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A METHOD FOR RANKING OF FUZZY NUMBERS USING NEW WEIGHTED DISTANCE

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Abstract- In this paper, the researchers proposed a modified new weighted distance method to rank fuzzy numbers. The modified method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques. The proposed model is studied for a broad class for fuzzy numbers and class of functions the membership of which is formed on the basis of the template $\mu(x)=\max(0.1-|x|^s)$. This article also used some comparative examples to illustrate the advantage of the proposed method.

Key Words : Ranking, Fuzzy numbers, Fuzzy distance, Defuzzification

1. INTRODUCTION

In many applications, ranking of fuzzy numbers is an important component of the decision process. In addition to a fuzzy environment, ranking is a very important decision making procedure. Since Jain [4, 5] employed the concept of maximizing set to order the fuzzy numbers in 1976, many authors have investigated various ranking methods. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [6], and more recently by Chen and Hwang [7]. Other contributions in this field include: an index for ordering fuzzy numbers defined by Choobineh and Li [8], ranking alternatives using fuzzy numbers studied by Dias [9], automatic ranking of fuzzy numbers using artificial neural networks proposed by Requena e.t al [10], ranking fuzzy values with satisfaction function investigated by Lee e.t al [11], ranking and defuzzification methods based on area compensation presented by Fortemps and Roubens [12], and ranking alternatives with fuzzy weights using maximizing set and minimizing set given by Raj and Kumar [13]. However, some of these methods are computationally complex and difficult to implement, and others are counterintuitive and not discriminating. Furthermore, many of them produce different ranking outcomes for the same problem. In 1988, Lee and Li [14] proposed a comparison of fuzzy numbers by considering the mean and dispersion (standard deviation) based on the uniform and the proportional probability distributions. Cheng [15] proposed the coefficient of variance (CV index), i.e. $CV=\sigma$ (standard error) $/|\mu|(mean), \mu\neq 0, \sigma>0$. In this approach, the fuzzy number with smaller CV index is ranked higher, therefore Cheng's CV index also contains shortcomings. To improve Murakami e.t al.'s method, Cheng [15] proposed the distance method for ranking fuzzy numbers; i.e., $R(A)=\sqrt{\bar{x}^2+\bar{y}^2}$. For any two fuzzy numbers A_i and A_j, if $R(A_i)<R(A_j)$, then $A_i<A_j$; if $R(A_i)=R(A_j)$, then $A_i=A_j$ and if $R(A_i)>R(A_j)$, then $A_i>A_j$. Moreover, the distance method contradicts the CV index in ranking some

fuzzy numbers.
Consider the three fuzzy numbers, $A=(0.2,0.3,0.5)$, $B=(0.17,0.32,0.58)$, $C=(0.25,0.4,0.7)$ from [15] . In Cheng's distance method, *R(A)=0.590, R(B)=0.604*, and *R(C)=0.662*, produce the ranking order $A \prec B \prec C$. From this result, the researchers can logically infer the ranking order of the images of these fuzzy numbers as $-A \succ -B \succ -C$. However, in the distance method, the ranking order remains $-A \prec -B \prec -C$. Obviously, the distance method also has shortcomings. Moreover, in [1] a method based on "Sign Distance" was introduced and a new method based on "Distance Minimization" was introduced by Asady e.t al.'s [3]. This method has some drawbacks, i.e., for all triangular fuzzy numbers $u=(x_0,\sigma,\beta)$ where $x_0=(\sigma-\beta)/4$ and also trapezoidal fuzzy numbers $u=(x_0,y_0,\sigma,\beta)$, such that $x_0+y_0=(\sigma-\beta)/2$, gives the same results. However it is clear that these fuzzy numbers do not place in an equivalence class. Recently, a new method based on "the left and the right spreads at some α-levels of trapezoidal fuzzy numbers" was introduced [2]. This method has some shortcoming too, because, for any two symmetric trapezoidal fuzzy numbers, gives equal ordering.
Having reviewed the previous methods, this article proposes here a method to use the concept of fuzzy distance, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambiguities resulted from the comparison of previous ranking.
The paper is organized as follows: In Section 2, we recall some fundamental results on fuzzy numbers. Proposed method for ranking of fuzzy numbers is in the Section 3. In this Section some theorems and remarks are proposed and illustrated. Discussion and comparison of this work and other methods are carried out in Section 4. The paper ends with conclusions in section 5.

2. BASIC DEFINITIONS AND NOTATIONS

The basic definitions of a fuzzy number are given in [18, 19, 20] as follows:
Definition 2.1. A fuzzy number A is a mapping $A(x):\Re\to[0,1]$ with the following properties:

1. A is an upper semi-continuous function on $\Re$,
2. $A(x)=0$ outside of some interval $[a_1,b_2]\subset\Re$,
3. There are real numbers a_2,b_1 such as $a_1\le a_2\le b_1\le b_2$ and
   - $A(x)$ is a monotonic increasing function on $[a_1,b_2]$,
   - $A(x)$ is a monotonic decreasing function on $[b_1,b_2]$,
   - $A(x)=1$ for all x in $[a_2,b_1]$.

Let $\Re$ be the set of all real numbers. The researchers assume a fuzzy number A that can be expressed for all $x\in\Re$ in the form

$$A(x)=\begin{cases} g(x) & when \quad x\in[a,b), \\ 1 & when \quad x\in[b,c], \\ h(x) & when \quad x\in(c,d], \\ 0 & otherwise. \end{cases} \qquad (1)$$

where a,b,c,d are real numbers such as $a<b\leq c<d$ and g is a real valued function that is increasing and right continuous and h is a real valued function that is decreasing and left continuous. The trapezoidal fuzzy number $A=(x_0,y_0,\sigma,\beta)$, with two defuzzifier x_0,y_0 and left fuzziness $\sigma>0$ and right fuzziness $\beta>0$ is a fuzzy set where the membership function is as

$$A(x)=\begin{cases} \frac{1}{\sigma}(x-x_0+\sigma) & x_0-\sigma\leq x\leq x_0, \\ 1 & x_0\leq x\leq y_0, \\ \frac{1}{\beta}(y_0-x+\beta) & y_0\leq x\leq y_0+\beta, \\ 0 & otherwise \end{cases}$$

Support function is defined as follows:

$$\operatorname{supp}(A)=\overline{\{x\mid A(x)>0\}}$$

where $\overline{\{x\mid A(x)>0\}}$ is closure of set $\{x\mid A(x)>0\}$.

In this paper, the researchers will always refer to fuzzy number A described by (1).

3. NEW APPROACH FOR RANKING OF FUZZY NUMBERS

In this section, the researchers will propose the ranking of fuzzy numbers associated with the metric D in F, that F denotes the space of fuzzy numbers. We will assume that the fuzzy number $A\in F$ is represented by means of the following LR-representation:

$$A=\bigcup_{\alpha\in[0,1]}(\alpha,A_\alpha)$$

where $\forall\alpha\in[0,1]$: $A_\alpha=[L_A(\alpha),R_\alpha(\alpha)]\subset(-\infty,+\infty)$. Here, $L:[0,1]\to(-\infty,+\infty)$ is a monotonically non-decreasing and $R:[0,1]\to(-\infty,+\infty)$ is a monotonically non-increasing left-continuous functions. The functions $L(.)$ and $R(.)$ express the left and right sides of a fuzzy number, respectively.

Definition 3.1. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number A:

$$I(A)=\int_0^1(cL_A(\alpha)+(1-c)R_A(\alpha))d\alpha, \qquad (2)$$

and

$$D(A)=\int_0^1 (R_A(\alpha)-L_A(\alpha))\, f(\alpha)\, d\alpha . \qquad (3)$$

Here $0 \le c \le 1$ denotes an "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function $f(\alpha)$ is nonnegative and increasing function on $[0,1]$ with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(\alpha)\, d\alpha = \frac{1}{2}$. The function $f(\alpha)$ is also called weighting function. In actual applications, function $f(\alpha)$ can be chosen according to the actual situation. In this article, in practical case, we assume that $f(\alpha)=\alpha$.

Definition 3.2. Let $\nabla : F \to \{-1, 1\}$ be a function that is defined as follows:

$$\forall A \in F \; : \; \nabla(A)=\begin{cases} 1 & when \quad I(A) \ge 0, \\ -1 & when \quad I(A) < 0. \end{cases} \qquad (4)$$

Remark 3.1. If $\inf \text{supp}(A) \ge 0$ or $\inf L_A(\alpha) \ge 0$, then $\nabla(A)=1$.

Remark 3.2. If $\sup \text{supp}(A) < 0$ or $\sup R_A(\alpha) < 0$, then $\nabla(A)=-1$.

Definition 3.3. For arbitrary fuzzy numbers A and B the quantity

$$TRD(A,B)=\sqrt{[I(A)-I(B)]^2+[D(A)-D(B)]^2} \,,$$

is called the TRD distance between the fuzzy numbers A and B.

It is easily proved that the TRD distance satisfies the following properties:

$$TRD(A,B) \ge 0,$$
$$A \approx B \;\Leftrightarrow\; TRD(A,B)=0,$$
$$TRD(A,B)+TRD(B,C) \ge TRD(A,C),$$
$$TRD(A,B)=TRD(B,A).$$

The membership function of $a \in \Re$ is $A_a(x)=1$, if $x=a$, and $A_a(x)=0$, if $x \ne a$. Hence if $a=0$, there is

$$A_0(x)=\begin{cases} 1 & when \;\; x=0, \\ 0 & when \;\; x \ne 0. \end{cases}$$

This article considers A_0 as a fuzzy origin and since $A_0 \in F$, so for each $A \in F$,

$$TRD(A,A_0)=\sqrt{[I(A)]^2+[D(A)]^2} \,. \qquad (5)$$

Definition 3.4. For each $A \in F$ and with "optimism/pessimism" coefficient equal to 0.5,

$$TR(A,A_0)=\nabla(A)\, TRD(A,A_0),$$

is called weighted distance.

The steps of TR algoritm are:

Step 1: Computing the left and right sides of each fuzzy number ($L(.)$ and $R(.)$ **).**

Step 2: Using Eqs. (2) and (3) to find weighted averaged and weighted width ($I(.)$ and $D(.)$ **).**

Step 3. Computing TRD between fuzzy numbers and $\nabla(.)$ by Eqs. (4) and (5).

Definition 3.5. For each arbitrary fuzzy numbers $A, B \in F$, define the ranking of A and B by TR on F, i.e.

$$TR(A, A_0) > TR(B, A_0) \quad \text{if and only if} \quad A \succ B,$$
$$TR(A, A_0) < TR(B, A_0) \quad \text{if and only if} \quad A \prec B,$$
$$TR(A, A_0) = TR(B, A_0) \quad \text{if and only if} \quad A \sim B.$$

This article formulates the order $\succeq$ and $\preceq$ as $A \succeq B$ if and only if $A \succ B$ or $A \sim B$, $A \preceq B$ if and only if $A \prec B$ or $A \sim B$.

This article considers the following reasonable axioms that Wang and Kerre [21] proposed for fuzzy quantities ranking.

Let TR be an ordering method, S the set of fuzzy quantities for which the method TR can be applied, and A a finite subset of S. The statement "two elements A and B in A satisfy that A has a higher ranking than B when TR is applied to the fuzzy quantities in A" will be written as "$A \succ B$ by TR on A" , "$A \sim B$ by TR on A", and "$A \succeq B$ by TR on A" are similarly interpreted. [21], the axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach TR are as follows:

A-1 For an arbitrary finite subset A of S and $A \in$ A ; $A \succeq B$.

A-2 For an arbitrary finite subset A of S and $(A, B) \in \mathrm{A}^2$; $A \succeq B$ and $B \succeq A$ by TR on A, this method should have $A \sim B$.

A-3 For an arbitrary finite subset A of S and $(A, B, C) \in \mathrm{A}^3$; $A \succeq B$ and $B \succeq C$ by TR on A, this method should have $A \succeq C$.

A-4 For an arbitrary finite subset A of S and $(A, B) \in \mathrm{A}^2$; inf supp(A) $>$ sup supp(B), this method should have $A \succeq B$.

A'- 4 For an arbitrary finite subset A of S and $(A, B) \in \mathrm{A}^2$; inf supp(A) > sup supp(B), this method should have $A \succ B$.

A- 5 Let S, S' be two arbitrary finite sets of fuzzy quantities in which TR can be applied and A and B are in $S \cap S'$. This method obtain the ranking order $A \succeq B$ on S' iff $A \succeq B$ on S.

A-6 Let A, B, $A+C$ and $B+C$ be elements of S. If $A \succeq B$ by TR on A, B, then $A+C \succeq B+C$ by TR on $A+C$ and $B+C$.

A'- 6 Let A, B, $A+C$ and $B+C$ be elements of S. If $A \succ B$ by TR on A, B, then $A+C \succ B+C$ by TR on $A+C$ and $B+C$.

Remark 3.3. The function TR has the properties (A-1), (A-2), ..., (A-5).

Remark 3.4. If $A \preceq B$, then $-A \succeq -B$.

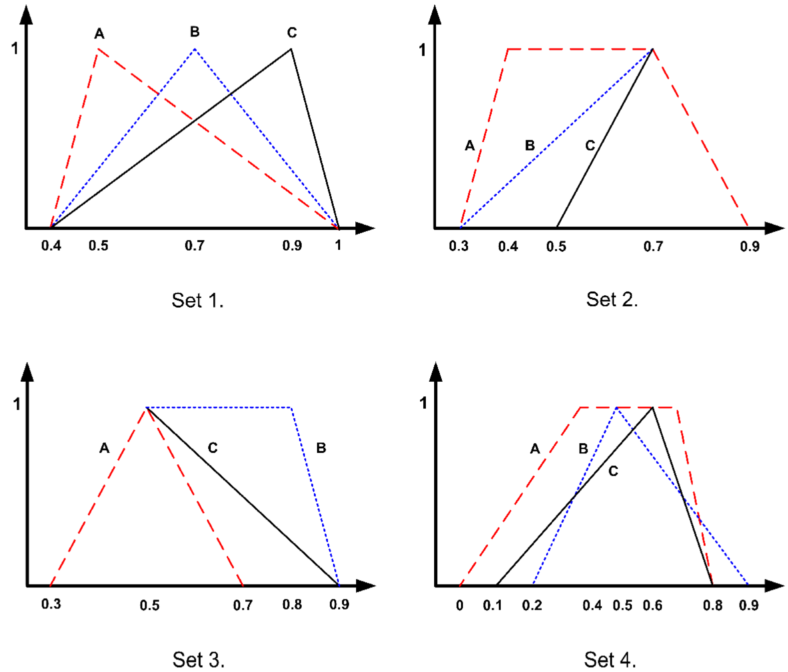
Hence, this approach can infer ranking order of the images of the fuzzy numbers.

4. NUMERICAL EXAMPLES

Now, the authors compare proposed method with the others in [8,22,24,25]. Throughout this section, we assum that $f(x) = x$, and "optimism/pessimism" coefficient is 0.5.

Example 1. Consider the following sets in [27]. (see Figs. 1)

Set 1: A=(0.5,0.1,0.5), B=(0.7,0.3,0.3), C=(0.9,0.5,0.1).
Set 2: A=(0.4,0.7,0.1,0.2) , B=(0.7,0.4,0.2), C=(0.7,0.2,0.2).
Set 3: A=(0.5,0.2,0.2), B=(0.5,0.8,0.2,0.1) (trapezoidal fuzzy number), C=(0.5,0.2,0.4).
Set 4: A=(0.4,0.7,0.4,0.1) (trapezoidal fuzzy number), B=(0.5,0.3,0.4), C=(0.6,0.5,0.2).
To compare with other methods authors refer the reader to Table 1.



Figs. 1

Table 1. Comparative results of Example 1.

Authors	Fuzzy number	Set1	Set2	Set3	Set4
Proposed method	A	0.6946	0.6087	0.5054	0.4630
	B	0.7826	0.6576	0.6562	0.5377
	C	0.8732	0.7031	0.5590	0.4770
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Sing Distance method with p=1	A	1.2000	1.1500	1.0000	0.0950
	B	1.4000	1.3000	1.2500	1.0500
	C	1.6000	1.4000	1.1000	1.0500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Sing Distance method with p=2	A	0.8869	0.8756	0.7257	0.7853
	B	1.0194	0.9522	0.9416	0.7958
	C	1.1605	1.0033	0.8165	0.8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Distance Minimization	A	0.6	0.575	0.5	0.475
	B	0.7	0.65	0.625	0.525
	C	0.9	0.7	0.55	0.525

Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Abbasbandy and Hajjari (Magnitude method)	A	0.5334	0.5584	0.5000	0.5250
	B	0.7000	0.6334	0.6416	0.5084
	C	0.8666	0.7000	0.5166	0.5750
Result		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$B \prec A \prec C$
Choobineh and Li	A	0.3333	0.5480	0.3330	0.5000
	B	0.5000	0.5830	0.4164	0.5833
	C	0.6670	0.6670	0.5417	0.6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Yager	A	0.6000	0.5750	0.5000	0.4500
	B	0.7000	0.6500	0.5500	0.5250
	C	0.8000	0.7000	0.6250	0.5500
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chen	A	0.3375	0.4315	0.3750	0.5200
	B	0.5000	0.5625	0.4250	0.5700
	C	0.6670	0.6250	0.5500	0.6250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	0.3000	0.2700	0.2700	0.4000
	B	0.3300	0.2700	0.3700	0.4200
	C	0.4400	0.3700	0.4500	0.4200
Results		$A \prec B \prec C$	$A \sim B \prec C$	$A \prec B \prec C$	$A \prec B \sim C$
Chu and Tsao	A	0.2990	0.2847	0.2500	0.2440
	B	0.3500	0.3247	0.3152	0.2624
	C	0.3993	0.3500	0.2747	0.2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	A	0.6000	0.5750	0.5000	0.4750
	B	0.7000	0.6500	0.6250	0.5250
	C	0.8000	0.7000	0.5500	0.5250
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \sim C$
Cheng Distance	A	0.7900	0.7577	0.7071	0.7106
	B	0.8602	0.8149	0.8037	0.7256
	C	0.9268	0.8602	0.7458	0.7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng CV uniform distribution	A	0.0272	0.0328	0.0133	0.0693
	B	0.0214	0.0246	0.0304	0.0385
	C	0.0225	0.0095	0.0275	0.0433
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$
Cheng CV proportional distribution	A	0.0183	0.0260	0.0080	0.0471
	B	0.0128	0.0146	0.0234	0.0236
	C	0.0137	0.0057	0.0173	0.0255
Results		$A \prec C \prec B$	$A \prec B \prec C$	$B \prec C \prec A$	$A \prec C \prec B$

Example 2. Let fuzzy numbers A and B described by the membership functions (Fig. 2)

$$A(x)=\begin{cases} g(x) & when\ x\in[0,10), \\ 1 & when\ x\in[10,11], \\ 12-x & when\ x\in(11,12], \\ 0 & otherwise \end{cases}$$

Where

$$g(x)=\begin{cases} 0.1\sqrt{x} & when\ x\in[0,9), \\ 0.7x^2-12.6x+57 & when\ x\in[9,10]. \end{cases}$$

And

$$B(x)=\begin{cases} \frac{1}{10}x & when\ x\in[0,10), \\ 1 & when\ x\in[10,11], \\ 12-x & when\ x\in(11,12], \\ 0 & otherwise\ . \end{cases}$$

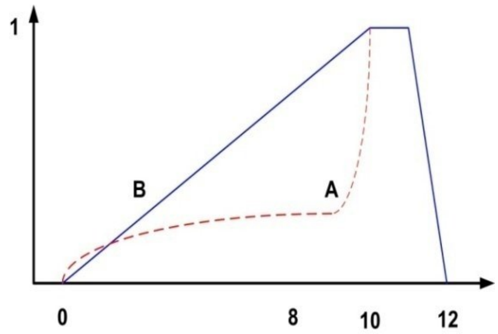


Fig. 2

By using this method $TR(A,A_0)=10.39$ and $TR(B,A_0)=8.57$. Thus, the ranking order is $B\prec A$. As you may see in Table 2, the results of Chu and Tsao method and Cheng distance method are unreasonable. The result of Sign Distance method with $p=1$ and with $p=2$ [1], are the same as this new approach.

Table 2. Comparative results of Example 2.

fuzzy number	Weigthed Distance (new approach)	Sign Distance with p=1	Sign Distance with p=2	Cheng Distance	Chu and Tsao
A	10.39	21.68	16.13	7.51	4.11
B	8.57	16.5	12.87	7.62	4.15
Results	$A\succ B$	$A\succ B$	$A\succ B$	$A\prec B$	$A\prec B$

Example 3. Consider the triangular fuzzy number $A=(2,1,3)$, and the general number, $B=(2,2,1,2)$, shown in Figure 3. The membership function of B is defined by

$$B(x)=\begin{cases}\sqrt{1-(x-2)^2} & when \quad x\in[1,2],\\ \sqrt{1-\frac{1}{4}(x-2)^2} & when \quad x\in[2,4],\\ 0 & otherwise.\end{cases}$$

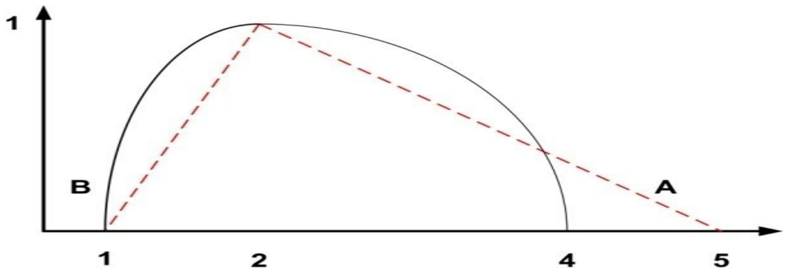


Fig. 3.

In Liou and Wang's ranking method, different rankings are produced for the same problem when applying different indices of optimism. In Sign Distance method with $p=1$, $d_p(A,A_0)=5$, $d_p(B,A_0)=4.78$, and with $p=2$, $d_p(A,A_0)=3.9157$, $d_p(B,A_0)=3.8045$, the ranking order $A\succ B$ is obtained. In Chu and Tsao ranking method, there is $S(A)=1.2445$ and $S(B)=1.1821$, therefore, $A\succ B$. By using this new approach, there is $TR(A,A_0)=2.5932$ and $TR(B,A_0)=2.5873$. Thus, the ranking order is $A\succ B$, too. Also, the result of Distance Minimization method, was similar to this method. Obviously, this method can also rank fuzzy numbers other than triangular and trapezoidal, and compared to Liou and Wang's method. Along with method of Chu and Tsao, this method produces a simpler ranking result.

Example 4. The two triangular fuzzy numbers $A=(3,2,2)$ and $B=(3,1,1)$ shown in Fig. 4, taken from paper [25]. To compare with other methods this article refers the reader to Table 3.

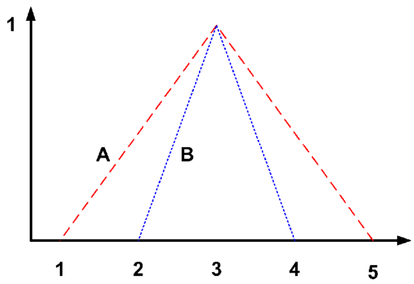


Fig. 4.

Table 3. Comparative results of Example 4

fuzzy number	new approach	Magnitude method	Sign Distance With p=1	Sign Distance with p=2	Distance Minimization	Chen Max-Min
A	3.073	3	6	4.546	3	0.5
B	3.0184	3	6	4.32	3	0.5
Results	$A \succ B$	$A \sim B$	$A \sim B$	$A \succ B$	$A \sim B$	$A \sim B$

In Table 3, $A \sim B$ is the results of Sign Distance method with p=1, Magnitude method, Distance Minimization and Chen method, which is unreasonable. The results of this method is the same as Sign Distance method with p=2, i.e., $A \succ B$.

All the above examples show that the results of this method are reasonable results. The proposed method can be overcome the shortcoming of "Magnitude" method and "Distance Minimization" method.

5. CONCLUSION

In this article, the researchers proposed a new weighted distance between the two fuzzy numbers and a ranking method for the fuzzy numbers. The new method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques. The calculations of the proposed method are simpler than the other approaches. This article also used comparative examples to illustrate the advantages of the proposed method.

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