**Supplementary Materials: Chemically Roughened, Sputtered Au Films with Trace-Loaded Manganese Oxide for both On-Chip and Off-Chip High Frequency Supercapacitors**

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**Calculation:**

**Three electrode measurement:**

The areal specific capacitance ($C_A$) based on CV is calculated by:

$$C_A = \frac{\int i(V) dV}{2A\Delta V}$$

where $\int i(V) dV$ is the integrated area of the CV curve, and $A$, $\Delta V$, $\nu$, are the electrode area, working potential range, scan rate.

**Two electrode full cell measurement:**

The areal specific capacitance ($C_A$) is calculated by:

$$C_A = -\frac{1}{2\pi f Z'' A}$$

The resistor-capacitor time constant ($\tau_{RC}$) is calculated by:

$$\tau_{RC} = -\frac{Z'}{2\pi f Z''}$$

The real or imaginary areal specific areal capacitance ($C'$, $C''$) are calculated by:

$$C' = -\frac{Z''}{2\pi f |Z|^2 A}$$

$$C'' = -\frac{Z'}{2\pi f |Z|^2 A}$$

$\tau_0$ is derived from the frequency at maximum $C''$

$$\tau_0 = \frac{1}{f_0}$$

where $f$ is the frequency, $A$ is the electrode area, $Z'$ and $Z''$ are the real and imaginary impedance, $f_0$ is the frequency at maximum $C''$. 

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