The Diversification Benefits of Including Carbon Assets in Financial Portfolios

Yinpeng Zhang, Zhixin Liu and Xueying Yu *

School of Economics and Management, Beihang University, Beijing 100191, China; sd_zyp@buaa.edu.cn (Y.Z.); LiuZhiXin@buaa.edu.cn (Z.L.)

* Correspondence: xueying@buaa.edu.cn; Tel.: +86-10-8233-8727

Academic Editor: Yongrok Choi
Received: 21 January 2017; Accepted: 14 March 2017; Published: 19 March 2017

Abstract: Carbon allowances traded in the EU-Emission Trading Scheme (EU-ETS) were initially designed as an economic motivation for efficiently curbing greenhouse gas emissions, but now it mimics quite a few characteristics of financial assets, and have now been used as a candidate product in building financial portfolios. In this study, we examine the time-varying correlations between carbon allowance prices with other financial indices, during the third phase of EU-ETS. The results show that, at the beginning of this period, carbon price was still strongly corrected with other financial indices. However, this connection was weakened over time. Given the relative independence of carbon assets from other financial assets, we argue for the diversification benefits of including carbon assets in financial portfolios, and building such portfolios, respectively, with the traditional global minimum variance (GMV) strategy, the mean-variance-OGARCH (MV-OGARCH) strategy, and the dynamic conditional correlation (DCC) strategy. It is shown that the portfolio built with the MV-OGARCH strategy far out-performs the others and that including carbon assets in financial portfolios does help reduce investment risks.

Keywords: carbon market; portfolio optimization; MV-OGARCH model; dynamic conditional correlation model

1. Introduction

Climate change has seriously challenged almost all human communities for their chance of sustainable development, and accordingly caused great concerns [1–3]. Among the various approaches to climate change mitigation, artificial carbon-trading markets are created to encourage self-motivated carbon abatement and facilitate the abatement efforts exchange among different emitters.

The European emission trading scheme (EU ETS) represents the most influential carbon market. It is the first and largest greenhouse gas emissions trading scheme in the world, and will remain so until the nationalized emission trading system in China is established in 2017 [4,5]. The scheme covers four industrial sections, including energy, ferrous metal production and processing, minerals, and a collective “other energy-intensive sectors”. It has experienced three trading periods, one from 2005 to 2007, one from 2008 to 2012, and one running now until 2020.

The single most important product traded on EU ETS are EU allowances (EUA), the carbon credits or emission permits allocated or auctioned to members of the scheme. Each EUA represents one ton of CO\(_2\) that the holder is allowed to emit [6]. Given that the sellers and buyers settle their transactions at a carbon price equal to the marginal abatement cost of CO\(_2\), the carbon market system could provide the most efficient way of emission mitigation. An annual allocation of 2298.5 million EUAs were issued during the first stage of EU-ETS, among them 95% were allocated for free and 5% through auctions. However, this cap arrangement was not properly adjusted against the demand for it, which made the EU ETS a non-binding scheme in terms of curbing carbon emission. In view of that,
the annual emission cap during the second stage shrank to 2086.5 million, further to 2039.2 million during the third stage (all EUAs expire by the end of April each year, when firms within the EU ETS must surrender EUAs equivalent to their emissions. For businesses that emit more carbon than their cap, they have to purchase extra EUAs, while those that manage to reduce their carbon emissions below their cap are entitled to sell their excess EUAs. However, EUAs cannot be saved for application in following phases.). With tightening emission caps from one stage to the next, the proportion of EUAs allocated through auctions had also been raised up, from 10% in the second stage to 50% in the third stage, which turns the carbon market to a more economic-incentive-driven one [7].

Now, emission allowances are traded like other financial assets in both spot and future markets. The most liquid spot carbon market is BlueNext [8] (unfortunately, the market closed its spot and derivatives trading permanently on 5 December 2012), which organizes over 70% of daily turnover transactions. The most liquid future market is Inter-Continental Exchange (ICE), which attracts 90% of the daily turnover. With large transaction volume and high price volatility, EU-ETS mimics the key characteristics of most financial markets, and it functions as a financial market on some occasions. In this paper, we treat the EU-ETS as a financial market and test the hypothesis that includes carbon assets in traditional financial portfolios would result in diversification benefits. The research is designed out of such a consideration: the relative independence of the carbon market from other financial markets may help further diversify financial portfolios and, accordingly, reduce investment risks, as compared to investment focusing only on traditional financial products. This hypothesis is confirmed in our study: an optimal portfolio built out of three typical strategies, i.e., the global minimum variance (GMV) model, the mean-variance OGARCH (MV_OGARCH) model, and the MV-dynamic conditional correlation model, all generate lower volatility when carbon assets are included. This finding may help investors further expand their asset pools and devise new investment strategies.

The remainder of this paper is organized as follows. Section 2 reviews the theoretical deduction and empirical examination about how the carbon market relates with traditional financial markets. Section 3 introduces the basic concepts and methods of our empirical testing, and Section 4 presents numerical results. Conclusions are offered in Section 5.

2. Literature Review

Theoretical deduction predicts that the price of carbon allowance is positively associated with oil prices [9], since both of them are subject to the influence of energy demand. An increase in the demand drives up energy price and induces boom in energy consumption, as well as carbon emission. This calls for more carbon allowances in the carbon market, and finally raises the carbon price. In addition, oil prices and stock prices are also connected at a theoretical level. As a major economy-wide input, an increase in the price of oil aggravates cost burdens of individual firms, either directly or indirectly, which, in turn, reverses the growth trend of the whole economy and depresses overall stock prices [10,11]. Since both carbon prices and stock prices are theoretically connected with oil price, a close connection between the first two prices may be assumed.

The theoretical connections among carbon, oil, and stock prices have been repetitively examined, but these examinations reached quite different, and sometimes controversial, findings. Positive correlations between carbon prices (usually represented by, EUA future price) and crude oil prices (usually represented by Brent future oil price) constitute the main conclusion of a few studies [12,13], but evidence of negative correlations also exist. For example, Hammoudeh and Nguyen [14] find that an increase in crude oil price is usually associated with a substantial drop in carbon price, given that the latter is sufficiently high. It is generally thought that a change in oil price, or other fossil energy prices, is the Granger cause of carbon price changes [15], rather than vice versa. However, more recent studies abandon the practice of connecting the oil market and carbon market under a simple and definitive correlation framework. In the pioneering work by Daskalakis, Psychoyios, and Markellos, they show that the spot carbon prices are characterized by
jumps and non-stationarity, and should be better approximated with geometric Brownian motion. In addition, due to prohibition of banking of carbon credits, only intra-phase future carbon contracts follow conventional insights on financial asset behaviors [16]. Scholars in this line argued that the connection between carbon and oil prices hinges on many external factors. The first leveraging factor is time. Yu and his colleagues [17] decompose the time series prices of crude oil and carbon credits into short-, medium-, and long-run series, and find that the two prices are not correlated at the original level because the short-run noise dominated their relationship. However, in the long-run, oil and carbon prices are linearly correlated. Price levels also leverage how oil and carbon prices connected with each other. With the threshold vector error-correction method, Peri and Baldi [15] point out that both oil and carbon prices follow a drift-less random walk when their absolute values are sufficiently close to each other, but they would tend to converge to equilibrium when diverging apart quite far. Given the conditional independence of carbon market from oil market, the potential diversification benefits of substituting oil futures with carbon credits in a financial portfolio has been proposed in recent studies [18,19].

While oil prices are connected with carbon prices on one hand, its changes also have a strong impact on stock markets on the other hand. It has been empirically confirmed that the oil price shock is always associated with economic recessions and stock market fluctuations, which may be reflected by either dampened stock returns or aggravated investment risks [20–22]. In-depth studies find that the direction and magnitude of such impacts are also leveraged by the states of an economy, for example, whether the economy is developing or developed, which phase of business cycles the economy is in, whether the economy is net importing or exporting oil, and whether the oil price shock is caused by changes from the demand or supply side [23–25]. New evidence also shows that the oil shock’s impacts on stock markets are most likely to be observed when the stock market is also under an extreme status, for example, financial crisis or contagion from the crisis [26,27].

Given the strong connections between carbon markets and oil markets, as well as that between oil markets and stock markets, the markets of carbon credits and stocks are also closely connected. Such connection can be commonly found around the world, although it is more volatile in the U.S. and Europe than in China [28]. This connection provides a solid theoretical foundation of using carbon assets as a financial tool in investment diversification [28,29], although the diversification benefits may vary across temporal and geographic scales. In a case study, Koleini [30] found that a typical New Zealand investor could readily reap the diversification benefits by investing in his national carbon credits. However, in the European market, such benefits were available only in the first stage of EU ETS and only when short-sale of carbon credits was allowed [31,32]. Another way that carbon trading directly influences a firms’ stock performance relates to the increased cash flow derived from free allocation of carbon assets. In Oestreich and Tsiakas’ empirical work, it is demonstrated that firms that received free carbon allowances in the first stage of EU ETS outperformed those that did not [33].

The connections among the three markets (carbon market, crude oil market, and stock market) have been confirmed in previous studies, but most of them focused on the first two phases of EU-ETS. As a supplement, we investigate the pattern of these connections in its third stage, i.e., from 2013 till now. Our work shows that after the major economic powers, like Canada, Japan, and New Zealand, quitting the Kyoto Protocol, the connections of the EU ETS market with other financial markets have been weakened, with the correlation coefficients stagnate around zero for most of the time during Phase III. The independence of carbon market from other financial markets provides investors an opportunity to diversify with carbon assets. Thus, we further build three optimal portfolios, respectively, with the global minimum variance (GMV) model, the mean-variance-OGARCH (MV-OGARCH) model, and the MV-dynamic conditional correlation (MV-DCC) model, to showcase the diversification benefits. The MV-OGARCH strategy consistently beats the other two strategies in our tests.
3. Methodology

We select the GMV model, the MV-OGARCH model, and the MV-DCC model to analyze the carbon market’s connections with other financial markets, and to build the optimal investment portfolios. This selection combines the traditional benchmark and nowadays mainstream strategies in building optimal investment portfolios out of multiple financial assets. This section reviews the basic concepts and optimization methods of the three models.

3.1. Global Minimum Variance Model

Portfolio selection is about how to allocate a fixed amount of capital over a number of available assets, with certain optimization targets, e.g., maximizing expected return or minimizing investment risk. The fundamental portfolio selection model was formulated by Markowitz [34] in 1952, and the concrete formulation could be seen in [34,35], which based asset selection on the balance between portfolios’ expected return and the corresponding variance, thus called mean-variance (MV) model. While the MV represents a mathematically perfect solution to asset selection, the large variance associated with the estimated of mean and variance matrix undermines its popularity [36–38]. Instead, empirical evidence suggests that some intuitive portfolios may out-perform the MV model, such as the global minimum variance (GMV) portfolio strategy and the naive portfolio with equal weights [39–41]. Since we target on assets’ dynamic correlations, GMV model is selected in this work, partly because it has a specific focus on estimates of the covariance matrix and partly because it is not sensitive to estimation error [42,43].

The GMV model can be specified as follows:

\[
\begin{align*}
\text{Min } & \quad x^T \Omega x \\
\text{subject to: } & \quad x^T \times 1 = 1 
\end{align*}
\]

In this model setting, we have n available assets, and their returns are correlated under the rule represented by a covariance matrix of \( \Omega \). Each element in \( \Omega \) represents the covariant coefficient of any pair assets’ returns and those on the diagonal are the variances of each asset’s return. The optimization target is to choose a weight vector \( x = (x_1, x_2, \ldots, x_n)^T \) that minimizes portfolio’s return variance (risk). The model is a convex quadratic programming which can be solved directly with the existing algorithm.

3.2. Mean-Variance-OGARCH Model

Most recent portfolio selection models are developed out of the MV model, including the MV-OGARCH. The MV-OGARCH model improved the traditional MV model by updating the algorithm for calculating the covariance matrix \( \Omega \). In the MV model, the covariance between asset i and j in \( \Omega \) is calculated with standard statistical method:

\[
\Omega_{ij} = \frac{1}{T-1} \sum_{k=1}^{T} (r_{ik} - \bar{r}_i)(r_{jk} - \bar{r}_j)
\]

where T is the number of trading days, \( r_{ik} \) is the return rate of asset i on day k, and \( \bar{r}_i \) is asset i’s average return over T trading days. In contrast, the covariance matrix in the MV-OGARCH model is calculated with the multi-GARCH method. The root GARCH model has been widely used in modeling the volatility of single time series [44,45]. When coupled with principle component methods, the original GARCH is extended to orthogonal GARCH (OGARCH) by Ding [46], Alexander and Chibumba [47], as well as Lam, et al. [48], to model the volatility of inter-correlated multivariate time series. Here we summarize the basic techniques of the OGARCH model. For more details, please refer to the work of Lam [48] and Luo [28].
At each time \( t \), suppose that we have a set of \( N \) assets that are traded over previous \( T \) days, and their daily return rates are summarized into the matrix of \( R_t \) with the dimensions of \( T \times N \). The principle components of \( R_t \) are denoted by \( P_t \) and are calculated as follows:

\[
P_t = R_t \times W_t
\] (3)

where \( W_t \) represents the orthogonal matrix for \( R_t^T R_t \) with its columns arranged in descending order by eigenvalues.

Then at time \( t \), the covariance matrix of \( R_t \) can be calculated as:

\[
OG_t = \text{var}(R_t) = W_t \text{var}(P_t) W_t^T = W_t D_t W_t^T
\] (4)

In the above expression, \( D_t \) is a diagonal matrix with its elements representing the conditional variances of columns in \( P_t \) predicted by GARCH (1, 1):

\[
P_{i,k} = c_i + \varepsilon_{i,k} \\
\sigma_{i,k}^2 = \alpha_0 + \alpha_1 \varepsilon_{i,k-1}^2 + \alpha_2 \sigma_{i,k-1}^2
\] (5)

where \( P_{i,k} \) is the value of the \( i \)-th principle component on date \( k \), \( c_i \) is a constant, \( \sigma_{i,k}^2 \) is the conditional variance of \( \varepsilon_{i,k} \), and \( \varepsilon_{i,k} = \sigma_{i,k} u_{i,k} \) with \( u_{i,k} \sim \mathcal{N}(0, 1) \) for \( k = 1, \ldots, T \). Substituting \( D_t \) with \( D_{t,h} \), the covariance matrix \( OG_{t,h} \) for a period of \( h \)-day ahead can be predicted as:

\[
OG_{t,h} = W_t D_{t,h} W_t^T
\] (6)

where \( D_{t,h} \) is a diagonal matrix with its elements as the \( h \)-day ahead conditional variance of each principal component forecasted by GARCH (1, 1).

Once we get the covariance matrix \( OG_t \), conditional correlation coefficient between security \( i \) and security \( j \) can be calculated as:

\[
\rho_{ij} = \frac{OG_{tij}}{\sqrt{OG_{tii} OG_{tjj}}}
\] (7)

where \( OG_{tij} \) is the component located at the crossing of the \( i \)-th row and the \( j \)-th column of \( OG_t \).

3.3. MV-DCC Model

The MV-DCC model developed by Engle [49] is an extension of the conditional correlation coefficient (CCC) method proposed by Bollerslev [50]. It provides a new approach to solve the problem of heteroscedasticity. The method captures the dynamic correlation, and is formulated as follows:

\[
r_t | \Omega_{t-1} \sim \mathcal{N}(0, H_t) \\
H_t = D_t R_t D_t
\] (8)

where \( r_t \) represents the \( k \)-th asset's return ratio, \( \Omega_{t-1} \) is the information set until time \( t - 1 \), \( H_t \) is the conditional covariance matrix, \( D_t \) is a \( k \times k \) diagonal matrix with the \( k \)-th elements representing conditional standard deviation of asset \( k \), and \( R_t \) is the conditional correlation matrix. According to Engle [49], \( R_t \) is defined as follows:

\[
R_t = Q_t^{-1} Q_t^{-1} \\
Q_t = (1 - \alpha - \beta) \overline{Q} + \alpha \xi_{t-1} \xi_{t-1}' + \beta Q_{t-1}
\] (9)

where \( \overline{Q} \) is the unconditional covariance matrix of the standardized residuals and \( \xi_t \) represents the standardized residuals. The model requires that \( \alpha \) and \( \beta \) are both non-negative, and \( \alpha + \beta < 1 \).
4. Empirical Results

4.1. Data

Our dataset consists of seven time series asset returns that, respectively, represent the behaviors of the EU-ETS market, global oil markets, and global stock markets. Specifically, these series include EUA credit future price in the EU ETS market, crude oil price in OPEC countries, and the most representative stock market indices from the U.S., the U.K., France, Germany, and China. The series are selected out of such considerations: (1) the EU ETS market represents the largest and most influential market facilitating carbon credit transactions around the world; (2) future price serves the function of price discovery in carbon markets by embodying information and transferring it to the spot markets [51]; (3) OPEC countries, as a whole, represent the dominating power in the world crude oil market, and oil price is highly influenced by OPEC decisions of production capacity adjustment, quota allocation, over-quota production, and oil stock adjustment [52]; and (4) stock markets in the selected countries have the largest transaction volume and are quite representative of worldwide stock market behaviors. Specifically, we use Financial Times Index from the U.K. stock market, S & P 500 index from the U.S. market, DAX index from the German market, BNP Paribas index from the French market, and Shanghai-Shenzhen 300 index from the Chinese market. All price/index series cover the period from 4 January 2013 to 27 September 2016, counting to 858 trading days. The data are retrieved from the Wind Database, a major financial database developed in China, with all missing data deleted.

Table 1 presents the statistical characteristics of the seven return series over the observation period. Some interesting patterns can be identified. At the beginning, carbon market return was quite volatile, reaching the high of 24.05% and dropping to the low of −43.47%. This phenomenon may be subject to the shock that several developed countries quit the Kyoto Protocol around this time. Meanwhile, other financial markets behaved quite smoothly, with the return rates ranging from −8% to 8%. Thus, we believe that the terrible impacts of financial crises had already been alleviated during the observation period. Further, we notice that (1) EUA future market was more volatile when compared with other markets, since the corresponding series had a much greater standard deviation; (2) the distribution of returns from investing in OPEC oil market was positively skewed, while all other series had fat tails; and (3) none of the series are normally distributed since the kurtosis of all of them exceed three.

Table 1. Descriptive statistical properties of the selected time series.

<table>
<thead>
<tr>
<th>Statistical (%)</th>
<th>EUA</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>England</th>
<th>Oil</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.04</td>
<td>0.05</td>
<td>0.02</td>
<td>0.03</td>
<td>0.01</td>
<td>−0.11</td>
<td>0.03</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>3.82</td>
<td>0.85</td>
<td>1.3</td>
<td>1.29</td>
<td>0.98</td>
<td>1.85</td>
<td>1.79</td>
</tr>
<tr>
<td>Skewness</td>
<td>−171.73</td>
<td>−36.21</td>
<td>−56.23</td>
<td>−45.38</td>
<td>−17.16</td>
<td>57.08</td>
<td>−85.97</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2614.72</td>
<td>626.42</td>
<td>668.50</td>
<td>519.10</td>
<td>574.26</td>
<td>747.82</td>
<td>753.42</td>
</tr>
<tr>
<td>Min</td>
<td>−43.46</td>
<td>−4.02</td>
<td>−8.38</td>
<td>−7.07</td>
<td>−4.78</td>
<td>−6.75</td>
<td>−9.15</td>
</tr>
<tr>
<td>Max</td>
<td>24.05</td>
<td>4.75</td>
<td>4.91</td>
<td>4.85</td>
<td>5.04</td>
<td>10.8</td>
<td>6.5</td>
</tr>
<tr>
<td>Medium</td>
<td>0</td>
<td>0.07</td>
<td>0.07</td>
<td>0.10</td>
<td>0.06</td>
<td>−0.14</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The seven time series are also somehow correlated. As shown in Table 2, carbon market return rates are positively related with both oil and stock market returns in the third phase of EU-ETS, which is consistent with observation from Phase II [28]. However, the magnitude of return rates’ correlation between carbon market and other financial markets, as well as between the oil market and China’s stock market are quite low, as shown in the first column of Table 2. Lower correlation implies that we may reap the diversification benefits by simultaneously investing in the carbon, oil, and stock markets.
Table 2. Correlation matrix for the selected time series (Pearson correlation coefficient).

<table>
<thead>
<tr>
<th></th>
<th>EUA</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>England</th>
<th>Oil</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUA</td>
<td>0.0867</td>
<td>0.5961</td>
<td>0.1241</td>
<td>0.1042</td>
<td>0.1064</td>
<td>0.0672</td>
<td>0.0331</td>
</tr>
<tr>
<td>US</td>
<td></td>
<td></td>
<td>0.1241</td>
<td>0.1042</td>
<td>0.1064</td>
<td>0.0672</td>
<td>0.0331</td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td></td>
<td>0.5961</td>
<td>0.5646</td>
<td>0.6053</td>
<td>0.1531</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.9347</td>
<td>0.8533</td>
<td>0.1407</td>
</tr>
<tr>
<td>England</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.8146</td>
<td>0.2763</td>
</tr>
<tr>
<td>Oil</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3324</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2. Dynamic Volatility under MV-OGARCH and MV-DCC Assumptions

The seven series’ dynamic variances (as represented by standard deviation here (since the magnitude of the variances is relatively small, we can determine the variation details when they are presented in the original format. Thus, we present the standard deviation here as a substitutive indicator of series volatility) are simulated with the GARCH method, as represented in Figure 1. Obviously, the EUA return is much more volatile compared to other traditional financial products. This is especially true during the first half period of our observation. The oil price is also subject to large uncertainty. In contrast, the performance of traditional stock markets is quite stable, especially those operated in the developed world.

![Figure 1. Dynamic standard deviation for the select assets.](image-url)

The dynamic pattern of how EUA is correlated with the other six series is also simulated with both MV-OGARCH and MV-DCC models, as presented in Figures 2 and 3. As seen in Figure 2, the MV-OGARCH correlation between the carbon market and other financial markets are quite strong at the beginning of the observation period, but it gets weakened over time and almost reaches zero. Although this trend somehow reverses after the 700th trading day, the correlation coefficient never exceeded 0.2 since then. Generally speaking, the EUA future market is relatively independent from other markets during the third phase of the EU ETS framework. The MV-DCC method provides a more
consistent estimation about EUA’s correlation with other assets. As indicated by the MV-DCC results, most correlations lie within the range of $[-0.05, 0.2]$, further confirming the condition that the return of carbon assets is relatively independent from other financial assets.

![Figure 2. Dynamic correlation among the selected assets with MV-OGARCH.](image1)

![Figure 3. Dynamic correlation among the selected assets with MV-DCC.](image2)
4.3. Portfolio Optimization Results

The relative independence of carbon market from other financial markets provides an opportunity to further diversify our investment portfolios by simultaneously investing across these markets. In this section, we build three investment strategies with all available assets in the carbon, oil, and stock markets, respectively with the GMV model, the MV-OGARCH model, and the MV-DCC model. We also test their performance.

Specifically, we separate the series into two parts, in-sample for optimization and out-sample for testing. The in-sample period runs from 4 January 2013 to 15 April 2016, containing 750 daily observations, and the out-sample period from 15 April 2016 to 27 September 2016, containing 107 observations. With the in-sample data, we estimate constant covariance of a market portfolio under the GMV framework for predicting out-sample performance. Quite differently, under the MV-OGARCH and MV-DCC framework, mean and covariance matrix are updated each day with the input of all trading information before the investment date, and the optimal market portfolio is rebalanced accordingly. If optimal investment weights are constrained in the rage of $[-1, 1]$ and if we adopt a risk-free asset with return rate of 0.01%, the GMV, the MV-OGARCH and the MV-DCC optimization strategies respectively generate the following allocation schemes that define the weights about how to assign capital among the typical investment assets in the seven markets considered in this work (Tables 3 and 4). Since both MV-OGARCH and MV-DCC involve dynamic adjustment of asset allocation among a portfolio, we report here two allocation schemes, one on the last day of the in-sample period and one on the first day of the out-sample period.

<table>
<thead>
<tr>
<th>Statistic (%)</th>
<th>EUA</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>England</th>
<th>Oil</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>0.0260</td>
<td>0.6422</td>
<td>-0.2386</td>
<td>0.0210</td>
<td>0.4358</td>
<td>0.0448</td>
<td>0.0679</td>
</tr>
<tr>
<td>MV-OGARCH</td>
<td>0.1623</td>
<td>0.1436</td>
<td>0.1296</td>
<td>0.1326</td>
<td>0.1397</td>
<td>0.1579</td>
<td>0.1333</td>
</tr>
<tr>
<td>MV-DCC</td>
<td>0.1133</td>
<td>0.55372</td>
<td>-0.1543</td>
<td>0.0600</td>
<td>0.1809</td>
<td>0.1289</td>
<td>0.1164</td>
</tr>
</tbody>
</table>

Table 3. The optimization scheme on the last day of the in-sample period.

<table>
<thead>
<tr>
<th>EUA</th>
<th>US</th>
<th>France</th>
<th>Germany</th>
<th>England</th>
<th>Oil</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>0.0260</td>
<td>0.6422</td>
<td>-0.2386</td>
<td>0.0210</td>
<td>0.4358</td>
<td>0.0448</td>
</tr>
<tr>
<td>MV-OGARCH</td>
<td>0.1341</td>
<td>0.1725</td>
<td>0.2009</td>
<td>0.1071</td>
<td>0.1748</td>
<td>0.1333</td>
</tr>
<tr>
<td>MV-DCC</td>
<td>0.1271</td>
<td>0.6273</td>
<td>-0.1681</td>
<td>-0.0140</td>
<td>0.1648</td>
<td>0.1238</td>
</tr>
</tbody>
</table>

Table 4. The optimization scheme on the first day of the out-sample period.

As indicated by our simulation results, a typical investor should hold shares of carbon credits in the third stage of EU ETS transactions. This finding contrasts Luo’s [28] conclusion that carbon assets should take a negative position in an optimal portfolio during the first stage of EU ETS. When taking the carbon asset as a diversification tool when building portfolios, the GMV, MV-OGARCH, and the MV-DCC methods produce different investment strategies. If an investor takes GMV rule, he should allocate most of his wealth on the U.S. and the U.K. stock portfolios, whereas the MV-DCC strategy would suggest allocation with a single largest share on the U.S. market portfolio and almost equal assignment on other assets. However, an investor following MV-OGARCH strategy would tend to equally assign his wealth on all the assets.

Tables 5 and 6 summarize the performance of the portfolios, respectively, built out of the GMV, the MV-OGARCH, and the MV-DCC strategies, both in sample and out-of-sample. Generally, the MV-OGARCH strategy out-performs both the GMV and the MV-DCC strategy. The out-sample average return rate generated under the MV-OGARCH strategy is as high as 0.08%, beating all of the selected market indices. The second-best strategy is GMV. We may reap an average return rate as high as 0.06% if this strategy is applied, a satisfactory return rate that also beats all market indexes. Interestingly, the MV-DCC is not a good option for us, since it only generates an out-sample return
rate of 0.03%, even lower than the U.S. stock market index. In other words, a typical investor would be better off by only investing in the U.S. stock market index portfolio, if the MV-DCC strategy is his only option. Considering the substantive transaction costs involved in daily adjustment, the MV-DCC strategy may be the last resort we would turn to. Thus, our result again confirms the empirical wisdom that dynamic optimization may not always beat its static counterparts.

<table>
<thead>
<tr>
<th>Statistic (%)</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Medium</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>0.0227%</td>
<td>−4.34%</td>
<td>4.80%</td>
<td>0.0663%</td>
<td>0.75%</td>
</tr>
<tr>
<td>MV-OGARCH</td>
<td>0.0288%</td>
<td>−4.59%</td>
<td>4.10%</td>
<td>0.0713%</td>
<td>0.92%</td>
</tr>
<tr>
<td>MV-DCC</td>
<td>0.0167%</td>
<td>−6.67%</td>
<td>4.36%</td>
<td>0.0510%</td>
<td>0.82%</td>
</tr>
</tbody>
</table>

Table 6. Performance of the out-sample series.

<table>
<thead>
<tr>
<th>Statistic (%)</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Medium</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>0.0553%</td>
<td>−2.44%</td>
<td>2.11%</td>
<td>0.0256%</td>
<td>0.69%</td>
</tr>
<tr>
<td>MV-OGARCH</td>
<td>0.0766%</td>
<td>−3.58%</td>
<td>1.82%</td>
<td>0.0585%</td>
<td>0.74%</td>
</tr>
<tr>
<td>MV-DCC</td>
<td>0.0332%</td>
<td>−2.62%</td>
<td>1.67%</td>
<td>−0.0234%</td>
<td>0.72%</td>
</tr>
</tbody>
</table>

4.4. Diversification Benefits

In this section, we test the performance of a portfolio with/without carbon market to further illustrate the diversification benefits of including carbon asset in a financial portfolio. As shown in Table 7, including carbon asset may not generate a higher portfolio return rate. This is understandable, since the EUA’s return rate per se is relatively lower than other financial products. However, including carbon assets does help reduce the volatility of an optimized portfolio, no matter which optimization strategy is utilized. Therefore, we could reap the diversification benefits as expected from the initial observation of asset correlations.

<table>
<thead>
<tr>
<th>Statistic (%)</th>
<th>GMV</th>
<th>MV-OGARCH</th>
<th>MV-DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0553%</td>
<td>0.0766%</td>
<td>0.0332%</td>
</tr>
<tr>
<td>Min</td>
<td>−2.44%</td>
<td>−3.58%</td>
<td>−2.62%</td>
</tr>
<tr>
<td>Max</td>
<td>2.11%</td>
<td>1.82%</td>
<td>1.67%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.0256%</td>
<td>0.0585%</td>
<td>−0.0234%</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.69%</td>
<td>0.74%</td>
<td>0.72%</td>
</tr>
</tbody>
</table>

Table 7. Portfolio performance with/without the carbon market.

5. Conclusions

In this paper, we focus on the time-varying relationships between the EU-ETS carbon market and other financial markets, including the oil market and stock markets in the U.S., the U.K., France, Germany, and China. MV-OGARCH and MV-DCC models are used to estimate the volatility connection of the third stage of EU ETS market and other financial markets. The results show that at the beginning of third stage of EU ETS, the return rates of investing in EUA credits are highly correlated with the return rates in other financial markets, but this connection gets weakened over time. The relative independence of carbon market reveals an opportunity to further diversify the existing optimal portfolio by adding carbon assets into the investment set. We build three optimal
portfolios, specifically considering the availability of carbon assets, one with GMV strategy, one with MV-OGARCH strategy, and one with MV-DCC strategy. The testing results show that the portfolio built out of the MV-OGARCH strategy significantly out-perform that built out of the GMV and MV-DCC strategies. In addition, including carbon allowance into financial portfolios does help diversify and reduce the investment risk, although portfolios built with carbon assets sometimes reap lower expected returns.

While our simulation results boost the obvious benefits of daily rebalancing with dynamic variance and correlation adjusting models, we ignore potential transaction costs involved in daily updating. When the updating costs are considered, our basic results would remain as they are, but our investment strategy recommendations may be different. As Blum and Kalai [53] pointed out, portfolios built with small transaction costs would always be preferred, given other conditions equal. They also emphasize that rebalancing should be carefully avoided when large commissions are involved. Della Corte et al. [54] further investigated the scenarios of rebalancing applications. They argue that the high-cost-strategy may still be profitable, if transaction costs take a percentage lower than the calculated threshold, but beyond that investors should stick to static strategies.

Acknowledgments: This work was supported by funding from the National Natural Science Foundation of China (Grant No. 71403011).

Author Contributions: Yinpeng Zhang, Zhixin Liu and Xueying Yu conceived and designed the experiments; Yinpeng Zhang collected and analyzed the data; Yinpeng Zhang and Zhixin Liu contributed analysis tools; Xueying Yu wrote the paper.

Conflicts of Interest: The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

References

24. Wang, Y.; Wu, C.; Yang, L. Oil price shocks and stock market activities: Evidence from oil-importing and oil-exporting countries. *J. Comp. Econ.* **2013**, *41*, 1220–1239. [CrossRef]
44. Papantonis, I. Volatility risk premium implications of GARCH option pricing models. *Econ. Model.* 2016, 58, 104–115. [CrossRef]

© 2017 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).