

Article

The Markovian Pattern of Social Deprivation for Mexicans with Diabetes

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Abstract: This paper aims to determine the Markovian pattern of the factors influencing social deprivation in Mexicans with Type 2 diabetes mellitus (DM2). To this end, we develop a methodology to meet the theoretical and practical considerations involved in applying a Hidden Markov Model that uses non-panel data. After estimating the latent states and ergodic vectors for diabetic and non-diabetic populations, we find that the long-term state-dependent probabilities for people with DM2 show a darker perspective of impoverishment than the rest of the Mexican population. In the absence of extreme events that modify the present probability structure, the Markovian pattern confirms that people with DM2 will most likely become the poorest of Mexico's poor.

Keywords: diabetes mellitus; social deprivation; Hidden Markov model; state-dependent probabilities; ergodic vectors

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1. Introduction

Diabetes is a chronic non-communicable disease that occurs when the pancreas does not produce enough insulin (type 1), or the body cannot effectively use the insulin it produces (type 2). Insulin is a hormone that allows glucose to enter cells and regulate it in the bloodstream. When this process is normal, beta cells stop producing insulin once blood glucose drops. In people with diabetes, the process does not happen this way because the immune system mistakenly destroys beta cells (type 1), or these cells stop releasing the amount of the hormone demanded by the body (type 2). After a latency period, excessive glucose accumulation in diabetic patients' blood leads to retinopathy, nephropathy, hypertension, amputation of limbs, cardiovascular and neurological disorders, and, in many cases, premature death. The disease's causes result from genetic and environmental factors, unhealthy lifestyles, and high-risk behaviors [1].

The number of diabetics in the world is alarming due to its growth rate. Between 1980 and 2014, their number nearly quadrupled, from 108 to 422 million, and cases are expected to reach 552 million by 2030 if preventive measures are not taken [2,3]. Of the total number of patients, 85–90% suffer from Type 2 diabetes mellitus (DM2), making it one of the leading international causes of morbidity, mortality, and lost labor force productivity. The economic costs associated with the disease are stratospheric worldwide, with \$376 billion spent in 2010, while the projected figure for 2030 is \$490 billion. This financial burden is heavy for low and middle-income countries, where 75% of people with diabetes are concentrated, and the projected growth of cases for the next 25 years is around 150% [2,3].

Mexico is a paradigmatic case of the panorama just described. According to IDF data [2,3], the country ranks sixth globally in the prevalence of diabetes, with 11.4 million people affected by the disease in 2012 and an estimated 17.5 million people affected by 2040.

As in the rest of the world, DM2 is the most important variant in the country, as it represents the second cause of mortality and the first in years of healthy life lost. With a prevalence rate of 14.8%, DM2 is also the leading cause of kidney failure, acquired blindness, and non-traumatic limb amputations that recently occurred in Mexico [4]. The direct and indirect costs associated with the disease amount, on average, to USD 1.12 billion annually, without considering the expenses derived from its complications [5]. In microeconomic terms, these figures are equivalent to 15% of health expenses by families or, if we consider regional disparities, a much higher percentage in the country's southern zones, where the disease has grown faster. According to Reference [4], DM2 growth rates in those areas were around 138%, compared to 32.5% for its northern counterpart between 1980 and 2000.

This document focuses on studying the environmental causes of DM2 in Mexico and aims to estimate the stationary probabilities of exposure to two populations' social deprivation: diabetics and non-diabetics. The idea is to differentiate the diabetics' probabilistic profile from that of the rest of Mexicans to understand their risk-factors' specific environmental conditions (very often associated with the metabolic syndrome). Among these factors, the literature highlights those linked to the direct causes of diabetes, such as sex, age, waist circumference, body mass index, levels of glucose, cholesterol, diastolic pressure, hypertension, family history, sedentary lifestyle, alcoholism, smoking, main intake and type, and regularity of diet [6].

The importance of the objective is unquestionable since, without knowledge of the social and economic context that determines lousy eating habits or deficient access to medical services, it is impossible to deepen our understanding of the direct causes of diabetes. For this reason, obtaining the stationary probabilities of social deprivation constitutes the first contribution of this document by offering a future scenario of diabetics' social conditioning. For this purpose, we adopt a Markovian approach because the Markov property perfectly mirrors DM2's dynamics. Like other phenomena such as the learning process, the spread of epidemics, or the pricing behavior of financial assets, its evolution depends critically on the present state's information. The random variables characterizing the DM2's evolution behave if they had "current absolute memory," meaning that all required to predict their next stages is an appropriate initial "state of the world" within a state space and a probability distribution as a rule of change. Past states of the world do not add relevant information to the present state in the prediction process. To illustrate this property, let us think of a contagion model in which the probability of being infected tomorrow depends entirely on today's transition probabilities.

As a chronic disease, DM2 is repetitive by nature, making it also susceptible to being studied using Markov models. These models allow a flexible sequencing of outcomes associated with the disease's progression or regression states through time [7]. Among those outcomes, the costs of therapies or the results of treatments stand out. Different variations of Markov models can evaluate the resulting transition probabilities to determine if such therapies have been successful, according to budget restrictions at some point.

The paper's second contribution is to meet the theoretical and practical considerations involved in the correct use of a Markovian approach. Most applied studies rarely address this critical issue. The proper use of any member of the Markov model's family requires specific justification and methodology. It is not indistinct to prefer one model over another to analyze the same phenomenon. Results can change dramatically. This paper addresses both types of considerations when the Hidden Markov model (HMM) uses non-panel data, and the process parameter is a binomial distribution. The idea of the methodology presented here is that there is no way to substantiate an adequate HMM application without fulfilling certain theoretical and practical prerequisites. If the HMM's results contradict the prerequisites or vice versa, both need a review. Our methodology's novelty lies in showing that the HMM and its prerequisites are part of the same process. One needs the other for a successful HMM application.

Finally, the third contribution relates to the paper's results. In some cases, Markov models' application intends to provide evidence for a practical exercise and, in other cases, to test a theoretical hypothesis. Examples of both situations are the forecast of disease prevalence rates and the efficient market hypothesis, in which it is assumed that the prices of financial assets follow the Markov property. Our paper falls in the second category and seeks to prove that diabetes expresses extreme poverty in Mexico. In particular, it reveals that, since the general population experiences mainly deprivation in health services and education, there is a greater probability that the number of diabetics in the country will soon increase.

The document consists of three additional sections and the conclusions. The second offers a brief review of the literature on the application of Markov models in the study of diabetes. The third focuses on the prerequisites for applying any HMM using non-panel data. The fourth section presents the HMM results: the optimal number of states, the ergodic vectors, the sensitivity analysis of the state-dependent probabilities to changes in risky behaviors (obesity, hypertension, alcohol consumption, and smoking), and a general discussion on the statistical analysis results. The conclusions summarize the main findings of the paper.

2. Literature Review

Markov models have been applied to the study of diabetes in a very similar fashion to the general works on the subject following two paths: that is, on the one hand, to investigate the direct causes of DM2 and, on the other hand, to explain the relationship between the disease and the living conditions of patients (for general works, see References [6,8–10]). Thus, we have authors who use continuous Markov models to detect states of progression and regression of diabetes in the face of lifestyle changes [1] or Markov and Blanket-type decision processes to predict, respectively, the influence of health management and other behavioral factors on the complications of DM2 [11,12]. Additionally, some studies use Markov models to predict the prevalence of diabetes according to specific sociodemographic characteristics [13] and multi-cohort models to evaluate the effects of clinical and social variables on the disease's evolution: normal, pre-diabetic, and diabetic [14].

These studies' main results are auspicious because the different variants of Markov models offer a dynamic picture of the causes of each health status patient that is hard to obtain by other means. With the support of clustering methods, the models can assist, for example, in the identification and treatment of groups of people exposed to high-risk behaviors (smoking or high-fat intake) in various stages of the disease, whether or not they have comorbidities [15]. Similarly, Markov models offer very accurate prevalence predictions throughout the health-disease process trajectory [16] and describe, quite diligently, the costs and benefits of a specific therapy on population groups with different sociodemographic characteristics [7].

In Mexico's particular case, the conclusions' scope is not so broad as in the international context due to these models' limited application. However, even so, the results are very significant because they not only identify similar patterns between the prevalence and incidence rates of DM2 by sex and age group [14] but also establish an inverse relationship between their state of deprivation social status, and the condition of being or not diabetic [17]. Specifically, in the last cited work, a relevant methodology is used for our purposes. Based on a list of social items, the authors construct three states of deprivation (lacking, moderate poor, and extreme poor). Using a hidden Markov model, they conclude that rural diabetics experience a greater probability of remaining moderate and extreme poor than non-diabetics. Despite the value of the method and the conclusion, the paper has limitations that have to do with the authors' information based on the Encuesta Nacional sobre Niveles de Vida de los Hogares (ENNVIH) [18]. This survey does not have an exhaustive battery of questions about diabetes or data from Mexican people without

deprivation. Therefore, the conclusion is limited to the lacking or poor population. In addition to this problem, the authors do not include the income variable in the state's construction, making the definition of moderate and extreme poor not comparable to that used by the government body in charge of measuring poverty [19].

We seek to overcome these limitations using the Encuesta Nacional de Salud y Nutrición (ENSANUT) [20] for more recent periods (between 2000 and 2018) and a four-state Markov model. Figure 1 shows the general model we use to make state diagrams for Markov chains. Given that the model's states are recurrent and the whole chain is irreducible, all the four categories into which we divide the population (non-deprived, lacking, moderate poor, and extreme poor) interact. Non-deprived diabetics, for example, can remain in the same state or turn into lacking diabetics or vice versa with some transition probabilities (arrows in the figure can go around the same circle or in both directions). As the matrices analyzed below fit this model, the diabetic and non-diabetic populations can interchangeably worsen or improve their social deprivation probabilities before reaching the ergodic values. There are no absorbing states.

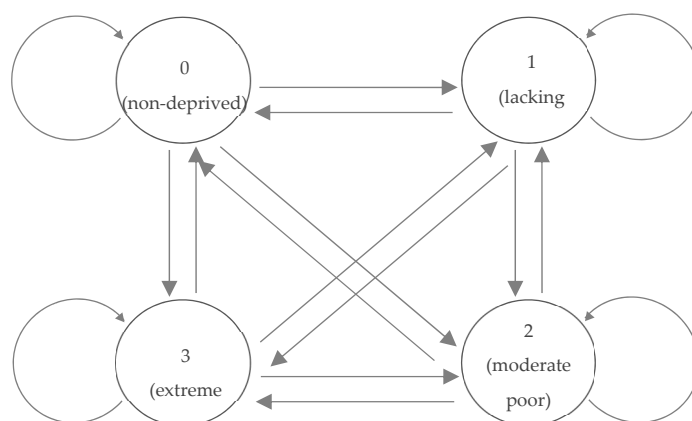


Figure 1. A four-state Markov model for diabetics and non-diabetics.

3. The Prerequisites for Applying the HMM Using Non-Panel Data

Figure 2 establishes that the exhaustive use of the HMM presupposes at least three methodological stages: one before its implementation and two during and after it. The first stage is related to the fulfillment of the HMM assumptions. Any application must comply with theoretical and practical considerations, even though most applied research overlooks it. The former reduces to show that the phenomenon analyzed behaves as a homogeneous Markov chain and the latter to constructing the cohort of individuals or households exposed to the phenomenon throughout the study period. The robustness of the HMM results critically depends on both considerations.

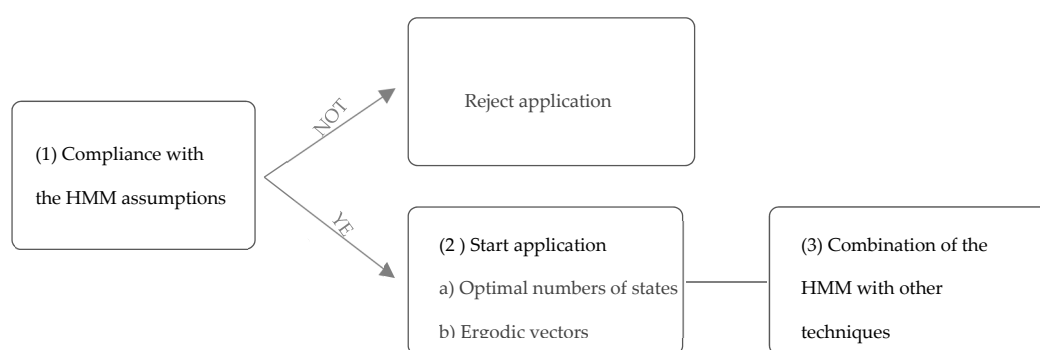


Figure 2. Methodological stages in the application of the HMM.

3.1. Theoretical Considerations in the Application of the HMM

Showing that social deprivation behaves as a one-step homogeneous chain requires weighing: (1) the deprivation modeling proposal, (2) the state-dependent probability distribution, and (3) the method of calculating the hidden states and matrices associated with a specific probability distribution. These three aspects constitute theoretical prerequisites for applying the HMM to the diabetic and non-diabetic populations.

The first point to highlight in (1) is the family of random variables and the justification of its dynamics as a Markov chain. Specifically, we define the random variable X as the “exposure to social deprivation” that maps the elements of the sample space $\Omega = \{E_0, \dots, E_6\}$ on a sequence of natural numbers $1, \dots, N$. The sample space is made up of the events that record exposure to no deprivation $\{E_0\}$ or one of the following six deprivations $\{E_1, \dots, E_6\}$ contained in Reference [20]: (1) educational lag, (2) null rights to medical services, (3) inadequate access to social security, (4) housing with little space and of low quality, (5) housing without public services, and (6) food insecurity. Since this paper is interested in the number of deprivations not using a weighting scheme to build a poverty index, each event has the same weight.

The sum of these events corresponds to the set of individuals, denoted by N , that make up the sample universe at the beginning of the survey dates (2000, 2006, 2012, 2016, and 2018). To allow changes in the structure of events over time, we assume that X has an inverse image for each subset of Borel $B \in \beta$ in a given sigma-field \mathfrak{F} . The sigma-field contains all possible combinations of the elements of Ω such that $X^{-1}(B) \in \mathfrak{F}$, where B is any numerical arrangement between the events.

We propose to model the dynamics of the variable X 's family as a Markov chain because Equations (1) and (2) fit the needs of the objective of the document entirely. In other words, the two equations that define a one-step homogeneous Markov chain $\{C_n\}$ allow us to obtain the probabilities of exposure to social deprivation from a list of deprivations of a fixed period of 2000–2018 (Equation (1)), given that the economic conditions are such that they do not change the probability's structure (Equation (2)). The list of deprivations serves as the basis for constructing a transition matrix among states belonging to a discrete state space E .

$$P\{X_{n+1} = s_j | X_0 = s_{i_0}, X_1 = s_{i_1}, \dots, X_n = s_{i_n}\} = P\{X_{n+1} = s_j | X_n = s_{i_n}\} \quad (1)$$

$$\forall s_{i_0}, \dots, s_{i_n} \in E$$

$$P\{X_1 = s_j | X_0 = s_i\} = P\{X_{n+1} = s_j | X_n = s_i\} \quad (2)$$

The statistical analysis performed in Section 3.2 supports this modeling proposal in studying social deprivation in Mexico. The results of that section confirm that the transition matrices based on observable data behave like an ergodic $\{C_n\}$. Both tables and figures back one of the assumptions required to apply the HMM: that of considering that an unobserved $\{C_n\}$ describes the phenomenon under study with an ergodic vector as its initial distribution. The property of ergodicity is inherent in such matrices because their states form a final class within a closed set C such that $\sum_{s_j \in C} P(s_i, s_j) = 1 \quad \forall s_i \in C$. These are regular matrices, whose vectors of stationary probabilities are estimated using Equation (3) or exponentiating the original matrix (also called the Chapman-Kolmogorov Equation (4), which calculates the number of stages to reach the ergodic values).

$$\delta(j) = \lim_{n \rightarrow \infty} P\{X_n = s_j | X_0 = s_i\} \quad (3)$$

$$P\{X_n = s_j | X_0 = s_i\} = P^n(s_i, s_j) \quad (4)$$

When it comes to aspect (2), it is essential to make explicit the procedure to obtain the state-dependent probabilities ${}_n\pi_{s_i}$. The whole process begins by proposing a set of probabilities distribution to generate a discrete random sequence $\{S_n\}$ [21]. The importance of correctly choosing this set is that $\{S_n\}$ takes specific value s with ${}_n\pi_{s_i}$ given

that $C_n = i$, depending on the distribution of the sequence considered. Hence, for the state-dependent probabilities to have a proper meaning in any study, a justification about the adopted conditional distribution must be offered. In our case, the binomial distribution choice is almost direct because the decision process is binary. It consists of determining the probability of success p_i (suffering a certain number of deprivations) or failure $1 - p_i$ (not suffering a certain number of deprivations). In this way, the ${}_n\pi_{s_i}$ are calculated according to Equation (5).

$$P(S_n = s | C_n = i) = {}_n\pi_{s_i} = \binom{n}{s} p_i^s (1 - p_i)^{n-s} \quad (5)$$

The state-dependent probabilities distribution (5) is ideal for binary variables, such as deprivations. Deprivations take the value of one when there is a certain number of them and zero in the opposite case. This way, equation (5) can assign defined values of ${}_n\pi_{s_i}$ to each of the four states unambiguously. Another possibility of generating the state-dependent probabilities is by using an activation function of a neural network. However, the advantages of that function over a binomial are not clear. In any case, it must be clear that the generating function of ${}_n\pi_{s_i}$ has to consider normal-binary and not rare events. For that reason, values of ${}_n\pi_{s_i}$ cannot admit, for instance, a Poisson distribution.

Finally, aspect (3) refers to estimating the state-dependent probability matrix Π , the hidden ergodic vector, and the binomial distribution parameters. On this point, it is worth noting that there are several computational methods. However, we opted for the Expectation-Maximization (EM) algorithm for the economy of its computation. This algorithm maximizes the conditional likelihood pseudo-function (6) in two stages. In the first stage (stage E), the algorithm estimates the parameters of the function $\theta^* = \theta_i$ and, in the second (stage M), it finds the maximum values of $\hat{\theta}$ for certain θ^* and selects the hidden states \hat{N}^* , Π , and the stationary vectors using information criteria. In this document, we set $\theta_i = 0.5$ and run the total program in the R library for Markov chains (<https://cran.r-project.org/web/packages/depmixS4/index.html>, accessed on 18 November 2020).

$$Q(\theta, \theta_0) = E_{\theta_0}[\sum_{t=1}^T \ln(P[X_t, S_t, \theta | \theta_0, X_{1:t-1}])], \quad (6)$$

where $Q(\theta, \theta_0)$ is the conditional expected value, θ_0 is the set of initial parameters (specifically, we consider $\frac{1}{n}$ as the initial value of the probabilities, in which n is the number of assumed states), X_t is the vector of selected conditional probabilities, and S_t , $t \in [1, 2, \dots, n]$ is the discrete sequence [22].

3.2. Practical Considerations in the Application of HMM: Data and Treatment Method

Among the practical prerequisites, the cohort of individuals exposed to the same phenomenon in equidistant periods stands out. Without panel data or cross-section information divided by equal time intervals, the HMM would be of little utility because the estimation of any phenomenon's dynamics would present strong biases. This paper uses Reference [20], which, unlike longitudinal surveys (such as the ENNVIIH), it is not a panel type that allows monitoring of the same households of diabetics over time. Instead, it is a cross-sectional survey in which demographic, nutritional, and health information involves different households in each wave (2000, 2006, 2012, 2016, and 2018).

ENSANUT [20] collects information from two questionnaires (one for health and one for nutrition), on-site measurements (anthropometry, blood pressure, hemoglobin, and capillary lead), and biological samples to explore undetected diseases. The sample units are households whose respondents are divided into children (at most ten-years-old), adolescents (11–19 years old), and adults (at least 20 years old). This paper's information mainly comes from the health questionnaire applied to the last group (see Table 1). Adults belong to the survey's universal sample for the health questionnaire consisting of 45,726 (in 2000), 48,304 (in 2006), 50,528 (in 2012), 9474 (in 2016), and 50,562 (in 2018) households spread across Mexico's 32 entities.

Table 1. The universe of individuals aged 20 years and over in different rounds.

| | Sample | |
|------|----------|-------------|
| | Diabetes | No Diabetes |
| 2000 | 2956 | 41,151 |
| 2006 | 2965 | 42,175 |
| 2012 | 4490 | 41,787 |
| 2016 | 972 | 7643 |
| 2018 | 5893 | 37,177 |

Source: Own elaboration based on ENSANUT 2000–2018.

Table A1 of Appendix A displays all the information included in this study. The table contains the definitions relevant to deprivations and variables considered in the health questionnaires for adults and in general. Particularly, we use the information from the section on preventive programs and medical diagnosis of chronic diseases (diabetes, hypertension, and obesity) of the adult health questionnaire and the general questionnaire's socio-demographic data.

With all this information, we build a pseudo-panel in three steps. First, following Reference [17,19], we group figures from Table 1 to the four states shown in Figure 1. These states are defined as follows: state 0 (non-deprived), state 1 (lacking or individuals with at least one deprivation, but with income above the minimum welfare line), state 2 (moderate poor or individuals with one or two deprivations and income below the minimum welfare line) and state 3 (extreme poor or individuals with three or more deprivations and income below the minimum welfare line). Data on incomes referred to as the minimum welfare line come from Reference [19]. According to Reference [17], the optimal size to model the population's deprivation that suffers at least one social deprivation in Mexico is three states. Therefore, if we add the non-deprived population, then the selected number is justifiable.

The second step consists of matching the observations using the nearest neighbor method proposed by Reference [23]. The variables chosen for matching are the sociodemographic ones listed in Table A1. The cohort resulting from this match comprises 8519 adults (876 diabetics and 7643 non-diabetics), who are precisely those who retain the characteristics most closely related throughout the period. Table 2 summarizes the basic statistics of the sample. Since most of the deprivations, variables, and risk-factors in the table are binary, their interpretation is easy. If we focus on the deprivations and risk-factor categories, we see that people with diabetes are more deprived and have more comorbidities than the rest of the population. For example, diabetics experience more educational lag (31% of them) and obesity (53% of them) than non-diabetics (29% and 45%, respectively) on average. Likewise, people with diabetes are less educated (4.43 years) than the rest (4.96 years) and register larger bounded coefficients of variation (BCV) in most of the categories. If we choose hypertension, we observe that diabetics' BCV are almost twice that of non-diabetics (0.85 vs. 0.46), which means that this comorbidity distribution is more dispersed for the first group. A small proportion of diabetic people suffer more hypertension than the rest, including both diabetics and non-diabetics.

Table 2. Basic statistics of the sample (2000–2018).

| Deprivations | ENSANUT 2000-2018 | | | | | | | | |
|--------------------------------------|------------------------|--------------------|-------|----------|--------------------|-------|-------------|--------------------|-------|
| | Full Sample | | | Diabetes | | | No Diabetes | | |
| | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV |
| Educational lag | 0.31 | 0.40 | 0.42 | 0.34 | 0.42 | 0.44 | 0.29 | 0.38 | 0.41 |
| Access to health services | 0.22 | 0.28 | 0.29 | 0.23 | 0.28 | 0.30 | 0.21 | 0.27 | 0.28 |
| Access to social security | 0.24 | 0.34 | 0.35 | 0.26 | 0.36 | 0.37 | 0.22 | 0.32 | 0.36 |
| Quality and spaces in the home | 0.14 | 0.32 | 0.36 | 0.15 | 0.34 | 0.38 | 0.14 | 0.31 | 0.34 |
| Access to basic services in the home | 0.39 | 0.19 | 0.20 | 0.40 | 0.19 | 0.21 | 0.37 | 0.18 | 0.18 |
| Access to food | 0.40 | 0.42 | 0.48 | 0.41 | 0.45 | 0.51 | 0.39 | 0.40 | 0.47 |
| Diabetes and some risk-factors | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV |
| | Diabetes | 0.11 | 0.29 | 0.63 | -- | -- | -- | -- | -- |
| | Obesity and overweight | 0.48 | 0.40 | 0.64 | 0.53 | 0.48 | 0.82 | 0.45 | 0.57 |
| Hypertension | 0.21 | 0.42 | 0.49 | 0.23 | 0.50 | 0.85 | 0.18 | 0.39 | 0.46 |
| Alcohol | 0.53 | 0.47 | 0.63 | 0.47 | 0.50 | 0.78 | 0.52 | 0.34 | 0.53 |
| Tobacco | 0.16 | 0.28 | 0.62 | 0.15 | 0.26 | 0.56 | 0.17 | 0.37 | 0.62 |
| Sociodemographic variables | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV | Mean | Standard Deviation | BCV |
| | Sex | 0.49 | 0.41 | 0.86 | -- | -- | -- | -- | -- |
| | Age | 46.32 | 16.66 | 0.33 | 48.34 | 16.15 | 0.28 | 45.65 | 17.33 |
| Years of education | 4.50 | 4.97 | 0.86 | 4.43 | 5.00 | 0.82 | 4.96 | 4.85 | 0.92 |
| Single | 0.25 | 0.37 | 0.69 | 0.25 | 0.37 | 0.69 | 0.24 | 0.37 | 0.66 |
| Married | 0.57 | 0.46 | 0.69 | 0.66 | 0.47 | 0.73 | 0.58 | 0.44 | 0.67 |
| Urban zone | 0.66 | 0.48 | 0.73 | 0.76 | 0.48 | 0.65 | 0.75 | 0.56 | 0.74 |

Source: Own elaboration based on ENSANUT [20] 2000–2018.

In the third step, we use the paired data and sample expansion factors from each wave of Reference [20] to interpolate individuals' sub-cohorts by the deprivation status throughout the period. The interpolation includes the construction of a cubic spline polynomial $SP(x)$ for each sub-cohort in the intervals $[x_j, x_{j+1}]$, $j = 0, 1, \dots, n - 1$ subject to the following terms:

- (1) $SP_j(x_j) = f(x_j)$ and $SP_j(x_{j+1}) = f(x_{j+1}) \forall j = 0, 1, \dots, n - 1$
- (2) $SP'_{j+1}(x_{j+1}) = SP'_j(x_{j+1}) \forall j = 0, 1, \dots, n - 1$
- (3) $SP''_{j+1}(x_{j+1}) = SP''_j(x_{j+1}) \forall j = 0, 1, \dots, n - 1$
- (4) $SP''(x_0) = SP''(x_n) = 0$

The intervals $[x_j, x_{j+1}]$ are partitions of the domain of x over the closed set $2000 = a \leq x_0 < x_1 < \dots < x_N \leq b = 2018$, whose range is composed of the $N + 1$ coordinates $(x_0, y_0), \dots, (x_N, y_N)$ of the curve $y_k = f(x_k)$; $k = 0, 1, \dots, N$. The curve yields the deprivals by states for each sub-cohort of adults.

Once the interpolated values have been obtained, finally, we take the equidistant points 2006, 2012, and 2018 to distribute the adult cohort in equal time intervals. In this way, we could convert independent cross-sectional data into an equidistant pseudo-panel in time, which is necessary to use the HMM adequately. Tables 3 and 4 present the exercise's final results. These tables are the basis for building the HMM as they record the observed transitions experienced by diabetics and non-diabetics among the four states during the sample period.

Table 3. Number of individuals by states of deprivation in 2006, 2012, and 2018.

| Year | State | Individuals | |
|------|-------|-------------|---------------|
| | | Diabetics | Non-Diabetics |
| 2006 | 0 | 189 | 1677 |
| | 1 | 234 | 2072 |
| | 2 | 184 | 1602 |
| | 3 | 269 | 2292 |
| 2012 | 0 | 172 | 1549 |
| | 1 | 227 | 2034 |
| | 2 | 179 | 1567 |
| | 3 | 298 | 2493 |
| 2018 | 0 | 166 | 1504 |
| | 1 | 225 | 2020 |
| | 2 | 181 | 1585 |
| | 3 | 304 | 2534 |

Source: Own elaboration based on ENSANUT [20] 2000–2018.

Table 4. A. Number of Mexican people with DM2 according to the transition matrix (2006–2018). B. Number of Mexican people without DM2 according to the transition matrix (2006–2018).

| A | | | | |
|----------|-----|-----|-----|------|
| States | 0 | 1 | 2 | 3 |
| 0 | 79 | 65 | 18 | 27 |
| 1 | 39 | 59 | 60 | 76 |
| 2 | 21 | 43 | 45 | 75 |
| 3 | 27 | 58 | 58 | 126 |
| B | | | | |
| States | 0 | 1 | 2 | 3 |
| 0 | 832 | 465 | 235 | 145 |
| 1 | 279 | 649 | 396 | 748 |
| 2 | 177 | 413 | 406 | 606 |
| 3 | 216 | 493 | 548 | 1035 |

Source: Own elaboration based on ENSANUT [20] 2000–2018.

An interesting point about Table 4A is that they can empirically validate our modeling proposal. The matrix form of each table makes it easier to check that observable deprivations behave like $\{C_n\}$. The procedure compares the observed matrices with those estimated between 2006 and 2018 for diabetics and non-diabetics. Table 5 shows the figures resulting from the comparison after using the maximum likelihood method for 90% confidence intervals (see References [24,25]). While the left part of each table contains the observed matrices between 2006 and 2012, the right part displays the estimated matrices for 2006–2018, which results from iterating the observed matrices one period forward. The meaning of estimated matrices is unique here, as it only represents a heuristic resource to test the hypothetical evolution of $\{C_n\}$. Multiplying the observed matrix in 2006–2012 by itself to obtain an estimate of the observed matrix for the sample period of 2006–2018 is an indirect method to verify the probabilities' structures of both matrices. If those structures are statistically different, then the two matrices do not belong to the same $\{C_n\}$.

Table 5. A. Transition probabilities for Mexican people with Type 2 diabetes mellitus (DM2) (2006–2018). B. Transition probabilities for Mexican people without DM2 (2006–2018).

A

| States | π_{i0} | π_{i1} | π_{i2} | π_{i3} | States | π_{i0} | π_{i1} | π_{i2} | π_{i3} |
|--------------------------------------|----------------------------|----------------------------|----------------------------|----------------------------|---------------------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| 0 | 0.3949 (0.3717, 0.4901) | 0.3265 (0.2712, 0.3671) | 0.1043 (0.1031, 0.1569) | 0.1742 (0.0956, 0.1826) | 0 | 0.4303 (0.4119, 0.4444) | 0.3197 (0.3045, 0.3338) | 0.1286 (0.1156, 0.147) | 0.1214 (0.1062, 0.1398) |
| 1 | 0.1588 (0.1268, 0.2346) | 0.2417 (0.2294, 0.3137) | 0.2507 (0.1237, 0.2881) | 0.3488 (0.3277, 0.4374) | 1 | 0.1949 (0.1775, 0.2089) | 0.2711 (0.257, 0.2852) | 0.1548 (0.1364, 0.1689) | 0.3792 (0.3608, 0.3965) |
| 2 | 0.1030 (0.0844, 0.1425) | 0.2345 (0.2163, 0.2965) | 0.2650 (0.2434, 0.3355) | 0.3976 (0.2965, 0.4245) | 2 | 0.1122 (0.096, 0.1285) | 0.2555 (0.2371, 0.2685) | 0.2888 (0.2747, 0.305) | 0.3435 (0.3283, 0.3586) |
| 3 | 0.0975 (0.0831, 0.1362) | 0.2111 (0.1934, 0.2709) | 0.2276 (0.2115, 0.2883) | 0.4638 (0.3576, 0.4775) | 3 | 0.1063 (0.0911, 0.1193) | 0.2300 (0.2116, 0.2473) | 0.2480 (0.2296, 0.2611) | 0.4157 (0.4016, 0.4298) |
| Observed Transition Matrix 2006–2012 | | | | | Estimated Transition Matrix 2006–2018 | | | | |
| Stationary vector | | | | | Stationary vector | | | | |
| π | 0.2121 | 0.2685 | 0.2001 | 0.3193 | π | 0.1936 | 0.2635 | 0.2088 | 0.3340 |
| Statistic | | | | | $\chi^2 = 23.184$ | | | | |
| | | | | | $p\text{-value} = 0.1089$ | | | | |

B

| States | π_{i0} | π_{i1} | π_{i2} | π_{i3} | States | π_{i0} | π_{i1} | π_{i2} | π_{i3} |
|--------------------------------------|----------------------------|----------------------------|----------------------------|----------------------------|---------------------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| 0 | 0.4652 (0.3896, 0.4843) | 0.2532 (0.2024, 0.3359) | 0.1523 (0.1102, 0.1959) | 0.1292 (0.1062, 0.1765) | 0 | 0.4173 (0.3907, 0.421) | 0.3270 (0.3035, 0.3359) | 0.1315 (0.1124, 0.147) | 0.1241 (0.1051, 0.1398) |
| 1 | 0.1279 (0.1097, 0.2122) | 0.2909 (0.2060, 0.3273) | 0.2043 (0.1864, 0.2678) | 0.3768 (0.3019, 0.3933) | 1 | 0.1982 (0.1819, 0.2111) | 0.2587 (0.2414, 0.2673) | 0.1574 (0.1386, 0.171) | 0.3857 (0.3652, 0.3944) |
| 2 | 0.1030 (0.0971, 0.1306) | 0.2345 (0.2014, 0.2707) | 0.2767 (0.2163, 0.3177) | 0.3859 (0.3477, 0.3936) | 2 | 0.1090 (0.0992, 0.1285) | 0.2481 (0.2414, 0.2707) | 0.2928 (0.2874, 0.3145) | 0.3502 (0.3466, 0.378) |
| 3 | 0.0898 (0.0533, 0.1236) | 0.2011 (0.1827, 0.2462) | 0.2543 (0.1796, 0.2664) | 0.4549 (0.4051, 0.5132) | 3 | 0.1010 (0.0933, 0.1225) | 0.2186 (0.2149, 0.2473) | 0.2357 (0.2317, 0.2631) | 0.4448 (0.4507, 0.4843) |
| Observed Transition Matrix 2006–2012 | | | | | Estimated Transition Matrix 2006–2018 | | | | |
| Stationary vector | | | | | Stationary vector | | | | |
| π | 0.2060 | 0.2616 | 0.1998 | 0.3326 | π | 0.1864 | 0.2552 | 0.2082 | 0.3502 |
| Statistic | | | | | $\chi^2 = 22.706$ | | | | |
| | | | | | $p\text{-value} = 0.1218$ | | | | |

Note: Number in round brackets are 95% simultaneous confidence intervals. Source: Own elaboration.

Goodness-of-fit tests reject the null hypothesis that the observed and estimated probability distributions are independent. That is, the tests show that deprivals behave as an ergodic homogeneous Markov chain $\{C_n\}$ of four states between 2006 and 2018 (see p -values and ergodic values calculated with (3) at the bottom of the two tables). What is valuable about this result is that since the X 's family behaves like $\{C_n\}$, states' information is sufficient to explain the dynamics of exposure to social deprivation. To predict X 's natural evolution, we only need states' transition probabilities.

It is important to note that our modeling proposal and, in general, the prerequisites for applying the HMM are valid under the assumption that $\{C_n\}$ has four states. The HMM finds a different optimal size for transition matrices, prerequisites should be re-checked to verify whether the HMM results are meaningful or not. A well-applied HMM is not possible without well-founded prerequisites. The HMM is a suitable tool to predict hidden regimes' behavior from observed data if and only if prerequisites are met [26]. One needs the other in a double-check process.

Since the HMM considers the data's underlying structure, there is no way to expect the same Table 5 information. Unlike direct (observed) Markov models, such as those behind the mentioned tables, where the states and transition probabilities are predetermined with the same coefficient of variation and no serial correlation, the HMM generates non-predetermined quantities (number of states, transition probabilities, and parameters values) with different means, variances, and correlation degrees. The HMM's variety of results is highly dependent on the probability distribution assumed. Hence, the greater realism of hidden models lies precisely in the sound foundation of the prerequisites.

4. Results

After meeting the theoretical and practical considerations, stage 2 of Figure 2 establishes that the first aspect to consider is the optimal number of states. Table 6 shows that,

following the Akaike (AIC) and Bayesian (BIC) information criteria, the optimal size generated by the EM algorithm is four states for the two populations (see the lower values of AIC and BIC). This result is relevant as it confirms that the division of states previously assumed in Table 5 is well-founded. In other words, the hidden data matches the number of observable states when using a binomial distribution. Therefore, we can safely assume that the hidden states reflect the size and meaning of observable data categories.

Table 6. A. The optimal number of states for the population without DM2. B. The optimal number of states for the population with DM2.

| A | | | | |
|--------------------------|------------------|----------------|---------|---------|
| Probability Distribution | Number of States | Log Likelihood | AIC | BIC |
| Binomial | 2 | −5902.84 | 1546.39 | 1555.71 |
| | 3 | −7698.52 | 1452.48 | 1547.46 |
| | 4 | −7705.17 | 1099.44 | 1255.94 |
| | 5 | −7321.83 | 1490.76 | 1641.78 |
| B | | | | |
| Probability Distribution | Number of States | Log Likelihood | AIC | BIC |
| Binomial | 2 | −6686.40 | 1424.62 | 1608.45 |
| | 3 | −7580.08 | 1487.91 | 1435.14 |
| | 4 | −8234.53 | 1232.34 | 1255.94 |
| | 5 | −6356.93 | 1514.81 | 1667.44 |

Source: Own elaboration.

Table 7A present the Π matrices associated with these states, which result from applying Equations (5) and (6) to observable data. They show that the probabilities of staying in the same state are higher in s_0 and s_3 than in s_1 and s_2 , which is indicative of the initial disparity in the social conditions of the country. However, when considering the remaining values of ${}_n\pi_{s_i}$, a general impoverishment profile emerges due to the potential migration of diabetics and non-diabetics to more deprived states. In particular, individuals who already experience some deprivations are more likely to become extreme poor than moderate. Likewise, those who do not suffer from DM2 and do not have any deprivation have significant probabilities of being lacking (0.3197), moderate poor (0.1286), and extreme (0.1214). The figures for people with DM2 are slightly higher.

Table 7. A. State-dependent probability matrices for the population without DM2. B. State-dependent probability matrices for the population with DM2.

| A | | | | | | |
|-------------------|------------|------------|------------|------------|-------|----------|
| States | π_{i0} | π_{i1} | π_{i2} | π_{i3} | Mean | Variance |
| 0 | 0.4303 | 0.3197 | 0.1286 | 0.1214 | NA | NA |
| 1 | 0.1949 | 0.2711 | 0.1548 | 0.3792 | 1.462 | 1.021 |
| 2 | 0.1122 | 0.2555 | 0.2888 | 0.3435 | 3.783 | 2.287 |
| 3 | 0.1063 | 0.23 | 0.248 | 0.4157 | 5.187 | 2.103 |
| Stationary vector | | | | | | |
| π | 0.1936 | 0.2635 | 0.2088 | 0.3340 | | |
| B | | | | | | |
| States | π_{i0} | π_{i1} | π_{i2} | π_{i3} | Mean | Variance |
| 0 | 0.4173 | 0.3270 | 0.1315 | 0.1241 | NA | NA |
| 1 | 0.1982 | 0.2587 | 0.1574 | 0.3857 | 1.443 | 1.223 |
| 2 | 0.1090 | 0.2481 | 0.2928 | 0.3502 | 3.525 | 2.653 |
| 3 | 0.1010 | 0.2186 | 0.2357 | 0.4448 | 5.021 | 2.437 |
| Stationary vector | | | | | | |
| π | 0.1864 | 0.2552 | 0.2082 | 0.3502 | | |

Source: Own elaboration.

In the absence of any event that alters the current probabilities' structure, the previous scenario likely becomes real in a period of six stages (or 24 years), which are the number of times the hidden matrices need to be exponentiated to reach the stationary values, according to Equation (4). Specifically, people with diabetes present a more impoverished probabilistic profile than those who are not ill due to their lower probability of being deprived (0.1864 vs. 0.1936) and their greater probability of becoming extreme poor (0.3502 vs. 0.3340). The upper values of the mean of the events in state 3 confirm this probabilistic scenario.

A remarkably similar pattern is observed when comparing these results with those of Table 5. The values of the ergodic vectors of the observed matrices for the two populations and the number of stages needed to reach them coincide, in general, with the quantities calculated by the HMM. Hence, the HMM conclusions find support in the data from the observed Markov matrices.

4.1. Statistical Differences between Diabetics and Non-Diabetics

An outstanding aspect of any Markov model is its flexibility to combine techniques that support results beyond its original scope. Following stage 3 of Figure 2, we present a couple of techniques to highlight diabetics' differences from the rest of the population. Table 8 reports the first technique's results, which involve a multivariate analysis of variance (MANOVA), for the two populations by the state of the chain and type of social deprivation (state zero is excluded).

The data in columns two to four represent the mean differences ($\mu_1 - \mu_2$) associated with each social deprivation. A positive value of ($\mu_1 - \mu_2$) indicates a greater exposure of people with diabetes to such deprivation. The figures show that people with diabetes experience a more significant educational lag and less access to quality services in health and social security than those who do not suffer from the disease in states 1 and 2 (in the latter, it only applies to the educational lag). Table 9 shows that these differences are especially significant in access to quality health services in state 1 since, in this case, people with diabetes experience greater exposure than non-diabetics in a percentage that ranges between 0.061% and 0.562%. It is followed in importance by the educational lag and poor access to social security services. There are no statistically significant differences between the two populations in the rest of the social deprivations, as shown by the p -values in Table 8 or the Bonferroni simultaneous intervals in Table 9.

Table 8. Multivariate analysis of variance for diabetic and non-diabetic populations by states and social deprivation (percentages).

| Deprivation | State 1 | | State 2 | | State 3 | |
|--------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| | $\mu_1 - \mu_2$ | <i>p</i> -Value | $\mu_1 - \mu_2$ | <i>p</i> -Value | $\mu_1 - \mu_2$ | <i>p</i> -Value |
| Educational lag | 0.215 | 0.000 | 0.136 | 0.013 | −0.129 | 0.274 |
| Access to health services | 0.271 | 0.001 | 0.114 | 0.345 | −0.293 | 0.102 |
| Access to social security | 0.285 | 0.000 | 0.290 | 0.435 | 0.439 | 0.761 |
| Quality and spaces in the home | −0.148 | 0.137 | 0.026 | 0.871 | −0.096 | 0.585 |
| Access to basic services in the home | 0.051 | 0.600 | 0.031 | 0.858 | 0.028 | 0.864 |
| Access to food | −0.002 | 0.372 | 0.015 | 0.245 | 0.067 | 0.049 |

Source: Own elaboration.

Table 9. Bonferroni simultaneous intervals for diabetic and non-diabetic populations by states and social deprivation (percentages) at a 95% confidence.

| Deprivation | State 1 | | State 2 | | State 3 | |
|--------------------------------------|-----------------|--|-----------------|--|-----------------|--|
| | | | | | | |
| Educational lag | (0.035, 0.396) | | (0.045, 0.316) | | (−0.31, 0.051) | |
| Access to health services | (0.061, 0.562) | | (−0.177, 0.405) | | (−0.583, 0.002) | |
| Access to social security | (0.197, 0.372) | | (−0.202, 0.378) | | (−0.351, 0.527) | |
| Quality and spaces in the home | (−0.238, 0.059) | | (−0.064, 0.116) | | (−0.186, 0.007) | |
| Access to basic services in the home | (−0.039, 0.064) | | (−0.019, 0.044) | | (−0.04, 0.015) | |
| Access to food | (−0.037, 0.034) | | (−0.021, 0.051) | | (−0.031, 0.103) | |

Source: Own elaboration.

To what extent do these deprivations and the adoption of certain risky behaviors affect the probability of being diabetic? To answer this question, we use the second technique, a multinomial logit model in which the independent variables are some of the direct causes of the disease (tobacco and alcohol consumption and overweight or obesity), and the dependent variable Y is defined as:

$$Y = \begin{cases} j & \text{if the person is diabetic and presents the deprivation } j \\ m + j & \text{if the person is not diabetic and presents the deprivation } j, \end{cases}$$

where m are social deprivations, $j = 1, 2, \dots, m$. Formally, the conditional probability is estimated using:

$$P[Y = i | X] = F(Y'_i),$$

where F is the logistic cumulative probability distribution, X is a vector of independent variables X_1, X_2, \dots, X_p , and Y'_i is a latent variable that determines the realization of the variable Y at a specific value $i = 1, 2, \dots, 2m$. Usually, the level of the variable Y'_i is estimated according to a linear specification of the form.

$$Y'_i = X\beta_i + \varepsilon_i,$$

in which β_i are the coefficients calculated for the i th category of variables X and ε_i is the random disturbance, $i = 1, 2, \dots, 2m$. The marginal effect of the variable X_k , $k = 1, 2, \dots, p$, on Y'_i is expressed as:

$$\frac{\partial P}{\partial X_k} = \frac{\partial F(Y'_i)}{\partial X_k}.$$

Table 10 shows the marginal effects of DM2's risk-factors on the probability of being diabetic or not being diabetic by social deprivation and the corresponding p -values and confidence interval at the 95% level. The data indicates that when the individual presents an educational lag, poor access to health services, and food insecurity, the marginal effects of direct causes are statistically significant, but not in the other cases. Thus, for example, if the individual experiences educational lag, then the direct causes increase the probability of acquiring diabetes by 0.38% (obesity or overweight), 0.326% (hypertension), 0.34%

(alcohol consumption), 0.22% (tobacco consumption), and a similar occurrence happens with the other two deprivals.

When interpreting the results in Table 10, one must be very cautious. They do not mean, for example, that educational lag conditions behavior in consumption of alcohol or tobacco. Rather, the results mean that adults with an educational lag have a greater probability of developing DM2 when they are obese, hypertensive, smoke, and drink alcohol. The specific contribution of these risky behaviors varies by type of deprivals.

Table 10. Marginal effects on the probability of being diabetic (multinomial logit model) by type of clinical risk and social deprivation.

| Risk-Factors | Coefficient | p-Value | Confidence Interval (95%) |
|--|-------------|---------|---------------------------|
| Lack due to educational lag | | | |
| Obesity and overweight | 0.380 | 0.000 | (0.171, 0.851) |
| Hypertension | 0.326 | 0.043 | (0.195, 0.485) |
| Alcohol | 0.340 | 0.000 | (0.127, 0.775) |
| Tobacco | 0.220 | 0.000 | (0.084, 0.501) |
| Lack of access to health services | | | |
| Obesity and overweight | 0.395 | 0.000 | (0.191, 0.879) |
| Hypertension | 0.298 | 0.004 | (0.205, 0.379) |
| Alcohol | 0.365 | 0.000 | (0.133, 0.835) |
| Tobacco | 0.249 | 0.000 | (0.115, 0.556) |
| Lack of access to social security | | | |
| Obesity and overweight | −0.165 | 0.635 | (−0.066, 0.267) |
| Hypertension | 0.054 | 0.987 | (−0.981, 0.789) |
| Alcohol | −0.541 | 0.426 | (−0.168, 0.595) |
| Tobacco | −0.327 | 0.525 | (−0.157, 0.565) |
| Lack of quality and spaces in the home | | | |
| Obesity and overweight | 0.287 | 0.280 | (−0.263, 0.837) |
| Hypertension | 0.184 | 0.296 | (−0.418, 0.745) |
| Alcohol | 0.157 | 0.242 | (−0.317, 0.631) |
| Tobacco | 0.208 | 0.271 | (−0.323, 0.739) |
| Lack of access to basic services in the home | | | |
| Obesity and overweight | 0.287 | 0.392 | (−0.481, 1.055) |
| Hypertension | 0.132 | 0.456 | (−0.181, 0.255) |
| Alcohol | 0.157 | 0.342 | (−0.514, 0.828) |
| Tobacco | 0.208 | 0.355 | (−0.488, 0.904) |
| Lack of access to food | | | |
| Obesity and overweight | 0.215 | 0.000 | (0.106, 0.207) |
| Hypertension | 0.279 | 0.008 | (0.163, 0.371) |
| Alcohol | 0.316 | 0.000 | (0.133, 0.261) |
| Tobacco | 0.278 | 0.000 | (0.087, 0.171) |

Source: Own elaboration.

4.2. Discussion

The previous results present similarities and differences concerning those reported by the literature, especially by Reference [17]. We agree with those authors that stationary probabilities of becoming extreme poor are higher in diabetics than in non-diabetics. These probabilities are associated with an educational lag and problems of access to medical services and social security. Nevertheless, we differ from them when they argue that differences between the two populations are significantly distinct in all the states and that there is no strong statistical correlation between social deprivals and the direct causes of DM2. As we make clear in Tables 8–10, people with DM2 experience more exposure to the three deprivals mentioned practically only in state 1, and that two of these deprivals, together with food insecurity, increase the probability of acquiring the disease when the individual consumes tobacco or alcohol and is obese and hypertensive.

Other studies addressing health or lifestyle intervention treatment predictors in people with diabetes also share these latter results. Using a machine learning approach, Seligman [27] found that social factors, such as education, are good predictors of DM2's direct causes. Derevitskii [15] states that smoking significantly affects DM2's complications in patients' trajectories analyzed as Markov chains. Sanchez [28] conclude that intervention in human behavior helps diabetic older adults improve their quality of life. Factors identifying human behavior (posture, nutritional habits, daily activity, duration, and location) are extracted from a hidden Markov model. Finally, Radcliff [29] uses Markov transition matrices to assert that nutrition education with behavioral coaching programs is effective and efficient in preventing or delaying DM2-associated consequences of obesity.

Unfortunately, these papers concentrate on studying the intermediate steps of a Markov chain and do not perform analyses on the ergodic values. This omission prevents us from comparing our results with other experiences. In this sense, the methodology used for constructing the stationary matrices in Tables 5 and 7 constitutes a novelty in the literature on diabetes. This methodology allows us to obtain the same stationary social deprivation pattern employing either direct (Table 5) or hidden matrices (Table 7). The differences between both types of ergodic matrices are insignificant, even though their transition probabilities' calculation uses different methods (maximum likelihood in the direct and the EM algorithm in the hidden model). Thus, we can safely conclude that *ceteris paribus*, Mexican people with diabetes will become extreme poor around 2050 because the prerequisites for applying the HMM coincide with the model's results.

5. Conclusions and Future Work

The paper develops a methodology to study social deprivation in diabetic and non-diabetic populations using HMM. The idea is to differentiate the probabilistic profile of the exposure to deprivation in both populations to understand the disease's economic and social context. For this, the paper proposes some theoretical and practical considerations, leading to implementing the model correctly. Compliance with these considerations is unavoidable for any HMM user.

The main conclusion from the statistical analyses is that, in the absence of events that alter the 2006–2018 period's probability structure, it is highly likely that people with DM2 have a greater probability of becoming extreme poor than the rest of the population. This probabilistic scenario combined with some risky behaviors, such as tobacco and alcohol consumption, hypertension, or obesity, increases the probability of acquiring DM2. The lack of medical supervision services or education to know how to exercise or eat properly makes low-income families a natural target for chronic diseases. Thus, creating a probabilistic scenario is essential for understanding the context that forces individuals to adopt risky behaviors for their health.

A future research agenda on the subject should include two critical aspects: the inclusion of new variables and the link between the direct causes of diabetes and social deprivation. The new variables to consider are sex, age, residence, and income deciles, due to their importance in explaining the new disease trends in Mexico. Adolescent and adult DM2 require different treatments because patients experience them differently by sex, urban and rural areas, and economic strata. For example, uneducated, extreme poor women have greater DM2 prevalence rates than extreme poor men in urban zones but not in some rural zones [17]. Additionally, it is necessary to perform an in-depth study of household members' cultural and genetic backgrounds to learn about their social propensity to diseases. Knowing the family's cultural environment or the parents' diabetic history is essential for understanding the intimate links between DM2 and social deprivation.

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Appendix A

Table A1. Variables and definitions used in this study (2000–2018).

| Variables | Definition |
|--|--|
| Educational lag * | An adult who does not have compulsory basic education (complete elementary and secondary education). |
| Access to health services * | An individual who does not have an affiliation to receive medical services from a public or private institution. |
| Access to social security * | The person who does not have employment benefits, AFORE (AFORE is the Retirement Savings Fund Administration System), or another retirement service. |
| Quality and spaces in the home * | A person who lives in a house whose floors, roofs, and walls are made up of waste material, cardboard sheet, mud or bark, reed, bamboo, palm, metallic or asbestos sheet, and the ratio of people per room is greater than 2.5. |
| Access to basic services in the home * | A person who gets the water from a well, river, lake, stream, pipe, or piped water is obtained by hauling it from another home, public tap, or hydrant. People who do not have a drainage service or the drainage is connected to a pipe that leads to a river, lake, sea, ravine, or crevasse also fall in this category. They usually do not have electricity, and the fuel they use to cook or heat food is firewood or charcoal. |
| Access to food * | People who present one or more food insecurity characteristics in the last three months. |
| Diabetes * | People with a diagnosis of DM2. |
| Obesity and overweight * | People with a body mass index above the healthy thresholds recommended by the World Health Organization and the Ministry of Health in Mexico. |
| Hypertension * | People with a diagnosis of high blood pressure or hypertension. |
| Alcohol * | People who consume alcohol above the median of the population. |
| Tobacco * | People who smoke cigarettes above the median of the population. |
| Sex * | Male or female (male takes number 1 and female 0). |
| Age | Age of the person. |
| Years of education | The number of years of education. |
| Marital status * | The person is single, married, divorced, widowed, or separated. |
| Rural zone * | An area with less than 2500 inhabitants. |
| Urban zone * | An area with more than 2500 inhabitants. |
| Federal entity | State of residence in Mexico. |
| Municipality | Municipality of residence in Mexico. |
| Location | Localities of residence in Mexico. |

Source: Own elaboration based on ENSANUT [20] (2018) and CONEVAL [30] (2016). Note. The asterisk* indicates that the variable is a dummy, taking the value 1 if the condition is met or 0 otherwise.

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