
Oliver Kunc and Felix Fritzen

Efficient Methods for Mechanical Analysis, Institute of Applied Mechanics (CE), University of Stuttgart, 70569 Stuttgart, Germany; kunc@mechbau.uni-stuttgart.de

* Correspondence: fritzen@mechbau.uni-stuttgart.de

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The authors wish to make a correction to Formula (42) of the paper [1]. The correct formula reads

\[
\overline{C}_{ijkl}(F) = \overline{C}_{ijkl}(R U) = \sum_{m,n=1}^{3} R_{im} \overline{C}_{mijnl}(U) R_{kn} \quad (i, j, k, l = 1, 2, 3).
\]  

(1)

Correspondingly, a correction to Equations (A1)–(A4) of Appendix A of [1] is now provided. To this end, Green’s strain tensor \( \overline{E} = \frac{1}{2}(\overline{F}^{-1} \overline{F}^T - I) \), the corresponding stored energy density function \( \overline{W}(\overline{E}) = \overline{W}(\overline{F}) \), the second Piola–Kirchhoff stress \( \overline{S} = \partial \overline{W}/\partial \overline{E} |_{\overline{F}} \), and the corresponding stiffness tensor \( \overline{C}^E = \partial^2 \overline{W}/(\partial \overline{E}^2) |_{\overline{F}} \) are introduced. Starting from the well-known relationship \( \overline{F} = \overline{F} \overline{S} \) between \( \overline{S} \) and the first Piola–Kirchhoff stress \( \overline{P} = \partial \overline{W}/\partial \overline{F} |_{\overline{F}} \) (see for instance [2]), we express the components of \( \overline{C} \) in terms of those of \( \overline{S} \) and of \( \overline{C}^E \):

\[
\overline{C}_{ijkl} = \frac{\partial^2 \overline{W}}{\partial \overline{F}_{ij} \partial \overline{F}_{kl}} = \frac{\partial \overline{S}_{ij}}{\partial \overline{F}_{kl}} = \sum_{m,n=1}^{3} \frac{\partial \overline{F}_{im}}{\partial \overline{F}_{kl}} \frac{\partial \overline{S}_{mj}}{\partial \overline{F}_{kl}} = \sum_{m=1}^{3} \left( \delta_{ik} \delta_{lm} \overline{S}_{mj} + \overline{C}_{im} \frac{\partial \overline{S}_{mj}}{\partial \overline{F}_{kl}} \right)
\]

(2)

\[
= \delta_{ik} \overline{S}_{lj} + \sum_{m,n=1}^{3} \overline{C}_{im} \frac{\partial \overline{S}_{mj}}{\partial \overline{F}_{kl}} \frac{\partial \overline{S}_{nj}}{\partial \overline{F}_{kl}}
\]

(3)

\[
= \delta_{ik} \overline{S}_{lj} + \sum_{m,n=1}^{3} \overline{C}_{im} \frac{\partial \overline{S}_{mj}}{\partial \overline{F}_{kl}} \frac{\partial \overline{S}_{nj}}{\partial \overline{F}_{kl}}
\]

(4)

\[
= \delta_{ik} \overline{S}_{lj} + \sum_{m,n=1}^{3} \overline{C}_{im} \frac{\partial \overline{S}_{mj}}{\partial \overline{F}_{kl}} \frac{\partial \overline{S}_{nj}}{\partial \overline{F}_{kl}}
\]

(5)

In the last step, the minor symmetry \( \overline{C}_{mjno} = \overline{C}_{mjon} \) has been exploited, and \( i, j, k, l = 1, 2, 3 \) above and throughout. From this, the inverse relation

\[
\overline{C}^{-1}_{ijkl} = - \left( \overline{U}^{-2} \right)_{ik} \overline{S}_{lj} + \sum_{m,n=1}^{3} \left( \overline{F}^{-1} \right)_{im} \overline{C}_{mijnl} \left( \overline{F}^{-T} \right)_{nk}
\]

(6)

can be derived. The fact that Green’s strain tensor is frame invariant, i.e., \( \overline{E}(\overline{R} \overline{U}) = \overline{E}(\overline{U}) \), implies that both the left hand side \( \overline{C}^E_{ijkl}(\overline{E}) \) and the second Piola–Kirchhoff stress \( \overline{S}_{ij} = \overline{S}_{ij}(\overline{E}) \) are independent of \( \overline{R} \). This is in contrast to \( \overline{C}^{-1}_{mnjl} = \overline{C}^{-1}_{mnjl}(\overline{R} \overline{U}) \) from which follows that

\[
\sum_{m,n=1}^{3} \left( \overline{F}^{-1} \right)_{im} \overline{C}_{mijnl}(\overline{R} \overline{U}) \left( \overline{F}^{-T} \right)_{nk} = \sum_{m,n=1}^{3} \left( \overline{U}^{-1} \right)_{im} \overline{C}_{mijnl}(\overline{U}) \left( \overline{U}^{-T} \right)_{nk}.
\]

(7)
By contraction of the indices \( i \) and \( k \) with the second index of \( T \) and the first index of \( T^T \), respectively, Equation (1) follows.

The above changes do not affect the scientific results.

References


2. Bertram, A. *Elasticity and Plasticity of Large Deformations*; Springer: Berlin/Heidelberg, Germany, 2008. [CrossRef]

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