar is [15] with a total of 196 citations. Some conclusions are established by bridging the different influential perspectives of the main works. We also highlight some possible challenges and perspectives for further research of the topic.

The paper is organized as follows. In Section 2, we describe the methodological approach used in this research. In Section 3, we formulate the questions that guide the review. In Section 4, we describe how the relevant work was identified. In Section 5, we develop a bibliometric analysis of the literature. In Sections 6 and 7, we summarize the review and present a discussion. Finally, in Section 8, we draw some conclusions with short comments about some possible challenges and perspectives for further research.

2. Research Methodology

In order to define the methodology supporting this research, we recall that there are at least three approaches related with the literature review: the bibliometric analysis, the systematic literature review, and narrative review [41,42]. The goal of a bibliometric analysis is to develop a quantitative research by applying statistical methods in order to evaluate several characteristics of specific bibliographic information like journals, research institutions, geographic location, and other characteristics [43]. The narrative literature review is developed to provide an overview of a large spectrum for some specific topic chosen by the author and is based on available literature on their particular interest, is descriptive, and written in a friendly readable format [44]. Meanwhile, the systematic literature review has two principal goals: to develop an extensive literature search with a very detailed process; and to give a critical evaluation of the selected literature. Moreover, the researchers who develop literature review recognize that the systematic reviews contain an explicit a priori strategy which is detailed and comprehensive, reducing the appraising when identify the relevant studies.

For the present study, our methodology is a combination of a systematic review and a bibliometric analysis. More precisely, firstly we develop a systematic review of the literature following the five steps introduced in [38]:

Step 1. Framing questions for a review.
Step 2. Identifying relevant work.
Step 3. Assessing the quality of studies.
Step 4. Summarizing the evidence.
Step 5. Interpreting the findings.

Particularly, in Step 2 we generate a list of references that was explored using a bibliometric analysis with particular well defined quantitative indicators in Step 3.

3. Framing Questions for a Review (Step 1)

We follow the discussion given by Benitti [45] to establish the following three research questions:

Question 1: What are the studies developed for teaching and learning of ordinary differential equations with a reported classroom experiences? What types of didactic methodologies have been used in those studies?
Question 2: What topics of ordinary differential equations have been explored in the previous studies?
Question 3: What are the results for the effectiveness of traditional and new didactic methodologies to teach and learning ordinary differential equations, as reported in previous studies?

4. Identifying Relevant Work (Step 2)

To answer our research questions, we drew on multiple resources to identify the topics of differential equations and the teaching methodology that were most mentioned in the papers. We proceed in several steps as is specified below (see Figure 1 for a summary):

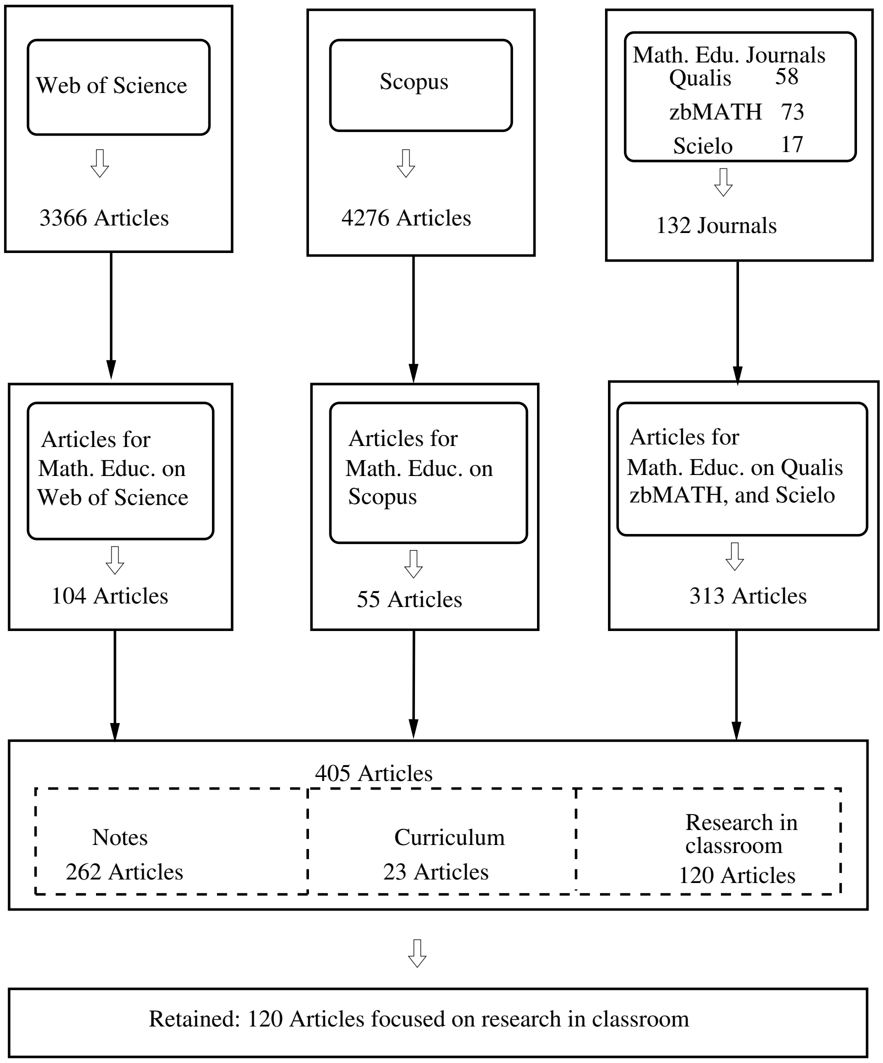


Figure 1. Schematic presentation of the process used for identify the relevant work. For specifications, consult the items (a) to (e) in Section 4.

(a) We have selected the following databases:
- Web of Science (https://clarivate.com/products/web-of-science accessed on 8 August 2020),
- Scopus (https://www.elsevier.com/solutions/scopus accessed on 8 August 2020),
- Qualis (http://qualis.capes.gov.br accessed on 8 August 2020)
- Zbmath (https://zbmath.org accessed on 8 August 2020), and
- Scielo (https://scielo.org accessed on 8 August 2020).

(b) In the case of Web of Science and Scopus, we have derived two major keywords to answer *Questions 1–3* and we have replaced them in the search engine of databases by some synonyms and some alternative terms, as specified below:

Ordinary differential equations. Differential equation; solution to differential equations; graphical interpretation; graphical solution; qualitative solutions; numerical solutions; analytic solutions; first order equations; higher order equations; Laplace transform; power series method; variable separable equation; reducible to variable separable equation; homogeneous equation; reducible to homogeneous equation; exact equation; reducible to exact equation; Bernoulli equation; linear equation; Ricatti equation; phase plane; isoclines; slope fields;

equilibrium; stability of solutions; initial value problems; boundary value problems; scalar equations; systems of equations; linear; nonlinear.

Didactic methodologies. Teaching methodologies; students' understanding and difficulties; interpretation of solutions; registers of representations; mathematical modeling; mathematical models; problem-based learning; problem solving; error analysis; mathematics teaching practices; real world situation; computational resources; mathematical application; classroom discourse; didactic of differential equations; critical discourse analysis.

More specifically the strings are given in Appendix A. First we searched the list of selected words in all fields of the search engine of databases, i.e., in titles, article keywords, abstracts, author, topic, and full paper text.

The search on Web of Science was restricted to all journals indexed to "Science Citation Index Expanded (SCI-Expanded)", "Social Sciences Citation Index (SSCI)", "Arts & Humanities Citation Index (A&HCI)", and "Emerging Sources Citation Index (ESCI)". We get a total of 342,179 publications. Then, we refined the results using the "Document Types" option by "article" and the option "Web of Science Categories" by "Social Sciences Mathematical Methods or Education Educational Research or Education Scientific Disciplines" generating a list of 3366 articles.

In Scopus, when restricting the search to Document Type "article", a total of 23,967 publications were found. Then, we refined the option "subject area" by selecting "psychology or "social sciences", getting a list of 4276 articles.

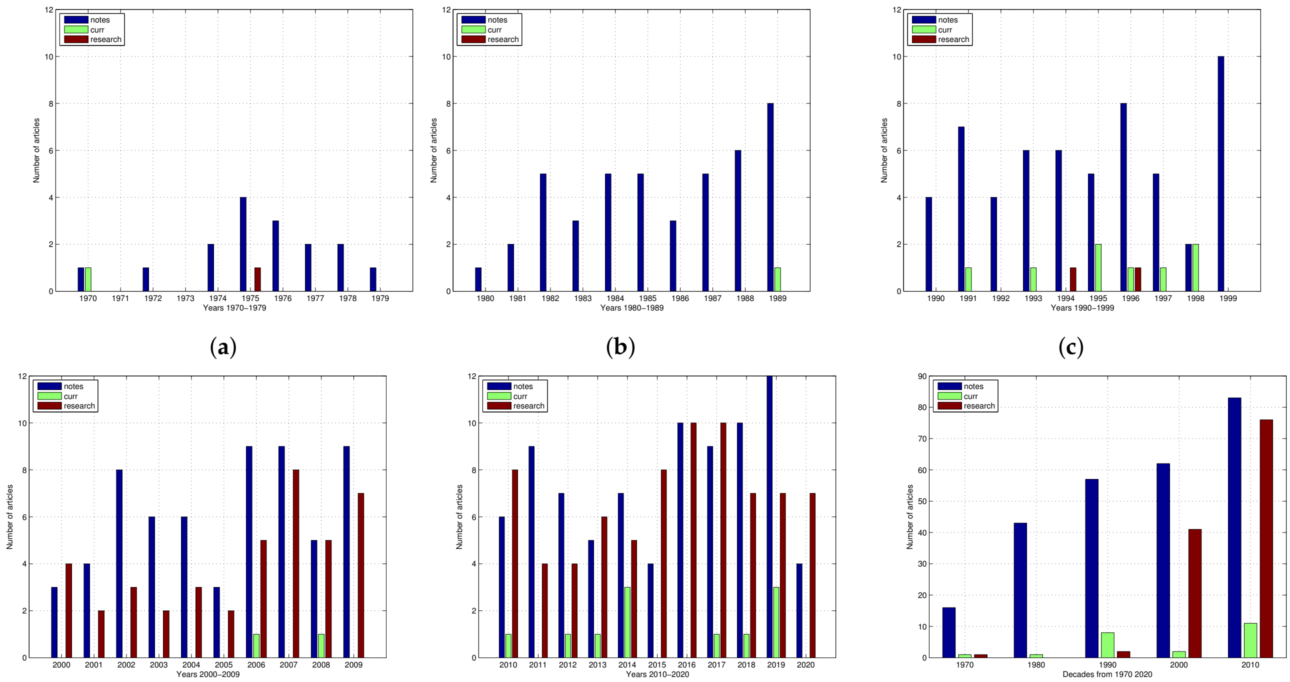
(c) In the case of Qualis, zbMATH, and Scielo. we selected the journals associated to Mathematics Education as specified below. In the database Qualis, we find that a total of 1434 journals are associated to quadrennium 2013–2016 and are classified as A1, A2, B1, B2, B3, B4, B5, and C in the evaluation area Teaching (ensino). Then, we selected a list of 58 journals associated with Mathematics Education, see Table 1. For zbMATH database, we used the list of journals suggested by Godino [46], where the author present a list of journals from zbMATH classified in two sections labeled as "Serie A" and "Serie B" journals. Moreover, in each category there are three groups or types of journals called A, B, and C, the total of journals of each serie and the corresponding types are summarized in Table 1. Now, from Scielo database we have selected a total of 17 journals associated with the scope in Mathematics Education. Thus, combining the three list of journals and deleting the duplicated ones, we get a list of 132 journals, see Table A1 in Appendix B.

(d) We examined the titles, abstract, and full paper text in the list of papers from Web of Science and Scopus generated in step (b). Then, we retained the paper if it was related to the teaching and learning of ordinary differential equations. After a careful examination, we have identified 104 and 55 articles from Web of Science and Scopus, respectively. Moreover, in the case of the selected journals of step (c), we have applied two types of searches: (i) we consulted the index of each volume of the journal from the years specified on the column labeled as "Years Consulted" in Table A1 and (ii) we have searched for key words in the search engines of each journal. As a result, a total of 313 articles were considered to be analyzed.

(e) Combining the three list of articles and deleting the duplicated ones, we get a list of 405 articles. Then, in order to focus our analysis on classroom methodologies, we classified the 405 articles in three types: *notes, curriculum, and research in classroom*. We consider that an article is a *note* or a *classroom note*, when there is a proposal for teaching some concepts related to differential equations, but there is not a specific didactic methodology or at least, it was never implemented in the classroom. In the class *curriculum*, we consider all works where the aim of the paper was the curriculum innovation proposal and there is not an specific application in the classroom. Meanwhile, we assume that a paper is of the type *research in classroom*, when there is a proposal to teach some topic of ordinary differential equations, there is an explicit didactic methodology, and also includes the implementation in the classroom with a well detailed report of the experience. Thus, by a revision of all 405 papers, we deduce that there were a total of 262, 23 and 120 articles belonging to types classroom notes, curriculum, and research in classroom, respectively. In Figure 2, we present a classification by year and by decade from 1970 to 2020. An isolated case, which is not presented in Figure 2, is the classroom note [47] published in 1913.

Table 1. Number of journals for Mathematics Education on Qualis and zbMATH databases, see also Appendix B.

Classification Qualis	A1	A2	B1	B2	B3	B4	B5	C	Total
J. of Math./Prob.	120	145	260	229	170	135	179	196	1434
J. Math. Educ.	15	15	9	1	8	4	3	3	58
Classification for zbMATH [46]	**A**	**B**	**C**	**Total**					
Serie A: Didact. of Math.	3	14	16	33					
Serie B: Related Areas	27	4	9	40					



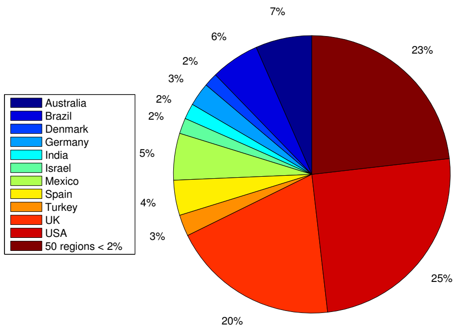
(**a**) (**b**) (**c**)

(**d**) (**e**) (**f**)

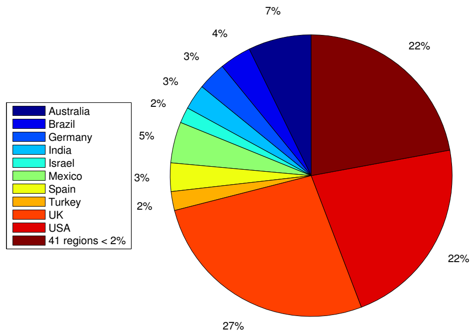
Figure 2. Number of articles from 1970 to 2020. Here the abbreviation notes, curr, and research are used for the types *notes*, *curriculum* and *research in classroom*, respectively. (**a**) Articles from 1970 to 1979. (**b**) Articles from 1980 to 1989. (**c**) Articles from 1990 to 1999. (**d**) Articles from 2000 to 2009. (**e**) Articles from 2010 to 2020. (**f**) Articles by decades from 1970 to 2020.

On the other hand, we also have identified and counted the geographic location declared by the authors in the corresponding affiliation of each article, see Figure 3. We registered the affiliations of each coauthor and then we counted all coincidences of a given region location. The regions with the highest number of records are United States of America (USA), United Kingdom (UK), and Australia with 110, 86, and 29 records, respectively. The ranking is followed by Brazil, Denmark, Germany, India, Israel, Mexico, Spain, and Turkey, which have between 6 and 29 records, see Figure 3a for percentages. Moreover, the following 50 regions have at most 6 records (less than 2%):

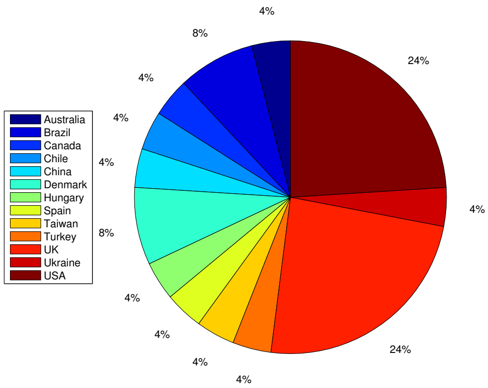
Argentina, Azerbaijan, Bahrain, Brunei, Canada, Chile, China, Colombia, Costa Rica, Cuba, Czechia, Ethiopia, France, Ghana, Grece, Holland, Hungary, Iceland, Iran, Iraq, Italy, Kenya, Lebanon, Libya, Lithuania, Malaysia, Netherlands, New Zeland, Nigeria, Norway, Perú, Poland, Portugal, Romania, Russia, Saudia Arabia, Serbia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spalj, Sweden, Switzerland, Taiwan, Ukraine, United Arab Emirates, and Uruguay.



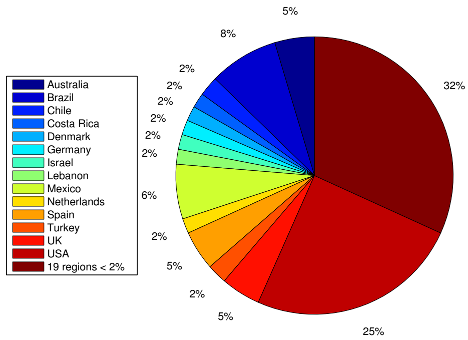
(**a**)



(**b**)



(**c**)



(**d**)

Figure 3. Percentages of the number of papers according to the geographic location declared by the authors in the corresponding affiliation of each article. We remark that the percentages are rounded off by its integer part, then apparently in (**a**) and (**d**) the total percentages are more than 100%. (**a**) Regions for authors with all types, (**b**) regions for authors with articles of *notes* type, (**c**) regions for authors with articles of *curriculum* type, (**d**) regions for authors with articles of *research in classroom* type.

In the case of *notes*, we find that UK (74 records), USA (61 records), and Australia (20 records) are the regions with the highest number of records. Brazil, Germany, India, Israel, Mexico, Spain, and Turkey, appear with more than 4 and less than 20 records. Moreover, we get that the following 41 regions have less than 4 records each, see Figure 3b:

Argentina, Azerbaijan, Bahrain, Brunei, Canada, China, Colombia Cuba, Denmark, Ethiopia, France, Ghana, Grece, Holland, Hungary, Iceland Iran, Italy, Kenya, Libya, Lithuania, Malaysia, Netherlands, New Zeland, Nigeria, Norway, Peru, Poland, Portugal, Russia, Saudia Arabia, Serbia, Singapore, Slovakia, South Africa, South Korea Spalj, Switzerland, United Arab Emirates, and Uruguay.

In the ranking for regions with publications related to *curriculum*, the first two places are for UK, USA, Brazil, and Denmark with a total of 6, 6, 2, and 2 records, respectively. Moreover, each of the following 9 regions: Australia, Canada, Chile, China, Hungary, Spain Taiwan, Turkey, and Ukraine, have associated 1 record, Figure 3c. Now, corresponding to articles of type *research in classroom*, the regions with highest number of registered affiliations are USA, Brazil, and Mexico with 43, 14, and 11 records, respectively. The ranking of *research in classroom* type regions is followed by Australia, Chile, Costa Rica, Denmark, Germany, Israel, Lebanon, Netherlands, Spain, Turkey, and UK with the percentages given in Figure 3d. Moreover, the following 19 regions appear with less than 2 records: Argentina, Colombia, New Zeland, South Korea, Sweden, Ukraine, Canada, Cuba, Czechia, France, Iran, Iraq, Malaysia, Norway, Romania, Singapore, Slovakia, Slovenia, and Taiwan.

Hereinafter, unless stated otherwise, **the retained list** or **the retrieved list** refer to the 120 papers which will be analyzed and are explicitly given by the following references: [1–23,27–34,39,48–135]. The other 285 articles (*notes* and *curriculum*) will be presented and analyzed in a forthcoming work by the authors.

5. Assessing the Quality of Studies (Step 3)

In this section, in order to assess the quality of the 120 articles of *research in classroom* type retrieved and selected in Section 4, we develop a bibliometric study by considering several characteristics to capture the impact of articles, authors, and journals. Amongst the literature characteristics and indicators, which are frequently used in bibliometric analysis, we consider the number of citations, the ranking of authors, the ranking of journals, and the geographic location [43]. Thus, we identify the following characteristics in the analyzed documents:

(i) *Total number of publications by geographic location.* In Section 4, we present initial information regarding the location of origin, which is declared in the affiliation of authors. The top three regions are USA, Brazil, and México with percentages of coincidences of 25%, 8%, and 5%, respectively; see Figure 3d.

(ii) *Total number of publications by year.* In Figure 2, we present six histograms considering all the articles (*notes*, *curriculum*, and *research in classroom*) decade by decade from 1970 to 2020. In Figure 2f, we can see an increase in the number of publications along the years. The first publications are from 1970s, one related to *research in classroom* and one related to *curriculum*, see Figure 2a. Regarding *research in classroom*, the first article retained is from 1975 and is unique in the 1970s. The 2010s have the largest number of publications with a record count of 76 papers, where 2016 and 2017 were the years with more publications with 10 records each year, see Figure 2f,e, respectively. The graphs also show how the research related to *curriculum* has evolved slowly.

(iii) *The most prolific journals.* There are 46 journals associated to the retained list of papers. Table 2 shows the 13 journals which are in the top four positions according to the number of published articles. The first three journals show a similarity in the declared scope, all of them are focused on teaching and learning mathematics. These journals publish research regarding learning and teaching mathematics for different scholar levels and particularly for undergraduate mathematics. These coincidences in the journals' aims are probably the reason which they have the most publications in the area of differential equations, which is traditionally a topic of undergraduate mathematics. Additionally, we found 33 journals with less than 3 publications each one, which are distributed as follows: 9 and 24 journals with 2 and 1 articles, respectively.

(iv) *Ranking of journals by the H index.* In Table 3 we show the top 11 retained journals according to the H index of SCImago Journal & Country Rank (https://www.scimagojr.com/ accessed on 1 September 2020), where the indicator SJR 2019 is also included, quartil, and subject area of those journals.

(v) *The most prolific authors.* Table 4 shows the most prolific authors in the retained list of *research in classroom* articles. The top author in the field is Chris Rasmussen with 12 articles (9.52%). Moreover, we observe that there are four authors from the USA which is naturally related with the higher impact of the research developed in the field by institutions from the USA, see Figure 3d.

(vi) *The impact of articles.* In Table 5, we show the top 10 articles, where the ranking is established by the number of citations reported in google scholar in September 2020. We observe that the research line introduced by Rasmussen and collaborators in the

2000s decade is one of the most prolific, since 8 of the top 10 articles are authored or coauthored by Chris Rasmussen.

Some additional bibliometric characteristics of the *research in classroom* articles, are the following: 92 articles are written in English, 18 in Spanish, and 10 in Portuguese, from which 3 articles [29,61,66] are applied for teaching and learning ordinary differential equations in high school students and the rest of articles (117) for undergraduate students.

Table 2. The top four journals according to the number of published articles.

Rank	Journal	Record Count	% of 120
1°	Teaching mathematics and its applications	16	13.33%
	International journal of mathematical education in science and technology	16	13.33%
2°	The journal of mathematical behavior	13	10.83%
3°	ZDM–Mathematics Education	6	5.00%
4°	Computer applications in engineering education	3	2.50%
	Educação matemática pesquisa	3	2.50%
	Educación matemática	3	2.50%
	Enseñanza de las ciencias: revista de investigación y experiencias didácticas	3	2.50%
	European journal of engineering education	3	2.50%
	International electronic journal of mathematics education	3	2.50%
	International journal of science and mathematics education	3	2.50%
	PRIMUS: problems, resources, and issues in mathematics undergraduate studies	3	2.50%
	Revista latinoamericana de investigación en matemática educativa	3	2.50%

Table 3. The top 11 journals according the H index reported by Scimago Journal & Country Rank, where particularly there are 7 journals in the subject area of Education. The quartil and subject area of Physical review special topics is not assigned yet.

Rank	Journal	H Index	SJR 2019	Quartil	Subject Area and Category
1°	American journal of physics	88	0.51	Q2	Physics and astronomy (miscellaneous)
2°	Journal of chemical education	77	0.47	Q2	Physics and astronomy (miscellaneous)
3°	Journal for research in mathematics education	74	2.92	Q1	Education
4°	Educational studies in mathematics	60	1.57	Q1	Education
5°	Journal of science education and technology	56	1.17	Q1	Education
6°	Advances in physiology education	55	0.52	Q2	Education
7°	International journal of engineering education	47	0.45	Q1	Engineering (miscellaneous)
8°	European journal of engineering education,	41	0.7	Q1	Engineering (miscellaneous)
	Physical review special topics-physics education research	41	–	–	–
9°	Journal of professional issues in engineering education and practice	37	0.45	Q2	Civil and structural engineering
10°	ZDM–Mathematics Education	36	1.08	Q1	Education
11°	International journal of science and mathematics education	35	0.9	Q1	Education
	Journal of mathematics teacher education	35	1.96	Q1	Education

Table 4. Authors with the highest number of articles in the retained list.

Author	Institution	Number of Articles
Chris Rasmussen	San Diego State University, USA	13
Matías Camacho-Machín	University of La Laguna, Spain	4
Samer Habre	Lebanese American University, Lebanon	4
Debasree Raychaudhuri	California State University, USA	4
Lourdes Maria Werle de Almeida	State University of Londrina, Brazil	3
Carolina Guerrero-Ortiz	Potificia Universidad Católica de Valparaiso, Chile	3
Karen Allen Keene	North Carolina State University, USA	3
Karen King	Michigan State University, USA	3
Oh Nam Kwon	Seoul National University, South Korea	3
José Arturo Molina-Mora	Universidad de Costa Rica, Costa Rica	3

Table 5. Top 10 articles followed by the number of citations in Google scholar.

Article Title and Reference	Number of Cites
New directions in differential equations: a framework for interpreting students' understandings and difficulties [15].	196
Advancing mathematical activity: a practice-oriented view of advanced mathematical thinking [15].	187
An inquiry-oriented approach to undergraduate mathematics [109].	186
Classroom mathematical practices in differential equations [125].	184
Teaching mathematical modeling through project work [53].	174
Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions [124].	171
Social and sociomathematical norms in an advanced undergraduate mathematics course [133].	160
Students' retention of mathematical knowledge and skills in differential equations [89].	136
Locating starting points in differential equations: a realistic mathematics education approach [16].	135
Classroom mathematical practices and gesturing [106].	106

6. Summarizing the Evidence (Step 4)

To approach the answer to the questions presented in Section 3, we gathered and selected the relevant information from the retained list of publications (see last paragraph of Section 4). In Table 6, a synthesis with focus on didactic methodology and topics taught or evaluated is showed. More details related to the didactic methodologies (traditional methodology, mathematical modeling, etc.) will be presented in Section 7. The articles with empty topic are those where the topic covered was not specified. Moreover, related to the question of the reported effectiveness of the new didactic methodologies in comparison with the traditional methodology, we found that few articles address explicitly this topic. From the list in Table 6, the following articles: [33,34,55,57,64,66,76,79,84,89,92,123,126,135] provide an explicit treatment of effectiveness.

Table 6. Summary of didactic methodologies and the topics of ordinary differential equations declared on the list of retained list of papers (see last paragraph of Section 4). Here Ref. is used for abbreviation of the reference number in the list of references.

Ref.	Didactic Methodology	Topics Taught or Evaluated
[1]	Traditional methodology	Scalar: first order, second order, orthogonal curves, existence and uniqueness theorem
[2]	Traditional methodology	Scalar: first order, second order, orthogonal curves, existence, and uniqueness theorem
[3]	Geometric and qualitative solutions, Active learning, Information and communication technology	Scalar: first order, applications to exponential decay problems
[4]	Geometric and qualitative solutions, Mathematical modeling, Information and communication technology	Scalar: Malthus model, logistic generalized
[5]	Mathematical modeling	Scalar: second order and applications to electronic circuits
[6]	Active learning	Scalar and systems: Laplace Transform
[7]	Active learning	
[8]	Mathematical modeling	Scalar: first order, applications to mixing problems, freefall problems
[9]	Mathematical modeling	Scalar: first order, applications to mixing problems, second order
[10]	Information and communication technology	Systems: Lotka–Volterra model
[11]	Information and communication technology	Systems: plane phase, linear system, qualitative behavior
[12]	Information and communication technology	Scalar: first order, slope fields, asymptotic behavior
[13]	Mathematical modeling, Information and communication technology	Scalar: first order, logistic generalized
[14]	Geometric and qualitative solutions, Active learning	Scalar: rate of change
[15]	Active learning	Scalar and systems: several topics
[16]	Active learning	Scalar and systems: several topics
[17]	Mathematical modeling	Scalar: first order, applications to electronic circuits
[18]	Information and communication technology	Scalar: first order, applications to electronic circuits
[19]	Information and communication technology	Scalar: first order, second order, graphical solution, Laplace transform
[20]	Mathematical modeling	Scalar: Malthus model, Verhulst model, equilibrium analysis.
[21]	Others	
[22]	Geometric and qualitative solutions Mathematical modeling	Systems: applications for asthma
[23]	Active learning	Scalar: first order
[27]	Information and communication technology	Systems: applications for Chemical reactions
[28]	Information and communication technology	Scalar: first order, second order, graphical solution, Laplace transform
[29]	Information and communication technology	Scalar: first order, freefall problems
[30]	Project-based learning	
[31]	Projects-based learning	Scalar: first order, applications to tumor growth, Gompertz model, graphical solutions, bifurcation
[32]	Information and communication technology	Scalar and systems: several topics
[33]	Information and communication technology	Scalar and systems: first order, Laplace transform, application to chemical reaction and control
[34]	Information and communication technology	Scalar and systems: several topics
[39]	Traditional methodology, Geometric and qualitative solutions	Scalar: first order, applications to exponential decay problems
[48]	Mathematical modeling	Scalar: Newton's law of cooling and laws for velocity, acceleration and volume

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[49]	Information and communication technology	Scalar: Verhulst model, generalized logistic.
[50]	Active learning	Scalar: first order, linear, Bernoulli
[51]	Active learning	Scalar: first order, applications to exponential decay problems
[52]	Projects-based learning	
[53]	Projects-based learning	Scalar and systems: populations model, linear system
[54]	Mathematical modeling	Scalar: first order, applications to mixing problems
[55]	Active learning	
[56]	Geometric and qualitative solutions	Scalar: first order
[57]	Traditional methodology	Scalar: linear higher order
[58]	Mathematical modeling	Systems: equilibrium
[59]	Traditional methodology	Scalar: first order
[60]	Information and communication technology	Scalar: applications to electronic circuits
[61]	Others	Scalar: First order
[62]	Others	
[63]	Mathematical modeling	Scalar: Malthus model
[64]	Traditional methodology, Geometric and qualitative solutions	Scalar: first order, second order, graphical solution, slopy fields
[65]	Mathematical modeling	
[66]	Active learning	Scalar: first order, second order, several applications (biomedical, scientific, and social-economic contexts)
[67]	Active learning	Scalar: First order, Verhulst equation
[68]	Mathematical modeling	Scalar and systems: linear, exponential, logistic, and ecology applications
[69]	Geometric and qualitative solutions	Scalar: first order
[70]	Geometric and qualitative solutions	Scalar: first order
[71]	Geometric and qualitative solutions, Active learning	Scalar and systems: first order, autonomous differential equations, slope fields, Lotka–Volterra models
[72]	Geometric and qualitative solutions	Scalar and systems: graphical solutions
[73]	Active learning, Information and communication technology	Scalar: first order, applications to mixing problems
[74]	Information and communication technology	Systems: first order, validation with real data
[75]	Information and communication technology	Scalar: first order
[76]	Traditional methodology	Scalar: first order, applications to kinetics
[77]	Others	Scalar: Laplace transform
[78]	Mathematical modeling	Systems: first order
[79]	Active learning	Scalar: first order, second order, slope fields, several applications
[80]	Traditional methodology	Scalar: first order
[81]	Active learning	
[82]	Traditional methodology	Scalar: first order, higher order
[83]	Others	
[84]	Geometric and qualitative solutions, Active learning	Scalar: first order, autonomous differential equations, slope fields
[85]	Active learning	Scalar: first order, Newton's law of cooling
[86]	Active learning	Systems: first order, linear, slope fields, Lotka–Volterra models
[87]	Active learning	Scalar: first order, autonomous differential equations, slope fields
[88]	Mathematical modeling	Systems: Lotka–Volterra model, phase plane, equilibrium solutions, phase trajectories

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[89]	Active learning	Scalar and systems: first order, second order, slope field, linear system
[90]	Projects-based learning	
[91]	Mathematical modeling	Scalar: first, order, Malthus model, Newton's law of cooling
[92]	Information and communication technology	Systems: second order, applications to vibration of a two-mass two-spring problems
[93]	Active learning	Scalar: first order, zombies models
[94]	Mathematical modeling, Information and communication technology	Scalar and systems: first order, freefall problems, generalized Lotka–Volterra
[95]	Active learning, Information and communication technology	Scalar: second order
[96]	Active learning	Scalar: first order, second order
[97]	Geometric and qualitative solutions	Scalar and systems: first order, Lotka–Volterra model, Euler methods
[98]	Mathematical modeling	Scalar: first, order, Malthus model, AIDS models
[99]	Information and communication technology	Scalar and systems: amplitude and phase in second order equations, linear phase portraits in linear system, Fourier coefficients, vibrations applications
[100]	Mathematical modeling, Information and communication technology	Laplace Transform, Newton's law of cooling
[101]	Information and communication technology, Projects-based learning	Systems: Lotka–Volterra model, pancreatitis model
[102]	Information and communication technology	Scalar: first order, applications to energy balance, chemical process and control fundamentals
[103]	Information and communication technology	Systems: applications to epidemics
[104]	Others	Scalar: first order
[105]	Others	Scalar: firs order
[106]	Active learning	
[107]	Active learning	Scalar and systems: Verhulst equation, bifurcation
[108]	Active learning	
[109]	Active learning	Scalar and systems: first order, slope fields, second order with spring-mass applications, linear systems, straight-line solutions, Lotka–Volterra models
[110]	Geometric and qualitative solutions, Active learning	
[111]	Active learning	Scalar: existence and uniqueness theorem of first order
[112]	Active learning	Scalar: concept of solution of first order equation
[113]	Active learning	Scalar: first order
[114]	Active learning	Scalar and systems: several topics
[115]	Mathematical modeling, Information and communication technology	Systems: applications to electronic circuits
[116]	Mathematical modeling	Scalar: second order, applications of Newton's second law
[117]	Others	Scalar and systems: second order, applications of Newton's second law
[119]	Mathematical modeling	Scalar: first order
[118]	Mathematical modeling	Scalar: first order
[120]	Information and communication technology	Scalar: first order, Laplace transform
[121]	Active learning	Scalar: first order, applications to electronic circuits
[122]	Information and communication technology	Scalar: first order, applications to molar and energy balances

Table 6. *Cont.*

Ref.	Didactic Methodology	Topics Taught or Evaluated
[123]	Traditional methodology, Information and communication technology	Scalar: definition of differential equations, graphical solution, applications.
[124]	Active learning	Scalar: first order, Malthus model
[125]	Active learning	Scalar: first order
[126]	Traditional methodology	Scalar: first order
[127]	Mathematical modeling	Scalar: first order
[128]	Mathematical modeling	Scalar: first order, Bernoulli's equation
[129]	Traditional methodology	Several topics
[130]	Mathematical modeling	Scalar: first order, population mathematical model
[131]	Traditional methodology	Scalar: first order, freefall problems
[132]	Active learning	Scalar: first order
[133]	Active learning	Scalar: first order
[134]	Mathematical modeling	Scalar and systems: first order, applications to exponential decay problems, Lotka–Volterra model
[135]	Projects-based learning	Several topics of noise and vibrations concepts

7. Interpreting the Findings (Step 5)

After gathering, filtering, synthesizing, and analyzing the main contributions of each paper of the retained list, in this section we address the answers to the framing questions introduced in Section 3.

7.1. Question 1: What Are the Studies Developed for Teaching and Learning of Ordinary Differential Equations with a Reported Classroom Experiences? What Types of Didactic Methodologies Have Been Used in Those Studies?

To answer this question, we recall Section 4 where we identified 405 articles which were classified in *notes* (262), *curriculum* (23), and *research in classroom* (120), see Figure 1. In the case of *notes* and *curriculum* types of articles, there are no reported empirical applications of classroom experiences. Thus, there are 120 articles with classroom experiences, which are explicitly specified at the end of Section 4 and in the first column of Table 6. Now, regarding the didactic methodologies, we have identified seven groups:

- the traditional teaching and learning methodology,
- graphical or qualitative and numerical approach of teaching,
- active learning methods,
- The mathematical modeling-based methodology,
- information and communication technology-based methodologies,
- project-based learning, and
- other methodologies,

Each classification is discussed below. There are many works that can be included in more than one classification, so we decided to include the paper in a group according to the aim declared by the authors.

7.1.1. The Traditional Teaching and Learning Methodology

The traditional teaching is focused in solving ordinary differential equations by applying algebraic or analytic methods, where solving means that we can find an explicit or implicit expression for the unknown function [69]. Those methods are characterized by being algorithmic, procedural, symbolic, and particularly related with a specific type of differential equation. For instance, the traditional teaching of first-order ordinary differential equations can be summarized in two steps: (i) the educator introduces the general form of the equation by writing the following two equivalent forms

$$\frac{dy}{dx} = f(x,y) \quad \text{or} \quad M(x,y)dx + N(x,y)dy = 0,$$

where f, M and N are given functions from $D \subset \mathbb{R}^2$ to $\mathbb{R}$, followed by the introduction of the classification as separable, homogeneous, exact, linear, Bernoulli and others, depending on the functions f, M, N, see Table 7; and (ii) the educator teaches the students their own algorithmic solution technique for each class of equation, where the algebraic manipulation and the integration of functions are essential techniques common to all classes. Two similar steps of teaching are also applied to higher-order ordinary differential equations and for first-order systems of differential equations. Thus, according to [123], the traditional approach to teaching differential equations consists of the use of a wide variety of algebraic or analytic methods for solving different type of problems.

Table 7. Typical classification of first-order ordinary differential equations.

Class	Properties of Functions f, M or N
Separable	$f(x,y) = h(x)g(y)$ with h and g real functions
Homogeneous	$f(\lambda x, \lambda y) = f(x,y)$ for all $\lambda \in \mathbb{R}$ and $(x,y) \in D$
Exact	$\partial_y M = \partial_x N$
Linear	$f(x,y) = p(x)y + q(x)$ with p and q real functions
Bernoulli	$f(x,y) = p(x)y + q(x)y^n$ with p, q real functions and $n \in \mathbb{R} - \{0,1\}$

The articles [1,2,39,57,59,64,76,80,82,123,126,129,131] address aspects related to the traditional approach to teaching ordinary differential equations. Such as, development of algebraic abilities, student's difficulties of learning, uses of different mathematical representations, among others. The articles [1,2] are in the boundary between traditional and new didactic methodologies of teaching and learning differential equations, since the author discusses the relationship between procedural and conceptual learning. In [57], the authors propose a didactic material to develop skills for solving non-homogeneous higher-order ordinary differential equations by the use of indeterminate coefficient and constant variation methods. In a broad sense, the didactic material proposed by the authors consist of a list of algebraic exercises to select the appropriate method and apply the corresponding algorithmic technique. In [59,82], the author's aim was to measure the undergraduate student's mathematical knowledge through several tests. Although, the authors do not give information about the pedagogical methodology used to teach ordinary differential equations, we observed that the questions in their tests evaluate the processes of finding solutions rather than evaluating the concepts. In the article [64], the authors discuss the prevalence of traditional teaching based on analytic methods and the slow incorporation of geometric methods, they argue that the incorporation of new teaching techniques require a new learning communication skills. A similar approach to [64] is presented in [80,129], where the authors establish a study to identify the difficulties of students to develop a conceptual understanding and to use symbolic representations, meanwhile, learning differential equations based on a procedural teaching. For their part, the authors of [39,123] introduced a widely documented discussion about the characteristics of traditional methods and describe the main disadvantages. In the papers [76,131], a new method to get an analytic solution of first order differential equations is proposed. In [126], the author investigates a mnemonic acronym designed for the pedagogy of first-order ordinary differential equations. The aim in this paper is to develop a critical analysis, and propose a pedagogical model with the potential to move mnemonics from being viewed as a particular tricks where learners repeat some information which they do not understand altogether; towards a deeper, more conscious experience where learners are fostered to think beyond the mnemonic.

On the other hand, several authors have developed a broad research and discussion related to the constrains of traditional learning of differential equations. Here we mention some of the main concerns reported in the literature: the students prefer to learn algebraic methods of solution because it gives them an exact answer, however, these methods present difficulties to converting symbolic information into graphical information and vice versa [72]; student learning with the traditional method is limited because it is focused on applying and mastering algebraic procedures [2]; the main difficulties of students are related with the unsuitable choice of the solution method or an incorrect integration [3]; and the students learning in traditional methodology present some difficulties to contextualize the concepts of ordinary differential equations because they are not able to interpret correctly the terminologies out of the algebraic meaning [2,119]. Consequently, the students develop misunderstandings and learning difficulties related to differential equations [15]. It is widely documented that traditional methods for teaching and learning of ordinary differential equations are not suitable for conceptual learning, and therefore other methodologies are required [1,16,69]. Aspects like the learning in different classroom environments, the design of instructional sequences of activities, and the prompting to rethink theoretical issues as graphical representations, mathematical modeling, and even social interactions, need a further theoretical and empirical investigation [15].

Even though the traditional method of teaching and learning ordinary differential equations has several disadvantages, specifically it is passive to develop concept learning, should not be discarded entirely, since the learning of differential equation concepts needs capability in calculus concepts and skills [136]. Moreover, any change in the teaching methodologies (lecture notes, worksheets, and demonstration materials) should be implemented carefully, considering that although the students may have knowledge on concepts and skills to work with functions, differentiation, integration, and graphical representation of the derivative function, they may be unable to utilize these resources in a differential equations course [3,96].

7.1.2. Qualitative and Numerical Approach to Teaching Differential Equations

As noted in various sources, the traditional teaching of ordinary differential equations has been focused in the teaching of analytic methods, however is also know that those methods are restricted to solve only few types of of equations. In the last decades, we have witnessed the incorporation of graphical and numeric solutions methods to the teaching of differential equations. The practice of these qualitative methods is becoming more frequent in the classroom due to its potential to approach solutions of several types of ordinary differential equations [39,40]. However, in practice, there are some drawbacks. For instance, the order and the non-linearity of the equation which does not permit the universal application of those methods. In our list, 14 articles are focused on exploring the teaching of graphical solution, qualitative behavior and numerical solution of ordinary differential equations [3,4,14,22,39,56,64,69–72,84,97,109]. In the articles related to the teaching of qualitative analysis of ordinary differential equations, the focus is mainly in the learning of several concepts like graphical solution, direction fields, stability, and increasing or decreasing behavior of the solution, interpretation of situations based on the behavior of solutions. Meanwhile the articles on numerical solution are focused to introduce the concept of numerical solution and the construction of the numerical solution by application of the standard schemes like Euler and Runge–Kutta.

There are some works related to qualitative approaches that deserve special mention [137–140]. These works were pioneers in the exploration of new teaching and learning methods for the teaching and leaning differential equations, but they do not appear with our search criteria. The works [137–139] are out of the selected databases where we looked (see Section 4, item (a)) and the work [140] belongs to *notes* type of articles.

In recent years, the list of papers about the teaching of graphical and numerical solution of ordinary differential equations has been increased by the incorporation of technology. Those articles will be presented below on the Section 7.1.5.

7.1.3. Active Learning Methods

In the literature, there is not a unique definition of active learning, although this term is frequently used to refer the classroom practices that engage students in learning activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking [141]. The active learning methods are student-centered teaching methodologies which provide the students the opportunity to participate in mathematical investigation or problem-solving groups, where they construct and share knowledge in communities while maintaining an appropriate feedback on their work from experts and peers. Several research studies conducted in the last years have evidenced that active learning environments developed for students present better performance and retention than traditional and passive teaching.

In the last decades, a great number of instructional strategies have been proposed to foster the "active learning" approach. For instance, the inquiry-based learning, problem-based learning, the collaborative learning, the flipped classroom, problem solving and modeling activities, thinking-based learning, competencies-based learning, etc. Particularly, in the case of the teaching ordinary differential equations, we found 36 works [3,6,7,15,16,23,30,50,51,55,64,66,67,71,73,79,81,84–87,89,96,106–114,121,124,125,132], which are organized as follows:

(a) **Inquiry-based learning.** The "inquiry-based learning" is one kind of active learning methodology with several implementations in math classroom and its particular form of implementation is the "inquiry-based instruction" [71]. The methodology of inquiry-oriented instruction consists of four main steps: the generation of ways for reasoning of students, the analysis of student contributions, the development of a shared understanding, and the connection of finding in the development of research tasks to standard mathematical language and notation. Thus, the inquiry-oriented instruction generates classroom environments where the students practice an authentic research mathematical activity meanwhile they discover mathematical concepts, answering to purposefully designed tasks.
The inquiry-based instruction for ordinary differential equations is researched in the following articles [15,16,71,79,81,87,89,106–110,124,125]. In [71], the author reports the findings about the students' work with concepts related to slope fields, horizontal and vertical translation of solutions, systems modeling species interaction, and graphical solution of scalar autonomous differential equations. The author concludes that several advantages are generated by the inquiry oriented environment. Particularly he pointed out the following results: the students showed a notable cognitive gain in understanding and thinking; through the intervention of the instructor guiding the discussion the students reinvented knowledge; and they expressed their satisfaction with the inquiry instruction environment. In [79], the authors focus on the teaching of slope direction fields and the conception of solutions. Through a quantitative analysis, they showed that the students were able to successfully identify direction fields when the ordinary differential equation was given in analytical form, matching the appropriate direction field and the solution curve. They also found that students improved their understanding of the concept of solution for an ordinary differential equation as a result of the inquiry oriented intervention. The authors claim that the training had a long-lasting impact. In [81], discourse analysis is used to study the students mathematical narratives when learning the basic concepts of ordinary differential equations in a inquiry-oriented classroom environment, particularly the student's positions and beliefs related to learning mathematics. The articles [15,16,87,89,106–110,125] are part of the line of research introduced by Chris Rasmussen and collaborators. These papers are mainly focused on studying the retention of mathematical knowledge, students reasoning with mathematical ideas, and conceptual understanding, in the context of learning differential equations. From these studies, the inquiry-oriented methodology stands out for its potential to facilitate the development of mathematical reasoning ability and fostering meaningful learning. With a different perspective,

in the article [124], the authors discuss the knowledge and capacity of the instructor to manage whole-class discussions concluding that the teacher's knowledge is a valuable component to be considered in the curricular reforms or in the classroom reforms under the inquiry-oriented perspective.

(b) **Problem-based learning.** The problem-based learning is an innovation of the pedagogical teaching and learning process which is learning student centered, promoting significant learning, and developing important skills and abilities which will be useful in the student's professional careers. The principle of problem-based learning is the use of problems as a starting point for the acquisition and integration of new knowledge [142]. The methodology is developed through students work in small groups where they participate in a cooperative learning experience with the aim to solve a problem proposed by the instructor, meanwhile they get a self-learning process. The self-learning process takes several steps like: read and analyze the problem, a focus group, make a list with the known and unknown facts about the problem, make a list of tasks to do, give a formal definition of the problem, get new information, and give a solution to the problem. From our list, 3 articles [55,73,93] are focused on the teaching and learning of ordinary differential equations under the problem-based learning methodology.

(c) **Other active learning methodologies.** Here we included other works related with research on active learning [3,6,7,23,30,50,51,64,66,67,84–86,96,111–114,121,132,133]. In [3,23,30,51,121], the authors apply the problem solving methodology. In [50], the authors develop a methodology based on the analysis of errors. In [6], the authors use the actions-processes-objects-schemas (APOS) theory. In [7], a competences-based methodology is used. In [64], a knowledge-guided based on discursive strategies is implemented. In [66], a guided small-group tasks perspective is applied. In [67], a methodology based on inquiry approach to learning in the context of community of practice theory is used. In [84], the authors compare the students performance when using three different methods for visualizing differential equations and their solutions, they also introduce a new method of visualization called Dynamic Method. In [85], a problem-centered methodology is used. In [86], the author presents a characterization of dynamic reasoning to improve student understanding in time related areas of mathematics. In [96], a discovery-based approached is applied for constructing the solutions of first and second-order linear ordinary differential equations and in [132] a learning methodology supported in embodied cognition and conceptual metaphors are discussed. Now, in the articles [111–114], innovative active learning methodologies are introduced in order to teach advanced topics of ordinary differential equations. For instance, in [111], the called framework of layers concepts–conditions–connectives–conclusions is presented, which was used to teach the interpretation and usage of existence and uniqueness theorems for ordinary differential equations.

The works related with the active methodologies of mathematical modeling, flipped classroom, and projects-based learning will be commented on in Sections 7.1.4–7.1.6, respectively.

7.1.4. The Mathematical Modeling Based Methodology

The mathematical modeling has a long history and a wide spectrum of applications in modern science. However, modeling is not defined in a unified single sense and, in the context of mathematics education, it has been conceptualized in a variety of ways, for instance as a process, a skill, and as a theory for student learning [8]. Over the last decades, research in mathematical modeling has increased highlighting several approaches to the teaching of mathematics and developing of students' modeling abilities. Mathematical modeling has become part of the educational standards in many institutions worldwide, being included in the curriculum of different scholar levels and careers from pedagogy, science, technology, and engineering. The researchers in mathematical modeling have emphasized different pedagogical goals as developing of modeling competencies through centered subject activities, orquestation of teaching and learning processes, developing of critical understanding of different situations, and students' motivation [143,144].

In the context of Mathematics Education, mathematical modeling has also been considered as a didactic methodology where we can find many approaches. Here we mention at least two of these: (i) research works motivated in curricular reasons and use some contextualized examples arising from validated mathematical models and, (ii) the papers that propose implementing mathematical modeling to involve the students in the treatment of real-world or life problems enhancing their career formation abilities [145]. Notice that in the case of (i) and (ii) the modeling can act as a vehicle for teaching mathematics or as content to be learned. This is, in the case (i), the modeling is a mean for attainment curricular contents and, in (ii), the modeling seeks first to nurture and enhance the ability of students to solve authentic real-world or life-like problems. In the case of (ii), the mathematical modeling process has been described as a cyclic process involving phases which are well discussed in [8,9,143,144,146]. A wide and documented discussion of meanings, approaches, priorities, challenges, and research perspectives associated with the mathematical modeling is presented in [145].

In the conceptualization of mathematical modeling cycle, there are several phases involving the process and sub-process of learning [146]. An example of the representation of the modeling process is presented in Figure 4 which was introduced by [147] and cited in [9]. The mathematical modeling is used to transit between two systems called the real world and the mathematical theories or representations. The process of mathematical modeling typically starts when the modeler has a question in the real world, which is referred as real-world situation on the diagram. Then, the modeler observes the situation mathematically by exploring the characteristics of the system which can be described by mathematical quantities and determine the relation between those quantities. After that, in the process known as mathematization or abstraction, the modeler considers some "conditions and assumptions" and replaces the real world by a mathematical entity (mathematical model) in terms of mathematical properties and parameters. The mathematical model is analyzed by applying the specific mathematical theory, deducing some mathematical conclusions which are transferred back to the real-world situation by examining if the conclusions of the mathematical model have a coherent answer to the original question. If the answer is ambiguous or has clear limitations, the modeler can repeat the cycle by considering new and more insightful observations and then improving the mathematical model.

Specifically, in the retained list, the articles [4,5,8,9,12,13,17,18,20,22,48,49,54,56,58, 63,65,68,74,78,88,91,94,98,100,101,115,116,118,119,127,128,130,134] are related to some approaches to the mathematical modeling for the teaching of ordinary differential equations. These works were developed between the years 2004 and 2019, with the exception of [78,130]. The inclusion of [78] in the list of mathematical model papers for teaching ordinary differential equations obey to the fact that the author introduced an example of a real-life problem which is analyzed by the application of ordinary differential equations. Meanwhile, in [130], the author addressed the teacher training and recommended to include tests questions to enhance students to experience higher thought levels. Particularly, he exemplified and analyzed a question related with mathematical models for describing population dynamics with ordinary differential equations. The rest of articles (i.e., the works from 2004 to 2019) have diverse and disperse approaches for mathematical modeling. However, we can distinguish some similar characteristics which allow the definition of the following four groups:

(a) **Development of skills for mathematical modeling.** We find some articles where the aim was to study the development of mathematical modeling abilities in order to solve real problem models by employing mathematical theory knowledge related to ordinary differential equations [8,17,20,54,63,65,68,88,91,100]. The papers [20,63] are focused on the teaching and learning of mathematical models, particularly in the construction and application of mathematical models through mathematical activities.

In [20], the authors present two activities, one of them is based on mathematical models already known in the literature of ordinary differential equations and, the other one is based on the treatment of quantitative information for a new situation, concluding that different approaches to mathematical modeling lead to different actions of the students. In [8], the author introduces the methodological tool "Modeling Transition Diagrams" for capturing and representing the individual modeling process which uses this tool to examine the mathematical thinking while the students participate in modeling activities. The authors of article [65] are interested in the experience of implementing a mathematical modeling course, they report that the students adopt different approaches to learn mathematical models and conclude that after the experience, the students appreciate mathematical models, and suggest the usage of mathematical modeling to engage students into higher level learning approaches. The authors of [68,88] report the results of an innovative approach for teaching mathematical modeling with emphases in topics of environment, ecology, and epidemiology. Particularly, in [88] the students were involved in the solution of real-life problems adjusted to their region, by using the mathematical modeling tools were encouraged to pay attention to environmental issues like survival and sustainability. The paper [91] is focused on how to use ordinary differential equations as a pedagogical strategy to introduce students to the concepts of mathematical modeling. The author of [100] presents an application of mathematical modeling as a contextualized activity in several topics of an integral calculus with a small introduction to some topics of ordinary differential equations. In [17], the author studies the transposition of the mathematical modeling process used by the experts into the learning and teaching of mathematical modeling for undergraduate students.

(b) **Modeling as pedagogical strategy to teach concepts of ordinary differential equations.** In these papers, the authors are focused on several topics of ordinary differential equations which are taught by using mathematical modeling. In a broad sense, the authors deduce several advantages in the teaching and learning process and also present some conclusions that promise a continuous development of mathematical modeling as a pedagogical methodology for the following years. Among the advantages pointed out by the authors, we highlight that mathematical modeling is a pedagogical methodology that promotes meaningful learning and, it is a significant and concrete alternative to the questioned traditional teaching. In this group of papers, we have include the following articles [9,54,94,127,134]. In [54] is presented a research about how mathematical modeling as teaching and learning methodology can provide meaningful learning for the students. In [9], the author develops a comparative study of two instructional approaches used in the teaching of ordinary differential equations for engineering students. In one classroom, decontextualized techniques are emphasized, while in the other one, the teaching is based on modeling principles. She concludes that mathematical modeling practice as an instructional approach is a technique that can be used to circumvent several cognitive obstacles identified in the learning of differential equations. The authors of [94] develop a preliminary study of the application of mathematical modeling as a pedagogical tool for teaching several concepts of applied mathematics, particularly the geometric solutions of scalar and systems of ordinary differential equations. In [127], the author is interested in the students' understanding when learn ordinary differential equations under the mathematical modeling perspective. She develops an analysis using the APOS theory and mainly concludes that the modeling stimulates discussion, reflection, and the construction of new processes, objects, and schemes. Based on the didactic engineering perspective, the authors of [134] present the results of experimenting mathematical modeling process as didactic methodology for teaching ordinary differential equations.

(c) **Language games, representations, and relations of mathematics with other sciences.** There are some papers paying attention to some aspects like the different language games developed by the students involved in modeling activities [48], the usage of registers of representation for making relationships between the context and elements in ordinary differential equations [13], and the role of mathematical modeling to establish a relation between mathematics and other sciences [4,5,98].

(d) **Modeling activities using ordinary differential equations to teach other concepts.** Other articles are focused on the study of mathematical models based on ordinary differential equations for teaching concepts of other areas of mathematics or even other disciplines. More precisely, in [22] a study where the students were involved in the learning of concepts like drug administration by using simulations of the mathematical was developed. This experience was supported on modeling drug administration regimes for asthma through systems of coupled differential equations. In [115], the authors are focused in the teaching of concepts from cardiovascular physiology by using an analogous mathematical model to electronic circuits. In [116], some concepts of mechanics are introduced to the students through modeling fighter pilot ejection. In [118,119], the authors study how students understand units and rate of change when working with ordinary differential equations. In [30], some concepts of physical dynamic systems like the stability using mathematical models based on ordinary differential equation systems are studied; and in [128] the authors study some concepts of fluid dynamics using models based on the Bernoulli equation.

The articles [12,18,74,101] will be commented on Section 7.1.5; and [54,134] are presented on Section 7.1.3 and [49,56] on Section 7.1.2.

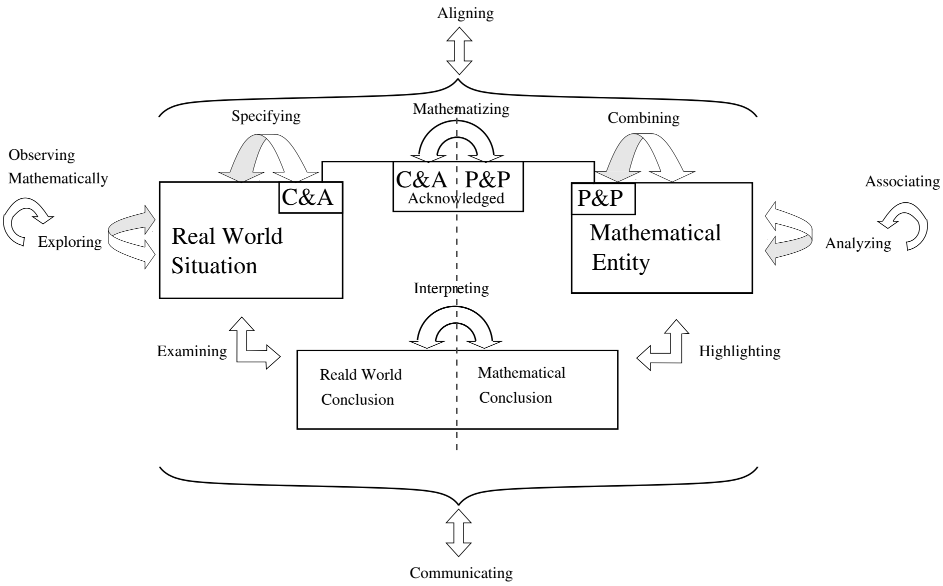


Figure 4. A diagram for the mathematical modeling cycle introduced in [147] and cited in [9]. The notations C&A and C&A are used for "conditions and assumptions" and "properties and parameters", respectively.

7.1.5. Information and Communication Technology-Based Methodologies

The increase of technology has challenged researchers worldwide to explore the roles technology plays and how transforms the teaching and learning of mathematics [148]. Particularly, in the case of ordinary differential equations, the information and communication technology has also become one of the essential hallmarks of contemporary educational landscape and several studies have been developed in the last years [32]. The studies of advantages, effectiveness, and other properties of technology are dynamic and have been constantly improved in recent years. For instance, an advantage of a simulation software as a learning platform is that students can solve more problems and develop abilities to achieve higher-level learning in less time than before when using traditional platforms [27].

The pedagogical methodologies based on the information and communication technology are diverse, including some learning activities like the following ones: the implementation of algorithms by writing computer codes, the analysis of some statements problems to be translated into a computer program, use of an specific software to solve problems or to learn some concepts, split a complex problem in a more small problems which integration permits the solution, conjecture some properties, and simulate the solutions in order to support the development of the proofs. Now, in the case of ordinary differential equations, it is well-known the existence of at least three approaches to solve an equation: the analytic, the qualitative, and the numeric solutions. With support on the information and communication technology, it is possible to implement pedagogic methodologies that address these approaches to the solution of ordinary differential equations. More precisely, from the retained list of papers, the articles related with information and communication technology are: [3,4,10–13,18,19,27–29,32–34,49,60,73–75,92,94,95,99–103,115,120,122,123], which can be arranged in three groups:

(a) **Computer algebra system.** The concept of computer algebra system is widely used to refer a type of software package that is used in learning some concepts by the manipulation of some appropriate mathematical formulae, and it is used in those cases where the algebraic, graphic, or algorithmic manipulations are tedious tasks with a low level of learning [149]. There are several papers focused in the usage of technological tools to find the analytic, numeric, or graphical solution of differential equations or even to analyze the qualitative behavior. Specifically, the articles [3,4,10–13,19,28,29,32,49,73,74,94,95,99–101,103,120,123] are related to the computer algebra system approach. In [28], the use of the software "Scientific Notebook" is studied to obtain the analytic and graphical solution of ordinary differential equations. The authors of [49] are focused on researching the teaching of differential equations through mathematical modeling in a computer enriched environment. In [29], it is reported a study where the students were encouraged to develop simulations of freefall problem by using a spreadsheet based on mathematical models. The authors study if the activities contribute to the mathematical, physical, and technological knowledge of students. The paper [3] discusses the cognitive process developed by students when participating in a teaching module for ordinary differential equations, which is based on problem solving and the usage of the VoyageTM200 calculator. The authors of [4,11] are interested in analyzing the different representations developed by students when learned ordinary differential equations using a computer algebra system as mediator. Indeed, in [4] some results about the application of spreadsheets and the HPGSolver software for visualizing and interpreting the properties of a given phenomenon arising in population dynamics are reported, and [11] contributes to study the connections between symbolic and graphical representations. The authors of [10,94] use the software Modellus to teach some properties of a Lotka–Volterra type system by using numerical simulations. In the research developed in [12,13], it is reported how the students were able to use several digital tools such as Excel, Derive, Wolfram-Alpha, Geogebra, to explore ordinary differential equations and their solutions. Particularly in [12], the students used an Applet to visualize and interpret the behavior of solutions of ordinary differential equations, some students' difficulties were found in this work; and in [13] the students were encouraged to use different digital tools as mentioned before and a computer package "GeomED" particularly designed to visualize and analyze the direction fields. In the research reported in [73] the software called STELLA was used to simulate the physical cascade system. In [74], the authors are focused on teaching mathematical models building for some given physical situations and in the numerical validation using technology. In [95], the authors use Maple to assist students in understanding the construction of analytic solution into the classroom. The authors of [99] present the experience of a project for teaching mathematics at the Massachusetts Institute of Technology and particularly present the result of a developed software called "mathlets" which was used for teaching concepts of dynamical systems. The author of [32,100,101] presents an experience of teaching several topics of calculus and ordinary differential equations using an integrated learning environment enriched with projects, mathematical modeling, and information and communication technology. In the article [103], some innovative ways to use free network computing laboratory called NCLab to the teaching of differential equations and applications are presented. In [120], the authors research how Maple helps the students in algebraic skills and construction of graphs, meanwhile the students learn some concepts related with the Laplace transform. The authors of [123] investigate the usage of Web-based simulations to learn ordinary differential equations. In [19], the authors studied the development of several mathematical thinking processes when the students learn ordinary differential equations using the software Maxima.

(b) **Simulation-based learning for teaching applications of ordinary differential equations.** There are some articles where the simulation-based learning or computer-assisted learning methodologies are used to teach the applications of ordinary differential equations to several areas like physics, biology, chemistry, or related areas. In those papers, the emphasis of teaching is given on concepts which are not included in a traditional course of differential equations. The numerical simulations are typically used to develop the understanding in the students by providing a visual animation and also for develop the intuition with respect to the change of some parameters, for instance, the initial conditions or the coefficients in an specific ordinary differential equations. The papers of this type are [18,27,60,75,92,102,115,122]. In [27], the authors review the traditional engineering textbooks and propose the computer simulations to teach the systems of ordinary differential equations arising in polymer molecular reaction dynamics. The authors of [60] are focused on the teaching several concepts of electric circuits theory by using some concepts of mathematical modeling, the Laplace transform, numerical simulations with MATLAB, and experiments. In [75], the aim was teaching some concepts of hydrostatic and atmospheric theories by using some mathematical models based on ordinary and partial differential equations and their simulation using spreadsheets. The authors of [92] are focused on helping to understand the applications of eigenvalue problems and develop a software using Visual BASIC for a simulation of solutions for the ordinary differential equations system modeling the problem of the two-mass two-spring physical system. The software simulates the vibration of the physical system, allowing the introduction by the user of some parameters such as the body masses and spring constants, solves the mathematical model, and shows on the screen the numerical and graphical results. In [102], it is reported the application of spreadsheet simulations to teach some topics of differential equations arising in a course of chemistry for undergraduate students. In [115], the authors propose the computer-based simulations to teach physiological processes like capacitance and resistance, and also suggest the introduction of those kind of teaching in undergraduate cardiovascular physiology courses. The authors of [18] study the simulation of electric circuits by using the construction of a physical laboratory model and a graphical calculator. In [122], the authors use Phyton to develop a software called REAJA, which is used for teaching some concepts in the undergraduate course of Chemical Processes.

(c) **Flipped classroom.** The pedagogical methodology called “flipped classroom” or “inverted classroom” has been widely used in the last decades to replace traditional lectures given in the classroom by an active learning. The main feature of this methodology is that the responsibility for learning the rest is on the learners, through the design of meaningful activities students have opportunities to control their own processes of leaning before the class. In principle, the activities may or may not be technology-based. However, the advances of information and communication technologies in the last years have increased individual instruction computer-based. The traditional lectures given in the classroom are temporally displaced by videos or similar resources which are previously available for students in a server, then the activities inside the classroom are developed on interactive groups of learning.

Particularly, in [33,34] the authors apply the flipped classroom to study the teaching of topics related to ordinary differential equations. In [33], the authors study the effectiveness of flipped classroom to develop skills related to the application of MATLAB/Simulink in the solution of ordinary differential equation mathematical models arising in a chemical course. Meanwhile, in [34], the authors combine the flipped classroom methodology with the cycle of mathematical model in order to study the introductory concepts of ordinary differential equations. In both works, supported on strong evidence, the authors conclude that the flipped classroom improves the active learning achievement of students.

Additionally, we observe that there are some papers in which digital tools are used without reporting particular results about the use of technology on their studies.

7.1.6. Project-Based Learning

According to the philosophy, concepts and examples of research projects in calculus are provided in [150], we can describe a research project as a multistep take-home assignment which is developed individually or in groups with a concerted effort in long period of time, for instance one or two weeks. The statements of the projects are carefully designed and include some parts expecting to get stuck even in the best students, such that the learners seek for help from their instructors, from whom receive hints, additional exercises, and supplementary readings. Moreover, the projects can be designed for different learning goals. Some projects consider real world problems in order to help the students to discover the applications of mathematics and their utility to study the affine sciences like physics, biology, chemistry, or engineering. One of the key goals when working with projects is to guide the learners to construct formal proofs by exploration of particular examples. For major details on project-based learning in calculus, we refer to [150].

Concerning the application of project-based learning in differential equations, we refer to the following articles from our retained list: [30,31,52,53,90,101,135]. The authors of [31] use mathematical projects arising in biology in the context of modeling tumor growth by differential equations. In [52,53], the authors combine the ideas of mathematical modeling and project-based learning methodologies to design projects to teach some concepts of ordinary differential equations. The authors argue that the project itself contributes to the development of students' competency for project work in science even in the introductory university courses. The authors of [90] are focused into researching the perceptions of the students when writing projects in the context of a differential equations course and conclude that the methodology is appropriate to develop some skills beyond the usual academic content of concepts and procedures. The students participating in the project recognized that they improved their capacity of scientific communication with each other when analyzing and solving real-life problems. An increase in their critical thinking was also observed. In [101], similar to [52,53], is also integrated modeling and project-based methodologies in the context of classroom environment based on the information and communication technology. The authors of [30] give a preliminary report of a series of projects applied in a course of ordinary differential equations. In [135], the author uses the methodology of projects to teach some concepts such as noise, vibration, and harshness, which are part of an undergraduate course in the mechanical engineering program. Particularly, the author studies the mathematical knowledge of students related to differential equations and linear algebra and evaluates the effectiveness of the methodology.

7.1.7. Other Methodologies

In the list of retained articles, we have that the works [21,61,62,77,83,104,105,117] are out of the groups presented before, although their topic of research is related to the teaching of ordinary differential equations and applications. However the didactic methodologies used are not explicitly presented or their goals are not precisely the teaching and learning ordinary differential equations in classroom experiences, for instance [21] is a review or [117] presents the results of a pilot research project.

7.2. Question 2: What Topics of Ordinary Differential Equations Have Been Explored in the Previous Studies?

From our retained list of 120 chosen articles, we can distinguish five groups for the topics covered in the teaching of differential equations:

(a) **Basic concepts of ordinary differential equation.** We refer to as basic concepts the definition of ordinary differential equation and their solutions. For instance, in [72], the author analyzed the answer of students to the question "What comes to your mind when you are asked to solve an ODE?" in two instants of a course, at the beginning and after the intervention. He found that firstly all students think about concepts related to the analytic solution and in the second two-thirds of students consider a change of their answers including some concepts related with the qualitative approach. A similar study was conducted in [69], where the answers of students to the following exam question were analyzed:

> *In your own words, define a differential equation. Explain what constitutes a solution to a differential equation. How can you represent geometrically a differential equation? Can the geometric representation of the differential equation help in sketching approximate solutions? In your opinion, how would you solve a differential equation?"* [69] (p. 654)

In the same study, the results of a semi-structured interview to the students who were asked six questions related with the definition of ordinary differential equation, the solution concept, the concept of geometric solution, and feeling of learning differential equations were also presented. In relation to the student construction of the concept solution a framework of four facets (context-entity-process-object) is introduced to analyze that type of constructions developed, see also [114]. The teaching of the concept of equilibrium solution in the case of scalar equations was investigated in [87]. More recently in [79], the authors research on the students conceptions about the solution of ordinary differential equations. Moreover, there are some works focused in the basic concepts related with graphical and numerical solution of an ordinary differential equation. In the case of graphical solution, researchers explore new ways for the students to interpret and give meaning to the information represented by a slope field. The initial value problem or Cauchy problem, autonomous differential equations, and the asymptotic behavior of solutions are also widely studied [12,71,84]. Regarding the numerical solution, the students have been introduced to learn the concepts of stability of the solution with respect to the initial condition and the coefficients of the equation by empirical examples [29].

Other concepts related with analytic solutions of first order (exact equations, linear, Bernoulli, etc.) and higher order (homogeneous, no homogeneous, coefficients variation, etc.) are treated in [9,19,57,64,79,82,89,95].

(b) **Biomathematical models.** There are several works that introduce some models arising in biomathematics which are based on differential equations. It is possible to find different types of population growth models, for example models from epidemics transmission. In those papers, the authors also pay attention to the introduction of qualitative analysis of solutions.

In the case of scalar models we have the articles [4,12,13,20,31,49,63,91,98,124], where the authors introduce the Malthus or Gompertz models and the Verhulst type models. Firstly, related with Malthus or Gompertz models, in [31] is presented research where the students are introduced in the study of population models according to:

$$\frac{dN}{dt} = rN, \tag{1}$$

$$N(0) = N_0, \tag{2}$$

contextualized to the case of $N(t)$ representing the density of carcinogenic cells of a tumor at the time t, with N_0 the measured initial density and r is a positive constant.

A similar topic of ordinary differential equations is also developed by [63,91,98,124]; particularly in [98] the authors study a model for disinfection and modify the assumption on r by considering that r is a negative constant. Now, concerning with Verhulst type models, in [20] the authors use the mathematical modeling to teach the population models of the form

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - p(N), \tag{3}$$

$$N(0) = N_0, \tag{4}$$

where $N(t)$ is the number of individuals at time t living in a given bounded region; r and K are positive constants used for the increasing rate and the caring capacity, respectively; $p(N)$ is the predation function; and N_0 is the initial population. The attention in [20] is reduced to predation function satisfying the properties $p(N) \to 0$ when $N \to 0$ and $p(N) \to \beta$ when $N \to \infty$, with β a positive number, for instance considering $p(N) = BN^2/(\alpha^2 + N^2)$ with α a positive constant. We notice that when $p(N) = 0$ the model (3)–(4) is reduced to the Verhulst or logistic equation, which is also treated by [49]. A similar model is taught by [4,12,13] where $p(N) = 3/2$ and $p(N) = 2$, respectively.

On the other hand, in the case of systems of differential equations, we have the Lotka–Volterra model in competence of species and epidemiology, which are treated by [10,71,86,88,94,97,101,109,134]. In [10], the authors use mathematical modeling for describing the transmission of Malaria to the humans by the female mosquitoes of the genus Anopheles, given by the following system

$$\frac{dX}{dt} = \frac{ap}{N}Y(N - X) - gX, \tag{5}$$

$$\frac{dY}{dt} = \frac{ac}{N}X(M - Y) - \nu Y, \tag{6}$$

$$X(0) = X_0, \tag{7}$$

$$Y(0) = Y_0, \tag{8}$$

where $X(t)$ is the number of infected humans in time t; $Y(t)$ is the number of (female) mosquitoes infected at time t; N is the total population of humans; M is the total population of mosquitoes; and a, c, p, g and ν are positive constants. The system (5)–(8) is a particular example of the wide class of the models well known as Lotka–Volterra like systems and is used to model competence of species, which are also treated by [71,86,88,94,97,101,109,134].

Other common topics covered by the articles in teaching biomathematical modeling are related to some advances in model design and mathematical analysis. In the case of mathematical modeling, the core of teaching is focused on the simplification of some biological phenomenon using mathematical concepts recognized by the group of students involved in the experience. Related with the mathematical analysis, the works draw attention to understanding the meaning of the equations in the biology context and to the characteristics of the behavior of the solutions. For instance, in [10] the students belong to a course in an undergraduate program in Biology. The students had a previous knowledge about the disease of malaria caused by a parasite of the genus Plasmodium from a female mosquitoes of the genus Anopheles and they also mastered some concepts of calculus. The research reports, that firstly the aim of the modeling design was to increase the relations that the students could build between calculus concepts and Biology elements. In addition, the most important simplifications associated to Biology were stated as follows: the period of incubation is discarded; the human natality and mortality are ignored; the progressive acquisition of immunity in humans is ignored; and infected mosquitoes will prevail infected until death. Then, precisely stating the variables and parameters and, considering the behavior of populations interactions students formulated the model given by (5)–(8). The main two dependent variables at time t are the infected humans and the infected (female) mosquitoes populations given by $X(t)$ and $Y(t)$, respectively. Two parameters to be considered are total population of humans and mosquitoes given by N and M, respectively. To deduce the equation (5), describing the change over time of population for infected humans by interaction with mosquitoes, it is assumed that and infected mosquito bites a health human with a certain probably and the sick persons are recovered. The factors $N - X$ and ap/N represent the health human and the number of bites given by a mosquito per unit of time a/N with a probability of health humans to be infected equal to p, respectively. Meanwhile, the recovered of infected humans is described by the term gX with g a parameter for the recovery rate. Similar arguments are used to deduce the Equation (6), mainly the term $(ac/N)X(M - Y)$ is the change of infected mosquitoes when a non-infected mosquito bites into an infected human in a unit of time a/N with a probability to be infected equal to c, and the term νY is the infected mosquitoes that die at mortality rate ν. Second, concerning the mathematical analysis of (5)–(8), the authors observe that the system is non-linear and prevents the students from achieving analytical solutions and allows them access to the solutions using the software Modellus. The students worked with Modellus were guided by a set of activities that strengthen the concepts of calculus like functions, tangent line, derivative, and maxima and minima.

(c) **Scalar-based models**. We have some work using mathematical models based on scalar differential equations to teach some concepts of differential equations.
For mathematical models based on first order scalar equations, we have four groups of articles. Firstly, we have the increasing (or decreasing) mathematical models based on an ordinary differential equation of the form

$$\frac{d\alpha}{dt} = k\alpha, \quad \alpha(0) = \alpha_0, \tag{9}$$

where k is a positive (or negative) constant, t is the time, and α is the measurement of some physical quantity such that the initial time is α_0. In [51], the authors propose five activities in the context of problem solving and guided discovery methodologies, where particularly the four labeled activities are contextualized to radioactive decay modeled by (9) with α the quantity of radium in a body which is decreasing in time. The radioactive decay in the context of mathematical modeling is also considered by the authors of [39] where α is the number of radioactive atoms. A close problem is the model for uranium decay $p'(t) = -0.0003p(t) + 0.3$ explored in [3], which is described as a variation of (9), with $p(t)$ the amount of mercury in a given reservoir at any instant of time t. Related with the increasing behavior we have the works Malthus or Gompertz type described in the Biomathematical models, see the works for (1)–(2). Moreover, in [76] the authors use a difference equation of the form

$$\frac{[A]_{t_2} - [A]_{t_1}}{t_2 - t_1} = -k\Big([A]_{t_2} - [A]_{t_1}\Big)^m, \quad k > 0,\ m > 0,$$

arising in kinetic reactions and introduce the teach of convergence of discrete models to continuous models of the form (10) or to teach the relation of difference and differential equations. A second group of works are [3,8,17,29,39,48,51,85,100,116,117,131,132], where the authors use mathematical models based on first order differential equations. Here we distinguish four types of mathematical models. Firstly, we have the well known "freefall mathematical model", which is given by a differential equation of the type

$$m\frac{dv}{dt} = mg - bv^2, \quad v(0) = v_0 \tag{10}$$

with m denoting the mass of a body, g is the acceleration due to gravity, b is a constant associated to air resistance, v_0 is the initial velocity of the body, t is the time, and the unknown v is the velocity of the body. In [29], the author uses numerical methods to simulate the solution of (10) in the case of vacuum ($b = 0$) and with air resistance ($b > 0$). Ref. [8] is focused on the research of mathematical thinking process when the students analyze and solve a freefall problem, and in [131] the authors are focused on the analytic solution of (10) by the variable separation method. Third, the model for describing "Newton's law of cooling" given by a differential equation of the form

$$MC\frac{d\theta}{dt} = -h(\theta - \theta_a), \quad \theta(0) = \theta_0, \tag{11}$$

where h is a positive constant called the convective cooling coefficient, θ_a represents the environment temperature of cooling medium, M is the mass of the body, C is the specific heat, and $\theta(t)$ is the unknown temperature of the body in a time t with known initial condition θ_0. The model of type (11) is treated in [39,85,91,100]. The fourth type of mathematical model is based on "Kirchoff and Ohm laws" given by

$$\frac{dU_c}{dt} + \frac{1}{RC}U_c = 0, \quad U_c(0) = E,$$

with RC as the constant for the resistance of the capacitor, the unknown U_c is the voltage in the capacitor, and E is the voltage of the capacitor at $t = 0$; this equation is studied in [17,18].

On the other hand, a second group of scalar models of second order are presented in [5,99], where the authors use mathematical models arising in electric circuits and vibration problems, respectively. Indeed, in [5] the authors consider the model

$$I''(t) + 2\lambda I'(t) + \omega^2 I(t) = 0, \quad I(0) = 2, \quad I'(0) = 0,$$

where the I is the current intensity crossing the circuit and in [99] the authors use an interactive software for explore the equation

$$x''(t) + bx'(t) + kx(t) = k\cos(\omega t), \quad x(0) = x_0, \quad x'(0) = x_1,$$

where b, k, ω, x_0 and x_1 are constants and x is the displacement of the mass from equilibrium in a spring-mass system. In the case of [5], the authors study physical concepts such as the inductance and resistance and in [99] the authors study some concepts of Mechanical Vibration Theory like amplitude and phase.

(d) **Systems based on mechanical theory**. The works [92,117] consider second order systems arising in Mechanical Vibration Theory. To be more precise, in [92] the authors consider a system modeling a two-mass two-spring vibration system of the following type

$$\frac{d^2}{dt^2}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -(k_1 + k_2)/m_1 & k_2/m_1 \\ k_2/m_2 & -k_2/m_2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

where m_1, m_2 are the masses of two bodies connected by two springs with constants k_1 and k_2 and fixed at the top and y_1 and y_2 are the displacement from the equilibrium of the bodies. Moreover, in [92] several concepts like amplitude, modes of vibration, period, and frequency are taught.

(e) **Other concepts**. There are some works focused on the teaching and learning of other topics of differential equations like the Theorems of existence and uniqueness [1,2,111,112], Laplace transform [6,19,120], and bifurcation concept [31,135].

7.3. Question 3: What Are the Results for the Effectiveness of Traditional and New Didactic Methodologies to Teach and Learning Ordinary Differential Equations, as Reported in Previous Studies?

The effectiveness of a new methodology is usually an implicit motivation. However, in a practical research, the aim of a specific paper is usually defined explicitly in terms of other topics which are considered relevant to study in order to improve the teaching and learning process. Then, given that the effectiveness is implicitly transversal to all articles proposing innovative didactic methodologies for ordinary differential equations, here the works where effectiveness was explicitly mentioned were included [33,34,55,57,64,66,76,79,84,89,92,123,126,135].

Concerning the evaluation of the effectiveness, we distinguish four groups of articles: (i) works where only the effectiveness of the new didactic methodology was evaluated [33,55,66,76,79,123,135]; (ii) works where only the effectiveness of the traditional didactic methodology was evaluated [57,126]; (iii) works comparing the traditional and the new didactic methodologies without introducing a measurement of each didactic methodology alone [89,92]; and (iv) works where the authors introduce a quantification of the effectiveness for each didactic methodology and also a comparison [34,64,84].

8. Conclusions

The followed research methodology allowed us to identify and analyze the papers addressing the teaching and learning of ordinary differential equations. We retrieved and reviewed 120 papers from 1970 to 2020 which are associated with Web of Science, Scopus, Qualis, ZbMath, and Scielo. We recognized the didactic methodologies pointed out in each paper. When doing this, the most explored concepts and topics associated to ordinary differential equations and the effectiveness of didactic methodologies reported by the authors were identified. We noticed an increase in research where the attention has been given to the design of new didactic methodologies which have also been strengthened by the development of digital tools. The research related to teaching and learning differential equations has transitioned from exploring elements associated to the teaching in traditional classrooms to the introduction of a qualitative and numerical approach, active learning methods, modeling, and use of technology, emphasizing the importance of student participation in their own learning. As a result of the nature of differential equations for describing several phenomena, it also stands out in research modeling and interdisciplinarity. It should be noted that the characterization presented is not unique and many papers could be organized in one or more category.

The most relevant features achieved of the present article are the identification of works that address the subject of teaching and learning of ordinary differential equations, the recognition of the most explored mathematical content, and the synopsis of teaching methodologies that have used to teach the topic over the years. However, through our review analysis, we have found that there are also some issues that have received little attention. For example, little evidence is found regarding the retention, in terms of learning and skills development, that students achieve after being involved in learning with a particular methodology, which requires considering the validation and improvement of the implemented methodologies. Another element to consider is the update of the university curriculum considering the research results that involve the new teaching methods and use of information and communication technologies (for instance, those indicated in Section 7.1.5) or the relevance of the processes involved in the transition from the learning of calculus to the learning of ordinary differential equations. In relation to the teachers who are normally in charge of teaching ordinary differential equations, the research does not give importance to the fact that in many cases they are engineers or mathematicians, without or a little knowledge of didactic. Then, it is necessary to pay attention to the desired knowledge (didactic, pedagogical and mathematical) that these teachers need to teach the subject, which will allow them to become aware of the learning difficulties that students may face. Teachers of ordinary differential equations still need to be encouraged to experiment and enrich their classes with different teaching methodologies to support the students developing knowledge to respond the challenges that the academic or work field demands of them. Therefore, more research is currently needed in the classroom, in relation to the use of technology, development of simulations, resources for online teaching, and interdisciplinary projects.

The research on the teaching of differential equations is an active area with an increasing number of articles in the last decade. However, there is still much to do toward addressing the challenges in teaching and learning differential equations. We set out three issues that need more detailed exploration. Firstly, we found that some advanced topics of ordinary differential equations are incipient developed in the research. For instance the teaching of the existence an uniqueness Theorems for scalar equations of first order are treated only in [1,2,111,112] and an introduction to bifurcation concept is presented only in [31,135]. However, in the reviewed references, there is not a treatment of other relevant concepts, techniques, and classic results associated to the study of qualitative behavior of solutions, and some properties of the solutions deduced from the qualitative behavior. To name a few concepts, the teaching of linear and non-linear equations is implicitly treated by some articles. The teaching of concepts as autonomous and not autonomous systems and the concepts around stability in non-linear systems are still open topics to research. The teaching of advanced techniques and results to study non-linear systems like Lyapunov functions, topological degree methods, and the Hartman–Grobmann theorem, are still open. We did not find research regarding the teaching of analysis of equilibrium points for nonlinear systems, the periodicity of solutions, and the asymptotic behavior of solutions. Thus, briefly, there is still open the didactic transposition of several topics of ordinary differential equations theory. Second, in the teaching of modeling from physical and biological problems, the topic of existence of positive solutions is uncovered yet. For instance, in [10] the authors do not consider as part of the set of activities the basic aspect of the biological phenomenon: the existence of positive solutions of the system (5)–(8). Thirdly, regarding the systematic literature review, our short-term goal is to analyze the remaining 285 articles (*notes* and *curriculum*) which were found in the search of references given in Section 4. Since in our actual analysis some representative works were excluded, we plan to extend our search to other indexations including books, book chapters, and theses.

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Appendix A. String Search Used in Web of Science and Scopus

The string search used in Web of Science is the following

ALL FIELDS: (“differential equation*” or “solution* to differential equation*” or “graphical interpretation” or “graphical solution*” or “qualitative solution*” or “numerical solution*” or “analytic solution*” or “first order equation*” or “higher order equation*") OR ALL FIELDS: ("Laplace transform" or "power series method" or "variable separable equation*" or "reducible to variable separable equation*" or "homogeneous equation*" or "reducible to homogeneous equation*" or "exact equation*" or "reducible to exact equation*" or "Bernoulli equation*") OR ALL FIELDS: ("linear equation*" or "Ricatti equation*" or "phase plane" or "isocline*" or "slope field*" or "equilibrium" or "stability of solution*" or "initial value problem*" or "boundary value problem*" or "scalar equation*" or "systems of equations" or " linear" or "nonlinear") AND ALL FIELDS: ("teaching methodologies" or "students' understanding and difficulties" or "interpretation of solutions" or "registers of representations" or "mathematical modeling" or "mathematical models" or "problem-based learning" or "problem solving") OR ALL FIELDS: ("error analysis" or "mathematics teaching practices" or "real world situation" or "computational resources" or "mathematical application" or "classroom discourse" or "didactic of differential equations" or "critical discourse analysis").

Meanwhile the string search for Scopus is given by

(TITLE-ABS-KEY ("differential equation" OR "solution* to differential equation*" OR "graphical interpretation" OR "graphical solution*" OR "qualitative solution*" OR "numerical solution*" OR "analytic solution*" OR "first order equation*") OR ALL ("higher order equation*" OR "Laplace transform" OR "power series method" OR "variable separable equation*" OR "reducible to variable separable equation*" OR "homogeneous equation*" OR "reducible to homogeneous equation*" OR "exact equation*") OR TITLE-ABS-KEY ("Bernoulli equation*" OR "linear equation*" OR "Ricatti equation*" OR "phase plane" OR "isocline*" OR "slope field*" OR "equilibrium" OR "stability of solution*" OR "initial value problem*") OR TITLE-ABS-KEY ("boundary value problem*" OR "scalar equation*" OR "systems of equations" OR " linear" OR "nonlinear") AND TITLE-ABS-KEY ("teaching methodologies" OR "students' understanding and difficulties" OR "interpretation of solutions") OR TITLE-ABS-KEY ("registers of representations" OR "mathematical modeling" OR "mathematical models" OR "problem based learning" OR "problem solving" OR "error analysis" OR "mathematics teaching"))

Appendix B. List of Journals from Qualis, zbMATH, Scielo, WOS, and Scopus Datbases

Table A1. List of journals from Qualis, zbMATH, and Scielo database. The notation A1, A2, B1, B2, B3, B4, B5, and C are the classification of Qualis. The notation AA, AB, and AC (or BA, BB, and BC) are used for journals considered in the Serie A (or Serie B) and types A, B, and C (or A, B and C) in the classification given by [46]. The "Journal code" is a abbreviated reference code of the corresponding journal which is introduced by citation convenience.

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
1	Academia journal of educational research	2315-7704	B3			2013-2020
2	Acta scientiae	2178-7727	A2			1999–2019
3	Actualidades investigativas en educación	1409-4703			SC	2011–2020
4	American mathematical monthly	0002-9890		BA		1894–2020
5	Applied measurement in education	0895-7347		BA		1988–2020
6	Australian journal of education	0004-9441		BC		1957–2019
7	BOLEMA : Boletim de educação matemática	1980-4415	A1	AA	SC	1985–2019
8	Boletim cearense de educação e história da matemática	2357-8661	B3			2014–2019
9	Boletim online de educação matemática	2357-724X	B1			2013–2019
10	Boletín das ciencias	0214-7807	B3			1988–2019
11	British educational research journal	6469-3118		BA		1975–2019
12	British journal of educational psychology	2044-8279		BA		1931–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
13	British journal of educational technology	1467-8535		BA		1970–2019
14	Child development	1467-8624		BA		1990–2020
15	Ciência & educação	1980-850X	A1		SC	1988–2019
16	Ciencia, docencia y tecnología	1851-1716			SC	2000–2019
17	Cognition	0010-0277		BB		1972–2020
18	Cognition and instruction	0737-0008		BA		1984–2020
19	Comparative education	0305-0068		BA		1964–2020
20	comparative education review	0010-4086		BA		1957–2020
21	Cpu-e. revista de investigación educativa	1870-5308			SC	2005–2020
22	Cuadernos de investigación educativa	1510-2432			SC	1997–2019
23	Economics of education review	0272-7757		BA		1981–2020
24	Educação e matemática: revista da associação de professores de matemática	0871-7222	B1			1987–2019
25	Educação matemática em foco	1981-6979	B3			2017–2019
26	Educação matemática em revista	2317-904X	A2			1983–2019
27	Educaçao matematica pesquisa	1516-5388	A2	AB		1999–2019
28	Educación	1019-9403			SC	1992–2020
29	Educación matemática	1665-5826		AC	SC	1989–2019
30	Educación y educadores	0123-1294			SC	1997–2019
31	Educar em revista	1984-0411	A1			1977–2019
32	Educational measurement: issues and practice	1742-3992		BB		1982–2020
33	Educational research	0013-1881		BA		1958–2020
34	Educational studies in mathematics	0013-1954	A1	AB		1968–2020
35	Educational technology research and development	1556-6501		BB		1953–2019
36	Educational technology: the magazine for managers of change in education	0013-1962		BC		1960–2017
37	Elementary school journal	0013-5984		BA		1914–2019
38	Em teia-revista de educação matemática e tecnológica iberoamericana	2177-9309	B1			2010–2019
39	Enseignement mathematique, l′	0013-8584		BC		2009–2019
40	Enseñanza de las ciencias	0212-4521	A1	BA		1983–2019
41	Ensino da matemática em debate	2358-4122	B4			2010–2019
42	Epsilon	2340-714X		AC		1984–2019
43	Estudios-centro de estudios avanzados. universidad nacional de córdoba	1852-1568			SC	1993–2019
44	Focus on learning problems in mathematics and science teaching	0272-8893		BC		1988–1991
45	For the learning of mathematics	0228-0671	A1	AB		1980–2017
46	Formação docente	2176-4360	B1			2009–2019
47	Hiroshima journal of mathematics education	0919-1720		AB		1993–2020
48	IEEE revista iberoamericana de tecnologias del aprendizaje	2255-5706	B3			2006–2012
49	Insegnamento della matematica e delle scienze integrate, l′	1123-7570		BC		1970–2020
50	Integración y conocimiento	2347-0658	C			2012–2020
51	Interciencia	0378-1844	A1			2009–2020
52	International electronic journal of mathematics education	2468-4945	C			2006–2020
53	International journal of engineering education	0949-149X	A1			1991–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
54	International journal of engineering research and applications	2248-9622	C			2011–2020
55	International journal of mathematical education in science and technology	0020-739X	A1	BB		1970–2020
56	International journal of science and mathematical education	1571-0068	A1			1970–2019
57	International statistical review	1751-5823		BA		1990–2020
58	Jornal internacional de estudos em educação matemática	2176-5634	A2			2009–2020
59	Journal for research in mathematics education	0021-8251		AA		1970–2020
60	Journal für mathematik-didaktik	0173-5322		AC		1980–2020
61	Journal of computers in mathematics and science teaching	0731-9258		BC		1981–2020
62	Journal of educational psychology	0022-0663		BA		2002–2020
63	Journal of educational research	0022-0671		BA		1920–2020
64	Journal of mathematics teacher education	1386-4416		AB		1998–2020
65	Journal of recreational mathematics	0022-412X		BC		1968–2014
66	Journal of research in science teaching	1098-2736		BA		1960–2020
67	Journal of statistics education	1069-1898		AC		1993–2015
68	Journal of the learning sciences	1050-8409		BA		1991–2020
69	Journal of urban mathematics education	2151-2612	B1			2008–2019
70	Learning and instruction	0959-4752		BA		1991–2020
71	Matemática e estatística em foco	2318-0552	B5			2013–2019
72	Matematica e la sua didactica, la	1120-9968		AC		2016–2020
73	Mathematical journal of interdisciplinary sciences	2278-9561	B5			2012–2020
74	Mathematics education research journal	0021-8251		AB		1989–2020
75	Mathematics in school	0305-7259		AC		1971–2014
76	Mathematics teacher	0025-5769		AC		1990–2020
77	Mathematics teaching	0025-5785		AC		1871–2020
78	Mathematics teaching in the middle school	1072-0839		AC		1994–2019
79	Mathematical thinking and learning	1098-6065		AB		1999–2020
80	Mediterranean journal for research in mathematics education	1450-1104		AB		2002–2020
81	Numeros	0212-3096		AC		1981–2020
82	Paradígma	1011-2251	A2		SC	1997–2019
83	Perspectivas da educação matemática	2359-2842	B1			2008–2019
84	Petit X	0759-9188		AC		1986–2007
85	Phi delta kappan	0031-7217		BA		2000–2020
86	Plot: mathematiques et enseignement	0397-7471		AC		1987–2017
87	PNA: revista de investigación en didáctica de la matemática	1887-3987	A2			2006–2020
88	Professor de matemática online	2319-023X	B4			2013–2019
89	Psychology in the schools	1520-6807		BA		1964–2020
90	Quadrante	2183-2838		AB		1992–2020
91	Recherches en didactique des mathematiques	0246-9367		AB		2000–2019
92	Redimat- revista de investigación en didáctica de las matemáticas	2014-3621	A2			2012–2020
93	REEC. revista electrónica de enseñanza de las ciencias	1579-1513	A2			2002–2019
94	Remat: revista eletrônica da matemática	2447-2689	B3			2015–2020

Table A1. *Cont.*

	Journal Title	ISSN	Qualis Class	zbMATH Class	Scielo	Years Consulted
95	Rematec. revista de matemática, ensino e cultura (ufrn)	1980-3141	B2			2006–2019
96	Rencimat	2179-426X	A2			2010–2019
98	Revemat : revista eletrônica de educação matemática	1981-1322	A2			2006–2020
99	Revista de ciência & tecnologia (unig)	1519-8022	B5			1995–2019
100	Revista de ciências da educação	2317-6091	B1			2012–2020
101	Revista de educação, ciências e matemática	2238-2380	A2			2011–2019
102	Revista de produção discente em educação matemática	2238-8044	B3			2012–2019
103	Revista digital de investigación en docencia universitaria	2223-2516			SC	2005–2019
104	Revista docência do ensino superior	2237-5864	B1			2011–2020
105	Revista electrónica de investigación educativa	1607-4041	A1			1999–2020
106	Revista electronica de investigacion en educacion en ciencias	1850-6666	A2		SC	2006–2019
107	Revista eureka sobre enseñanza y divulgación de las ciencias	1697-011X	A1			2004–2020
108	Revista iberoamericana de educación superior	2007-2872			SC	2010–2020
109	Revista internacional de aprendizaje en ciencia, matemáticas y tecnología	2386-8791	B3			2014–2019
110	Revista latinoamericana de investigación en matemática educativa	2007-6819	A2	AA	SC	1997–2020
111	Revista mexicana de investigación educativa	1405-6666			SC	1996–2020
112	School effectiveness and school improvement	0924-3453		BA		1990–2020
113	School psychology quarterly	2578-4218		BA		1986–2020
114	School science and mathematics	1949-8594		BC		1901–2020
115	Science education	1098-237X		BA		2001–2020
116	Science journal of education	2329-0897	B4			2013–2020
117	Sociology of education	0038-0407		BA		2004–2020
118	Statistics education research journal	1570-1824		AB		2002–2020
119	Suma	1130-488X		AC		1988–2019
120	Teaching and teacher education	0742-051X		BA		1985–2020
121	Teaching children mathematics	1073-5836		AC		1954–2019
122	Teaching mathematics and its applications	0268-3679	A1			1982–2020
123	Thai journal of mathematics	1686-0209	B4			2003–2020
124	The college mathematics journal	0746-8342		BC		1984–2020
125	The electronic journal of mathematics & technology	1933-2823	B1			2007–2020
126	The journal of mathematical behavior	0732-3123	A1	AB		1994–2020
127	Uniciencia	1011-0275			SC	1984–2020
128	Unión revista iberoamericana de educación matemática	1815-0640		AC		2005–2019
129	Uno. revista de didactica de las matematicas	1133-9853		AC		1994–2019
130	Young children	0044-0728		BA		1964–2001
131	Zentralblatt fur didactic der mathematik	1863-9690	A1	AB		1997–2021
132	Zetetiké	2176-1744		AB		1993–2020

Table A2. List of journals associated to WOS and Scopus databases which appear when we search articles related with the teaching and learning of ordinary differential equations by applying the strings given in Appendix A and are not included in the list of Table A1.

Journal Title	ISSN	Journal Title	ISSN
Advances in physiology education	1043-4046	International journal of research in undergraduate mathematics education	2198-9745
American journal of physics	0002-9505	Journal of chemical education	0021-9584
Biochemistry and molecular biology education	1470-8175	Journal of professional issues in engineering education and practice	1052-3928
CBE-Life sciences education	1931-7913	Journal of science education and technology	1059-0145
Computer applications in engineering education	1061-3773	Mathematics teaching-research journal online	2573-4377
Computers & education	0360-1315	Physical review-physics education research	2469-9896
Education for chemical engineers	1749-7728	PRIMUS: problems, resources, and issues in mathematics undergraduate studies	1051-1970
Eurasia journal of mathematics science and technology education	1305-8215	Research in mathematics education	1479-4802
European journal of engineering education	0304-3797	Research in science & technological education	0263-5143
European journal of physics	0143-0807	Resonance-journal of science education	0971-8044
Global journal of engineering education	1328-3154	Revista brasileira de ensino de fisica	1806-1117
IEEE transactions on education	0018-9359	Revista cientifica	0124-2253
Information technologies and learning tools	2076-8184	Revista conrado	1990-8644
Interdisciplinary science reviews	0308-0188	Revista electronica de humanidades educacion y comunicacion social	1856-9331
International journal for technology in mathematics education	1744-2710	Revista publicando	1390-9304
international journal of education and information technologies	2074-1316	Teaching of mathematics	1451-4966
International journal of electrical engineering education	0020-7209	The American mathematical monthly	0002-9890
International journal of engineering pedagogy	2192-4880	The physics teacher	0031-921X
International journal of mechanical engineering education	0306-4190	The Turkish online journal of educational technology	2146-7242
International journal of psychosocial rehabilitation	1475-7192		

References

1. Arslan, S. Do students really understand what an ordinary differential equation is? *Int. J. Math. Educ. Sci. Technol.* **2010**, *41*, 873–888. [CrossRef]
2. Arslan, S. Traditional instruction of differential equations and conceptual learning. *Teach. Math. Its Appl.* **2010**, *29*, 94–107. [CrossRef]
3. Camacho, M.; Perdomo, J.; Santos, M. Newblock Conceptual and cognitive processes in the introduction of ordinary differential equations through problem solving. *Ensen. Las Cienc. Rev. De Investig. Exp. Didácticas* **2012**, *30*, 9–32.
4. Camacho-Machín, M.; Guerrero-Ortiz, C. Identifying and exploring relationships between contextual situations and ordinary differential equations. *Int. J. Math. Educ. Sci. Technol.* **2015**, *46*, 177–195. [CrossRef]
5. Chaachoua, H.; Saglam, A. Modelling by differential equations. *Teach. Math. Its Appl.* **2006**, *25*, 15–22. [CrossRef]
6. Cordero, F.; Miranda, E. El entendimiento de la transformada de laplace: Una epistemología como base de una descomposición genética. *Rev. Latinoam. De Investig. Matemática Educ.* **2002**, *5*, 133–168.
7. Curia, L.; Pérez, M.; Lavalle, A. Evaluation of the contents of differential equations through mathematical competences. *Rev. ElectrÓnica Humanidades Educación Comun. Soc.* **2018**, *26*, 9–27.
8. Czocher, J.A. Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking. *Math. Think. Learn.* **2016**, *18*, 77–106. [CrossRef]
9. Czocher, J.A. How can emphasizing mathematical modeling principles benefit students in a traditionally taught differential equations course?. *J. Math. Behav.* **2017**, *45*, 78–94. [CrossRef]
10. da Silva, D.; Borba, M.C. The role of software modellus in a teaching approach based on model analysis. *ZDM Math. Educ.* **2014**, *46*, 575–587.
11. Dana-Picard, T.; Kidron, I. Exploring the phase space of a system of differential equations: Different mathematical registers. *Int. J. Sci. Math. Educ.* **2008**, *6*, 695–717. [CrossRef]

12. Guerrero-Ortiz, C.; Camacho-Machín, M.; Mejía-Velasco, H.R. Difficulties experienced by students in the interpretation of the solutions of ordinary differential equations that models a problem. *Ensen. Las Cienc. Rev. Investig. Exp. Didácticas* **2010**, *28*, 341–352. [CrossRef]
13. Guerrero-Ortiz, C.; Mejía-Velasco, H.R.; Camacho-Machín, M. Representations of a mathematical model as a means of analysing growth phenomena. *J. Math. Behav.* **2016**, *42*, 109–126. [CrossRef]
14. Rasmussen, C.; Keene, K. Knowing solutions to differential equations with rate of change as a function: Waypoints in the journey. *J. Math. Behav.* **2019**, *56*, 100695. [CrossRef]
15. Rasmussen, C.L. New directions in differential equations: A framework for interpreting students' understandings and difficulties. *J. Math. Behav.* **2001**, *20*, 55–87. [CrossRef]
16. Rasmussen, C.L.; King, K.D. Locating starting points in differential equations: A realistic mathematics education approach. *Int. J. Math. Educ. Sci. Technol.* **2000**, *31*, 161–172. [CrossRef]
17. Rodríguez, R. Teaching and learning modelling: The case of differential equations. *Rev. Latinoam. Investig. Matemática Educ.* **2010**, *13*, 191–210.
18. Rodríguez-Gallegos, R.; Quiroz-Rivera, S. The role of technology in the process of mathematical modeling for teaching differential equations. *Rev. Latinoam. Investig. Matemática Educ.* **2016**, *19*, 99–124.
19. Zeynivandnezhad, F.; Bates, R. Explicating mathematical thinking in differential equations using a computer algebra system. *Int. J. Math. Educ. Sci. Technol.* **2017**, *49*, 680–704. [CrossRef]
20. Almeida, L.M.; Kato, L.A. Different approaches to mathematical modelling: Deduction of models and studens' actions. *Int. Electron. J. Math. Educ.* **2014**, *9*, 3–11.
21. Alves de Oliveira, E.; Camargo, S.B. Ensino e aprendizagem de equações diferenciais: Um levantamento preliminar da produção científica. *Teia Rev. Educ. Matemática e Tecnológica Iberoam.* **2013**, *4*, 1–24.
22. András, S.; Szilágyi, J. Modelling drug administration regimes for asthma: A romanian experience. *Teach. Math. Its Appl.* **2010**, *29*, 1–13. [CrossRef]
23. Blass, L.; Huguenim, A.F.; Irala, V.B.; da Silva, V. O estudo de equações diferenciais através da aplicação do perfil logarítmico do vento. *Teia Rev. Educ. Matemática Tecnológica Iberoam.* **2019**, *10*, 1–18. [CrossRef]
24. Dreyer, T. P. *Modelling with Ordinary Differential Equations*; CRC Press: Boca Raton, FL, USA, 1993
25. Edelstein-Keshet, L. *Mathematical Models in Biology*; Classics in Applied Mathematics, 46; Society for Industrial and Applied Mathematics (SIAM): Philadelphia, PA, USA, 2005.
26. Segel, L.A.; Edelstein-Keshet, L. *A Primer on Mathematical Models in Biology*; Society for Industrial and Applied Mathematics (SIAM): Philadelphia, PA, USA, 2013.
27. Al-Moameri, H.H.; Jaf, L.A.; Suppes, G.J. Simulation approach to learning polymer science. *J. Chem. Educ.* **2018**, *95*, 1554–1561. [CrossRef]
28. Anderson, M.; Bloom, L.; Mueller, U.; Pedler, P. Enhancing the teaching of engineering differential equations with scientificnotebook. *Int. J. Eng. Educ.* **2000**, *16*, 73–79.
29. Benacka, J. Numerical modelling with spreadsheets as a means to promote stem to high school students. *Eurasia J. Math. Sci. Technol. Educ.* **2016**, *12*, 947–964. [CrossRef]
30. Vajravelu, K. Innovative strategies for learning and teaching of large differential equations classes. *Int. Electron. J. Math. Educ.* **2018**, *13*, 91–95. [CrossRef]
31. Beier, J.C.; Gevertz, J.L.; Howard, K.E. Building context with tumor growth modeling projects in differential equations. *PRIMUS Probl. Resour. Issues Math. Undergrad. Stud.* **2015**, *25*, 297–325. [CrossRef]
32. Molina, J.A. Experience in the incorporation of ict in teaching differential equations applied. *Rev. Iberoam. Educ.* **2015**, *69*, 79–96.
33. Li, X.; Huang, Z. An inverted classroom approach to educate matlab in chemical process control. *Educ. Chem. Eng.* **2017**, *19*, 1–12. [CrossRef]
34. Yong, D.; Levy, R.; Lape, N. Why no difference? a controlled flipped classroom study for an introductory differential equations course. *PRIMUS Probl. Resour. Issues Math. Stud.* **2015**, *25*, 907–921. [CrossRef]
35. Sierpinska, A. Some Reflections on the Phenomenon of French didactique. *J. Für-Math. Didakt.* **1995**, *16*, 163–192. [CrossRef]
36. Durán, P.A.; Marshall, J.A. Mathematics for biological sciences undergraduates: A needs assessment. *Int. J. Math. Educ. Sci. Technol.* **2019**, *50*, 807–824. [CrossRef]
37. Bibi, A.; Syed, S.; Abedalaziz, N.; Ahmad, M. Teaching and Learning of Differential Equation: A Critical Review to Explore Potential Area for Reform Movement. *Int. J. Innov. Multidiscip. Field* **2017**, *3*, 225–235.
38. Khan, K.S.; Kunz, R.; Kleijnen, J.; Antes, G. Five steps to conducting a systematic review. *J. R. Soc. Med.* **2003**, *96*, 118–121. [CrossRef] [PubMed]
39. Dullius, M.M.; Araujo, I.S.; Veit, E.A. Teaching and learning of differential equations with graphical, numerical and analytical approach: An experience in engineering courses. *BOLEMA Bol. Educ. Matemática* **2011**, *24*, 17–42.
40. Dullius, M.M. Enseñanza y Aprendizaje en Ecuaciones Diferenciales con Abordaje Gráfico, Numérico y Analítico. Ph.D. Thesis, Universidad de Burgos, Burgos, Spain, 2009.
41. Valenzuela, J.R.; Flores, M. *Fundamentos de Investigación Educativa (Tres Volúmenes) (eBook)*; Editorial Digital Tecnológico de Monterrey, Monterrey, México, 2012.
42. Murphy, C.M. Writing an effective review article. *J. Med. Toxicol.* **2012**, *8*, 89–90. [CrossRef] [PubMed]

43. Narin, F.; Hamilton, K.S. Bibliometric performance measures. *Scientometrics* **1996**, *36*, 293–310. [CrossRef]
44. Ferrari, R. Writing narrative style literature reviews. *Med. Writ.* **2015**, *24*, 230–235. [CrossRef]
45. Benitti, F.B.V. Exploring the educational potential of robotics in schools: A systematic review. *Comput. Educ.* **2012**, *58*, 978–988. [CrossRef]
46. Godino, J. Perspective of the Didactics of Mathematics as a Technoscientific Discipline. Available online: http://www.ugr.es/local/jgodino (accessed on 1 July 2020). (In Spanish)
47. Running, T. A Graphical Solution of the Differential Equation of the First Order. *Am. Math. Mon.* **1913**, *20*, 279–281. [CrossRef]
48. Alvim, B.N.P.; de Almeida, L.M.W. Linguistic appropriation and meaning in mathematical modeling activities. *BOLEMA Bol. Educ. Matemática* **2019**, *33*, 1195–1214.
49. Andresen, M. Modeling with the software 'derive' to support a constructivist approach to teaching. *Int. Electron. J. Math. Educ.* **2007**, *2*, 1–15.
50. Barbarán, J.J.; Fernéz, J.A. The analysis of errors in the solution of ordinary differential equations. a methodology to develop mathematical competence. *Ensenanza Las Cienc. Rev. De Investig. Exp. Didácticas* **2014**, *32*, 173–186.
51. Barros, A.A.; Laudares, J.B.; de Miranda, D.F. A resolução de problemas em ciências com equações diferenciais ordinárias de 1^a e 2^a ordem usando análise gráfica. *Educ. Matemática Pesqui.* **2014**, *16*, 323–348.
52. Blomhøj, M.; Kjeldsen, T.H. Project organised science studies at university level: Exemplarity and interdisciplinarity. *ZDM Math. Educ.* **2009**, *41*, 183–198. [CrossRef]
53. Blomhøj, M.; Kjeldsen, T.H. Teaching mathematical modelling through project work. *ZDM Math. Educ.* **2006**, *38*, 163–177. [CrossRef]
54. Borssoi, A.H.; de Almeida, L.M.W. Modelagem matemática e aprendizagem significativa: Uma proposta para o estudo de equações diferencias ordinárias. *Educ. Matemática Pesqui.* **2004**, *6*, 91–121.
55. Brito, L.; Farias, P.H.; Cardoso, J.V.; Ribeiro, R. Teaching of ordinary differential equations using the assumptions of the pbl method. *Int. J. Eng. Pedagog.* **2020**, *10*, 7–20.
56. Bukova-Güzel, E. An examination of pre-service mathematics teachers' approaches to construct and solve mathematical modelling problems. *Teach. Math. Its Appl.* **2011**, *30*, 19–36. [CrossRef]
57. Camunga, A.; Batard, L.F. Developing skills for selecting the most appropriate method for the solution of superior order ordinary differential equations. *Rev. Conrado* **2017**, *13*, 82–88.
58. Canu, M.; de Hosson, C.; Duque, M. Students' understanding of equilibrium and stability: The case of dynamic systems. *Int. J. Sci. Math. Educ.* **2016**, *14*, 101–123. [CrossRef]
59. Carr, M.; Prendergast, M.; Breen, C.; Faulkner, F. How well do engineering students retain core mathematical knowledge after a series of high threshold online mathematics tests?. *Teach. Math. Its Appl.* **2017**, *36*, 136–150. [CrossRef]
60. Carstensen, A.-K.; Bernhard, J. Student learning in an electric circuit theory course: Critical aspects and task design. *Eur. J. Eng. Educ.* **2009**, *34*, 393–408. [CrossRef]
61. Darlington, E. What benefits could extension papers and admissions tests have for university mathematics applicants? *Teach. Math. Its Appl.* **2015**, *34*, 179–193. [CrossRef]
62. Darlington, E.; Bowyer, J. Engineering undergraduates' views of a-level mathematics and further mathematics as preparation for their degree. *Teach. Math. Its Appl.* **2016**, *36*, 200–216. [CrossRef]
63. de Almeida, L.M.W. Considerations on the use of mathematics in modeling activities. *ZDM Math. Educ.* **2018**, *50*, 19–30. [CrossRef]
64. Dehesa de Gyves, N. Discursos en los registros algebraico y geométrico de las ecuaciones diferenciales ordinarias. *Educ. Matemática* **2006**, *18*, 123–148.
65. Flegg, J.A.; Mallet, D.G.; Lupton, M. Students' approaches to learning a new mathematical model. *Teach. Math. Its Appl.* **2013**, *32*, 28–37. [CrossRef]
66. Gijsbers, D.; de Putter-Smits, L.; Pepin, B. Changing students' beliefs about the relevance of mathematics in an advanced secondary mathematics class. *Int. J. Math. Educ. Sci. Technol.* **2020**, *51*, 87–102. [CrossRef]
67. Goodchild, S.; Apkarian, N.; Rasmussen, C.; Katz, B. Critical stance within a community of inquiry in an advanced mathematics course for pre-service teachers. *J. Math. Teach. Educ.* **2020**. [CrossRef]
68. Gruenwald, N.; Sauerbier, G.; Narayanan, A.; Klymchuk, S.; Zverkova, T. Applications in unusual contexts in engineering mathematics: Students' attitudes. *Math. Teach. Res. J. Online* **2010**, *4*, 53–67.
69. Habre, S. Investigating students' approval of a geometrical approach to differential equations and their solutions. *Int. J. Math. Educ. Sci. Technol.* **2003**, *34*, 651–662. [CrossRef]
70. Habre, S. Improving understanding in ordinary differential equations through writing in a dynamical environment. *Teach. Math. Its Appl.* **2012**, *31*, 153–166. [CrossRef]
71. Habre, S. Inquiry-oriented differential equations: A guided journey of learning. *Teach. Math. Its Appl.* **2019**, *39*, 201–212. [CrossRef]
72. Habre, S. Exploring students' strategies to solve ordinary differential equations in a reformed setting. *J. Math. Behav.* **2000**, *18*, 455–472. [CrossRef]
73. Hansen, D.; Cavers, W.; George, G.H. Use of a physical linear cascade to teach systems modelling. *Int. J. Eng. Educ.* **2003**, *19*, 682–695.
74. Heck, A. Bringing reality into the classroom. *Teach. Math. Its Appl.* **2009**, *28*, 164–179. [CrossRef]

75. Herman, R. Spreadsheet physics: Examples in meteorology and planetary science. *Am. J. Phys.* **2009**, *77*, 1124–1129. [CrossRef]
76. Hernández, L. Solving simple kinetics without integrals. *J. Chem. Educ.* **2016**, *93*, 669–675. [CrossRef]
77. Holmberg, M.; Bernhard, J. University teachers' perspectives on the role of the laplace transform in engineering education. *Eur. J. Eng. Educ.* **2017**, *42*, 413–428. [CrossRef]
78. Humble, S. Rolling and spinning coin: A level gyroscopic processional motion. *Teach. Math. Its Appl.* **2001**, *20*, 18–24. [CrossRef]
79. Hyland, D.; Van Kampen, P.; Nolan, B.C. Introducing direction fields to students learning ordinary differential equations (odes) through guided inquiry. *Int. J. Math. Educ. Sci. Technol.* **2019**. [CrossRef]
80. Hyland, D.; van Kampen, P.; Nolan, B.C. Outcomes of a service teaching module on odes for physics students. *Int. J. Math. Educ. Sci. Technol.* **2018**, *49*, 743–758. [CrossRef]
81. Ju, M.-K.; Kwon, O.N. Ways of talking and ways of positioning: Students' beliefs in an inquiry-oriented differential equations class. *J. Math. Behav.* **2007**, *26*, 267–280. [CrossRef]
82. Kamps, H.J.L.; Van Lint, J.H. A comparison of a classical calculus test with a similar multiple choice test. *Educ. Stud. Math.* **1975**, *6*, 259–271. [CrossRef]
83. Karakok, G. Making connections among representations of eigenvector: What sort of a beast is it? *ZDM Math. Educ.* **2019**, *51*, 1141–1152. [CrossRef]
84. KarimiFardinpour, Y.; Gooya, Z. Comparing three methods of geometrical approach in visualizing differential equations. *Int. J. Res. Undergrad. Math.* **2018**, *4*, 286–304. [CrossRef]
85. Kaw, A.K.; Yalcin, A. Problem-centered approach in a numerical methods course. *J. Prof. Issues Eng. Pract.* **2008**, *134*, 359–364. [CrossRef]
86. Keene, K.A. A characterization of dynamic reasoning: Reasoning with time as parameter. *J. Math. Behav.* **2007**, *26*, 230–246. [CrossRef]
87. Keene, K.A.; Rasmussen, C.; Stephan, M. Gestures and a chain of signification: The case of equilibrium solutions. *Math. Educ. Res. J.* **2012**, *24*, 347–369. [CrossRef]
88. Klymchuk, S.; Zverkova, T.; Gruenwald, N.; Sauerbier, G. Increasing engineering students' awareness to environment through innovative teaching of mathematical modelling. *Teach. Math. Its Appl.* **2008**, *27*, 123–130. [CrossRef]
89. Kwon, O.N.; Rasmussen, C.; Allen, K. Students' retention of mathematical knowledge and skills in differential equations. *Sch. Sci. Math.* **2005**, *105*, 227–239. [CrossRef]
90. Latulippe, C.; Latulippe, J. Student perceptions of writing projects in a university differential-equations course. *Int. J. Math. Educ. Sci. Technol.* **2013**, *45*, 1–11. [CrossRef]
91. Laudares, B.J.; de Miranda, D.F. Investigando a iniciação à modelagem matemática nas ciências com equações diferenciais. *Educ. Matemática Pesqui.* **2007**, *9*, 103–120.
92. Lee, W.-P.; Lu, M.-S. A digital simulation of the vibration of a two-mass two-spring system. *Comput. Appl. Eng. Educ.* **2010**, *18*, 563–573. [CrossRef]
93. Lewis, M.; Powell, J.A. Modeling zombie outbreaks: A problem-based approach to improving mathematics one brain at a time. *PRIMUS Probl. Resour. Issues Math. Stud.* **2016**, *26*, 705–726. [CrossRef]
94. Liberatti, S.; da Silva, D. Mathematical modeling and analysis of mathematical models in mathematics education. *Acta Sci.* **2012**, *14*, 260–275.
95. Maat, S.M.; Zakaria, E. Exploring students' understanding of ordinary differential equations using computer algebraic system (cas). *Turk. Online J. Educ. Technol.* **2011**, *10*, 123–128.
96. Mallet, D.G.; McCue, S.W. Constructive development of the solutions of linear equations in introductory ordinary differential equations. *Int. J. Math. Educ. Sci. Technol.* **2009**, *40*, 587–595. [CrossRef]
97. Marrongelle, K. The function of graphs and gestures in algorithmatization. *J. Math. Behav.* **2007**, *26*, 211–229. [CrossRef]
98. Melendez, B.; Bowman, S.; Erickson, K.; Swim, E. An integrative learning experience within a mathematics curriculum. *Teach. Math. Its Appl.* **2009**, *28*, 131–144. [CrossRef]
99. Miller, H.R.; Upton, D.S. Computer manipulatives in an ordinary differential equations course: Development, implementation, and assessment. *J. Sci. Educ. Technol.* **2008**, *17*, 124–137. [CrossRef]
100. Molina-Mora, J.A. Mathematical modeling as a didactic strategy for calculus teaching. *Uniciencia* **2017**, *31*, 19–36. [CrossRef]
101. Molina-Mora, J.-A. ICT-projects-modeling based experience for teaching of systems of differential equations. *Uniciencia* **2015**, *29*, 46–61.
102. Nachtigalova, I.; Finkeova, J.; Krbcova, Z.; Souskova, H. A spreadsheet-based tool for education of chemical process simulation and control fundamentals. *Comput. Appl. Eng. Educ.* **2020**, *28*, 923–937. [CrossRef]
103. Padmanabhan, S.; Solin, P. Enhancing student learning of differential equations through technology. *Int. J. Technol. Math. Educ.* **2017**, *24*, 207–216.
104. Pino-Fan, L.R.; Font, V.; Gordillo, W.; Larios, V.; Breda, A. Analysis of the meanings of the antiderivative used by students of the first engineering courses. *Int. J. Sci. Math. Educ.* **2018**, *16*, 1091–1113. [CrossRef]
105. Quinn, D.; Aarão, J. Blended learning in first year engineering mathematics. *ZDM Math. Educ.* **2020**, *52*, 927–941. [CrossRef]
106. Rasmussen, C.; Stephan, M.; Allen, K. Classroom mathematical practices and gesturing. *J. Math. Behav.* **2004**, *23*, 301–323. [CrossRef]

107. Rasmussen, C.; Zandieh, M.; King, K.; Tepo, A. Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Math. Think. Learn.* **2005**, *7*, 51–73. [CrossRef]
108. Rasmussen, C.; Marrongelle, K. Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *J. Res. Math. Educ.* **2006**, *37*, 388–420
109. Rasmussen, C.; Blumenfeld, H. Reinventing solutions to systems of linear differential equations: A case of emergent models involving analytic expressions. *J. Math. Behav.* **2007**, *26*, 195–210. [CrossRef]
110. Rasmussen, C.; Kwon, O.N. An inquiry-oriented approach to undergraduate mathematics. *J. Math. Behav.* **2007**, *26*, 189–194. [CrossRef]
111. Raychaudhuri, D. A layer framework to investigate student understanding and application of the existence and uniqueness theorems of differential equations. *Int. J. Math. Educ. Sci. Technol.* **2007**, *38*, 367–381. [CrossRef]
112. Raychaudhuri, D. Dynamics of a definition: A framework to analyse student construction of the concept of solution to a differential equation. *Int. J. Math. Educ. Sci. Technol.* **2008**, *39*, 161–177. [CrossRef]
113. Raychaudhuri, D. Adaptation and extension of the framework of reducing abstraction in the case of differential equations. *Int. J. Math. Educ. Sci. Technol.* **2013**, *45*, 35–57. [CrossRef]
114. Raychaudhuri, D. A framework to categorize students as learners based on their cognitive practices while learning differential equations and related concepts. *Int. J. Math. Educ. Sci. Technol.* **2013**, *44*, 139–1256. [CrossRef]
115. Ribaric, S.; Kordas, M. Teaching cardiovascular physiology with equivalent electronic circuits in a practically oriented teaching module. *Adv. Physiol. Educ.* **2011**, *35*, 149–160. [CrossRef]
116. Robinson, G.; Jovanoski, Z. Fighter pilot ejection study as an educational tool. *Teach. Math. Its Appl.* **2010**, *29*, 176–192. [CrossRef]
117. Rooch, A.; Junker, P.; Härterich, J.; Hackl, K. Linking mathematics with engineering applications at an early stage - implementation, experimental set-up and evaluation of a pilot project. *Eur. J. Eng. Educ.* **2016**, *41*, 172–191. [CrossRef]
118. Rowland, D.R. Student difficulties with units in differential equations in modelling contexts. *Int. J. Math. Educ. Sci. Technol.* **2006**, *37*, 553–558. [CrossRef]
119. Rowland, D.R.; Jovanoski, Z. Student interpretations of the terms in first-order ordinary differential equations in modelling contexts. *Int. J. Math. Educ. Sci. Technol.* **2004**, *35*, 503–516. [CrossRef]
120. Ruiz, L.; Gallardo, P.C.; Del Rivero, S. Deficient prerequisites with mathematical software in new concepts: Laplace transform. *Rev. Mex. Investig. Educ.* **2016**, *21*, 349–383.
121. Sabag, N. The effect of integrating lab experiments in electronic circuits into mathematic studies—A case study. *Res. Sci. Technol. Educ.* **2017**, *35*, 427–444. [CrossRef]
122. Sawaki, R.V.; Tannous, K.; Filho, J.B.F. Development of an educational tool aimed at designing ideal chemical reactors. *Comput. Appl. Eng. Educ.* **2020**, *28*, 459–476. [CrossRef]
123. Slavit, D.; LoFaro, T.; Cooper, K. Understandings of solutions to differential equations through contexts, web-based simulations, and student discussion. *Sch. Sci. Math.* **2002**, *102*, 380–390. [CrossRef]
124. Speer, N.M.; Wagner, J.F. Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *J. Res. Math. Educ.* **2009**, *40*, 530–562. [CrossRef]
125. Stephan, M.; Rasmussen, C. Classroom mathematical practices in differential equations. *J. Math. Behav.* **2002**, *21*, 459–490. [CrossRef]
126. Tisdell, C.C. On mnemonic instruction and the shields acronym in the pedagogy of first-order differential equations. *Teach. Math. Its Appl.* **2019**, *38*, 74–84. [CrossRef]
127. Trigueros, M. Vínculo entre la modelación y el uso de representaciones en la comprensión de los conceptos de ecuación diferencial de primer orden y de solución. *Educ. Matemática* **2014**, *25*, 207–226.
128. Vega-Calderon, F.; Gallegos-Cazares, L.; Flores-Camachoc, F. Conceptual difficulties in understanding the bernoulli's equation. *Rev. Eureka Sobre Ensenanza Divulg. Las Ciencias* **2017**, *14*, 339–352.
129. Villar, M.T.; Llinares, S. Análisis de errores en la conceptualizacion y simbolizacion de ecuaciones diferenciales en alumnos de química. *Educ. Matemática* **1996**, *8*, 90–101.
130. Vinner, S. A different test and different result analysis— An example from a calculus exam. *J. Für-Math. Didakt.* **1994**, *15*, 311–326. [CrossRef]
131. Wittmann, M.C.; Black, K.E. Mathematical actions as procedural resources: An example from the separation of variables. *Phys. Rev. Spec. Top. Phys. Educ. Res.* **2015**, *11*, 1–13. [CrossRef]
132. Wittmann, M.C.; Flood, V.J.; Black, K.E. Algebraic manipulation as motion within a landscape. *Educ. Stud. Math.* **2013**, *82*, 169–181. [CrossRef]
133. Yackel, E.; Rasmussen, C.; King, K. Social and sociomathematical norms in an—Advanced undergraduate mathematics course. *J. Math. Behav.* **2000**, *19*, 275–287. [CrossRef]
134. Zang, C.M.; Fernández, G.A.; León, M.N. Reflexiones sobre la implementación de problemas de modelado para la construcción y resignificación de objetos matemáticos vinculados a las ecuaciones diferenciales. *UNION Rev. Iberoam. Educ. Matemática* **2015**, *42*, 150–165.
135. Zhu, N. Embedding active learning and design-based projects in a noise and vibration course for the undergraduate mechanical engineering program. *Int. J. Mech. Eng. Educ.* **2020**. [CrossRef]

136. Czocher, J.A.; Tague, J.; Baker, G. Where does the calculus go? An investigation of how calculus ideas are used in later coursework. *Int. J. Math. Sci. Technol.* **2013**, *44*, 673–684. [CrossRef]
137. Artigue, M.; Gautheron, V. *Systemes Differentiels: Etude Graphique*; Cedic: Paris, France, 1983.
138. Artigue, M. Une recherche d' ingenierie didactique sur l'enseignement des equations differentielles en primer cycle universitarie. *Irem Univ. Paris* **1989**, *107*, 284–309.
139. Hernandez A. Obstáculos en la Articulacion de los Marcos Numérico, Gráfico y Algebraíco en Relación con las Ecuaciones Diferenciales. Ph.D. Thesis, Cinvestav, Mexico City, Mexico, 1995
140. Blanchard, P. Teaching differential equations with a dynamical systems viewpoint. *Coll. Math. J.* **1994**, *25*, 385–393. [CrossRef]
141. CMBS. 2016. Available online: http://www.cbmsweb.org/archive/Statements/Active_Learning_Statement.pdf (accessed on 1 September 2020).
142. Barrows, H.S. A Taxonomy of problem-based learning methods. *Med. Educ.* **1986** *20*, 481–486. [CrossRef]
143. Leung, F.; Stillman, G.; Kaiser, G.; Won, K. (Eds.) Mathematical modelling education in east and west. In *International Perspectives on the Teaching and Learning of Mathematical Modelling*; Springer: Dordrecht, The Netherlands, 2021.
144. Stillman, G.; Blum, W.; Kaiser, G. (Eds.) Mathematical Modelling and Applications. In *ICTMA 17, International Perspectives on the Teaching and Learning of Mathematical Modelling*; Springer: Dordrecht, The Netherlands, 2017.
145. Galbraigth, P. Models of Modelling: Genres, Purposes or Perspectives. *J. Math. Model. Appl.* **2012**, *1*, 3–16.
146. Borromeo, R. Theoretical and empirical differentiations of phases in the modelling process. *ZDM Math. Educ.* **2006**, *38*, 86–95.
147. Zbiek, R.M.; Conner, A. Beyond motivation: Exploring mathematical modeling as a context for deepening students' understandings of curricular mathematics. *Educ. Stud. Math.* **2006**, *63*, 89–112. [CrossRef]
148. Hoyles, C. Transforming the mathematical practices of learners and teachers through digital technology. *Res. Math. Educ.* **2018**, *20*, 209–228. [CrossRef]
149. Artigue, M. Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *Int. J. Comput. Math. Learn.* **2002**, *7*, 245–274. [CrossRef]
150. Cohen, M.S.; Gaughan, E.D.; Knoebel, A.; Kurtz, D.S.; Penegelley, D.J. *Student Research Projects in Calculus;* Mathematical Association of America: Washington, DC, USA, 1991