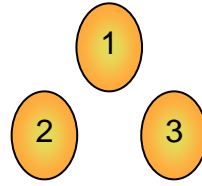
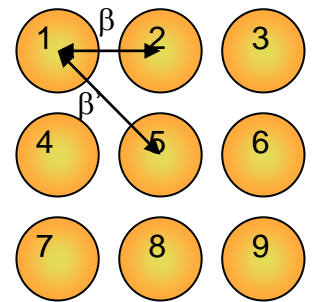


Supplementary Materials

$$\begin{vmatrix} \omega_0 - \omega & \beta & \beta \\ \beta & \omega_0 - \omega & \beta \\ \beta & \beta & \omega_0 - \omega \end{vmatrix} = 0$$



$$\begin{vmatrix} \omega_0 - \omega & \beta & 0 & \beta & \beta & 0 & 0 & 0 & 0 \\ \beta & \omega_0 - \omega & \beta & \beta & \beta & \beta & 0 & 0 & 0 \\ 0 & \beta & \omega_0 - \omega & 0 & \beta & \beta & 0 & 0 & 0 \\ \beta & \beta & 0 & \omega_0 - \omega & \beta & 0 & \beta & \beta & 0 \\ \beta & \beta & \beta & \beta & \omega_0 - \omega & \beta & \beta & \beta & \beta \\ 0 & \beta & \beta & 0 & \beta & \omega_0 - \omega & 0 & \beta & \beta \\ 0 & 0 & 0 & \beta & \beta & 0 & \omega_0 - \omega & \beta & 0 \\ 0 & 0 & 0 & \beta & \beta & \beta & \beta & \omega_0 - \omega & \beta \\ 0 & 0 & 0 & 0 & \beta & \beta & 0 & \beta & \omega_0 - \omega \end{vmatrix} = 0$$



$$\begin{vmatrix} \omega_0 - \omega & \beta & 0 & \beta & 0 & 0 & 0 & 0 & 0 \\ \beta & \omega_0 - \omega & \beta & \beta & \beta & 0 & 0 & 0 & 0 \\ 0 & \beta & \omega_0 - \omega & 0 & \beta & \beta & 0 & 0 & 0 \\ \beta & \beta & 0 & \omega_0 - \omega & \beta & 0 & \beta & \beta & 0 \\ 0 & \beta & \beta & \beta & \omega_0 - \omega & \beta & 0 & \beta & \beta \\ 0 & 0 & \beta & 0 & \beta & \omega_0 - \omega & 0 & 0 & \beta \\ 0 & 0 & 0 & \beta & 0 & 0 & \omega_0 - \omega & \beta & 0 \\ 0 & 0 & 0 & \beta & \beta & 0 & \beta & \omega_0 - \omega & \beta \\ 0 & 0 & 0 & 0 & \beta & \beta & 0 & \beta & \omega_0 - \omega \end{vmatrix} = 0$$

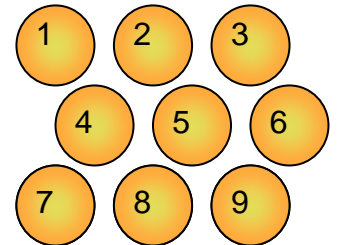


Figure S1. Examples of secular determinant of various NPs system geometry.