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Low Carbon-Oriented Optimal Reliability Design with Interval Product Failure Analysis and Grey Correlation Analysis

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Abstract: The problem of large amounts of carbon emissions causes wide concern across the world, and it has become a serious threat to the sustainable development of the manufacturing industry. The intensive research into technologies and methodologies for green product design has significant theoretical meaning and practical value in reducing the emissions of the manufacturing industry. Therefore, a low carbon-oriented product reliability optimal design model is proposed in this paper: (1) The related expert evaluation information was prepared in interval numbers; (2) An improved product failure analysis considering the uncertain carbon emissions of the subsystem was performed to obtain the subsystem weight taking the carbon emissions into consideration. The interval grey correlation analysis was conducted to obtain the subsystem weight taking the uncertain correlations inside the product into consideration. Using the above two kinds of subsystem weights and different caution indicators of the decision maker, a series of product reliability design schemes is available; (3) The interval-valued intuitionistic fuzzy sets (IVIFSs) were employed to select the optimal reliability and optimal design scheme based on three attributes, namely, low carbon, correlation and functions, and economic cost. The case study of a vertical CNC lathe proves the superiority and rationality of the proposed method.

Keywords: sustainable development; low carbon; reliability optimization; product failures; grey correlation; interval numbers; uncertainty

1. Introduction

Numerous environmental issues have recently become considerably more threatening, such as global warming and toxic hazes. In particular, the energy crisis and carbon emissions have become two critical and universal concerns which have generated additional loads in the cycles within the natural ecosystem and restricted the sustainable development of the whole world [1–3]. The manufacturing industry generates great financial fortunes, but also produces massive amounts of carbon emissions and waste resources. The problem of large amounts of carbon emissions and resource wastes caused by the manufacturing industry is a wide concern across the world, and how to reduce the carbon emissions and resource wastes of the manufacturing industry has become one of the primary questions in the modern manufacturing industry. The thorough study of technologies and methodologies for green product design has important theoretical meaning and practical value.

Reliability tends to be related to the safety of products and its operating performance under specified conditions [4]. Reliability allocation aims at reasonably planning the reliability of individual elements to optimize the utilization of resources and ensure the design functions of products [5,6]. In other words, once a specified reliability target for a system has been determined, the reliability value of every subsystem must be appropriately calculated and balanced [7,8]. The quality of products has been greatly enhanced in modern society, and product complexity and functionality have also increased greatly with the enormous development of science and the tough competition of the market. Products are also now required to be beneficial to environmental protection [9].

Before the processing and manufacturing of products, product quality design with assured product reliability apportionment is mandatory, and optimal reliability design for modern products is a complicated problem. Most traditional reliability optimal design methodologies cannot meet the requirements of long-term and responsible sustainable development for the modern product.

Firstly, most traditional reliability optimal design methodologies fail to consider the carbon emissions of products. Global warming is induced mainly by the growing amount of carbon emissions which can be processed with quantity calculations and have a major influence on the global climate balance [10,11]. To realize sustainable development, every country is now trying to implement policies for the reduction of carbon emissions. Unfortunately, sustainable product development is overlooked by conventional reliability apportionment theorists, and previous studies on carbon emissions mainly focused on carbon emissions and climate change, emission calculations, emission decomposition factors, and emissions projections [12], and the field combining carbon emissions and product reliability apportionment is under-researched.

For instance, Jung et al. [13] performed an investigation of the effect factors for carbon emissions and provided a case study of a college campus. Robinson et al. [14] revealed that carbon emissions increased more and more if the velocity of the engine increased and the load of the engine improved. Shi and Zhao [15] suggested that carbon emission reduction should highlight how to dominate the size of the economy and how to increase the energy density. Wang et al. [16] proposed that we would have the confidence to achieve the goal of emission reduction related to energy consumption if a series of necessary and reasonable actions were taken. Most scholars did not adequately take carbon emissions into consideration when they were allocating the reliability for the subsystems included in products.

It is universally acknowledged that optimal reliability design is a key step for the preliminary design of important and high-technology products. This has a significant influence on a product's carbon emissions. Failures of important and high-technology products have a latent influence on the physical security, economic profits, and the harmony between human beings and nature. Hence, product failure is an inevitable aspect of optimization design based on reliability. Usually, the effect of product failure is measured by the failure frequency and failure severity [17,18], and some scholars recently [19–23] handled the failure effect in product reliability optimal design with risk priority number (RPN) information produced by the qualitative analysis and quantitative calculation of failure mode effects analysis (FMEA). FMEA is defined as an analysis model to recognize various failure modes occurring in a system and the corresponding impacts on the whole product function. It has been widely employed to guarantee the safety and functionality of different domains, but it has two defects. (1) Product failure has a crucial influence on the design and development of products, but the relationship between the product failure analysis and carbon emissions is neglected. Once a product fails, the malfunctioned subsystem would be repaired or dumped, which will result in additional carbon emissions. The amount of carbon emissions caused by these product failures is determined by the failure severity, failure frequency and carbon emissions of the relative subsystems in processing; (2) FMEA classifies product severity and product failure occurrence by crisp numbers, but they cannot reflect and deal with the uncertainty data.

In addition, modern products usually have several functions that are facilitated by the mutual correlations among subsystems to satisfy variable customer demands. The subsystem with a high correlation degree is more important than the subsystem with a low correlation degree. The former should be apportioned with higher reliability than the latter. Products with optimal reliability design contain many different subsystems and the correlations among them are abstract and complex, so the grey correlation analysis is often used to determine the subsystem weight given their mutual correlations [24–27]. For many engineering problems, such as observation distortion, resource limitations, system complexity, and so on, it is difficult to collect sufficient information at the early design phase which tends to cause uncertainty in modeling real industrial products. Therefore, product designers often need to manage the treatment of all kinds of uncertainties in reliability optimal design [28–31]. Bayesian models and fuzzy theories are basic approaches for handling uncertainties [32–34]. Bayesian models rely on subject-reported uncertainty which can result in illogical calculation [35]. Fuzzy theories require product designers to determine the optimal membership functions in the early design stage, but this is impossible in engineering applications. To solve these problems, interval analysis has been proposed [36]. Lu et al. [37] applied interval analysis to the use and management of land resources containing multifarious uncertainties; the numerical example of Suzhou illustrated its advantages in reliability optimal design with uncertainties. Wang et al. [38] pointed out that the introduction of interval numbers into the product reliability redundancy optimization in the product concept design phase was of important practical value to speed up the product development and reduce the economic cost of product engineering. In order to reasonably conduct product reliability analysis and optimal design, interval analysis is often combined with mixed uncertainty, probability, fuzzy theory, and grey system theory in the optimal reliability design [24,39,40].

Thus, it can be concluded that, to reduce the carbon emissions and promote the sustainable development of the manufacturing industry, the problems discussed above in the traditional reliability optimal design methods must be solved. Therefore, carbon emissions brought about by product failures, uncertainty, and correlation among subsystems all need to be considered in modern product reliability optimal design. Thus, a novel reliability optimal design model with interval product failure analysis and interval grey correlation analysis is proposed in this paper.

This paper has four sections. In the next section, the framework and detail operators of the proposed methodology are described. In Section 3, the superiority and rationality of the proposed model are verified by the case study of a modern CNC lathe. Section 4 presents a summary and the future development directions of the proposed method.

2. Variables, Data, Framework and Method Specification

2.1. Framework Specification

As illustrated in Figure 1, the proposed methodology for product optimal reliability design contains three major phases.

Phase 1. Relevant data are prepared. The target system reliability is determined based on market demands, and experts on the target product are invited to give their assessment of the carbon emissions based on the product failure analysis and degrees of correlation among subsystems. The assessment information is expressed as interval numbers within the internal [0, 10]. To avoid a distortion of the optimal reliability design results caused by high uncertainty, the width of the interval numbers must be restricted to being within three.

Phase 2. The subsystem weight taking into account the carbon emissions and their mutual correlations is calculated based on the product failures analysis and the interval grey correlation analysis, respectively. With their integration, the revised subsystem weight is available. It should be noted that the optimal design result varies with the caution indicator of the decision maker. Therefore, a series of reliability design schemes can be obtained.

Phase 3. A fuzzy-multiple-attributes decision-making (MADM) method by the accuracy function of interval-valued intuitionistic fuzzy sets (IVIFSs) is adopted [41–43] to select the optimal reliability optimal design scheme so that it can provide effective guidance to obtain low-carbon, safe and economical products.

More detail about these operations can be found in Sections 2.2 and 2.3.

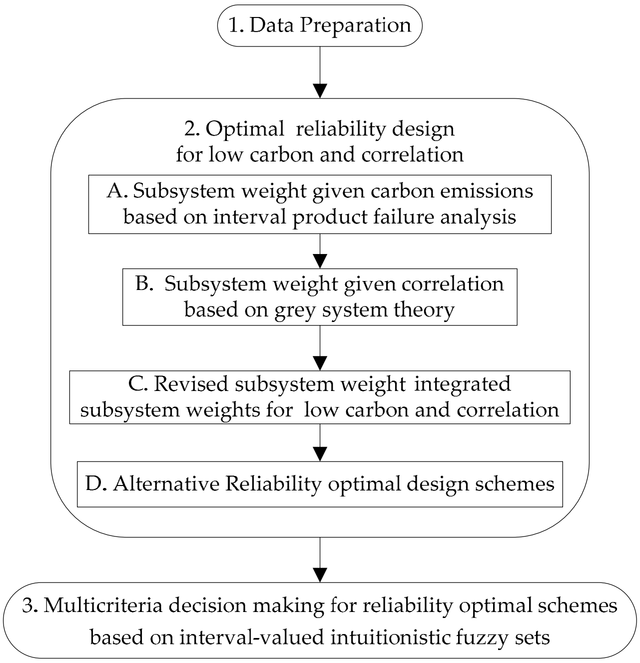

Figure 1. The general framework of the proposed reliability optimal design method.

2.2. Definition of Variables and Model of Low Carbon-Oriented Reliability Optimal Design Specification

2.2.1. Basic Arithmetic of Interval Fuzzy Numbers

The primitive form of an interval number can be expressed as $[a, b]$. In particular, an interval number is essentially a real number when $a = b$. The basic arithmetic of interval numbers must be introduced to conduct the interval analysis operator [44].

$[a, b]$ and $[c, d]$ denote two interval numbers, where a and c, b and d represent left-end and right-end points, respectively. Then, the basic interval arithmetic is expressed as Equations (1)–(4). In addition, let $A = [z, x]$ be an interval number, then its defuzzified value can be denoted as A and can be calculated by Equation (5), where v is the caution indicator of the decision maker. When $v = 0, 0.5$ and 1, decision makers are in the most radical condition, the most peaceful condition and the most cautious condition, respectively.

$$[a, b] + [c, d] = [a + c, b + d] \tag{1}$$

$$[a, b] - [c, d] = [a - d, b - c] \tag{2}$$

$$[a, b]\,[c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)] \tag{3}$$

$$[a, b]/[c, d] = [a, b][1/d, 1/c], \text{ where } cd > 0 \tag{4}$$

$$A = [(z + x) + (2v - 1)(x - z)]/2 \tag{5}$$

2.2.2. Interval Product Failure Analysis Considering Carbon Emissions

The product failure analysis and its carbon emissions are combined to obtain the product subsystem reliability design weight given the carbon emissions. From the discussion in Section 2, we can see that the traditional FMEA cannot fully adapt to the uncertainty in the optimal reliability design. Moreover, it overlooks the carbon emissions induced by the relative product failures. To overcome these difficulties, the traditional FMEA is improved to include an interval failure model, effect, critical and carbon emission analysis. Let subsystem i whose carbon emissions in processing are denoted as CE_i^*, have N_i failure modes with carbon severity S_{ij}^*, and frequency O_{ij}^*. Subsystem carbon emissions in processing, CE_i^*, mainly includes the carbon emissions brought about by its materials (determined by their types and weight) and its processing technology. The carbon severity S_{ij}^* represents the assessment of the carbon emission cost to maintain the failed subsystem. Frequency O_{ij}^* represents the quantitative measure of the related product failure mode. Evaluation data is expressed through the internal numbers between [0, 10]. In addition, to avoid a distortion of the reliability optimal design result caused by high uncertainty, the width of the interval numbers is restricted to being within 3.

Because this section aims to apportion high reliability to the subsystems with high carbon emissions brought about by the corresponding product failures, the evaluation standards are as follows. For carbon severity ranking, the lower the carbon emission cost to maintain the given failure mode, the higher the value of the related evaluation data. For occurrence ranking, the lower the frequency of the given failure mode, the higher the value of the related evaluation data. For carbon emissions in processing, the lower the carbon emissions induced by the subsystem's materials and processing technologies, the higher the value of the related evaluation data.

Suppose the product is composed of n subsystems. The carbon emissions for product failure of the ith subsystem is denoted as T_i^*, and it can be obtained from Equations (1)–(4) and Equation (6). Then, the interval vector of subsystem carbon emissions for product failure, $H^*= (T_1^*, T_2^*, \ldots, T_n^*)$, as well as its defuzzified value, $H = (T_1, T_2, \ldots, T_n)$, can be calculated by Equation (5), where v is the caution indicator of the decision maker and $v \in [0,1]$. The normalized optimal reliability design vector for a subsystem considering carbon emissions, $w^{\&} = (w_1^{\&}, w_2^{\&}, \ldots, w_n^{\&})$ can be calculated by Equation (7), where the $w_i^{\&}$ determines the weight of the ith subsystem given the carbon emissions for reliability optimal design.

$$T_i^* = \left(\sum_{j=1}^{j=j_i} S_{ij}^* \times O_{ij}^* \right) \times CE_i^* \tag{6}$$

$$w_i^{\&} = \frac{T_i}{\sum\limits_{i=1}^{n} T_i}; \tag{7}$$

2.2.3. Interval Grey Correlation Analysis

Due to the lack of basic data and the existence of uncertain information, such as the expert evaluation data, etc., as well as the neglect of the correlations among product subsystems, it was difficult to apply the conventional reliability optimal design theories to advanced modern products. To solve this problem, the interval grey correlation analysis based on expert experiences and knowledge is adopted to revise the normalized subsystem weight of reliability optimal design considering carbon emissions based on the improved product failure analysis. The core of this section is accessing the correlation degree among product subsystems by using the interval grey decision-making and grey relative analysis method. Then, high reliability should be apportioned to subsystems with high correlation degrees. Suppose the target product is composed of n subsystems, then we can establish the interval evaluation matrix for the correlations among subsystems. This is shown in Table 1.

Table 1. Interval evaluation matrix for correlation among product subsystems.

Expert	**Subsystem and the Relative Evaluation Data**			
	S_1	S_2	$\cdots$	S_n
E_1	$a_{11}{}^*$	$a_{12}{}^*$	$\cdots$	$a_{1n}{}^*$
E_2	$a_{21}{}^*$	$a_{22}{}^*$	$\cdots$	$a_{2n}{}^*$
$\cdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
E_m	$a_{m1}{}^*$	$a_{m2}{}^*$	$\cdots$	$a_{mn}{}^*$

Based on the grey correlation theory, select the subsystem group and individual subsystem to be the system's feature sequence and behavioral sequence, respectively. Compare the system's feature sequence and behavioral sequence to get their correlation degree. It is apparent that the subsystem with the high correlation degree is important to the whole product.

(1) Determining the feature sequence and behavioral sequence of the product's subsystem.

The feature sequence of the subsystem group is represented as $U_O = \{u_{o1}, u_{o2}, \ldots, u_{oi}\}$, where $u^*_{oi} = \max\{a^*_{i1}, a^*_{i2}, \ldots, a^*_{in}\}$ ($I = 1, 2, \ldots, m$ and m is the number of experts invited). The behavioral sequence of the kth subsystem is determined by the kth column vector in Table 1 and represented as $U_K = \{u_{k1}, u_{k2}, \ldots, u_{km}\}$, where $u_{ki} = a_{ki}$ ($k = 1, 2, \ldots, n$, and n is the number of product subsystems).

(2) Calculation of Subsystem Weight Considering Grey Correlation.

The grey correlation coefficient of the ith subsystem given the kth expert can be obtained by Equation (8).

$$\xi_i(k) = \frac{\min_i \min_k \Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}{\Delta_i(k) + \rho \max_i \max_k \Delta_i(k)}; \tag{8}$$

where ρ is the discriminating coefficient. $\rho \in [0, 1]$, and its value should be determined before the calculation. In this paper, suppose ρ to be 0.5. Based on the feature sequence and behavioral sequence of product subsystem given the kth expert, the absolute residual sequence, $\Delta(k)$, can be available. Then, the column vectors of the absolute residual sequence $\Delta_i(k)$ are selected to calculate the grey correlation coefficients with Equation (8).

The grey correlation degree reflects the advantages of the behavior sequence compared to the feature sequence. It can be obtained from Equation (9). The reliability optimal design weight of the ith subsystem is available from Equation (10).

$$r_i = \sum_{k=1}^{m} \xi_i(k) \Big/ m \tag{9}$$

$$w_i^G = r_i \Big/ \sum_{i=1}^{n} r_i \tag{10}$$

2.2.4. Low Carbon-Oriented Reliability Optimal Design Schemes

The low carbon-oriented subsystem weight for reliability optimal design, w_i^*, can be calculated with the above subsystem weight given carbon emissions and their mutual correlations, w_i^c and w_i^G, respectively, as shown in Equation (11). The final reliability of the ith subsystem, R_i, can be expressed by Equation (12), where is R the target reliability of the whole system.

$$\omega_i^* = \frac{\omega_i^{\&} \times \omega_i^G}{\sum_{i=1}^{m} \omega_i^{\&} \times \omega_i^G} (i = 1, 2, \ldots, n) \tag{11}$$

$$R_i = R^{w_i^*} \tag{12}$$

2.3. Multicriteria Decision-Making of Reliability Optimal Design Schemes

The proposed model has integrated the improved product failure analysis considering the uncertainty carbon emissions and interval grey correlation analysis of the uncertainty correlation of product subsystems. Thus, it is superior to the conventional reliability optimal design models in improving the product's carbon emissions and functioning. However, different reliability optimal design schemes can be obtained with different caution indicators of the decision maker. Besides, because the main purpose of enterprises is to make a profit, the economic cost is also an important attribute of product reliability optimal design. Therefore, it is necessary to find the optimal scheme among the different reliability design optional schemes according to three attributes including low carbon B_1, correlation and functioning B_2, as well as economic cost B_3. IVIFSs are good at dealing with the uncertainties in multiple attribute decisions, and this scheme is good at selecting the most suitable ones among a succession of decision options based on various actual attributes in practical engineering. Therefore, in this section, an appropriate and simple accuracy function of the IVIFSs is utilized to select the optimal reliability design scheme.

2.3.1. Preliminaries

(1) Basic Concepts [41].

Y and E denote a complete range of objects and all closed subintervals of the interval [0, 1], respectively, and then an IVFS, F in Y, can be expressed as $F = \{\langle y, e_F(y), c_F(y)\rangle : y \in Y\}$ with three constraint conditions shown in Equation (13).

$$\begin{cases} e_F : Y \to E[0,\ 1] \\ c_F : Y \to E[0,\ 1] \\ \sup(e_F(y)) + \sup(c_F(y)) \in [0,\ 1] \end{cases} \tag{13}$$

where $e_F(y)$ and $c_F(y)$ are the interval numbers representing the degree of membership and non-membership of y to F, respectively, whose inferior and superior end points can be expressed as $e_F(y) = \left[{e_F}^I(y), {e_F}^S(y)\right]$ and $c_F(y) = \left[{c_F}^I(y), {c_F}^S(y)\right]$.

In this way, the fundamental form of an IVIFS can be described as follows:

$$F = \left\{\left\langle y, \left[{e_F}^I(y), {e_F}^S(y)\right], \left[{c_F}^I(y), {c_F}^S(y)\right]\right\rangle : y \in Y\right\}, \tag{14}$$

where ${e_F}^S(y) + {c_F}^S(y) \in\ [0,\ 1]$.

Then, ${\phi_F}^S$ represents the hesitation degree of the fact that y belongs to F, and its inferior and superior end points can be expressed with Equation (15)

$$[{\phi_F}^I, {\phi_F}^S] = \left[1 - {e_F}^S - {c_F}^S, 1 - {e_F}^I - {c_F}^I\right] \tag{15}$$

To express this more succinctly, an IVIFS is represented by $F = ([f_1, f_2], [f_3, f_4])$ with the constraint conditions that the interval numbers $[f_1, f_2]$ and $[f_3, f_4]$ belong to the interval [0, 1].

(2) Concrete Weighted Average Operators

F_j (j = 1, 2, . . . , n) is a series of IVIFSs whose weight can be expressed as ω_j with the constraint conditions that $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$. The weighted averages of arithmetic and geometry can be calculated with Equations (16) and (17), respectively.

$$g_a(F_i) = \left(\left[1 - \prod_{j=1}^{n}(1 - {e_{F_j}}^I(y))^{w_j}, 1 - \prod_{j=1}^{n}(1 - {e_{F_j}}^S(y))^{w_j},\right], \left[1 - \prod_{j=1}^{n}(1 - {c_{F_j}}^I(y))^{w_j}, 1 - \prod_{j=1}^{n}(1 - {c_{F_j}}^S(y))^{w_j}\right]\right) \tag{16}$$

$$g_g(F_i) = \left(\left[\prod_{j=1}^{n}({e_{F_j}}^I(y))^{w_j}, \prod_{j=1}^{n}({e_{F_j}}^S(y))^{w_j},\right], \left[1 - \prod_{j=1}^{n}(1 - {c_{F_j}}^I(y))^{w_j}, 1 - \prod_{j=1}^{n}(1 - {c_{F_j}}^S(y))^{w_j}\right]\right) \tag{17}$$

(3) Appropriate Accuracy Function of IVIFNs.

An appropriate and simple accuracy function of IVIFNs with the consideration of their hesitancy degree [41] is more effective in the decision-making process than in most traditional numerical computation models [42–44]. The accuracy function of the F is represented by $G(F) \in [0,\ 1]$ and it is obtained by Equation (18).

$$G(F) = \frac{f_1 + f_2 \times (1 - f_1 - f_3) + f_2 + f_1 \times (1 - f_2 - f_4)}{2}, \tag{18}$$

2.3.2. MADM

An appropriate and simple accuracy function of IVIFNs [41] is adopted to select the optimal reliability deign scheme so that it can provide effective guidance to obtain low-carbon, safe and economical products. Assume that there are s reliability design options $F = \{F_1, F_2, \cdots, F_s\}$ and q decision attributes $B = \{B_1, B_2, \cdots, B_q\}$. The weight of the attribute B_j ($j = 1, 2, \ldots, q$) is denoted by ω_j, where $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1$. The feature of reliability design option F_i can be represented as follows.

$$F_i = \left\{ \left\langle y, \left[e_{F_i}{}^I(y), e_{F_i}{}^S(y) \right], \left[c_{F_i}{}^I(y), c_{F_i}{}^S(y) \right] \right\rangle : B_j \in B \right\} \tag{19}$$

For $j = 1, 2, \ldots, s$ and $i = 1, 2, \ldots, q$, the corresponding constraint conditions can be expressed as

$$\begin{cases} e_{F_i}{}^I(y) + e_{F_i}{}^S(y) \in [0,1] \\ c_{F_i}{}^I(y) + c_{F_i}{}^S(y) \in [0,1] \\ e_{F_i}{}^I(y) \ge 0 \\ c_{F_i}{}^I(y) \ge 0 \end{cases} \tag{20}$$

The fuzzy value for attribute B_j is represented with Equation (21).

$$\delta_{ij} = \left[\left(\left[\delta^1{}_{ij}, \delta^2{}_{ij} \right], \left[\delta^3{}_{ij}, \delta^4{}_{ij} \right] \right) \right], \tag{21}$$

where $\left[\delta^1{}_{ij}, \delta^2{}_{ij}\right]$ is the degree that the corresponding reliability design option F_i meets the requirements of attribute B_j. $\left[\delta^3{}_{ij}, \delta^4{}_{ij}\right]$ is the degree that the alternative corresponding reliability design option F_i does not meet the requirements of attribute B_j. The expert evaluation for the decision-making of optimal reliability design is represented as the matrix $\delta = \left(\delta_{ij}\right)_{m \times n}$.

Utilize Equation (16) to calculate the value of accuracy function for the option F_i given the reliability optimal design attributes, $G(F_i)$, to judge which is the optimal design scheme. The higher the value of the $G(F_i)$, the better the performance of the reliability design option F_i.

3. Case Study

3.1. Low Carbon-Oriented Reliability Apportionment

In this section, the numerical example of a computerized numerical controlled (CNC) lathe is presented to demonstrate the potential application of the proposed method. CNC lathes are widely used in the manufacturing industry. In particular, precise and advanced CNC machines account for 65% of the whole equipment investment. Hence, the investigation of CNC lathes for the optimal reliability design is very important [19,20,45]. The target reliability for the vertical CNC lathe is required to be 0.875. As shown in Figure 2, it is divided into five subsystems. They are: the main transmission system (S_1), the electrical system (S_2), the feed system (S_3), the detecting system (S_4), and the hydraulic system (S_5); The main components and parts of these subsystems are shown in Table 2. Table 3 shows the improved product failure analysis for carbon emissions of the vertical CNC lathe.

The interval carbon emissions for product failure of the ith subsystem T_i^* and the interval vector of subsystem carbon emissions for product failure H can be obtained with the use of Equations (1)–(6). Let v be 0.2, 0.4, 0.6 and 0.8, respectively. Then, the defuzzified vector of subsystem carbon emissions for product failure, H^*, and the normalized vector of subsystem carbon emissions for product failure, $w^{\&}$, can be calculated by Equation (7).

H = ([660.8, 1054], [328.5360, 594.6420], [308.2320, 675.5800], [195.8580, 387.2580], [456.1460, 761.7720]).
$H^*(v = 0.2)$ = [739.4400, 381.7572, 381.7016, 234.1380, 517.2712];
$H^*(v = 0.4)$ = [818.0800, 434.9784, 445.1712, 272.4180, 578.3964];
$H^*(v = 0.6)$ = [896.7200, 488.1996, 528.6408, 310.6980, 639.5216];
$H^*(v = 0.8)$ = [975.3600, 541.4208, 602.1104, 348.9780, 700.6468];
$w^{\&}(v = 0.2)$ = [0.3313, 0.1673, 0.1653, 0.1039, 0.2322];
$w^{\&}(v = 0.4)$ = [0.3203, 0.1723, 0.1743, 0.1067, 0.2265];
$w^{\&}(v = 0.6)$ = [0.3131, 0.1786, 0.1846, 0.1085, 0.2183];
$w^{\&}(v = 0.8)$ = [0.3048, 0.1825, 0.1900, 0.1131, 0.2096].

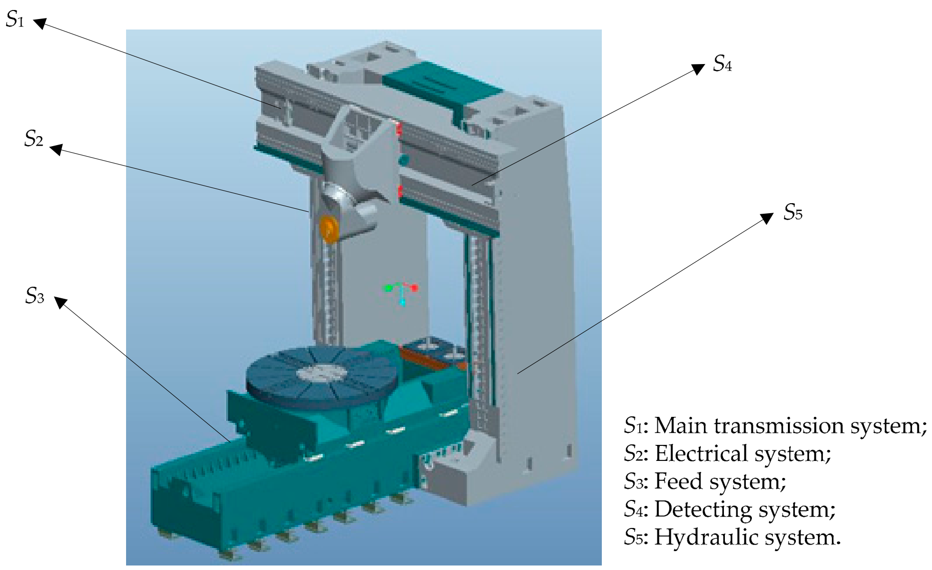

Figure 2. Subsystem structure of the vertical computerized numerical controlled (CNC) lathe.

Table 2. Main components and parts of the vertical CNC lathes' subsystems.

ID	Main Components and Parts
S_1	axle, bearing, gear, bed, column, beam, base, workbench
S_2	electromotor, transformer, magnetic valve, fuse
S_3	worm gear, guideway, bearing
S_4	multispeed revolver, rotary inductosyn, grating measuring device, pulse encoder
S_5	hydraulic pump, hydraulic valve, hydraulic cylinder, tank, hydraulic motor, filter

Table 3. The improved product failure analysis for carbon emissions of the vertical CNC lathe.

ID	Failure Mode	S_{ij}^*	O_{ij}^*	CE_{ij}^*
S_1	Gear worn out Transmission inaccuracy	[6.2, 7.3] [4.2, 5.6]	[7.7, 8.6] [6.4, 7.8]	[9.1, 9.9]
S_2	Insufficient power Relay or solenoid valve failure	[5.4, 7.2] [4.1, 4.6]	[3.7, 4.6] [5.4, 6.7]	[7.8, 9.3]
S_3	Positioning inaccuracy rapid movement failure	[4.6, 6.3] [4.5, 6.1]	[5.1, 7.0] [4.3, 5.8]	[7.2, 8.5]
S_4	Low sensitivity of detection Lose of safe-guard ability	[5.7, 6.6] [4.2, 4.7]	[3.7, 4.4] [2.5, 3.4]	[6.2, 7.9]
S_5	Leak oil contamination and pump jam	[6.6, 7.5] [5.4, 6.8]	[4.3, 5.6] [5.4, 6.7]	[7.9, 8.7]

3.2. Reliability Design Considering Correlation among Subsystems

Five experts were invited to give their assessment about the correlation degree among subsystems based on the interval grey system theory, and the evaluation data is listed in Table 4. The first step is to calculate the interval grey evaluation matrix.

Table 4. Expert evaluation on correlation among the CNC lathe subsystems.

Expert	Subsystem				
	S_1	S_2	S_3	S_4	S_5
1	[5.3, 6.4]	[8.2, 9.6]	[5.6, 7.2]	[7.9, 8.5]	[5.5, 7.2]
2	[5.5, 6.2]	[7.8, 8.4]	[6.2, 6.7]	[7.2, 8.1]	[5.4, 7.1]
3	[5.1, 6.9]	[8.2, 9.9]	[5.9, 7.5]	[9.2, 9.9]	[5.3, 6.8]
4	[5.7, 6.6]	[6.9, 8.7]	[5.5, 6.9]	[7.6, 9.0]	[6.0, 7.5]
5	[5.1, 6.3]	[7.9,8.4]	[6.3, 7.1]	[8.2, 9.6]	[5.9, 6.6]

Let v be 0.6. Based on Table 1, the interval feature sequences about the subsystems of the CNC lathe and its defuzzified value can be obtained as U = {[8.2, 9.6], [7.8, 8.4], [9.2, 9.9], [7.6, 9.0], [8.2, 9.6]} and U^* = {9.04, 8.16, 9.62, 8.44, 9.04}, respectively. The defuzzified sample evaluation matrix for the correlation among the CNC lathe subsystems can be calculated, as shown in Table 5, and the absolute residual sequences are obtained as shown in Table 6.

Table 5. Defuzzified sample evaluation matrix about the CNC lathe subsystems.

Expert	Subsystem				
	S_1	S_2	S_3	S_4	S_5
1	5.96	9.04	6.56	8.26	6.52
2	5.92	8.16	6.50	7.74	6.42
3	6.18	9.46	6.86	9.62	6.20
4	6.24	7.98	6.34	8.44	6.90
5	5.82	8.21	6.78	9.04	6.32

Table 6. Absolute residual sequences about the CNC lathe subsystems.

Expert	Subsystem				
	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5
1	3.08	0	2.48	0.78	2.52
2	2.24	0	1.66	0.42	1.74
3	3.44	0.16	2.76	0	3.42
4	2.20	0.46	2.10	0	1.54
5	3.22	0.83	2.26	0	2.72

It can be known that $\min\limits_i \min\limits_k \Delta_i(k) = 0$, $\max\limits_i \max\limits_k \Delta_i(k) = 3.44$, $i = 1, 2, \ldots 5$, $k = 1, 2, \ldots 5$. Let the identification coefficient ρ be 0.5. From Equation (8), the correlation coefficients of the CNC lathe subsystems are obtained as shown in Table 7.

Table 7. Correlation coefficients of the CNC lathe subsystems.

Expert	$\xi_1(k)$	$\xi_2(k)$	$\xi_3(k)$	$\xi_4(k)$	$\xi_5(k)$
1	0.3583	1	0.4095	0.6880	0.4057
2	0.4343	1	0.5089	0.8037	0.4971
3	0.3333	0.9149	0.3839	1	0.3346
4	0.4388	0.7890	0.4503	1	0.5276
5	0.3482	0.6745	0.4322	1	0.3874

With Equation (9), the correlation coefficient vector of the CNC lathe subsystems can be calculated as r = [0.3826, 0.8757, 0.4370, 0.8983, 0.4305]. From Equation (10), the normalized subsystem weight vector for reliability optimal design considering their correlations is obtained as $w^G(v = 0.6)$ = [0.1265, 0.2896, 0.1445, 0.2970, 0.1424]. Similarly, let v = 0.2, 0.4, and 0.8. Then we can obtain $w^G(v = 0.2)$ = [0.1575, 0.2351, 0.1792, 0.2657, 0.1625], $w^G(v = 0.4)$ = [0.1405, 0.2596, 0.1562, 0.2859, 0.1578], and $w^G(v = 0.8)$ = [0.1132, 0.3019, 0.1330, 0.3136, 0.1383].

3.3. Reliability Optional Schemes Considering Low Carbon and Correlation among Subsystems

From Equations (11) and (12), the low carbon-oriented reliability optimal design schemes, which are based on the product failure analysis for carbon emissions and interval grey correlation analysis for correlations among subsystems, can be obtained as shown in Table 8.

Table 8. Low carbon-oriented reliability optimal design schemes.

Caution Indicator of the Decision Maker	Reliability of Subsystem				
	R_1	R_2	R_3	R_4	R_5
$v = 0.2$	0.9633	0.9722	0.9790	0.9804	0.9733
$v = 0.4$	0.9677	0.9679	0.9804	0.9780	0.9743
$v = 0.6$	0.9712	0.9626	0.9805	0.9766	0.9774
$v = 0.8$	0.9746	0.9598	0.9814	0.9739	0.9787

3.4. Multiple Attribute Decision-Making for Reliability Apportionment Schemes

The product designer needs to select the optimal reliability design scheme from a series of options according to the attributes including the low carbon B_1, correlations and functions B_2, as well as the economic cost B_3. Take the optional schemes obtained in Table 8 as objectives, the expert evaluation information for the decision-making of reliability optimal design is obtained, as shown in Table 9.

Table 9. Expert evaluation for the decision-making of reliability optimal design.

Optional Scheme	B_1	B_2	B_3
F_1	([0.60, 0.70], [0.16, 0.25])	([0.45, 0.49], [0.44, 0.51])	([0.46, 0.60], [0.30, 0.39])
F_2	([0.59, 0.62], [0.31, 0.37])	([0.52, 0.58], [0.35, 0.41])	([0.41, 0.70], [0.21, 0.28])
F_3	([0.50, 0.58], [0.36, 0.41])	([0.58, 0.65], [0.27, 0.34])	([0.31, 0.44], [0.28, 0.42])
F_4	([0.41, 0.49], [0.45, 0.50])	([0.62, 0.71], [0.13 ,0.25])	([0.51, 0.62], [0.25, 0.34])

The weighted averages of arithmetic and geometry of these optional reliability design schemes can be calculated with Equation (16) and (17), respectively.

$g_a(F_1)$ = ([0.5203, 0.6282] [0.2866, 0.3075]), $g_a(F_2)$ = ([0.5124, 0.6405] [0.2882, 0.3515]),
$g_a(F_3)$ = ([0.4659, 0.5575] [0.3116, 0.3967]), $g_a(F_4)$ = ([0.5038, 0.5993] [0.3146, 0.3919]);
$g_g(F_1)$ = ([0.4990, 0.5998] [0.3094, 0.3925]), $g_g(F_2)$ = ([0.4946, 0.6314] [0.2924, 0.3555]),
$g_g(F_3)$ = ([0.4880, 0.5495] [0.3045, 0.3910]), $g_g(F_4)$ = ([0.5061,0.5998] [0.2893, 0.3721]).

Then, the accuracy function of the optional schemes based on the two kinds of weighted averages are obtained with the Equations (18)–(21). They are shown as follows.

$G(g_a(F_1))$ = 0.6352, $G(g_a(F_2))$ = 0.6424, $G(g_a(F_3))$ = 0.5844, $G(g_a(F_4)$ = 0.6077;
$G(g_g(F_1))$ = 0.6084, $G(g_g(F_2))$ = 0.6335, $G(g_g(F_3))$ = 0.5801, $G(g_g(F_4)$ = 0.6214.

It can be found that the value of $G(g_a(F_2))$ is higher that these of $G(g_a(F_1))$, $G(g_a(F_3))$ and $G(g_a(F_4))$, and the value of $G(g_g(F_2))$ is higher that these of $G(g_g(F_1))$, $G(g_g(F_3))$ and $G(g_g(F_4))$. Therefore, the optional scheme F_2 is the final reliability optimization design scheme.

4. Conclusions

With the improved product failure analysis for carbon emissions, the proposed method apportions high reliability to subsystems with high carbon emissions brought about by product failures. This method is beneficial in reducing the total carbon emissions of products. With the interval grey correlation analysis, the proposed method takes the correlations among subsystems into consideration and advocates to apportion high reliability to subsystems with a high degree of correlations. This is an important step to guarantee the functioning of the whole product. In addition, the uncertainties in reliability optimal design are handled by the adoption of interval numbers in these steps. Therefore, the proposed method is superior to most traditional reliability optimal design methods because it has taken the carbon emissions brought about by product failures, uncertainties, and correlations among product subsystems into account. Meanwhile, the process of interval numbers contains an important parameter, the caution indicator of the decision maker, which results in a series of reliability optimal design schemes. Given that the reliability optimal design is a MADM problem, the IVIFS is introduced to select the optimal reliability design scheme according to three attributes, including: low carbon, correlations and functions, and economic cost. Thus, the final optimal reliability design scheme is conductive to balance the carbon emissions, functioning and economic cost of products, and this is of great significance to promote the sustainable development of products. The superiority and rationality of the proposed method is illustrated by the case study of a vertical CNC lathe for reliability optimal design. The proposed method can be applied to more large-scale mechanical and electrical equipment, for instance, air separation machines, coal mining machines and vibrating screens, to reduce carbon emissions and promote the sustainable development of the total equipment manufacturing industry.

In addition, this proposed reliability optimal design method is not suitable for use in certain operating conditions without the accuracy and competency of subjective expert evaluation. Thus, the collection of qualified evaluation data is of great significance. In particular, compared with traditional optimal design, the proposed reliability optimal design is prone to causing inaccurate reliability optimal design results at the design phase without accurate expert evaluation. Furthermore, this method usually requires a large number of calculations. Given the efficiency and time costs, improving computational efficiency has become another urgent issue.

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