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A GEOMETRICAL OPTIMIZATION PROBLEM ASSOCIATED WITH FRUITS OF POPPY FLOWER

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Abstract- Inspired from the poppy fruit (*Papaver rhoeas* L.), a geometrical optimization problem is posed. The aim is to minimize the surface area for a given volume. The poppy fruit geometry is selected as the optimization geometry. The mathematical problem is solved using calculus. The optimum solutions obtained from mathematical model are contrasted with measurements of the fruit. A good agreement with difference in areas less than 1% in most of the cases is observed between the results.

Keywords- Surface Area Optimization, Geometry of Poppy Fruit, Calculus, *Papaver rhoeas* L.

1. INTRODUCTION

Optimization principles are successfully applied to many biological problems [1]. In Chapter 6 of the mentioned reference, an excellent discussion of the power as well as limitations of the theory with respect to biological applications was discussed in detail. The critical questions are "What should be optimized?" and "What are the design constraints?" If the problem is not posed properly, the optimization may lead to a wrong answer.

Here, in this study, a geometric optimization problem inspired from the poppy (*Papaver rhoeas* L.) fruit is formulated. The poppy fruit geometry can be approximated by an attachment of a paraboloid and a cone. For a prescribed volume, the optimization problem is to minimize the surface area. Since the fruit contains wet seeds and a milky juice inside, dehydration is thought to be the major problem. Dehydration depends directly on the magnitude of surface area among other factors such as the permittivity of the shell material. For a specific material then, area minimization would be helpful in retaining the wetness inside the capsule.

The volume of the paraboloid and attached cone is calculated and fixed. The area is then calculated and differentiated to find the ratios for maintaining a minimum area. Results are compared with direct length measurements of the fruit. The length ratios of the optimum solutions and measurements agree well and the difference in calculated and measured areas are less than 1% in most of the cases.

It is beyond the scope of this study to give a general review of studies on poppy flowers. Instead, some examples are given. A description of the plant, its history, distribution throughout the world, biology, its uses as a medicine and weediness is discussed in detail by Mitich [2]. Germination ecology of four Papaver taxa including *Papaver rhoeas* L. is investigated in three different artificial climates by Karlsson and Milberg [3]. The geotropic response of a poppy flower stalk was studied by Kohji et al. [4]. Red bowl-shaped flowers including *Papaver rhoeas* L. was studied with respect to beetle pollination in the Mediterranean region by Dafni et al. [5]. A study was made to find whether the nodding of the flower stalk in *Papaver Rhoeas* L. immediately after its formation was triggered by the weight of its flower bud or by positive georeaction by Kohji et al. [6]. Signalling and the cytoskeleton of pollen tubes of *Papaver rhoeas* L. were investigated by Snowman et al. [7]. Growth of Pollen Tubes of *Papaver rhoeas* L. was discussed by Franklin-Tong et al. [8]. The genetical control of self-incompatibility in *Papaver rhoeas* L. was presented by Lawrence et al. [9]. As can be seen from the examples, although many different features of the poppy flower have already been investigated in detail, to the best of the author's knowledge, there is no study on geometric optimization problem of poppy fruits.

2. MATHEMATICAL FORMULATION

The geometry of a united paraboloid and cone is given in Figure 1.

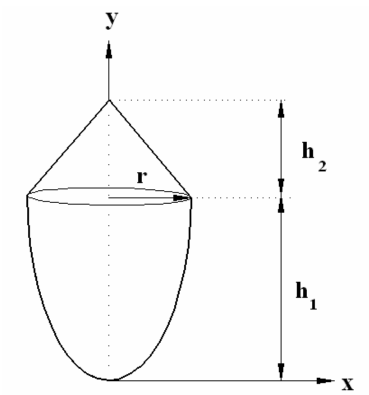

Figure 1. Volume consisting of a paraboloid and a cone

The total volume is

$$V_0 = \pi r^2 \left(\frac{h_1}{2} + \frac{h_2}{3} \right) \tag{1}$$

The total area can be calculated from calculus [10]

$$A = \pi r^2 \left\{ \frac{r^2}{6h_1^2} \left[\left(1 + 4\frac{h_1^2}{r^2} \right)^{3/2} - 1 \right] + \left(1 + \frac{h_2^2}{r^2} \right)^{1/2} \right\} \tag{2}$$

Since V_0 is a prescribed volume, the radius can be expressed in terms of the constant volume and heights

$$r^2 = \frac{V_0}{\pi\left(\frac{h_1}{2} + \frac{h_2}{3}\right)} \tag{3}$$

and then substituted into (2)

$$A = \frac{6V_0}{h_1(3+2k)}\left\{\frac{V_0}{\pi h_1^3(3+2k)}\left[\left(1+\frac{2\pi h_1^3}{3V_0}(3+2k)\right)^{3/2} - 1\right] + \left(1+\frac{\pi h_1^3 k^2}{6V_0}(3+2k)\right)^{1/2}\right\} \tag{4}$$

where

$$k = \frac{h_2}{h_1} \tag{5}$$

The optimization is searched with respect to the height ratio parameter k. Differentiating area with respect to k, equating to zero, substituting the volume from (1) and arranging yields

$$-\frac{2}{3\alpha^2}\left[\left(1+4\alpha^2\right)^{3/2} - 1\right] - 2(1+\alpha^2k^2)^{1/2} + 2(1+4\alpha^2)^{1/2} + 3\alpha^2 k(1+k)(1+\alpha^2k^2)^{-1/2} = 0 \tag{6}$$

where

$$\alpha = \frac{h_1}{r} \tag{7}$$

For a given α, the height ratio k can be calculated numerically by root finding techniques from (6). If r is small, from Figure 1, the object would be a slender object and the area would become large. Conversely, if r is large, then the object is squat and very wide increasing its surface area. Hence, the optimum solution found should be a minimum solution.

3. APPLICATION TO POPPY FRUIT

A photo of the poppy flower (*Papaver rhoeas* L.) and its fruit is shown in Figure 2. The cross section of the fruit is given in Figure 3. As shown by the approximating lines in Figure 4, the bottom of the fruit can be approximated by a paraboloid and the top by a cone. Seven sample fruits are taken for each three different stages of the flower (See Figure 5).

Figure 2. Poppy flower and its fruit

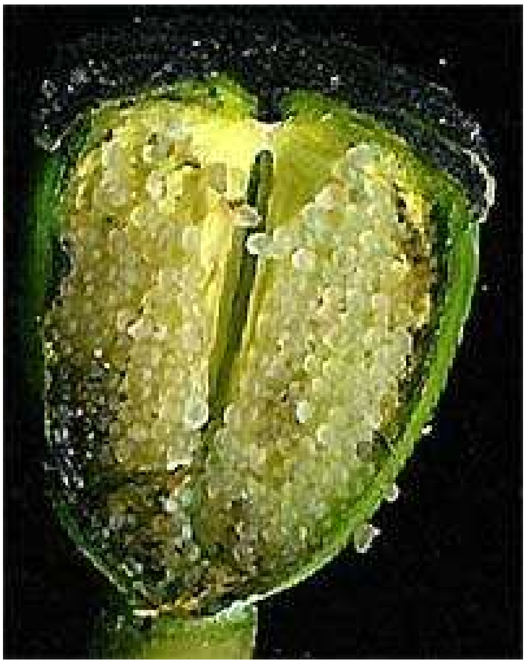

Figure 3. Cross section of the fruit

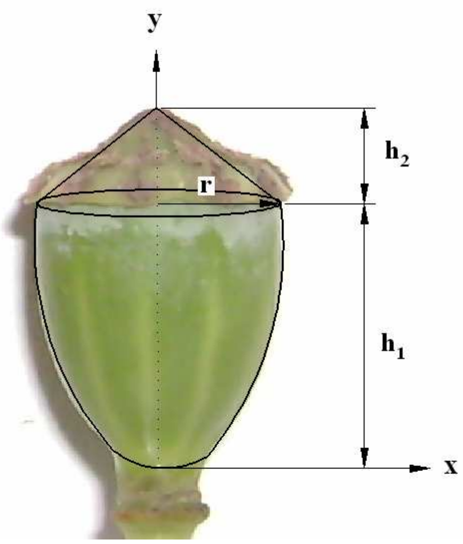

Figure 4. Approximation of the fruit volume by a paraboloid and a cone

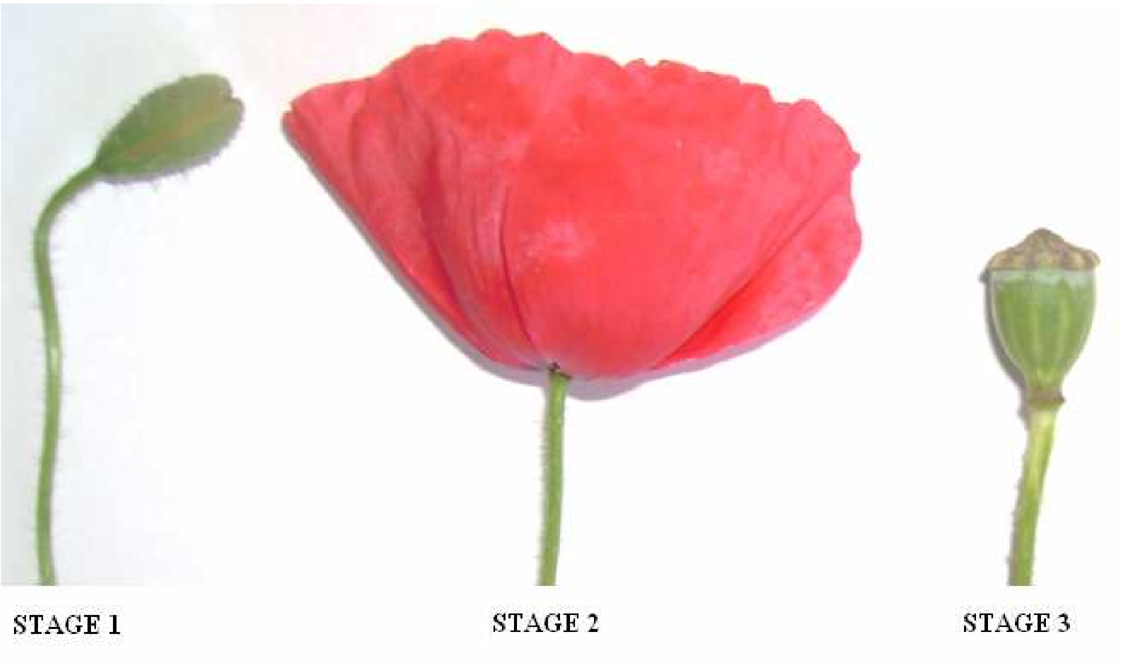

Figure 5 Three stages of the flower: Stage I Bud; Stage II Flower with Leaves, Stage III Flower with Leaves Fallen

Stage I corresponds to bud, Stage II corresponds to flower with leaves and Stage III corresponds to flower with leaves fallen. For each of the specimens, fotographs are taken and processed by Motic Images software and the lengths are measured from these images for improved precision. The parameters $\alpha=h_1/r$ and $k=h_2/h_1$ are then calculated from the measured quantities.

For the given α values, the optimum mathematical solutions for height ratios k are calculated from equation (6). To compare the difference in measured and calculated areas, the following formula is used

$$\% \text{ Difference in Area} = \frac{A_m - A_c}{A_c} x100 \qquad (8)$$

where A_m is the measured area and A_c is the calculated area from the optimum solution. Results are given in Tables 1-3 for all three stages of the flower. In most of the cases, the measured and optimum height ratios are close to each other. The percentage differences in areas in most cases are less than 1%.

Table 1. Comparison of Measured and Calculated Data for a Fruit at Stage I

Specimen	h_1/r	h_2/h_1 (measured)	h_2/h_1 (optimum)	% Area Difference
1	2.36	0.33	0.37	0.15
2	2.40	0.29	0.36	0.33
3	3.06	0.19	0.25	0.42
4	2.55	0.3	0.33	0.15
5	2.28	0.35	0.38	0.10
6	2.43	0.34	0.35	0.04
7	2.34	0.33	0.37	0.13

Table 2. Comparison of Measured and Calculated Data for a Fruit at Stage II

Specimen	h_1/r	h_2/h_1 (measured)	h_2/h_1 (optimum)	% Area Difference
1	2.27	0.35	0.39	0.15
2	2.38	0.33	0.36	0.08
3	2.55	0.32	0.33	0.04
4	2.52	0.18	0.33	1.38
5	2.22	0.38	0.40	0.06
6	2.85	0.29	0.28	0.12
7	2.47	0.21	0.34	1.13

Table 3. Comparison of Measured and Calculated Data for a Fruit at Stage III

Specimen	h_1/r	h_2/h_1 (measured)	h_2/h_1 (optimum)	% Area Difference
1	2.43	0.28	0.35	0.36
2	2.22	0.29	0.40	0.60
3	3.01	0.25	0.26	0.02
4	2.54	0.29	0.33	0.19
5	2.15	0.36	0.42	0.21
6	2.37	0.32	0.36	0.19
7	2.66	0.25	0.31	0.29

It is mathematically well known that for a fixed volume, the minimum surface area corresponds to that of a sphere. However, in the case of poppy fruit, it is believed that an attached paraboloid and cone volume is better suited to the design constraints of the flower. Therefore, an area minimization problem is posed for this specific geometry and the solution is a local minimum, not a global minimum. Note also that the paraboloid and cone assembly is only an approximation of the real volume. However, our results show that both the volume approximation and optimum criterion selected is well suited to the biological problem considered.

4. CONCLUDING REMARKS

The shape of the poppy fruit is formed to minimize dehydration problem. As outlined by calculations, for a given volume, the surface area is a minimum among the many possible shapes. Minimum area means minimum dehydration from the inside of the fruit.

5. REFERENCES

1. R. McNeill Alexander, Optima for Animals, Princeton University Press, Princeton, New Jersey, 1996.
2. L. W. Mitich, Corn Poppy (*Papaver rhoeas* L.), Weed Technology, 14, 826-829, 2000.
3. L. M. Karlsson and P. Milberg, A comparative study of germination ecology of four *Papaver* taxa, Annals of Botany, 99, 935-946, 2007.
4. J. Kohji, H. Hagimoto and Y. Masuda, Georeaction and elongation of the flower stalk in a poppy, *Papaver rhoeas* L., Plant and Cell Physiology, 20, 375-386, 1979.
5. A. Dafni, P. Bernhardt, A. Shmida, Y. Ivri and S. Greenbaum, Red bowl-shaped flowers: convergence for beetle pollination in the Mediterranean region, Israel Journal of Botany, 39, 81-92, 1990.
6. J. Kohji, K. Nishitani and Y. Masuda, A Study on the mechanism of nodding initiation of the flower stalk in a poppy, *Papaver Rhoeas* L., Plant and Cell Physiology, 22, 413-422, 1981.
7. B. N. Snowman, A. Geitmann, S. R. Clarke, C. J. Staiger, F. Christopher, H. Franklin, A. M. C. Emons and V. E. Franklin-Tong, Signalling and the cytoskeleton of pollen tubes of *Papaver rhoeas*, Annals of Botany, 85 (Supplement A), 49-57, 2000.
8. V. E. Franklin-Tong, B. K. Drobak, A. C. Allan, PAC. Watkins and A. J. Trewavas, Growth of pollen tubes of *Papaver rhoeas* is regulated by a slow-moving Calcium wave propagated by Inositol 1,4,5-Trisphosphate, The Plant Cell, 8, 1305-1321, 1996.
9. M. J. Lawrence, M. Afzal and J. Kenrick, The genetical control of self-incompatibility in *Papaver rhoeas*, Heredity, 40, 239–253, 1978.
10. G. B. Thomas and R. L. Finney, Calculus and Analytical Geometry, Addison-Wesley, Massachusetts, 1984.