**Article**

**Efficiency Analysis of Lignite Mining Operations Using Production Stochastic Frontier Modeling**

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**Abstract:** This paper proposes a stochastic frontier model for measuring both technical and environmental performance at the mine level by using a translog production function. The Kardia Field openpit lignite mine of the Greek Public Power Corporation (PPC), S.A. is the topic of the case study. Efficiency ratings are derived over a long period of time using annual operating data, and in addition, the determinants of inefficiency are established by means of the technical inefficiency effects model. In the light of the results, there is a strong correlation between technical and environmental efficiency; the results are validated by those produced by data envelopment analysis (DEA). In addition, the stripping ratio is identified as the statistically significant determinant of performance. The proposed framework could be used as an instrument to measure the efficiency of lignite mining operations and to identify the drivers of performance.

**Keywords:** stochastic frontier analysis; technical efficiency; environmental efficiency; technical inefficiency effects model

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**1. Introduction**

Performance measurement in mining is carried out by deriving metrics such as productivity and efficiency [1]. Compared to frontier approaches (i.e., stochastic frontier analysis (SFA) [2,3], data envelopment analysis (DEA) [4]) mining productivity analysis, including measures such as partial and multifactor productivity, it has some limitations. SFA and DEA are the two most established methods in the literature. The key differences between these approaches are that SFA is parametric (i.e., it is important to choose a production function form) and accounts for noise, whereas DEA is non-parametric and does not take noise into account. The benefits and drawbacks of these approaches and their use as a tandem in mining are discussed by Tsolas [5].

Production system efficiency is calculated as the ratio observed to the optimum values of either outputs or inputs. The pertinent models are output-oriented or input-oriented, and the ratio of observed output to maximum potential output or the ratio of observed input to minimum potential input is defined as productive or technical efficiency. In the literature, environmentally-adjusted efficiency models have emerged in a new research area. These new models come from the above frontier approaches (i.e., SFA and DEA) by introducing an additional pollution variable into the analysis, either as an input or as a weakly disposable bad output [6]. Within the context of SFA, environmental efficiency can be calculated as the ratio of minimum to actual environmentally detrimental (bad) input using the input minimization perspective. In this view, environmental performance refers to a bad input that has detrimental effects on the environment [7]. Two approaches [8] can be used to identify the drivers (i.e., environmental factors outside of the production function) of inefficiency, namely: (i) the one-step approach whereas the environmental factors are integrated into SFA; and (ii) the two-step approach, in which SFA is combined with Tobit model that is used to regress the estimated inefficiency against the environmental factors. Since the use of SFA and Tobit regression in sequential stages has certain disadvantages
and the results produced would not be free of bias, the one-step approach is adopted in the current research.

Mining activities pose environmental issues such as the depletion of non-renewable resources and harm the ecosystem. Environmental effects include air, soil, water, and noise pollution, impacts on natural ecosystems and levels of groundwater, and a visual impact on the landscape. Therefore, the essence of mining activities calls for methods to evaluate the performance of the mines, taking environmental considerations into account.

When low-cost compliance with environmental regulations is pursued by producers in developing countries, mining production is characterized as an operation where reduced average production costs are correlated with increased average environmental costs. Producers in developed countries are innovating or acquiring new technologies in order to boost the trade-off between the average environmental and production cost. This leads to the statement that in order to achieve the best environmental results, production (i.e., technical) efficiency must be present [9].

The use of SFA to estimate environmental efficiency in the case of Dutch dairy farming has been suggested in the related literature by Reinhard et al. [7,10]. In the present paper, this approach was applied at the mine level to derive, in addition to technical efficiency, environmental performance scores for the Greek Public Power Corporation’s (PPC’s) Kardia Field opencast lignite mine. As for technical (i.e., overall) efficiency, the use of all inputs was assessed, while the focus was on one environmentally bad input in order to derive environmental efficiency. The selected inputs include traditional production factors such as labor and energy and one bad environmental input that was regarded as conventional input (i.e., the amount of excavated overburden that is dumped both ex-pit and in-pit). In terms of environmental efficiency, an input-oriented measurement of the efficiency of the environmentally bad input is said to concentrate on only the amount of excavated overburden. This measurement varies from the technical (i.e., overall) measurement of efficiency, where the contribution of each input to the estimate of efficiency was treated equally.

The aim of this paper was to assess the ability of SFA to analyze the technical and environmental performance of lignite mining in Greece. The research questions to be addressed by the current study are: (i) Is there a correlation between technical and environmental performance? (ii) What are the explanatory variables that can help explain some of the variations in technical efficiency? (iii) Are the results validated by comparing them to those obtained by other methods?

This paper adds to the research field in many aspects. First, the nature of mining operations necessitates methods for assessing mine performance that takes environmental factors into account. In response to this, the current study employs SFA to measure technical and environmental performance at the mine level, as well as to analyze whether the two metrics are correlated. Second, this analysis compares the results with those obtained using Tyteca’s [11] proposed environmentally modified DEA models. Finally, this study provides some empirical evidence on the impact of specific mine characteristics on inefficiency.

This paper is organized as follows. Section 2 includes a literature review of the relevant works, the methods, and the data set used in the analyses. Section 3 describes how the proposed SFA model was implemented to measure mine efficiency and identify its determinants; SFA efficiency scores were compared with the efficiency scores of the related environmentally-adjusted DEA models, and the results are discussed. Section 4 concludes.

2. Materials and Methods

2.1. Literature Review

Four types of models are listed in the related literature that deals with integrating environmental effects into traditional frontier-based performance analysis [12,13]: “Environmentally adjusted production efficiency” (EAPE) models, “frontier eco-efficiency” (FEE) models, and models based on the materials balance theory and the exergy balance approach. The EAPE models are standard frontier-based (i.e., SFA or DEA) environmentally
adjusted models and the FEE models connect economic and ecological outcomes. The third category includes models that integrate the materials balance concept into frontier-based environmental adjusted models, and the fourth category includes models that use the exergy balance approach.

The environmental performance modeling in the EAPE models contains desirable and undesirable (bad) outputs. The bad outputs, either as outputs or as environmentally harmful inputs, are modeled as additional variables. The DEA-based works of Färe et al. [14], Tyteca [11,15], and the SFA-based works of Reinhard et al. [7,10] lie in this research strand. Färe et al. [14] integrated environmental impact as a bad output and propose hyperbolic DEA for environmental performance calculation. Tyteca [15] provided the DEA framework for environmental performance assessment and proposes three models, namely: the undesirable output (UO) model, the input-undesirable output (IUO) oriented model, and the normalized undesirable output model. Tyteca [11] derived environmental efficiency ratings based on the above models for a sample of U.S. fossil fuel-fired electric power plants. For a survey of DEA use in energy and environmental studies, the reader can consult Zhou et al. [16].

In the literature, there has been a lot of discussion on how to better model pollution using frontier approaches. One common idea is that pollution and production are complementary outputs, allowing pollution to be viewed as an input [7,10]. This modeling assumption is based on the empirical finding that there is often a positive correlation between pollution and intended output. A competing approach treats pollution as a weakly disposable or unintended output [14,15]. Weak disposability means that pollutants (i.e., undesirable outputs) may be minimized by reducing the amount of desirable output. In the SFA context, bad outputs are used as inputs in the analysis [7,10] or are separately modeled [17]. The former approach uses a single frontier to estimate technical efficiency and then uses parameter estimates from the SFA model to estimate environmental efficiency. The latter method allows both technical and environmental efficiency to be defined and calculated separately. Technical efficiency refers to a desirable output frontier, and environmental efficiency refers to a bad output frontier [18].

The directional distance function (DDF) can be used in multi-output settings. The DDF is based on the existence of a curve of multi-dimensional production possibilities curve, so it is possible to calculate the distance to it in several directions. The DDF efficiency measure is based on a path vector and allows for the expansion of desirable outputs as well as the reduction in undesirable outputs or inputs [19]. Both parametric and non-parametric approaches can be used to estimate the DDF [20].

The works of Wu [8] and Koop and Tole [21], respectively, are the first works in mining for the measurement of technical and environmental efficiency at the industry level using SFA. In order to measure technical efficiency and investigate the causes of inefficiency in the Chinese coal industry, Wu [8] used traditional SFA to measure technical efficiency and investigates the causes of inefficiency in the Chinese coal industry, and Koop and Tole [21] use environmentally adapted Bayesian SFA to measure firm environmental output in the global gold mining industry. Other recent conventional SFA studies that deal with aspects of efficiency at the industry level are the works by Burhop and Lübbers [22], Akinboade et al. [23], Shi [24], Shi, and Grafton [25], and Syed et al. [26]. SFA was also used in the utility sector to determine the environmental or technical efficiency of electric utilities [27,28]. Tsolas [1] proposed a framework for mine-level performance evaluation to estimate mine efficiency and identify the drivers of inefficiency for the Kardia field lignite mine of the Greek PPC, S.A.

SFA is superior in terms of the noise problem, while DEA is superior in terms of input and output specification. Enhanced DEA models with bootstrapping can get insight into the bias and uncertainties involved in the estimations, while in SFA, uncertainty is taken into account [29]. This provides justification for the use of SFA in the present study as a starting point. Most of the above traditional SFA studies concentrated on the level of the mining industry with the exception of Tsolas’s [1] work and mainly referred to technical
and economic efficiency with the exception of Koop and Tole’s [21] work that was based on environmentally-adjusted SFA modeling; hence there is a lack of mine-level environmental-adjusted SFA studies to measure both technical and environmental efficiency. The current study aims to fill this gap by using a proposed framework to analyze both technical and environmental performance at the Kardia Field lignite mine, as well as to define the drivers of mine inefficiency.

This study improves on Tsolas [1] by using an environmentally modified translog functional form, which is commonly used to estimate the stochastic production frontier. The inclusion of overburden in opencast mine evaluation for the first time in this study is a major development, as it allows for the estimation of a production function at the mine level and efficiency (technical and environmental) analyses. Compared to previous DEA mining studies [5], the current work also supports the use of DEA and SFA in tandem. It should be noted that the choice of the translog form is important in order to provide additional details into analysis by the environmental efficiency measure [7].

2.2. Methods—Stochastic Frontier Analysis

2.2.1. The Measurement of Technical Efficiency

The error component (EC) model of Battese and Coelli [30] and the technical inefficiency effects (TIE) model of Battese and Coelli [31] are the most commonly used methods in efficiency measurement using SFA [32].

A lignite mine is assumed to use two classes of inputs $x$ and $z$ to produce an output $y(y \in R_+)$, where $x(x \in R_+)$ is the vector of conventional inputs (e.g., labor and energy) and $z(z \in R_+)$ is the vector of inputs that have negative impacts on the environment (i.e., bad inputs). In line with the EC model, the stochastic production function, which, in its general form, expresses the transformation of inputs to mine output for a period of time, is given by:

$$y_i = f(x_i, z_i, \beta, \alpha, \delta) \exp(\varepsilon_i)$$

(1)

where $\beta, \alpha, \delta$ are vectors of unknown parameters; $\varepsilon_i = v_i - u_i$ is the error term; $u_i$ is an independently and identically distributed as half normal asymmetric non-negative error term that is assumed to account for technical production inefficiency; $v_i$ is an independently and identically distributed symmetric random error, independent of $u_i$; and $I = 1, \ldots, I$ is the number of observations (i.e., yearly activities of the same mine).

Since $u_i$ is estimated by the maximum likelihood (ML) method, the output-oriented technical efficiency (TE) is derived by:

$$TE_i = \frac{y_i}{f(x_i, z_i, \beta, \alpha, \delta)\exp(v_i)} = \exp(-u_i)$$

(2)

The prediction of technical efficiencies is based on the conditional expectation of $\exp(-u_i)$, given $\varepsilon_i$. The predictor of technical efficiency of the $i$th observation is [30]:

$$\hat{TE}_i = E[\exp(-u_i)|\varepsilon_i]$$

(3)

Technical efficiency reflects a producer’s ability to obtain the maximum output from a given set of inputs as well as the technical characteristics of the production system.

The maximum attainable output for each input level is represented by the production frontier. Technical efficiency (i.e., $u_i = 0$) or inefficiency is achieved by operating on or below the frontier, respectively.

The production function $f(\bullet)$ in (1) is approximated by a translog function, and the translog stochastic production function is given by:

$$ \ln y_i = \beta_0 + \sum_{k=1}^{K} \beta_k \ln x_{ki} + \sum_{m=1}^{M} a_m \ln z_{mi} + \frac{1}{2} \sum_{k=1}^{K} \sum_{n=1}^{N} \beta_{kn} \ln x_{ki} \ln x_{ni} $$

$$ + \frac{1}{2} \sum_{m=1}^{M} \sum_{h=1}^{H} a_{mh} \ln z_{mi} \ln z_{hi} + \sum_{k=1}^{K} \sum_{m=1}^{M} \delta_{km} \ln x_{ki} \ln z_{mi} + \varepsilon_i $$

(4)
where \(y\) is the output; \(x\) and \(z\) are the conventional inputs and environmentally detrimental inputs, respectively; \(\epsilon_i = v_i - u_i\) is the error term; \(\beta_{kn}, \alpha_{mh}, \delta_{km}\) are unknown parameters; \(k = 1, \ldots, K, n = 1, \ldots, N\) is the number of conventional inputs and \(m = 1, \ldots, M, h = 1, \ldots, H\) is the number of detrimental inputs of the \(i\)-th yearly activity, and \(i = 1, \ldots, I\).

A well-behaved translog output function (e.g., monotonic, strictly convex) was assumed as in some previous studies [7].

The current study’s aim was to find explanatory variables that can help clarify some of the technical efficiency variations, in addition to providing efficiency measures. The TIE model seemed to be the most suitable model. The function that explains the inefficiency scores in this model is calculated in a single stage, avoiding the inconsistency issue that the two-stage method has. The efficiency measurement method used in the TIE model is based on an extension of the EC model that assumes the technical inefficiency effects, \(u_i\), are represented by a linear relationship. The term, \(u_i\), can be replaced by a linear function of explanatory variables reflecting producer-specific characteristics [31]:

\[
u_i = w_o + \sum_{p=1}^{p} w_p s_{pi} + \epsilon_i
\]  

where \(s_{pi} (p = 1, \ldots, P)\) are producer specific explanatory variables associated with technical inefficiency; \(w_o, w_p\) are parameters to be estimated; and \(\epsilon_i\) is an independently and identically distributed random variable truncated at zero from below.

After combining (4) and (5), the resulting model is estimated by a single-equation estimation procedure using the ML method. The effects of the explanatory variables, based on Battese and Coelli [31], can be estimated using the FRONTIER computer program [33] and the R package Frontier [34] (see also [35]).

2.2.2. The Measurement of Environmental Efficiency

For the measurement of environmental efficiency, Reinhard et al. [7] proposed to set \(u_i = 0\) and then replace all bad inputs \(z\) in Equation (4) by \(\phi z\), where \(\phi\) is environmental efficiency \(EE (EE = \phi)\) because assuming strict monotonicity, \(EE\) implies technical efficiency [10]. As a result, the reduction in bad inputs at the production frontier is adjusted for statistical noise \(v_i\). This measure is more in line with the practical demands of mine producers, whose economic goal is to expand output to the production frontier and reduce bad inputs to optimal levels.

Reinhard et al. [7,10] showed that:

\[a(\ln EE_i)^2 + b_i \ln EE_i + u_i = 0 \iff EE_i = \exp\left(\frac{-b_i \pm \sqrt{b_i^2 - 4au_i}}{2a}\right)\]  

where:

\[a = \frac{1}{2} \sum_{m=1}^{M} \sum_{h=1}^{H} \alpha_{mh}\]  

\[b_i = \sum_{m=1}^{M} \alpha_m + \frac{1}{2} \sum_{m=1}^{M} \sum_{h=1}^{H} \alpha_{mh}(\ln z_{mi} + \ln z_{hi}) + \sum_{k=1}^{K} \sum_{m=1}^{M} \delta_{km} \ln x_{ki}\]  

Although for each yearly activity, one metric indicates environmental efficiency, Equation (6) has two solutions. However, a technically efficient firm (i.e., \(u_i = 0\)) is assumed to be necessarily environmentally efficient (i.e., \(\ln EE_i = 0\)) only if the \(+\sqrt{\cdot}\) formula is used [7].
2.3. Data Set

The data used in the current research were annual operational figures of the Kardia Field lignite mine over the 1984–2006 period. The (desirable) production \( Q \) is the amount of lignite produced by the mine as stated in tonnes. Conventional inputs for lignite production include labor and electrical energy, while overburden is considered an environmentally detrimental input. Mine reports are also the source of information on inputs. Man-hours are used to quantify labor \( L \). Electrical energy \( E \) is measured in kWhs and refers to the energy consumed by bucket wheel excavators. Since capital in the form of bucket wheel excavators operating hours has no positive correlation with lignite production, it has been excluded from the analysis. Overburden \( O \) is the single detrimental input that is calculated in bank cubic meters. The uneconomical material that must be extracted in order to access the underlying ore body is known as overburden. The overburden is dumped ex-pit and in-pit when there is enough dump space, and the total amount dumped in- and ex-pit was taken into account in the study. More details on input-output data are provided by Tsolas [1].

The treatment of the overburden excavation as an input can be clarified in two ways. First, as in Pittman [36], a reduction in the overburden implies a reduction in the use of conventional inputs and, as a result, a reduction in excavated lignite output. Therefore, since an increase in traditional inputs is associated with an increase in output, overburden excavation is positively correlated with the output of the lignite. Second, while the overburden excavation as an indicator of the unconventional input is measurable, the repercussions of the overburden cannot be measured so readily. Overburden is viewed as an input variable in the mine production function for these two purposes.

In DEA modeling, overburden removal is viewed as an output from opencast mining other than coal output. Under the weak disposability assumption, overburden is treated as a non-discretionary output [37], as undesirable output [38], or as a production input [39]. Although there is no agreement among researchers on how to handle overburden removal in DEA modeling, the fact that the case study involves the evaluation of the same mine justifies the decision to use it as a detrimental input in SFA. If the assessment includes a variety of different mines with different geological characteristics, this treatment would certainly pose some concerns. In the current study, restricting the analysis to one mine, its yearly activities are presumed to be homogeneous in terms of geological conditions (i.e., the same mine is evaluated against its environment), and since management can decrease overburden, it is treated as an input.

Equipment capability and availability, seam geometry, and stripping ratio (SR) all affect the rate of ore production in surface mines [40]. Furthermore, since dumping overburdened waste material in a cost-effective manner is a major challenge for open-pit mine operations, the emphasis is on optimizing earth-moving operations and minimizing the size of the waste dump’s footprint [41].

Due to the availability of data on mine-specific variables, SR, energy productivity, the capital to labor ratio (CLR), and the age (i.e., vintage) of the mine were used in the current study as candidate explanatory variables that impact efficiency. The amount of excavated overburden was linked to mine pit development, and the excavated rate was linked to the SR, which was the ratio of overburden extracted per ton of lignite mined. Since overburden excavation produces no revenue, reducing the SR is an efficient way to minimize waste rock production. However, in order to lower the SR, the slope should be as steep as possible while staying stable [42]. For more on the SR, the interested reader is referred to Kennedy [43].

The energy productivity of the material handling, which is performed mainly by a conveyor system, was also used as a mine-specific variable. The energy productivity was measured in this study as the ratio of total materials conveyed to conveyor energy consumed. In mine hauling operations, the amount of product (e.g., the tonnage of material) is often used as a proxy for useful work performed, and the energy consumed in haulage
is often used as a measure of energy input [44]. The energy productivity reflects the effectiveness of earth-moving operations.

According to the literature [1], CLR, which captures the impact of factor intensity, should have a positive relationship with performance because the higher the CLR, and therefore the degree of mechanization, the better the performance should be. When a mine reaches the end of its life cycle, the age of the mine is expected to have a negative relationship with performance since production levels will be affected by the mine’s age.

Based on the above, the current research examined SR, energy productivity, CLR, and mine age in order to identify the inefficiency drivers by formulating the following hypotheses: Performance is negatively impacted by SR, while energy productivity is positively impacted, CLR is positively impacted, and mine age is negatively impacted.

A summary of variable definitions is provided in Table 1.

Table 1. Variable definitions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Abbreviation</th>
<th>Definition</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>Labor</td>
<td>L</td>
<td>Input and output variables</td>
<td>man-hours</td>
</tr>
<tr>
<td>Electrical energy</td>
<td>E</td>
<td>Electrical energy consumed by bucket wheel excavators</td>
<td>kWhs</td>
</tr>
<tr>
<td>Overburden</td>
<td>O</td>
<td>Uneconomical material removed to reach the underlying ore body</td>
<td>bank m³</td>
</tr>
<tr>
<td>Output</td>
<td>Q</td>
<td>Annual lignite production</td>
<td>tonnes</td>
</tr>
<tr>
<td>Stripping ratio</td>
<td>SR</td>
<td>Ratio of overburden extracted per ton of lignite mined</td>
<td>bank m³/tonne of lignite</td>
</tr>
<tr>
<td>Energy productivity</td>
<td>EP</td>
<td>Ratio of total materials conveyed to conveyor energy consumed</td>
<td>bank m³/kWh</td>
</tr>
<tr>
<td>Capital-labor ratio</td>
<td>CLR</td>
<td>Ratio of operating hours of bucket-wheel excavators to total man-hours</td>
<td>hours of bucket-wheel excavators/man-hours</td>
</tr>
<tr>
<td>Age of mine</td>
<td>AGE</td>
<td>Age of mine, measured in years since it first began operations</td>
<td>years</td>
</tr>
</tbody>
</table>

3. Results

3.1. SFA Results

The translog production function to be estimated using the TIE model of Battese and Coelli [31] is given by:

$$
\ln y_i = \beta_0 + \beta_1 \ln L_i + \beta_2 \ln E_i + a_1 \ln O_i + \frac{1}{2} \beta_{11} \ln^2 L_i + \frac{1}{2} \beta_{22} \ln^2 E_i + \beta_{21} \ln E_i \ln L_i + \frac{1}{4} \alpha_{11} \ln^2 O_i + \delta_{11} \ln L_i \ln O_i + \delta_{21} \ln E_i \ln O_i + w_0 + w_1 \text{SR}_i + w_2 \text{CLR}_i + w_3 \text{AGE}_i + \epsilon_i
$$

(9)

where, $\beta_0, \beta_1, \beta_2, a_1, \delta_{11}, \delta_{21}, w_0, w_1, w_2, w_3$ are unknown parameters; $\ln y_i$ is the logarithm of output ($Q$); $\ln L_i, \ln E_i$ are the logarithms of conventional inputs ($L, E$); $\ln O_i$ is the logarithm of environmentally detrimental input ($O$); $\text{SR}_i$ is the stripping ratio; $\text{CLR}_i$ is the capital-labor ratio; $\text{AGE}_i$ is the age of the mine; $\epsilon_i$ is an independently and identically distributed random variable truncated at zero from below; and $i = 1, \ldots, I; I$ is the number of observations.

It should be noted that the results for energy productivity as an explanatory variable were unsatisfactory, so it was removed from the analysis. The results of the ML estimates for the parameters of the above translog production function are depicted in Table 2.

Dependent variable: Logarithm of lignite production; L: labor; O: overburden; E: electrical energy consumed; SR: stripping ratio; CLR: capital-labor ratio; AGE: age of the mine; $\sigma^2 = \sigma_u^2 + \sigma_v^2$; $\gamma = \sigma_u^2 / \sigma_v^2$; $\sigma_u$, $\sigma_v$ are the standard deviation of $u$ (error term that accounts for technical inefficiency) and $v$ (symmetric random error), respectively; Number of obs. = 23; ML estimates and their standard errors, $z$- and $p$-values have been produced by the R package Frontier [34].
Table 2. Specification of the SFA model.

| Variable     | Parameter | Coefficient | Standard Error | z-Value | Pr (>|z|) |
|--------------|-----------|-------------|----------------|---------|----------|
| Constant     | $\beta_0$ | 53.74       | 0.98           | 54.59   | 0.000    |
| $\ln L$      | $\beta_1$ | -10.89      | 0.98           | -11.07  | 0.000    |
| $\ln E$      | $\beta_2$ | 39.34       | 0.91           | 43.32   | 0.000    |
| $\ln O$      | $a_1$     | -79.01      | 0.92           | -85.98  | 0.000    |
| 0.5 $\ln^2 L$| $\beta_{11}$ | 14.31       | 1.00           | 14.36   | 0.000    |
| 0.5 $\ln^2 E$| $\beta_{22}$ | -12.50      | 0.48           | -25.97  | 0.000    |
| $\ln E \ln L$| $\beta_{21}$ | -21.64      | 0.64           | -33.92  | 0.000    |
| 0.5 $\ln^2 O$| $a_{11}$   | 5.12        | 0.75           | 6.79    | 0.000    |
| $\ln L \ln O$| $\delta_{11}$ | 29.53       | 0.80           | 36.72   | 0.000    |
| $\ln E \ln O$| $\delta_{21}$ | 9.14        | 0.59           | 15.48   | 0.000    |

Inefficiency model

| Parameter | Coefficient | Standard Error | z-Value | Pr (>|z|) |
|-----------|-------------|----------------|---------|----------|
| Constant1 | $w_0$       | -0.17          | 0.66    | -0.26    | 0.796    |
| SR        | $w_1$       | 0.21           | 0.10    | 2.11     | 0.035    |
| CLR       | $w_2$       | -0.04          | 1.00    | -0.04    | 0.964    |
| AGE       | $w_3$       | -0.01          | 0.03    | -0.22    | 0.825    |

Variance parameters

| $\sigma^2$ | 0.05 | 0.04 | 1.33 | 0.183 |
| $\gamma$   | 0.97 | 0.28 | 3.46 | 0.001 |

Log Likelihood function = 12.79

The gamma parameter ($\gamma$) may be used to monitor for inefficiency in the model since it measures the relative proportion of variability attributable to inefficiency. It indicates the percentage of output variance due to technical efficiency, ranging from 0 to 1, with a value close to 1 implying that a random component of inefficiency contributes significantly to the production system [31]. The estimated value of $\gamma$ is 0.97, implying that technical inefficiency accounts for 97 percent of total output variability, while measurement errors and variables not incorporated in the stochastic frontier model account for the remaining 3 percent (i.e., $1 - \gamma = 0.03$).

The signs of the coefficients of the SFA model are not all consistent with expectations. In particular, the coefficient of electrical energy had a positive sign, but the coefficients of other inputs (i.e., labor and overburden) had negative signs. In regards to energy, this result implies that it is a key factor of lignite production, and therefore, any increase in this input will yield positive returns. The negative sign of labor indicates that the value of the production function is non-increasing with respect to labor input which means that efficient labor usage will improve performance. This also applies to the overburden in terms of performance improvement. The second-order and interaction term coefficients were all statistically significant. It was obvious that all inputs had a significant influence on output, either partially or in the form of quadratic and interaction. The interaction coefficients of overburden with labor and electrical energy were both significantly positive, suggesting complimentary relationships between overburden and labor as well as energy (i.e., more labor and energy input raises the productivity of overburden). Labor and electrical energy, on the other hand, have a substitute relationship (i.e., more energy input lowers the productivity of labor).

Table 2 also shows that in explaining technical (overall) inefficiency, the SR coefficient is statistically significant, and moreover, it had the expected positive sign. It was noted that a positive SR coefficient means that annual activities with higher SR values appear to have had a higher level of inefficiency (i.e., they were more inefficient).
the predicted negative sign, the CLR coefficient was not statistically significant. The age of
mine did not seem to have a negative impact on performance, but the relevant coefficient
was not statistically significant.

Results concerning the technical and environmental efficiency estimates are reported
in Table 3. It should be noted that the output-oriented TE was estimated econometrically,
whereas EE was calculated using Equation (6).

Table 3. Technical and environmental efficiency. Descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>TE</th>
<th>EE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>0.4668</td>
<td>0.9939</td>
</tr>
<tr>
<td>Max.</td>
<td>0.9720</td>
<td>0.9998</td>
</tr>
<tr>
<td>Mean</td>
<td>0.7763</td>
<td>0.9978</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.1477</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

TE: technical efficiency; EE: environmental efficiency.

The mine technical efficiency varied from 0.467 to 0.972, with an average of 0.776
(Table 3). The estimated mean technical efficiency was lower than that found by Tsolas [1].
Since technical efficiency is based on output maximization, the results indicate that the
mine could increase lignite production by about 29 percent by better utilizing its resources
if technical inefficiency is completely removed. This can be explained as follows: Maximum
output equaled $1.29 \cdot \text{real output} = \text{real output}/0.776$, since the average efficiency (i.e., the
ratio of real lignite output to maximum output) was 0.776.

Environmental efficiency ranged from as low as 0.994 to as high as about 1 with an
average of 0.998 (Table 3). The estimated mean environmental efficiency suggests that the
observed quantity of lignite production could have been maintained by using the observed
values of other inputs while generating 0.2 percent ($1 - 0.998$) less overburden.

3.2. Validation of Results

The compatibility of technical efficiency and environmental efficiency is investigated
by Spearman’s rank correlation coefficients. (Table 4). The SFA-generated technical and
environmental efficiency scores were strongly correlated; Spearman’s rank correlation
coefficient = 0.973. This implies that yearly activities with higher technical efficiency were
more likely to have higher environmental efficiency as well.

Table 4. Spearman’s rank correlation coefficients of the efficiency measures yielded by SFA and DEA.

<table>
<thead>
<tr>
<th></th>
<th>TE-SFA</th>
<th>EE-SFA</th>
<th>UO-DEA</th>
<th>IUO-DEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE-SFA</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EE-SFA</td>
<td>0.973</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UO-DEA</td>
<td>0.767</td>
<td>0.758</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>IUO-DEA</td>
<td>0.765</td>
<td>0.761</td>
<td>0.823</td>
<td>1.000</td>
</tr>
</tbody>
</table>

TE-SFA: Technical efficiency yielded by SFA; EE-SFA: Environmental efficiency yielded by SFA; UO-DEA: Efficiency yielded by UO DEA model; IUO-DEA: Efficiency yielded by IUO DEA model.

It is worth noting that although the aim of this study was to employ the SFA for the
measurement of both technical and environmental efficiency, DEA, a competing method
to SFA, was also performed for the benefits of robustness. Selected DEA models were
used for the validation of SFA-based efficiency scores. Tyteca [15] proposed two DEA
models: the UO and the IUO, both of which treated overburden as an undesirable output
and included it as an input to the model. The UO model is based on the perspective of
input minimization and is in line with the principle of the methodological framework
employed here for environmental performance derivation. The IUO model is based on
the perspective of input minimization and is more preferable for the derivation of overall
efficiency to traditional DEA models. The two DEA models employed were based on the
assumption that the undesirable outputs are weakly disposable, which seems realistic in
the case of lignite mining. The interested reader is directed to Tyteca [11,15] for more details on environmentally adjusted DEA models and weak disposability of undesirable outputs. The DEA results are available upon request from the author.

The SFA-generated environmental efficiency scores are correlated with the efficiency scores derived from the UO DEA model; Spearman’s rank correlation coefficient = 0.758. They are also correlated with the efficiency scores derived from the IUO DEA model for the technical efficiency scores provided by SFA; Spearman’s coefficient of rank correlation = 0.761 (Table 4).

4. Conclusions

4.1. Contribution of the Study

Several enhancements to opencast mine efficiency evaluation were proposed by this study. The first enhancement is the inclusion of an overburden in the assessment, which allowed the production function of the Kardia field mine in Greece to be estimated. The second contribution is to establish an improved approach to the derivation of both technical and environmental efficiency and to examine the drivers of inefficiency. This new approach to mine efficiency evaluation will provide credit for more environmentally sustainable balances in the traditional input and overburden composition of the mine. To the best of the author’s knowledge, this is the first attempt to use SFA to evaluate mine efficiency by integrating overburden in the input side of a production function together with conventional mine inputs. The final contribution concerns the validation of SFA-based efficiency ratings using selected DEA models.

4.2. Key Conclusions

The current study is a first step toward evaluating mine-level technical and environmental efficiency. SFA modeling was used with data from the Kardia field mine. By integrating one bad input, namely overburden, into the analysis, the approach of Reinhard et al. [7,10] is expanded. The current study answers a set of research questions raised in the first section. According to the empirical results for Question (1) there is a strong correlation between SFA-based technical and environmental efficiency. In response to Question (2), the findings support the hypothesis that SR as identified as the statistically significant determinant of performance. In regards to Question (3), the SFA-based efficiency results were validated by those produced by DEA. It should be noted that while consistent with prior findings and hypotheses in literature, empirical findings cannot be generalized beyond the case study’s geographical context.

In general, the mine’s environmental efficiency was higher than its technical efficiency, indicating that the mine, on average, follows good environmental practices while retaining a sufficient technical efficiency rating. It is preventable that the results and findings of the analyses can be used by policymakers and regulators.

4.3. Implications

Efficiency evaluations are used by regulators in industries, such as mining, to analyze and compare regulated firms and to formulate policies. Regulators should consider not only technical but also environmental efficiency when developing policies related to mine efficiency. Moreover, along with an abundance of DEA applications in measuring efficiency, regulators may be keenly interested in relevant case studies in the stochastic frontier setting. In addition, managers may use SFA-based metrics not only internally but also in reporting on corporate social responsibility.

4.4. Future Research

More precise on-site measurement of environmental pressures such as contaminants in the air, soil, and water, as well as their effect on biodiversity, will greatly support future research into environmental efficiency, regardless of methodological approach. A much richer analysis would be possible if such data were combined with more detailed
mine operational and technical data (e.g., footprint of the waste dump, hauling distance). Furthermore, the findings of such a study will provide policymakers and regulators with far more accurate information.

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