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Review of Critical Point Searches and Beam-Energy Studies †

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Abstract: I give an introductory overview of the search for the critical point in the phase diagram of QCD in the context of heavy-ion collisions from a theoretical perspective. The focus is on static and dynamic critical physics and the corresponding properties of particle number fluctuations as relevant examples of promising observables for signatures of a critical point in beam-energy scan experiments.

Keywords: heavy-ion collisions, QCD phase diagram, critical phenomena

1. Introduction

The collision of heavy ions at ultrarelativistic energies produces the hottest and densest matter on earth. This has opened the possibility to study strongly interacting matter under the most extreme conditions, resembling the state of matter microseconds after the big bang or the interior of neutron stars. By varying the beam energy in such collision, the net baryon density in the fireball can be adjusted. This way, the transition from the deconfined quark-gluon plasma (QGP) to confined hadronic matter can be probed in the plane of temperature and baryon chemical potential. A detailed understanding of the corresponding phase diagram of QCD is still an open fundamental problem of particle and nuclear physics.

At vanishing baryochemical potential it has been established by lattice gauge theory that, instead of a phase transition, there is a crossover from the QGP to hadronic matter at a pseudocritical temperature of $T_c(\mu_B = 0) = (156.5 \pm 1.5)$ MeV [1,2]. This is in strikingly good agreement with the freeze-out temperature extracted from statistical model fits to hadron yields in $\sqrt{s} = 2.76$ TeV central Pb-Pb collisions measured in ALICE at the LHC [3]. It is worth emphasizing that the definition of the pseudocritical temperature is not unique and its precise value is very sensitive to the underlying definition [4]. Owing to the notorious sign problem, state-of-the-art simulations of QCD on the lattice are restricted to vanishing chemical potential. Functional continuum methods, such as the functional renormalization group or Dyson-Schwinger equations, do not suffer from the sign problem and can be used to study QCD from first-principles, but quantitative results are not available to date [5]. Furthermore, only the regimes of very high T and/or μ_B are accessible by perturbation theory [6]. Hence, information on the phase diagram in the region $0 < \mu_B \lesssim 3$ GeV from first-principle QCD computations are unavailable so far. Low-energy effective models of QCD have proven to provide valuable information about qualitative features of QCD also at finite μ_B. A prevailing prediction of many QCD-inspired model calculations is the existence of a point in the phase diagram where the transition is of second order, separating the crossover from a first-order transition. For recent results with three quark flavors see [7–9]. This critical endpoint (CEP) has numerous interesting features that have the potential to be observable in heavy-ion experiments. Most remarkable is the emergence of universal behavior in the vicinity of the CEP. This gives us the opportunity to confirm the existence of a true phase transition between the QGP and hadronic matter. This is crucial for our understanding of the underlying symmetries of QCD and the formation of matter around us.

In the following, I will outline the basics of critical physics and how it can be probed in heavy-ion collisions. For the sake of brevity, the explicit discussion is limited to particle number fluctuations. They serve as prime examples to illustrate the physics relevant for the search of the CEP.

2. Statics

We start with assuming that the system is in thermal equilibrium. The conventional description of second-order phase transitions, due to Landau, is based on minimizing the free energy, e.g.,

$$F(\sigma) = \alpha\,\sigma^2 + \beta\,\sigma^4 + \ldots \,. \tag{1}$$

σ is an order parameter field associated with the slow (long-range) modes of the system. In case higher-order terms in σ (denoted by the dots) can be neglected, β has to be positive to ensure a stable vacuum. The vacuum state of the theory is given by the value of the order parameter field at the minimum of F. If we think of α as being a continuous function of an external parameter, say, the temperature T, then there might be a second-oder phase transition at a critical temperature T_c with $\alpha(T_c) = 0$ and $\alpha(T) \gtrless 0$ for $T \gtrless T_c$, separating the symmetric from the symmetry broken phase. For example, if σ is a one-component real field, F is symmetric under sign flip $\sigma \to -\sigma$, i.e., the cyclic group Z_2 is the symmetry group. This corresponds to the well-known Ising model.

A central physical quantity in this context is the correlation length ξ. It can be thought of as the distance two particles have to be pulled apart in order to become decorrelated, $\langle\sigma(0)\sigma(r)\rangle \sim e^{-r/\xi}$ for $r \to \infty$. Since the correlation length is directly related to the mass parameter of the free energy, $\xi \sim 1/\sqrt{\alpha}$ [10], it diverges at a second-order phase transition, $\xi(T \to T_c) \to \infty$. These long-range correlations at (and close) to T_c lead to a collective behavior of the system. As a result, the microscopic details of the theory become irrelevant. This gives rise to the notion of *universality*. Its magic lies in the fact that, even though we likely never solve QCD exactly, near a second order phase transition the free energy is indeed of the form (1). In its modern and most developed form, this is described by the theory of critical phenomena formulated in terms of the renormalization group [11]. In this language, a second-order phase transition occurs at an infrared fixed point of the renormalization group flow of the system. Only very few relevant parameters remain while most details of the theory are washed out during the evolution towards the fixed point. The relevant parameters are determined by the *critical exponents*. For instance, the correlation length close to the phase transition is controlled by the distance from the critical temperature and the positive exponent ν, $\xi \sim (T - T_c)^{-\nu}$. The crucial observation is that the values of the critical exponents are fixed by the symmetries and the spacetime dimensionality of the system rather than all its potentially complicated microscopic details. Hence, microscopically very different systems can be grouped into universality classes characterized by the same critical exponents.

Since the QCD phase transition occurs at finite temperature, the relevant dimensions are the three spatial dimensions, $d = 3$. For the chiral phase transition, the relevant symmetry is the global flavor symmetry. In case of massless up, down and strange quarks and assuming the presence of the axial anomaly at T_c, it is given by independent left- and right-handed flavor rotations, $SU(3)_L \times SU(3)_R$. However, the chiral phase transition of QCD is expected to be of first order for any μ_B in this case and thus no critical phenomena occur [12,13]. If we consider only up and down quarks to be massless, the symmetry is $SU(2)_L \times SU(2)_R \sim O(4)$ and QCD is in the $3d$ $O(4)$ universality class in this limit. Since in reality all quarks have different finite masses, the only symmetry left is the Z_2 sign flip symmetry of the order parameter of the chiral phase transition, the chiral condensate $\sigma \sim \langle\bar{q}q\rangle$. So, remarkably, QCD close to the CEP shares the same universal properties as the $3d$ Ising model.

To search for the CEP in heavy-ion collisions, we perform beam-energy scans. The beam energy $\sqrt{s}$ basically fixes a trajectory through the phase diagram where the temperature decreases with time and the system evolves approximately along isentropes. Hence, for a given species of incident nuclei the phase diagram is probed in the T–μ_B plane. In order to have the chance to observe the CEP in a beam-energy scan there should be at most two relevant parameters in its vicinity. Otherwise it would be highly unlikely to hit just the right values of more than two relevant parameters with only two tuning parameters T and μ_B. In general, the phase structure depends on more than only two parameters. For instance, the location of the phase boundary is sensitive to isospin and strangeness chemical potentials, e.g., [14,15]. In some sense we have to be lucky enough that the existence of a CEP is robust against variations of parameters that are difficult to control in heavy-ion collisions.

The main question is now how critical fluctuations can manifest themselves in experimental results. While in theory one can compute order parameters directly, the challenge is to identify hadronic observables which inherit signatures of a second order phase transition. A prominent example are non-Gaussian event-by-event fluctuations of particle multiplicities [16,17]. They can be accessed through generalized susceptibilities which can be obtained from the pressure P and are related to the connected correlators of net-baryon number N_B,

$$\chi_n = T^{n-4}\,\frac{\partial^n P(T,\mu_B)}{\partial\mu_B^n} = \frac{1}{VT^3}\langle (N_B - \langle N_B\rangle)^n\rangle_{\text{connected}}\,. \tag{2}$$

To remove the volume dependence, ratios of susceptibilities such as the scaled kurtosis $\kappa\sigma^2 = \chi_4/\chi_2$ with the variance $\sigma^2 = VT^3\chi_2$ (not to be confused with the order parameter) are considered. The key finding in [17] is that particle number correlations directly couple to the order parameter σ and are therefore sensitive to the correlation length in a universal manner, $\chi_n \sim \xi^{n(5-\eta)/2-3}$, where η is a critical exponent ($\eta \approx 0.0363$ for $3d$ Ising). Due to the limited size of the fireball the correlation length cannot grow to infinity, but higher order susceptibilities carry clearer signals of critical physics in finite systems. The connection to experiment is established by the fact that event-by-event net-particle distributions $N_{\text{events}}(N_{\text{net-particle}})$ can be measured in beam-energy scans, e.g., [18]. Since net-proton number fluctuations carry the same critical singularity as net-baryons [19,20], one can measure net protons, define their probability distribution, $P(N_P) = N_{\text{events}}(N_{\text{P}})/N_{\text{events}}$, and extract proton number correlations from experimental data via $\langle (N_P - \langle N_P\rangle)^n\rangle = \sum_{N_P}(N_P - \langle N_P\rangle)^n P(N_P)$. Hence, the net-proton event-by-event distribution carries signatures of critical physics; its moments are directly sensitive to ξ.

One central property of universality is *scaling*. For illustration, imagine we had some way of determining the correlation length as a function of the beam energy, $\xi(\sqrt{s})$. Then one could rescale the measured susceptibilities according to $\tilde{\chi}_n(\sqrt{s}) = \chi_n(\sqrt{s})/\xi(\sqrt{s})^{n(5-\eta)/2-3}$. For beam energies that probe the critical region, i.e., where the evolution of the system passes the CEP sufficiently close such that universal physics dominate, the corresponding rescaled susceptibilities $\tilde{\chi}_n$ would lie on top of each other. Such a scaling would be an unambiguous signature of the existence of the CEP. Another, more tractable consequence of universality is non-monotonic behavior of the moments of the net-proton distribution as a function of $\sqrt{s}$ such that their sign may change at the CEP [21,22]. Indeed, the results of the kurtosis from the first phase of BES at RHIC are very promising [23]. However, the experimental situation is far more complex than the clean equilibrium critical physics discussed here. There are various other effects which might significantly influence the measured net-proton moments. Conceptually most obvious are non-equilibrium effects related to the expansion of the fireball.

3. Dynamics

Assume the QGP created in a heavy ion collision cools through the CEP during its expansion. The system tries to relax to thermal equilibrium through interactions between its constituents. Equilibrium can be achieved if the relaxation rate $1/\tau_{\text{rel}}$ is larger than the rate of change of the system due to expansion and cooling, $1/\tau_{\text{exp}}$. Heuristically speaking, if the interactions are short ranged, the system equilibrates faster since relaxation of the constituents is only influenced by their immediate neighborhood. However, since the correlation length becomes very large close to the CEP, so does the relaxation time. This phenomenon is known as *critical slowing down*. Eventually $\tau_{\rm rel} > \tau_{\rm exp}$, so the system is bound to fall out of equilibrium close to the CEP. As a result, the correlation length is significantly suppressed compared to its equilibrium part and its maximum does not occur exactly where the system passes the CEP, but at a later point in the evolution [24]. This retardation or memory effect can have significant influence on the signatures of the CEP.

Luckily, the static critical phenomena described above can be generalized to *dynamic critical phenomena* [25]. The corresponding universality classes depend on the equilibrium universality class and the stochastic dynamics imposed on the system. They are typically described by the time evolution of slow modes, i.e. order parameters and hydrodynamic modes (conserved quantities). We mentioned that the order parameter couples to the baryon number. As it turns out, σ relaxes much faster than $n_B = N_B/V$, leaving the conserved baryon number density as the prevailing slow mode. The expansion of the system leads to additional slow hydrodynamic modes. It has been shown in [26] that the CEP belongs to the same universality class as the liquid-gas phase transition of a pure fluid, called model H in the classification of [25]. With this knowledge, one can immediately infer that the relaxation time diverges as $\tau_{\rm rel} \sim \xi^z$, with the dynamic critical exponent $z \approx 3$. Furthermore, also the shear viscosity is singular at the CEP, but with a very small critical exponent. As in the static case, QCD dynamics in the vicinity of the CEP is governed by universal physics which facilitate model-independent predictions without knowledge of the full solution to QCD.

In order to understand the experimental signatures of the CEP, the realistic modeling of the non-equilibrium dynamics is essential. While universal properties help to simplify the problem immensely, many details still depend on non-universal quantities. Very similar to ordinary hydrodynamics, solving the evolution equations for the slow modes requires knowledge of the equation of state and various transport coefficients. Even in the critical regime governed by universality, the map from the parameters of the Ising model to those of QCD is non-universal. The same is true for the critical temperature and critical chemical potential as well as the size of the critical region. First studies in this context have been carried out, e.g., in [27–31]. A general framework to combine the bulk hydrodynamic evolution with critical fluctuations has recently been presented in [32]. First results show that the net-baryon number fluctuations discussed above are indeed strongly affected by non-equilibrium effects [29,30]. The non-monotonic behavior of the scaled kurtosis $\kappa\sigma^2$ and in particular its sign, which has been argued to be indicative of a CEP, can be significantly altered due to the memory/retardation effects already observed in [24]. Universality manifests itself in this non-equilibrium situation in scaling with respect to two emerging scales $\tau_{\rm KZ}$ and $l_{\rm KZ}$. This generalizes the equilibrium situation with only one relevant scale, the equilibrium correlation length $\xi_{\rm eq}$. These scales emerge since relaxational dynamics effectively freeze out when $\tau_{\rm rel} \geq \tau_{\rm exp}$ close to the CEP. Critical fluctuations can then be expressed in terms of the correlation length and the relaxation time at $\tau_{\rm rel} = \tau_{\rm exp}$. This Kibble-Zurek scaling [33–35] has been applied to particle number correlations in [36]. Hence, also in the dynamic case scaling could be used to identify signatures of the CEP.

4. Concluding Remarks

Heavy-ion collisions offer the unique opportunity to probe the phase diagram of QCD over a broad range of temperatures and chemical potentials. I tried to argued that particle number correlations are promising observables for the search of a critical point. Understanding universality and critical physics both in and out of equilibrium is of major importance for the interpretation and prediction of experimental signatures of the CEP.

In closing, I have to mention that there are various “non-critical” effects that influence the measured particle number correlations. Since particle numbers are exactly conserved in heavy-ion collisions, fluctuations can only be observed within limited acceptance regions in the detector. Understanding the acceptance dependence of fluctuation observables and finding “optimal” acceptance windows for the observation of a CEP are crucial [37–39]. Off-diagonal cumulants could provide valuable insights regarding the role of particle number conservation [40]. Detector efficiency effects have to be taken into account as well [41,42]. Hence, it takes a combined effort of theory and experiment to identify and understand signatures of the CEP.

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