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Alpha Beta Risk and Stock Returns—A Decomposition Analysis of Idiosyncratic Volatility with Conditional Models

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Abstract: The variance of stock returns is decomposed based on a conditional Fama–French three-factor model instead of its unconditional counterpart. Using time-varying alpha and betas in this model, it is evident that four additional risk terms must be considered. They include the variance of alpha, the variance of the interaction between the time-varying component of beta and factors, and two covariance terms. These additional risk terms are components that are included in the idiosyncratic risk estimate using an unconditional model. By investigating the relation between the risk terms and stock returns, we find that only the variance of the time-varying alpha is negatively associated with stock returns. Further tests show that stock returns are not affected by the variance of time-varying beta. These results are consistent with the findings in the literature identifying return predictability from time-varying alpha rather than betas.

Keywords: conditional model; time-varying alpha and betas; alpha beta risk; idiosyncratic volatility; stock returns

JEL Classification: G12; G32

1. Introduction

In an influential study, Ang et al. (2006) found that high idiosyncratic risk, defined as the standard deviation of the residuals from the Fama and French (1993) model, predicts low individual stock returns. This finding is puzzling because the modern portfolio theory by Markowitz (1952) suggests that investors should be compensated only for systematic risk in equilibrium. In addition, Merton (1987) and Malkiel and Xu (2002) argue that idiosyncratic risk is positively related to expected stock returns when portfolios are not perfectly diversified. However, the negative relation between idiosyncratic risk and stock returns is confirmed in other studies by Ang et al. (2009); and Hou and Loh (2016). All of these studies use an unconditional Fama–French three-factor model to estimate idiosyncratic risk. However, if the unconditional asset pricing model fails to account for important nature of the parameters such as a time-varying property, the negative relation may be misleading since the estimate of idiosyncratic risk is not purely idiosyncratic (i.e., it contains certain components that may exhibit systematic patterns in predicting stock returns).

2. Overview

In this paper, we decompose total risk based on a conditional Fama–French three-factor model. Using time-varying alpha and betas in this model, we find that four additional terms—the variance of alpha, the variance of the interaction between time-varying component of beta and its respective factors, plus two covariance terms—are the components included in the idiosyncratic risk estimated using an unconditional model. The main task of this paper is to study the relation between those components and stock returns, and how they affect the relation between idiosyncratic risk and stock returns.

The predictability of returns has been widely studied (e.g., Balvers et al. (1990), Patelis (1997), Avramov and Wermers (2006), Avramov et al. (2011), and Banegas et al. (2013), among others). Since Keim and Stambaugh (1986), conditional asset pricing models that incorporate instrumental variables have been studied to predict future stock returns. These models, which contain time-varying alpha or time-varying betas, provide new insight into explaining stock returns. For example, Jagannathan and Wang (1998), Ferson and Harvey (1999), and Lettau and Ludvigson (2001)—all of whom use macroeconomic variables as instrumental variables—show that a conditional model performs better than its unconditional counterpart. Moreover, Avramov and Chordia (2006) use an optimal portfolio strategy derived from mean-variance theory and demonstrate that the conditional model provides predictability for stock returns that is associated with the time-varying property of the model. They find that the optimal portfolio performs better when accounting for time-varying alpha, which implies that the return predictability is determined by the time-varying alpha. The question still remains whether this predictability comes from the additional sources of risk that are linked to the framework of conditional pricing models (i.e., the time-varying alpha) or from the time-varying betas. While the stock return variance has been extensively studied in the literature (see, for example, Barclay et al. (1990), Bollen (1998), Bekaert and Wu (2000), and Blitz and Vliet (2007), among others) and an increasing number of studies have investigated the relation between variance and return (e.g., French et al. (1987), Campbell and Hentschel (1992), Glosten et al. (1993), Braun et al. (1995), Duffee (1995), Dennis et al. (2006), Bollerslev et al. (2009), and Carr and Wu (2009)), few studies has been conducted from the perspective of idiosyncratic risk components. Due to the fact that the volatility of alpha is a component of idiosyncratic risk in an unconditional model and that the volatility of alpha may be an additional source of risk that comes with a conditional model, we label it "alpha risk" in this study. Since time-varying betas also play an indirect role in the components of idiosyncratic volatility, we define "beta risk" as the volatility of time-varying betas. Although Brooks et al. (1998) also study time-varying beta risk, this study is different in that we use conditional models following Ferson and Harvey (1999) rather than generalized autoregressive conditional heteroscedasticity (GARCH) and Kalmam filter approaches.

We find that the volatility of alpha can predict average stock returns, but the volatility of betas fails to do so in general. It is possible that the predictability is driven by the instrumental variable because the alpha and betas evolve directly with the macroeconomic variables. In a conditional model, macroeconomic variables are usually selected from those that predict stock returns. We then explore the negative relation between idiosyncratic risk and stock returns by controlling for these additional terms. Therefore, we also explore how the alpha beta risk interacts with idiosyncratic volatility in determining average stock returns. Last, we investigate whether the return predictability from the alpha risk is affected by the time-varying alpha itself. Supporting Avramov and Chordia (2006), we find that the time-varying alpha predicts average stock returns, but that this relation does not impact the relation between idiosyncratic risk and stock returns.

The findings of this paper are three-fold: (1) alpha risk predicts average stock returns at both the portfolio level and the firm level, which is not true for beta risk in general; specifically, stocks with higher alpha risk will have lower future returns; (2) alpha risk predicts returns for both small and medium stocks with a low book-to-market ratio, but does not predict returns for large stocks; and (3) the return predictability from alpha risk at the firm level is not driven by idiosyncratic risk, macroeconomic variables, or the time-varying alpha itself.

3. Alpha Beta Risk

In this section, we theoretically demonstrate why alpha beta risk is associated with stock returns. We adopt the decomposition approach similar to Campbell et al. (2001) and Xu and Malkiel (2003). First, we have the following relation:

$$Var(R_{i,t}) = Var(R_{M,t}) + Var(r_{i,t}) + 2Cov(R_{M,t},\ r_{i,t}) \tag{1}$$

where $Var(R_{i,t})$ is the total risk of stock i. $R_{M,t}$ is the systematic element of the stock's return and $r_{i,t}$ is its idiosyncratic return component. Because by definition $Cov(R_{M,t},\ r_{i,t}) = 0$, the idiosyncratic risk is $Var(r_{i,t}) = Var(R_{i,t}) - Var(R_{M,t})$.

We consider a generalized factor model that contains N risk factors. It is fitted to individual stocks as follows:

$$R_{i,t} = \alpha_i + \beta_i' F_t + \varepsilon_{i,t} \tag{2}$$

where F_t is an $\mathrm{N} \times 1$ vector that contains the N risk factor at time (t) and β_i' is a $1 \times \mathrm{N}$ vector that contains the corresponding factor loading for stock (i). Based on Equations (1) and (2), we decompose the variance of stock returns under an unconditional model as follows:

$$Var(R) = Var\big(\alpha + \beta' F\big) + Idio = Var\big(\beta' F\big) + \ Idio \tag{3}$$

where $Idio \equiv Var(\varepsilon)$. We omit the subscripts (i) and (t) for simplicity.

Considering that the alpha and betas are time-varying conditional on macro-economic state variables, we assume that both consist of a constant component and a time-varying component as follows:

$$\alpha_t = \alpha_0 + \alpha_1 \times f(z_{t-1}) \tag{4}$$

$$\beta_t = \beta_0 + \beta_1 \times f(z_{t-1}) \tag{5}$$

where $f(z_{t-1})$ is a scalar of a function of the macroeconomic state variable that drives the change in alpha and beta over time. β_0 and β_1 are $\mathrm{N} \times 1$ vectors. Note that Equations (4) and (5) nest the unconditional version of the alpha and betas, where α_1 and β_1 are assumed to be equal to zero. In the circumstances of Equations (4) and (5), the variance of stock returns can be decomposed as follows:

$$\begin{aligned} Var(R) = Var(\alpha) + Var(\beta' F) + 2Cov(\alpha, \beta' F) + \ Idio_c \\ = Var(\alpha) + Var((\beta_0{}' + \beta_1{}' \times f(z_t))F) + 2Cov(\alpha, \beta' F) + Idio_c \\ = Var(\alpha) + Var(\beta_0{}' F) + Var(\beta_1{}' \times f(z_t)F) \\ + 2Cov(\beta_0{}' F, \beta_1{}' \times f(z_t)F) + 2Cov(\alpha, \beta' F) + Idio_c \end{aligned} \tag{6}$$

where $Idio_c$ is the idiosyncratic volatility from a conditional model. Comparing Equations (3) and (6), we find that the variance of stock returns consists of four additional terms: the variance of alpha, the variance of the interaction between the time-varying component of beta and the respective factors, and two covariance terms. In an unconditional model, $\beta_1 = 0$ and then $\beta = \beta_0$; comparing Equation (3) with Equation (6) gives:

$$Idio = Var(\alpha) + Var\big(\beta_1{}' \times f(z_t)F\big) + 2Cov\big(\beta_0{}' F, \beta_1{}' \times f(z_t)F\big) + 2Cov\big(\alpha, \beta' F\big) + Idio_c \tag{7}$$

Equation (7) shows that the four additional terms are the components of idiosyncratic risk that occur when people fail to account for the conditional pricing model with the time variation of alpha and betas. If the time-varying conditional model is the true model, then these four components represent systematic risk but are mistakenly included in the idiosyncratic risk category.

4. Methodology and Data

The theoretical results above imply that the time variation of both alpha and betas plays a role in the components of idiosyncratic volatility. In this study, we focus on the volatility of the time-varying alpha and betas. While alpha risk is defined as the standard deviation of the time-varying alpha in one month, estimated from a regression of daily stock returns on the three Fama–French factors, we define the beta risk as the standard deviation of the beta for each factor. Since we use a conditional Fama–French three-factor model, we calculate the beta risk for Fama-French three factors- MKT, SMB, and HML[1]- separately. It is common practice to use macroeconomic variables to predict stock returns (e.g., Breen et al. (1989), Chen (2009), among others). Our conditional pricing model is in line with Ferson and Harvey (1999). The time-varying alpha and betas are generated from a linear function with respect to the instrumental variable. This model is similar to that used by Avramov and Chordia (2006), except that they consider the additional conditional time-variation of the risk premium. The model used by Ferson and Harvey (1999) explicitly models the time-variation of alpha and betas; thus, it is sufficient for studying the risk associated with time-varying alpha and betas.

Using an approach that differs from the optimal mean-variance portfolio strategy used in Avramov and Chordia (2006), we sort stocks directly into portfolios by alpha (beta) risk and use a long-short strategy to study the yield from this strategy. Stocks are sorted based on the characteristics demonstrated at the end of the previous month. Portfolios are held for one month and then portfolio returns are calculated. We also use regression tests at the firm level by regressing stock returns on alpha (beta) risk and firm characteristics.

We choose the data sample from the Center for Research in Security Prices (CRSP) common shares traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ from July 1963 to December 2015. The daily and monthly stock returns are from the CRSP. A stock should have available monthly accounting data from Compustat. Following French et al. (1987) and Schwert (1989), daily volatilities are converted into a monthly version by multiplying by the square root of the number of trading days in that month. In each month, a stock should have at least 15 daily returns. Following Ferson and Harvey (1999), we use four state variables:

TERM is calculated as the spread between 10-year and 1-year Treasury bond yields (e.g., Campbell and Vuolteenaho (2004); Adrian and Franzoni (2009)).

Default is the spread between Moody's Baa and Aaa corporate bond yield (e.g., Avramov and Chordia (2006)).

DIV is the dividend yield on a value-weighted CRSP market portfolio.

T-bill is the short-term risk-free rate, as measured by the secondary market rate of 3-month Treasury bills.

Data on bond yields and Treasury bill rates are downloaded from the Federal Reserve Bank of St. Louis website. The dividend yield on the value-weighted CRSP market portfolio is collected directly from the CRSP. To avoid the potential spurious effect of outliers, observations with monthly returns higher than the 99.75 percentile or lower than the 0.25 percentile are deleted. Similar to Ang et al. (2006), we measure the idiosyncratic volatility of stock i at month t as the standard deviation of the residuals from a regression of daily stock returns in month $t-1$ on the three Fama–French factors (which are obtained from Kenneth R. French's website).

5. Empirical Results

Table 1 reports the descriptive statistics of the alpha beta risk estimated from a conditional Fama–French three-factor model for the period of July 1963 to December 2015. Alpha risk (volatility

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[1] MKT is the excess return on the market minus the one-month Treasury bill rate. SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios; HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios.

of alpha) is defined as the standard deviation of time-varying alpha from the daily stock returns in month $t-1$. Beta risk (volatility of beta) is the standard deviations of time-varying beta MKT, beta SMB, and beta HML. Monthly risk is calculated by multiplying the standard deviation by the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. The alpha beta risk is reported as a percentage.

Table 1. Summary Statistics.

	Mean	S.D.	5%	25%	Median	75%	95%	N
Vol_Alpha	3.89	1.65	2.07	2.74	3.48	4.64	6.99	2,334,870
Vol_Bmkt	8.07	4.44	2.93	4.87	7.14	10.13	16.75	2,335,918
Vol_Bsmb	12.11	5.75	4.96	8.13	11.08	14.95	22.28	2,335,918
Vol_Bhml	14.74	7.32	6.31	9.98	13.06	17.83	27.67	2,335,918

Both the mean and median of alpha risk are smaller than that of the beta risk. While the magnitude of the beta HML risk comes with the largest mean and standard deviation, the beta MKT has the smallest mean and standard deviation. The average beta SMB risk is three times as large as the alpha risk and one and half times as large as the beta MKT risk.

Table 2 reports the returns of the stock portfolios that are sorted by size and alpha risk. Stocks are first sorted into five quintile portfolios, and then each size quintile is further sorted into five quintiles based on the alpha risk. As a result, equally weighted returns of 5×5 portfolios are obtained. Under each size quintile, we calculate the yield by buying the portfolio with the highest alpha risk and short selling the one with the lowest alpha risk. By controlling size, we also report the Jensen's alpha from the capital asset pricing model (CAPM) and the Fama–French three-factor model. The Newey–West adjusted t-statistics are in parentheses. For size quintiles 1 to 4, the long-short trading strategy yields negative returns that are significantly different from zero. No such relation is observed for portfolios under large size quintile 5. The largest negative yield is observed under size quintile 2 with -0.87 and this yield becomes much smaller as the size increases. The last column of Table 2 reports the portfolio returns that are sorted by alpha risk. All stocks are sorted into 5 quintiles based on the alpha risk. By only considering the alpha risk, we use a long-short trading strategy and obtain -0.32 as the yields. The results from Jensen's alpha are consistent with the yields. This evidence confirms that alpha risk is negatively related with stock returns at the portfolio level.

Table 2. Sort by Size and Vol_Alpha.

Vol Alpha Quintiles	Size Quintiles					All Stocks
	1-Small	2	3	4	5-Large	
1-Low	1.02	1.15	1.22	1.21	1.07	1.13
2	1.16	1.19	1.25	1.23	1.04	1.21
3	1.01	1.09	1.19	1.25	1.08	1.20
4	0.99	1.06	1.16	1.12	0.98	1.13
5-High	0.51	0.28	0.57	0.80	0.83	0.80
High-Low	−0.51 ***	−0.87 ***	−0.65 ***	−0.42 ***	−0.24	−0.32 **
	(−4.19)	(−7.41)	(−5.39)	(−3.32)	(−1.68)	(−2.20)
CAPM Alpha	−0.63 ***	−1.03 ***	−0.80 ***	−0.59 ***	−0.43 ***	−0.49 ***
	(−5.46)	(−9.93)	(−7.34)	(−5.48)	(−3.43)	(−3.85)
FF3 Alpha	−0.65 ***	−1.03 ***	−0.77 ***	−0.52 ***	−0.36 ***	−0.51 ***
	(−5.92)	(−11.14)	(−8.08)	(−5.79)	(−3.26)	(−5.46)

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

The results of the two-way sorting by size and beta MKT risk are shown in Table 3. By controlling for size, we long the portfolio with the highest beta MKT risk and short the one with the lowest beta MKT risk. Similar to the results for the alpha risk, the trading strategy yields are significantly negative across the different size quintiles. The size quintile 2 yields the most negative spread, and this spread decreases as the size increases. When all stocks are sorted solely by the beta MKT risk, this negative long-short trading spread is marginally significant. This result implies that on a portfolio level, the negative relation between the beta MKT risk and stock returns is weak but becomes stronger when size is controlled.

Table 3. Sort by Size and Vol_Bmkt.

Vol Bmkt Quintiles	Size Quintiles					All Stocks
	1-Small	2	3	4	5-Large	
1-Low	0.93	1.16	1.22	1.21	1.04	1.06
2	1.17	1.22	1.23	1.21	1.05	1.22
3	1.04	1.15	1.20	1.28	1.06	1.23
4	1.00	0.94	1.09	1.17	1.03	1.13
5-High	0.54	0.31	0.61	0.78	0.75	0.75
High-Low	−0.39 ***	−0.85 ***	−0.61 ***	−0.43 **	−0.29 **	−0.29 *
	(−3.35)	(−7.04)	(−5.30)	(−3.52)	(−2.07)	(−1.96)
CAPM Alpha	−0.51 ***	−1.01 ***	−0.76 ***	−0.61 ***	−0.48 ***	−0.44 ***
	(−4.72)	(−9.96)	(−7.25)	(−5.71)	(−3.98)	(−3.56)
FF3 Alpha	−0.58 ***	−1.03 ***	−0.73 ***	−0.52 ***	−0.41 ***	−0.48 ***
	(−5.80)	(−11.35)	(−7.67)	(−5.50)	(−3.80)	(−5.32)

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

Table 4 reports similar evidence for the negative relation between stock returns and the beta SMB risk. When stocks are sorted only by the beta SMB risk, the spread yielded from the long-short trading strategy is not significantly different from zero. As size is further controlled in two-way sorting, the negative long-short trading spread is only significant for size quintiles 2 to 4 and is much weaker for both small-size quintiles and large-size quintiles. Overall, the negative relation is somewhat significant for certain size quintiles when size is controlled for. The negative relation between the beta HML risk and stock portfolio returns is reported in Table 5. The one-way sorting by the beta HML risk yields a negative spread of −0.30, which is significant at the 5% level. By further controlling for the size, we find a significantly negative relation between the beta HML risk and stock returns. Jensen's alphas from the CAPM and the Fama–French three-factor model provide consistent evidence. This negative relation is weaker for stocks in large size quintiles.

Table 4. Sort by Size and Vol_Bsmb.

Vol Bsmb Quintiles	Size Quintiles					All Stocks
	1-Small	2	3	4	5-Large	
1-Low	0.84	1.00	1.22	1.23	1.08	1.13
2	1.08	1.19	1.23	1.26	1.05	1.21
3	1.08	1.17	1.19	1.20	1.02	1.20
4	0.99	0.99	1.15	1.18	1.03	1.17
5-High	0.60	0.38	0.60	0.76	0.77	0.89
High-Low	−0.24 *	−0.63 ***	−0.62 ***	−0.47 ***	−0.31 **	−0.23
	(−2.07)	(−5.30)	(−5.25)	(−3.92)	(−2.17)	(−1.59)
CAPM Alpha	−0.35 ***	−0.78 ***	−0.77 ***	−0.64 ***	−0.48 ***	−0.38 ***
	(−3.25)	(−7.47)	(−7.26)	(−5.96)	(−3.88)	(−3.07)
FF3 Alpha	−0.40 ***	−0.79 ***	−0.74 ***	−0.56 ***	−0.41 ***	−0.41 ***
	(−3.88)	(−8.31)	(−7.86)	(−6.09)	(−3.65)	(−4.54)

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

Table 5. Sort by size and Vol_Bhml.

Vol Bhml Quintiles	Size Quintiles					All Stocks
	1-Small	2	3	4	5-Large	
1-Low	1.02	1.25	1.26	1.21	1.05	1.20
2	1.11	1.13	1.19	1.27	1.06	1.21
3	1.04	1.04	1.26	1.22	1.02	1.20
4	0.96	0.93	1.02	1.10	1.06	1.14
5-High	0.55	0.41	0.64	0.76	0.74	0.88
High-Low	−0.48 ***	−0.84 ***	−0.62 ***	−0.46 ***	−0.31 **	−0.30 **
	(−4.16)	(−7.01)	(−5.29)	(−3.61)	(−2.17)	(−2.07)
CAPM Alpha	−0.60 ***	−1.00 ***	−0.77 ***	−0.64 ***	−0.50 ***	−0.46 ***
	(−5.76)	(−9.83)	(−7.32)	(−5.90)	(−4.10)	(−3.71)
FF3 Alpha	−0.63 ***	−1.00 ***	−0.74 ***	−0.56 ***	−0.43 ***	−0.49 ***
	(−6.24)	(−11.13)	(−8.34)	(−5.96)	(−3.85)	(−5.38)

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

Based on the evidence from the one-way and two-way sorting, the negative relation between the alpha risk and stock portfolio returns is quite significant. While the negative relation between the beta risk and stock portfolios returns is not as significant as that between the alpha risk and stock portfolio returns, it becomes more evident when size is controlled for at the portfolio level. The yield from the long-short trading strategy is larger when stocks are sorted by the alpha risk rather than by the beta risk. One-way sorting by the beta MKT risk and beta SMB risk shows that the relation between the beta risk and stock returns is not robust compared to the alpha risk.

Since the relation between the alpha risk and stock portfolio returns is more significant than that for the beta risk, we further investigate this relation by triple sorting stocks into $3 \times 3 \times 3$ portfolios by size, book-to-market ratio, and alpha risk. We first sort all stocks into three size portfolios, and then each size portfolio is sorted into three portfolios by the book-to-market ratios. Finally, each size-book-to-market portfolio is divided into three alpha risk portfolios. Again, we conduct the long-short trading strategy and calculate the spread of the portfolio returns. Panel A in Table 6 reports the results for low book-to-market portfolios. The negative relation is significant across the three different size portfolios. It shows that the alpha risk has the strongest negative relation with portfolios from small-cap and low book-to-market stocks. Panel B shows the portfolio returns and trading strategy yields from medium book-to-market stocks. Although the negative relation is still significant for small-cap and mid-cap stocks, the magnitude of the relation is small compared to the portfolios of the same size under low book-to-market stocks in Panel A. Furthermore, the relation becomes even smaller as shown in Panel C for high book-to-market stocks. Overall, there seems to be little relation between the alpha risk and stock portfolio returns for large-cap stocks. Based on the Jensen's alphas from the CAPM and the Fama–French three-factor model, the negative relation between the alpha risk and stock portfolio returns is more evident for small-cap stocks, especially those with low book-to-market ratios.

Table 6. Triple sort by size, book-to-market, and vol_alpha.

Vol_Alpha	Panel A			Panel B			Panel C		
	Low Book-to-Market			Medium Book-to-Market			High Book-to-Market		
	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap	Small Cap	Mid Cap	Large Cap
Low	0.66	0.94	1.05	1.14	1.26	1.07	1.37	1.38	1.20
Medium	0.50	0.96	1.00	1.12	1.25	1.11	1.44	1.49	1.24
High	−0.09	0.37	0.81	0.61	0.98	1.02	0.88	1.08	1.19
High-Low	−0.75 ***	−0.57 ***	−0.24 ***	−0.53 ***	−0.28 ***	−0.05	−0.49 ***	−0.31 ***	−0.01
	(−7.19)	(−7.14)	(−2.76)	(−4.74)	(−3.06)	(−0.68)	(−4.57)	(−3.07)	(−0.08)
CAPM Alpha	−0.84 ***	−0.65 ***	−0.34 ***	−0.64 ***	−0.40 ***	−0.14 **	−0.59 ***	−0.42 ***	−0.13 *
	(−8.33)	(−8.37)	(−4.19)	(−6.19)	(−4.87)	(−2.13)	(−5.82)	(−4.80)	(−1.82)
FF3 Alpha	−0.86 ***	−0.63 ***	−0.31 ***	−0.65 ***	−0.42 ***	−0.12 *	−0.64 ***	−0.47 ***	−0.15 **
	(−9.10)	(−8.57)	(−4.21)	(−6.44)	(−5.65)	(−1.90)	(−6.53)	(−6.03)	(−2.35)

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

To this point, we have shown a strong negative relation between the alpha risk and stock returns and a weaker relation between the beta risk and stock returns at the portfolio level. It is interesting to investigate the relation between the alpha beta risk and stock returns at the firm level. If a negative relation between the alpha beta risk and stock returns exists at the firm level, investors can use it to predict stock returns.

Table 7 provides the results of the firm-level Fama and MacBeth (1973) regressions of monthly stock returns on the alpha beta risk for the period of July 1963 to December 2015. Alpha risk (volatility of alpha) is defined as the standard deviation of time-varying alpha from daily stock returns in month $t-1$. Beta risk (volatility of beta) is the standard deviations of time-varying beta MKT, beta SMB, and beta HML. Monthly risk is calculated by multiplying the standard deviation by the square root of the number of trading days for each month. A stock should have at least 15 daily returns in a month to be included in the risk estimation. Beta is the regression coefficient of the past two years of monthly returns on market returns. ME and B/M are the size and book-to-market ratio in Fama and French (1992). Ret $(-2, -7)$ is the compound gross return from month $(t-7)$ to $(t-2)$. TURN and CVTURN are the average volume turnover and coefficient of variance of TURN calculated over the past 36 months in Chordia et al. (2001). The idiosyncratic volatility (Idio_VOL) is the standard deviation of the regression residuals of daily stock returns in month $t-1$. The Newey–West adjusted t-value is reported in parentheses. To avoid the effect of possibly spurious outliers, all explanatory variables below the 0.5 (above 99.5) percentile are set equal to the 0.5 (99.5) percentile.

Table 7. Firm-level cross-sectional regressions.

	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Beta	0.55**	0.56**	0.57**	0.57**	0.56**
	(2.32)	(2.35)	(2.39)	(2.40)	(2.34)
Ln(ME)	−0.19 ***	−0.19 ***	−0.18 ***	−0.20 ***	−0.16 ***
	(−4.74)	(−4.92)	(−4.73)	(−5.52)	(−4.28)
Ln(B/M)	0.16 ***	0.16 ***	0.16 ***	0.15 ***	0.16 ***
	(3.07)	(3.06)	(3.12)	(3.02)	(3.05)
Ret(−2, −7)	0.86***	0.86***	0.89***	0.86***	0.86***
	(5.16)	(5.01)	(5.08)	(5.18)	(5.12)
Ln(TURN)	−0.25***	−0.24***	−0.26***	−0.22***	-0.24***
	(−5.01)	(−4.92)	(−5.13)	(−4.76)	(−4.93)
Ln(CVTURN)	−0.38 ***	−0.37 ***	−0.38 ***	−0.36 ***	−0.38 ***
	(−6.43)	(−6.44)	(−6.56)	(−6.29)	(−6.46)
VOL_Alpha	−0.04 ***	−0.04 ***	−0.03 ***	−0.03 ***	−0.04 ***
	(−4.62)	(−5.90)	(−4.08)	(−4.93)	(−5.78)
VOL_Bmkt		0.005	0.007	0.005	0.004
		(1.06)	(1.55)	(1.38)	(0.98)
VOL_Bsmb		0.002	0.003	0.003	0.002
		(0.66)	(1.06)	(1.04)	(0.68)
VOL_Bhml		−0.007 ***	−0.007 **	−0.005 *	−0.007 **
		(−2.66)	(−2.56)	(−1.99)	(−2.49)
Alpha			−0.02 ***		
			(−8.41)		
Idio_VOL				−0.03***	
				(−4.09)	
Z (TERM)					−3.53
					(−1.29)
VOL_Z					1.35
					(0.02)
Adj. R2	0.10	0.11	0.12	0.12	0.11

*, **, and *** denote the statistical significance level at 10%, 5%, and 1%, respectively.

By controlling for size, book-to-market, momentum, and liquidity, model 1 demonstrates that the alpha risk is significantly negatively associated with stock returns. As the alpha risk increases by one percent, the expected stock return will drop by 0.04 percent. When the beta risk is added in model 2, only the beta HML risk is negatively related to stock returns. Although the relation is significant, the magnitude is much smaller (-0.007), being only one tenth of the relation between the alpha risk and stock returns. Since the literature shows that the time-varying alpha predicts stock returns, we control for it in model 3 to see if the relation between the alpha beta risk and stock returns changes. The Newey–West *t*-statistics for the alpha risk and beta HML risk show that the relation is not driven by the time-varying alpha. Idiosyncratic volatility is added in model 4. While the return predictability from the alpha risk is unchanged, the beta HML risk is marginally related to stock returns. This implies that the negative relation between the beta HML risk and stock returns is affected by the negative relation between idiosyncratic volatility and stock returns. Since the alpha beta risk is associated with the macroeconomic risk from the macroeconomic variables, one concern about the negative relation between the alpha risk and stock returns is that this relation may be driven by the macroeconomic variables themselves. The state variable, term spread, and its volatility are controlled for in model 5. The macroeconomic variables and their volatility do not affect the negative relation between the alpha risk and stock returns.

In sum, the negative relation between the alpha risk and stock returns is significant at both the firm level and the portfolio level. This relation is not driven by the beta risk, the time-varying alpha itself, idiosyncratic volatility, macroeconomic variables, or the volatility of the macroeconomic variables. On the other hand, the negative relation between the beta HML risk and stock returns is affected by idiosyncratic volatility. There is no relation between stock returns and the beta SMB risk or beta MKT risk.

6. Conclusions

In this paper, we provide a theoretical framework that shows four additional terms that are components of idiosyncratic risk when people fail to account for the conditional pricing model with the time variation of alpha and betas. If the time-varying conditional model is the true model, then these four components represent systematic risk but are mistakenly included in the category of idiosyncratic risk. Both alpha risk and beta risk play a role as a part of the additional terms. Empirical evidence shows that alpha risk is negatively related to stock returns at the portfolio level. This negative relation is less significant between beta risk and stock portfolio returns. Small-cap stocks and those with low book-to-market ratios are more affected by this negative relation between the alpha risk and stock returns.

We also analyze the relation between the alpha beta risk and stock returns at the firm level. The results of the cross-sectional regressions demonstrate that the negative relation between the beta HML risk and stock returns is driven by idiosyncratic volatility and that the negative relation between the alpha risk and stock returns is independent from beta risk, idiosyncratic volatility, time-varying alpha itself, macroeconomic variables, and the volatility of the macroeconomic variables. This negative relation is robust at both the firm level and the portfolio level.

While we mainly focus on the alpha beta risk in this study, future research could be done on how the covariance components of idiosyncratic volatility relate to stock returns. This work is related to the literature about covariance risk and stock portfolio management.

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