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Risk-Averse Facility Location for Green Closed-Loop Supply Chain Networks Design under Uncertainty

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Abstract: With the increasing attention given to environmentalism, designing a green closed-loop supply chain network has been recognized as an important issue. In this paper, we consider the facility location problem, in order to reduce the total costs and CO$_2$ emissions under an uncertain demand and emission rate. Particularly, we are more interested in the risk-averse method for providing more reliable solutions. To do this, we employ a coherent risk measure, conditional value-at-risk, to represent the underlying risk of uncertain demand and CO$_2$ emission rate. The resulting optimization problem is a 0-1 mixed integer bi-objective programming, which is challenging to solve. We develop an improved reformulation-linearization technique, based on decomposed piecewise McCormick envelopes, to generate lower bounds efficiently. We show that the proposed risk-averse model can generate a more reliable solution than the risk-neutral model, both in reducing penalty costs and CO$_2$ emissions. Moreover, the proposed algorithm outperforms and classic reformulation-linearization technique in convergence rate and gaps. Numerical experiments based on random data and a ‘real’ case are performed to demonstrate the performance of the proposed model and algorithm.

Keywords: green closed-loop supply chain; facility location; CO$_2$ emission; risk-averse decision; McCormick envelopes

1. Introduction

Closed-loop supply chain (CLSC) has been recognized as a trend in creating a sustainable supply chain management paradigm. CLSC integrates the forward and reverse logistics and accomplishes a complete flow from supply to recovery. This emerging schema has contributed considerably to reducing waste [1,2]. Despite significant profits from CLSC, a derived challenge should be pointed out, namely that the structure of CLSC is more complicated than the traditional supply chain network. The structural complexity of CLSC increases considerably because of new plants (e.g., recovery station, remanufacturing plant etc.) being established. This leads to another prominent issue; the emission of greenhouse gases is increasing, especially CO$_2$ [3–5]. Therefore, it’s necessary to consider CO$_2$ emissions in CLSC design to ensure the environmental performance of CLSC.

Facility location plays a key and core role in CLSC planning. Usually, it’s very costly to build facilities, and cannot be easily modified when a facility is set-up. Thus, it’s a crucial issue in supply chain design to select a proper facility location solution [6]. With respect to CLSC, where
the collection facility or remanufacturing plant is located can lead to significant differences in some main components of the cost structure, e.g., day-to-day transportation efficiency, costs and the volume of CO\(_2\) emissions [3,7].

In practice, some key parameters associated with CLSC design are usually uncertain, which leads to difficulty in solving problems [8]. For example, the rate of CO\(_2\) emission is greatly affected by traffic, which is usually changing all the time. To improve the reliability of the location solution under uncertainty, stochastic programming-based models have been widely used, e.g., Snyder et al. [9], Lu et al. [10], Cui et al. [11], Liu, Liu, Zhu, Chu and Man [8] and Wang et al. [12]. However, the flexibility of these models may be limited when parameters change significantly. More recently, several works were able to represent the uncertainty by robust optimization, which is able to obtain a reliable solution by introducing an uncertainty set addressing the data ambiguity [13]. However, a robust facility location is criticized because it always generates an overly conservative solution in the worst-case scenario [14,15].

In light of the results by Berman et al. [16], Mak and Shen [17] and Yu et al. [18], a risk-averse model is able to improve the reliability of the facility location solution. Moreover, it’s also less conservative compared to that by robust optimization. Additionally, decision makers are more likely to be risk-averse than risk-neutral or risk-seeking under uncertainty, according to Mak and Shen [17] and Yu, Haskell and Liu [18]. Thus, increasing attention has now been paid to the risk-averse facility location problem. However, almost all related works are interested in the facility location of traditional supply chain design, and very rarely, works consider the issue of green CLSC to reduce total costs, as well as CO\(_2\) emissions.

Particularly, most of the existing literature aimed to obtain an optimal solution based on the assumption that the decision-maker is risk-neutral, which has been falsified by many recent works. Additionally, most works on supply chain design reduced CO\(_2\) emission from operational aspects, and few papers considered it at the point of strategic facility location planning. Since the supply chain network is fundamentally established based on facility location networks, it would be more effective to take measures to reduce CO\(_2\) emissions when locating facilities.

To this end, in this paper, we aim to propose a risk-averse approach for building a green closed-loop supply chain from the strategic planning stage, i.e., a facility location problem. We aim to control the total cost spent in opening facilities and operational day-to-day transportation, meanwhile ensuring emissions of CO\(_2\) are under a certain level. We formulate the problem based on the basic model proposed in Yu et al. (2017). By incorporating a coherent risk measure, i.e., conditional value-at-risk (CVaR), we develop a risk-averse model with a 0-1 mixed integer nonlinear programming, which is usually challenging to solve. Then, a class of piecewise McCormick envelopes is proposed to obtain lower bounds efficiently. We show that this is more tractable than traditional reformulation-linearization techniques with a single McCormick envelope. Additionally, we demonstrate the performance of the proposed model by a numerical case, and show it can provide more reliable solutions than the traditional risk-neutral model, and is less conservative than the robust model.

We make the following main contributions to the literature:

- We propose a risk-averse model, based on conditional value-at-risk, to design facility location of green closed loop supply chain networks. The method is ongoing from a strategic view to reduce the emission of CO\(_2\), which differs from existing works that focus on controlling emissions in the operational aspect. It has been proved that the proposed risk-averse model can reduce more CO\(_2\) emissions under certain conditions. Moreover, the increase in total costs is much less than that of the robust approach.
- The proposed model is a 0-1 mixed integer nonlinear programming, which is nonconvex and challenging to solve. To do this, we develop partition-dependent reformulation-linearization techniques McCormick envelopes to further decompose the feasible region in classic RLT
and obtain tighter lower bounds. We show that the performance of the proposed algorithm outperforms commonly used solvers.

The remainder of this paper is organized as follows: Section 2 reports the literature review. Section 3 presents a problem description and models the problem by CVaR for the risk-averse approach. We design the solution algorithm in Section 4. A numerical example is expressed in Section 5. Finally, some conclusions are presented in Section 6.

2. Literature Review

There are many works that consider the facility location problem under uncertainty. However, related literature on green facility location problem is much less. In this section, we selected some of the most relevant literature to our paper. We searched for relevant papers using the keywords ‘green supply chain’, ‘green location’, ‘risk-averse’, ‘CO₂ emission’ or their combinations via Web of science and google scholar. Only 34 relevant papers were found. Here we selected some high-quality papers for the literature review.

2.1. Stochastic Facility Location Problem under Uncertainty

Cui, Ouyang and Shen [11] study a reliable facility location problem considering uncertain facility disruptions. A risk neutral 0-1 mixed integer nonlinear programming is proposed and a Lagrangian relaxation algorithm is developed to solve the model. Li and Ouyang [19] develop a continuum approximation model to take the correlation among adjacent facility disruptions into consideration for facility location planning. Pishvae et al. [20] develop a bi-objective, i.e., maximizing the responsiveness and minimizing the total costs, mixed-integer programming for reverse logistics network design, which is solved by a non-dominated gene algorithm. Doyen et al. [21] propose a two-stage stochastic programming to minimize total costs in the context of humanitarian relief logistics and a Lagrangian Heuristic Method is adopted to solve the problem. Wang et al. [12] consider a stylized facility location problem in a continuous plane under disruption risks, and an improved Voronoi-diagram-based algorithm is proposed to solve the model.

It’s noted that parameters in mixed-integer programming are often difficult to obtain, due to the uncertainty. For example, it’s hard work to collect the full information of the customer demands in practice and confirm an exact value for every individual customer. In this case, it becomes more challenging to model the uncertainty for the location problem.

Talaei, Moghaddam, Pishvae, Bozorgi-Amiri and Gholamnejad [13] model the facility location for a multi-product closed-loop green supply chain network by a mixed-integer linear programming, and a robust fuzzy programming approach is used to reduce the rate of carbon dioxide emission. Pishvae et al. [22] develop a robust stochastic programming model for CLSC design under several uncertain scenarios. Pishvae, Rabbani and Torabi [22] propose a bi-objective mathematical programming model to minimize the total cost and maximize the supply chain social responsibility, of which a robust programming is used to cope with uncertain parameters. Wu et al. [23] consider the robust/soft-capacitated 2-level facility location problems. They propose a primal-dual based-approximation algorithm for the robust model which explores some open facilities in the optimal solution.

2.2. Green Closed Loop Supply Chain Design

Rad et al. [24] propose an integrated mathematical programming model for multi-period, multi-product and capacitated closed loop green supply chains, where location-allocation facilities, transportation mode, technology type and carbon dioxide emissions are considered together. Nurjanni et al. [25] propose a multi-objective optimization mathematical model to minimize overall costs and carbon dioxide emissions, when setting the supply chain. Ghechi et al. [26] propose a two-stage stochastic programming model for the design of an integrated green biodiesel supply
chain network, as an environmentally friendly mixed-integer linear programming, multi-period and multi-product model. Soleimani et al. [27] develop a fuzzy model for designing a green closed-loop supply chain consisting of suppliers, manufacturers, distribution centers, customers, warehouse centers, return centers and recycling centers. A genetic algorithm has been used to solve their model under multiple scenarios with different aspects. Rezaee et al. [28] propose a two-stage stochastic programming model to design a green supply chain in a carbon trading environment that incorporates uncertainty in carbon price and product demand. Deng et al. [29] present a multi-criteria group decision making model for effectively evaluating the performance of green supply chain management (GSCM) practices under uncertainty in an organization.

We find that these works on green closed loop supply chain design share a common risk-neutral setting. That means the decision maker is assumed to be risk-immune under uncertainty, which is usually unrealistic in practice. Thus, in this paper, we will incorporate the risk preference of the decision-maker in designing green closed-loop supply chain networks.

As for the algorithm used in these existing works, most of them solve the model via widely used commercial solvers like Cplex. Additionally, to solve large-scale cases, some customized algorithms are also proposed. As most of these models are mixed integer linear programming, the exact algorithms, like branch-and-bound, are also popular. In this paper, we will see that the resulting model is a mixed integer nonlinear programming. The commonly used methods are branch-and-cut and classic RLT. Because the branch-and-cut algorithm has been packaged in Cplex, we will compare the performance of our algorithm with Cplex and classic RLT in the following numerical section.

3. Problem Description and Model Formulation

3.1. Problem Description

A general CLSC network may consist of a set of facilities of manufacturing $J_M$, remanufacturing $J_{RM}$, distribution $J_D$, and collection $J_C$. The manufacturing and remanufacturing facilities produce products or remanufacture returned ones. These products will be transported to distribution centers for getting into the markets $I$. We needed to design a facility network that can combine all these nodes, with minimum total costs and CO$_2$ emissions.

We first list here assumptions and notations used throughout the paper, as shown in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>Set of markets or customers, indexed by $i$</td>
</tr>
<tr>
<td>$J$</td>
<td>Set of potential facilities, indexed by $j$</td>
</tr>
<tr>
<td>$J_M, J_{RM}, J_D, J_C$</td>
<td>Set of potential manufacturing, remanufacturing, distribution, and collection facilities</td>
</tr>
<tr>
<td>$f_j$</td>
<td>Fixed setup cost for facility $j$</td>
</tr>
<tr>
<td>$s_{ij}$</td>
<td>Unit transportation cost from node $i$ to node $j$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Demand of customer $i$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Probability of customer $i$'s demand</td>
</tr>
<tr>
<td>$e_{kg}$</td>
<td>CO$_2$ emissions from travelling between every pair of nodes $(k, g) \in I \times J$</td>
</tr>
<tr>
<td>$p_{kg}$</td>
<td>Probability of CO$_2$ emissions from facility $k$ and customer $g$ to satisfy customer demand</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>Penalty costs for undersupplying customer $i$</td>
</tr>
<tr>
<td>$Q_j$</td>
<td>Capacity of facility $j$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_j$</td>
<td>Binary variable and 1 means the facility $j$ is opened</td>
</tr>
<tr>
<td>$Y_{ij}$</td>
<td>Binary variable and 1 if facility $j$ serves customer $i$</td>
</tr>
<tr>
<td>$u_{ij}$</td>
<td>Amounts of product transported from facility $i$ to customer $j$</td>
</tr>
</tbody>
</table>

We let the set of customers $I$ be indexed by $I$, and a set of candidate facility locations $J = \{J_M, J_{RM}, J_D, J_C\}$, be indexed by $j$. If facility $j$ is opened, then a fixed setup cost $f_j$ will occur. We denote the demand of market $i$ by $d_i$. The transportation cost of a unit of demand between each pair of nodes is denoted by $s_{ij}$. In case there are surplus amounts at every customer, they should be collected.
and remanufactured to reduce wastes. In case an undersupply exists from facility $j$, penalty costs $\eta_i$ for every customer should be charged. Without loss of generality, we only consider single products or commodities in this problem, and it can be directly extended to the model with multi-products. In this problem, due to the ever-changing markets, the demand is uncertain and follows a probability $\theta_i$ for market $i$ according to the conventional approach of representing stochastic events [28]. In addition, because of the uncertainty in traffic conditions, CO$_2$ emissions are also stochastic and assumed to follow the probability distribution $P_{kg}$. To model the problem, we also present the following necessary assumptions:

(i) Customer demands are mutually independent.
(ii) There are no other competitors in every area.
(iii) Each customer can only be served by a single facility.
(iv) The unit transportation cost between every node pair is identical.

3.2. Model Formulations

In this section, we will present our main model of a risk-averse facility location of green CLSC design. To begin, we first review some ingredients of CVaR and traditional models.

3.2.1. Conditional Value-at-Risk

CVaR has been a plausible coherent risk measure in representing the risk preference of a decision-maker, as well as hedging against risk under uncertainty [30,31]. It has been widely used in the fields of finance, commutation systems, grids etc. Given a random variable $x \in X$, let $F_X$ be the cumulative distribution function, then we have the definition of value-at-risk (VaR) at risk level $\alpha \in [0, 1)$ as

$$\vartheta_\alpha(x) \triangleq \inf \{ x : F_X(x) \geq \alpha \}$$

(1)

Because VaR is not able to account for the events at the tail of the distribution, it is not coherent and convex. CVaR is then proposed to accommodate the issue. It is defined as

$$\text{CVaR}_\alpha(x) \triangleq (1 - \alpha)^{-1} \int_0^1 \vartheta_x(x)(d\alpha)$$

(2)

as the conditional value-at-risk at level $\alpha \in [0, 1)$. It has been shown that $\text{CVaR}_\alpha(x)$ is a law invariant coherent risk measure [31].

For the computational recipe [32], we develop a convenient representation in terms of an optimization problem, i.e.,

$$\text{CVaR}_\alpha(x) \triangleq \inf_{\eta \in \mathbb{R}} \left\{ \eta + (1 - \alpha)^{-1} \mathbb{E}[x - \eta]_+ \right\},$$

(3)

which is a piecewise linear function and can be solved efficiently.
3.2.2. Basic Model for Facility Location Problem

We first present a basic facility location model for the classic risk neutral facility location problem of CLSC. Let \((X, Y, U)\) be the feasible solution of the problem, and \(X = (X_i)_{i \in I}\), \(Y = (Y_{ij})_{i \in I, j \in J}\), \(U = (u_{ij})_{i \in I, j \in J'}\) and the feasible region can be defined by \(\Omega \triangleq (X, Y, U)\). As for the traditional problem, where the decision-maker is assumed to be risk-neutral, the optimization model can be formulated as a bi-objective 0-1 mixed integer stochastic programming:

\[
\begin{align*}
\text{(B)} \quad & \min_{X, Y, U \in \Omega} \sum_{i \in I} f_i X_i + \sum_{i \in I} \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i \left( d_i - \sum_{j \in J} u_{ij} \right) \right) \\
& \min_{X, Y, U \in \Omega} \sum_{i \in I} \sum_{j \in J} p_{ij} c_{ij} Y_{ij} \\
& \sum_{j \in J} Y_{ij} = 1, \quad \forall i \in I, \quad (4c) \\
& \sum_{i \in I} Y_{ij} u_{ij} \leq Q_j X_j, \quad \forall j \in J, \quad (4d) \\
& u_{ij} \geq 0, \quad \forall i \in I, j \in J, \quad (4e) \\
& X_j, \ Y_{ij} \in \{0, 1\}, \quad \forall i \in I, j \in J. \quad (4f)
\end{align*}
\]

Here, the objective function (4a) is to minimize the total expected total costs, including the fixed costs \(\sum_{i \in I} f_i X_i\), the expected transportation costs \(\sum_{i \in I} \sum_{j \in J} \theta_i s_{ij} d_i Y_{ij}\) and the expected undersupplying penalty costs \(\sum_{i \in I, j \in J} \theta_i \pi_i \left( d_i - \sum_{j \in J} u_{ij} \right) Y_{ij}\). Note that the transportation costs contain two aspects, i.e., the day-to-day supply cost and the transit cost from the customer to the collection or remanufacturing plants. Please note that \(\left( d_i - \sum_{j \in J} u_{ij} \right) = \max \left\{ 0, d_i - \sum_{j \in J} u_{ij} \right\}\). This operator makes the objective function nonlinear. Then we introduce auxiliary variables \(M_i\) combining with two constraints to equivalently obtain the linearized objective function:

\[
\begin{align*}
\min_{X, Y, U \in \Omega} \sum_{i \in I} f_i X_i + \sum_{i \in I} \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i \right) \\
M_i \geq 0 \quad (5b) \\
M_i \geq d_i - \sum_{j \in J} u_{ij} \quad (5c)
\end{align*}
\]

The objective function (4b) aims to minimize the total expected CO\(_2\) emissions under uncertainty, which reflect the property of a green CLSC. Additionally, constraints (4c) mean that every customer should be served by only one facility, regardless of whether it is for the supply or collection. Constraints (4d) ensure that only an opened facility can provide service for customers, and the total supply amounts must not exceed the capacity of every facility. Constraints (4e) and (4f) define the decision variables.

Problem (B) is a risk-neutral model under uncertainty and is still nonconvex because of the bilinear item \(Y_{ij} M_i\). We will show that the following risk-averse model shares the same complexity with Problem (B).

3.2.3. Risk-Averse Model Based on CVaR

In this section, we turn to the main contribution of the paper and present a risk-averse model that provides more reliable solutions for the facility location problem of CLSC.
According to the definition of CVaR, we aim to control the risk of the uncertainty of the problem, i.e., demand and CO₂ emission. As for the random demand, the transportation cost and penalty cost can be related to the basic model, and so we let \( \rho_i(Y,U) \) be the corresponding risk measure by CVaR, then

\[
\rho_i(Y,U) = \min_{\eta_i \in \mathbb{R}} \left\{ \eta_i + (1 - \alpha)^{-1} \left[ \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i - \eta_i \right) \right]_+ \right\}
\]

denotes the CVaR of the transportation and penalty cost for every customer or market under uncertainty. Note that \( \rho_i(Y,U) \) is a risk diversification approach since we aim to control the risk at every market. Additionally, we can also formulate the CVaR from another view, i.e., risk pooling, as

\[
\rho(Y,U) = \min_{\eta \in \mathbb{R}} \left\{ \eta + (1 - \alpha)^{-1} \left( \sum_{i \in I} \sum_{j \in J} \theta_i Y_{ij} \left[ s_{ij} d_i + \pi_i M_i - \eta \right] \right)_+ \right\}
\]

Analogously, we can write the CVaR of CO₂ emissions in the two following ways,

\[
\phi_{ij}(Y,U) = \min_{\kappa_{ij} \in \mathbb{R}} \left\{ \kappa_{ij} + (1 - \beta)^{-1} p_{ij} (e_{ij} Y_{ij} - \kappa_{ij})_+ \right\} \quad (8a)
\]

\[
\phi(Y,U) = \min_{\kappa \in \mathbb{R}} \left\{ \kappa + (1 - \beta)^{-1} \left( \sum_{i \in I} \sum_{j \in J} p_{ij} e_{ij} Y_{ij} - \kappa \right)_+ \right\} \quad (8b)
\]

where \( \phi_{ij}(Y,U) \) is used to denote the risk of CO₂ emission between each pair of nodes, which can be noted as the risk diversification. \( \phi(Y,U) \) aims to control the risk of CO₂ emission of the entire network, i.e., risk pooling, because it focuses on the total CO₂ emission across all arcs.

Then, we can now formulate the risk-averse model (R-D) for the facility location problem of green CLSC design. We firstly incorporate the risk measure by risk diversification, and another risk pooling model (R-P) can be obtained by directly replacing the corresponding items.

\[
(R-D) \begin{array}{l}
\min_{X,Y,U \in \Omega, \eta \in \mathbb{R}} \sum_{j \in J} f_j X_j + \sum_{i \in I} \left\{ \eta_i + (1 - \alpha)^{-1} \left[ \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i - \eta_i \right) \right]_+ \right\} \\
\min_{(X,Y,U) \in \Omega, \kappa_{ij} \in \mathbb{R}} \sum_{i \in I} \sum_{j \in J} \left\{ \kappa_{ij} + (1 - \beta)^{-1} p_{ij} (e_{ij} Y_{ij} - \kappa_{ij})_+ \right\}
\end{array}
\]

and

\[
(R-P) \begin{array}{l}
\min_{X,Y,U \in \Omega, \eta \in \mathbb{R}} \sum_{j \in J} f_j X_j + \left\{ \eta + (1 - \alpha)^{-1} \left( \sum_{i \in I} \sum_{j \in J} \theta_i Y_{ij} \left[ s_{ij} d_i + \pi_i M_i - \eta \right] \right)_+ \right\}
\min_{(X,Y,U) \in \Omega, \kappa \in \mathbb{R}} \left\{ \kappa + (1 - \beta)^{-1} \left( \sum_{i \in I} \sum_{j \in J} p_{ij} e_{ij} Y_{ij} - \kappa \right)_+ \right\}
\end{array}
\]

where \( \Omega = (X,Y,U) \) denotes the feasible region defined by

\[
\Omega = \left\{ (X,Y,U) \left| \left( \sum_{j \in J} Y_{ij} = 1, \forall i \in I \right) - (X_i, Y_{ij} \in \{0,1\}, \forall i \in I, j \in J), (M_i \geq 0) - (M_i \geq d_i - \sum_{j \in J} u_{ij}) \right. \right\}
\]

Note that Problems (R-D) and (R-P) are bi-objective optimization programing, which is also difficult to handle. Many methods exist for coping with the multi-objective functions. An approach is to set a fixed weight to each objective and obtain a linear stochastic bi-objective programming, which
is able to ensure produce Pareto solutions. However, how to choose a proper weight for each objective is acknowledged as a challenging issue, which is worse when the objective contains uncertainty. Thus, it is not appropriate for Problems (R-D) and (R-P) to use the method.

In this paper, in light of [33], we present an alternative formulation that relaxes one of the bi-objectives into constraints, and forms a constrained stochastic optimization problem. That’s to say, we set one of the bi-objectives as the goal to minimize, e.g., the total cost, and let the other one, i.e., (E-R-P) are totally the same solutions as Problems (R-D) and (R-P).

3.3. Model Analysis

In this section, we provide some properties of the proposed risk-averse models. Particularly, we’re more interested in the relationship of the risk-averse models to classic risk-neutral models, in regards to both objective value and optimal solution.

Proof. For any two nodes $i$ and $i'$, according to the definition of CVaR, we have

$$\rho_i(Y, U) + \rho_{i'}(Y, U) = \eta_i + \eta_{i'} + \frac{1}{1-\alpha}E[(C_i - \eta_i)_+] + \frac{1}{1-\alpha}E[(C_{i'} - \eta_{i'})_+]$$

where $C_i = \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij}d_i + \pi_i M_i \right)$. Here, the above sum of CVaR indicates that we aim to control the risk arising from every individual market. Because $(C_i - \eta_i)_+ + (C_{i'} - \eta_{i'})_+ \geq (C_i - \eta_i + C_{i'} - \eta_{i'})_+$, let $\eta = \eta_i + \eta_{i'}$, then we have

$$\rho_i(Y, U) + \rho_{i'}(Y, U) = \eta + \frac{1}{1-\alpha}E[(C_i - \eta_i)_+] + \frac{1}{1-\alpha}E[(C_{i'} - \eta_{i'})_+]$$

$$\geq \eta + \frac{1}{1-\alpha}E[(C_i - \eta_i)_+ + (C_{i'} - \eta_{i'})_+]$$

$$\geq \eta + \frac{1}{1-\alpha}E[((C_i + C_{i'}) - (\eta_i + \eta_{i'}))_+]$$

$$\geq \min_{\eta} \eta + \frac{1}{1-\alpha}E[((C_i + C_{i'}) - \eta)_+]$$

$$= \rho(Y, U)$$
Here, $\rho(Y, U)$ refers to the risk lying in the entire supply network. Following the inductive Ptolemaic way, we have $\sum_{i \in I} \rho_i(Y, U) \geq \rho(Y, U)$. Analogously, we can obtain that $\sum_{i \in I} \phi_i(Y, U) \geq \phi(Y, U)$. □

Proposition 1 indicates that focusing on each individual market or customer will cause more costs, and controlling the risk on each link will lead to higher levels of CO$_2$ emission.

Next, we will see that the risk-averse models can be transformed into the classic risk-neutral model under certain conditions.

**Proposition 3.** For $\alpha, \beta \in [0, 1)$, $\forall i \in I$, if $\text{VaR}_\alpha(Y, U) = 0$, $\text{VaR}_\beta(Y, U) = 0$, then both Problems (R-D) and (R-P) are turn to Problem (B).

**Proof.** If $\text{VaR}_\alpha(Y, U) = 0$, $\text{VaR}_\beta(Y, U) = 0$, according to the definition of CVaR, we have

$$\rho_i(Y, U) = \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i \right),$$

thus,

$$\sum_{i \in I} \rho_i(Y, U) = \sum_{i \in I} \theta_i Y_{ij} \left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i \right).$$

Similarly, $\sum_{i \in I} \phi_i(Y, U) = \sum_{i \in I} \sum_{j \in J} p_{ij} e_{ij} Y_{ij}$.

Thus, the CVaR of the uncertainty takes the same form as the expected value, which shows that the risk-averse models become risk-neutral. □

**Corollary 1.** When $\alpha = \beta = 0$, $\forall i \in I$, both Problems (R-D) and (R-P) become Problem (B).

**Proof.** If $\alpha = \beta = 0$, we have $\text{VaR}_\alpha(Y, U) = 0$, $\text{VaR}_\beta(Y, U) = 0$, according to Proposition 2, we complete the proof. □

**Proposition 4.** For $\alpha, \beta \in [0, 1)$, $\forall i \in I$, if $\alpha \to 1, \beta \to 1$, we have Problems (R-D) and (R-P) become the maximum risk model or robust model.

**Proof.** If $\alpha \to 1$, according to the definition of CVaR and VaR, the corresponding uncertain demand should take the maximum value to make sure the distribution is equal to 1. This case is also called the worst-case in robust optimization. If $\beta \to 1$, we have a similar statement. □

Based on the aforementioned results, we see that the solution of the risk-averse model is more reliable than the traditional risk-neutral model, but less conservative than robust optimization.

### 4. Solution Algorithm

In this section, we turn to the solution algorithm of the above risk-averse models. Note that Problem R-D and Problem R-P share common properties in structure, so we explain the following algorithm with respect to R-D. That is to say, the algorithm is suitable for solving both Problem R-D and Problem R-P, with the same performance. Based on the equivalent transformation, we see that Problem (E-R-D) can generate the same solution as Problem (R-D). Thus, we begin to develop the algorithm by using the form of Problem (E-R-D).

It’s noted that the objective function (18) is still nonconvex because of the nonlinear items

$$\left( \sum_{j \in J} s_{ij} d_i + \pi_i M_i - \eta_i \right)_+ \quad \text{and} \quad (e_{ij} Y_{ij} - \kappa_{ij})_+,$$

and the inner bilinear item, $Y_{ij} M_i$, a product of the integer
decision variable and continuous decision variable. These nonlinear items make the model quite hard to solve.

To begin, we first make an equivalent reformulation by introducing two dummy variables, $H_i$ and $W_{ij}$, and combining them with two groups of corresponding constraints:

$$H_i \geq \sum_{j \in J} s_{ij} d_i + \pi_i M_i - \eta_i$$  \hspace{1cm} (13)

$$H_i \geq 0$$  \hspace{1cm} (14)

$$W_{ij} \geq e_{ij} Y_{ij} - \kappa_{ij}$$  \hspace{1cm} (15)

$$W_{ij} \geq 0$$  \hspace{1cm} (16)

Then we have following optimization model that only contains the bilinear item, $Y_{ij} H_i$:

$$(L-E-R-D) \min_{X, Y, U \in \Omega, \eta_i, \kappa_{ij}, w_{ij} \in \mathbb{R}} \sum_{j \in J} f_j X_j + \sum_{i \in I} \left\{ \eta_i + (1 - \alpha)^{-1} \sum_{j \in J} \theta_{ij} Y_{ij} H_i \right\}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in J} \left\{ \kappa_{ij} + (1 - \beta)^{-1} p_{ij} W_{ij} \right\} \leq \varepsilon$$  \hspace{1cm} (17)

To handle the bilinear item, $Y_{ij} H_i$, several approaches have been proposed in the literature. The reformulation-linearization technique (RLT) is now one of the most widely used methods because of the operational convenience. But the most glaring omission of traditional RLT is that it may lead to bad low bound because of the rough approximation to the nonconvex function.

Here, we firstly present an improved RLT with decomposed McCormick envelopes. It further decomposes the region based on classic RLT, and we can obtain a more delicate approximation to the nonconvex item.

The bilinear term, $Y_{ij} H_i$, leads to a nonlinear optimization problem. In light of the RLT, we can obtain an LP relaxation when replacing the bilinear term by introducing an auxiliary variable, $w_{ij} = Y_{ij} H_i$, combing the following four constraints,

$$\begin{cases}
    w_{ij} \leq H_i, \\
    w_{ij} \leq Y_{ij}, \\
    w_{ij} \geq 0, \\
    w_{ij} \geq H_i + Y_{ij}.
\end{cases}$$  \hspace{1cm} (18)

Then, we have the relaxed optimization programming:

$$(L-L-E-R-D) \min_{X, Y, U \in \Omega, \kappa_{ij}, w_{ij} \in \mathbb{R}} \sum_{j \in J} f_j X_j + \sum_{i \in I} \left\{ \eta_i + (1 - \alpha)^{-1} \sum_{j \in J} \theta_{ij} w_{ij} \right\}$$

$$\text{s.t.} \sum_{i \in I} \sum_{j \in J} \left\{ \kappa_{ij} + (1 - \beta)^{-1} p_{ij} w_{ij} \right\} \leq \varepsilon.$$  \hspace{1cm} (19)

Problem (L-L-E-R-D) is a 0-1 mixed integer programming, which can be solved using commercial software like CPLEX. We can observe that the corresponding solution is feasible in the new problem, obtained by replacing bilinear terms with $w_{ij} = Y_{ij} H_i$, but the results of $w_{ij}$, $H_i$ and $Y_{ij}$ do not follow $w_{ij} = Y_{ij} H_i$ strictly. Thus, the new optimization problem can be seen as a relaxation and lower bound of the original version. That’s to say, it’s necessary to find a way to tighten the relaxation.

To do this, we further decompose the current relaxed region for every variable, making the bilinear term into $n$ disjoint subregions. We achieve this by introducing new binary variables to specify and select the optimal one. It can be seen that the proposed decomposed RLT formulation is capable of obtaining the global optimal solution under a certain number of partitions.
We let \( H_{in}^{UB} \) and \( H_{in}^{LB} \) denote the upper and lower bounds of variable \( H_i \), respectively, for partition \( n \). In case the value of \( w_{ij} \) falls within the partition, we now define a new binary variable \( \lambda_{ijn} = 1 \), and we have the McCormick envelopes hold. Accordingly, we obtain a piecewise McCormick relaxation problem in the generalized disjunctive program proposed by [34], who proved that the solutions obtained by the partition-dependent parameters \( H_{in}^{UB} \) and \( H_{in}^{LB} \) in the four constraints inside the disjunction can be tighter than that bounded only by \( H_i^{UB} \) and \( H_i^{LB} \), as shown in Figure 1.

\[
\sum_{i} \sum_{j} \sum_{n} \left\{ \begin{array}{l}
\lambda_{ijn} \in \{0, 1\}, \\
w_{ij} \geq H_i^{LB} y_{ijn} + H_i^{UB} y_{ijn} - H_i^{LB} y_{ijn} \\\nw_{ij} \geq H_i^{LB} y_{ijn} + H_i^{UB} y_{ijn} - H_i^{UB} y_{ijn} \\
w_{ij} \leq H_i^{LB} y_{ijn} + H_i^{UB} y_{ijn} - H_i^{LB} y_{ijn} \\\nw_{ij} \leq H_i^{LB} y_{ijn} + H_i^{UB} y_{ijn} - H_i^{LB} y_{ijn} \\
y_{ijn}^{LB} = y_{ijn}^{LB} + \frac{y_{ij}^{UB} - y_{ij}^{LB}}{N} \\
y_{ijn}^{UB} = y_{ijn}^{UB} + \frac{y_{ij}^{LB} - y_{ij}^{UB}}{N} \\
H_i^{LB} \leq H_i \leq H_i^{UB} \\
y_{ijn}^{LB} \leq y_{ij} \leq y_{ijn}^{UB}
\end{array} \right. \tag{20}
\]

Thus, the linearized optimization problem for the location problem can be rewritten as:

\[
(L-L-L-E-R-D) \quad \min_{X,Y,U} \sum_{i,j} f_i x_j + \sum_{i,j} \left\{ \eta_i + (1 - a)^{-1} \sum_{j} \theta_i w_{ij} \right\}
\]

s.t. \[
\sum_{i,j} \left\{ k_{ij} + (1 - \beta)^{-1} p_{ij} w_{ij} \right\} \leq \epsilon,
\]

As for above problem, it can be easily turned to be pure convex optimization, which can be easily solved.

5. Numerical Experiments

In this section, we turn to the numerical experiments to show the performance of the proposed method. Particularly, we are more interested in differences between the proposed model and the classic risk-neutral model. Additionally, we will show the computational performance of our solution algorithm based on an improved RLT (IRLT) and some solvers like CPLEX and the classic RLT (CRLT). Here we have two classes of data to be used. The first class is a set of random data to illustrate the convergence and gap of the algorithms. And the second class is a case of a CLSC for a copiers firm,
which related to the location of facilities for producing, collection and remanufacturing. Note that all tests are conducted on a PC with Intel Core i5-4570 and 8 GB RAM, using 64-bit Windows 7.

5.1. Algorithm Performance Using a Random Data Set

A random data set is firstly generated to verify the performance of the proposed algorithm. We assume that there exist several instances, with the number of customers \( I = \{50, 100, 150\} \), the number of candidate locations \( J = \{10, 20, 30, 50\} \), the fixed cost of construction of the facility at a potential location \( f_j = 100 \), the customer demands \( d_i \) randomly generated from \([50, 65, 80]\), the unit transportation cost is 5 and the \( \text{CO}_2 \) emission follows a normal distribution with mean 0.18 Kg/Km and deviation 0.05 Kg/Km. The risk level \( \alpha = \beta = 0.95 \). Here, we do not need to specify the specific function of each facility, because we focus on the convergence rate of the algorithm, which has nothing to do with the function.

We report the results of the algorithms in Table 2. The second columns, labeled ‘Total costs’, show the optimal results for total costs by the risk-averse model obtained by each algorithm. The third columns labeled ‘\( \text{CO}_2 \) emission’ show the optimal results for the volume of \( \text{CO}_2 \) emission. The forth column labeled ‘No. of facilities’ reports the number of opened facilities obtained by each algorithm. The fifth columns labeled “CPU Times” present the solution times for each algorithm to obtain the optimal value. As shown in Table 2, the proposed IRLT based on decomposed McCormick envelopes, yields optimal results with tighter gaps, compared with CPLEX and CRLT for all size instances. Moreover, with the size of instance increasing, the gaps become larger. In order to increase clarity, we also illustrate the difference in Figure 2a. This comparison shows that the proposed improved RLT outperforms the classic RLT and widely-used solver CPLEX on the convergence rate.

Table 2. Performance of proposed algorithm.

<table>
<thead>
<tr>
<th>Instances</th>
<th>Total Cost</th>
<th>( \text{CO}_2 ) Emission</th>
<th>No. of Facilities</th>
<th>CPU Times (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IRLT</td>
<td>CPLEX</td>
<td>CRLT</td>
<td>IRLT</td>
</tr>
<tr>
<td>50–10</td>
<td>2597.8</td>
<td>2671.1</td>
<td>2599.1</td>
<td>623.1</td>
</tr>
<tr>
<td>50–20</td>
<td>2598.5</td>
<td>2787.3</td>
<td>2618.5</td>
<td>680.9</td>
</tr>
<tr>
<td>100–20</td>
<td>3616.4</td>
<td>3921.3</td>
<td>3744.4</td>
<td>1112.9</td>
</tr>
<tr>
<td>100–30</td>
<td>3698.5</td>
<td>3949.2</td>
<td>3727.5</td>
<td>1218.4</td>
</tr>
<tr>
<td>150–30</td>
<td>4740.6</td>
<td>5177.6</td>
<td>4943.3</td>
<td>2012.8</td>
</tr>
<tr>
<td>150–50</td>
<td>4791.1</td>
<td>5228.7</td>
<td>4994.5</td>
<td>2511.6</td>
</tr>
</tbody>
</table>

With respect to the solution time, we see that the proposed IRLT is slightly faster than CPLEX and the CRLT for smaller cases, e.g., 50–10 and 50–20. However, when the size increases, we see that the differences in solution time become obvious. Note that in the comparison of the proposed IRLT to CPLEX, our algorithm shows faster convergence time, over 254%, compared with CPLEX for instance 150–50. Moreover, it is also 151% faster than CPLEX in the same case. These results indicate that the proposed IRLT greatly outperforms CPLEX and CRLT. We report the performance in Figure 2b.
obtain the optimal value. As shown in Table 2, the proposed IRLT based on decomposed McCormick envelopes, yields optimal results with tighter gaps, compared with CPLEX and CRLT for all size instances. Moreover, with the size of instance increasing, the gaps become larger. In order to increase clarity, we also illustrate the difference in Figure 2a. This comparison shows that the proposed improved RLT outperforms the classic RLT and widely-used solver CPLEX on the convergence rate.

Table 2. Performance of proposed algorithm.

<table>
<thead>
<tr>
<th>Instances Total Cost</th>
<th>CO2 Emission</th>
<th>No. of Facilities</th>
<th>CPU Times (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRLT</td>
<td>CPLEX</td>
<td>CRLT</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------</td>
<td>-------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>50–10</td>
<td>2597.8</td>
<td>2671.1</td>
<td>623.1</td>
</tr>
<tr>
<td>50–20</td>
<td>2598.5</td>
<td>2787.3</td>
<td>680.9</td>
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<td>100–20</td>
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<td>3698.5</td>
<td>3949.2</td>
<td>1218.4</td>
</tr>
<tr>
<td>150–30</td>
<td>4740.6</td>
<td>5177.6</td>
<td>2012.8</td>
</tr>
<tr>
<td>150–50</td>
<td>4791.1</td>
<td>5228.7</td>
<td>2511.6</td>
</tr>
</tbody>
</table>

Figure 2. Comparison of IRLT to CPLEX and CRLT: (a) Fraction of gaps of the optimal value in total costs by each algorithm; (b) Fraction of increase of CPU times by each algorithm.

5.2. Algorithm Performance Using a ‘Real’ Data Set

In this section, we further show the performance of the proposed model, especially in the solution of controlling risk compared to the risk-neutral model. Here we use the data based on an electronic printer supply chain design problem. The firm builds the network to manufacture, sell and collect its used printers and remanufacture them. Here, the goal is to minimize the total cost, as well as the CO2 emission. Required parameters are shown in Table 3.

Table 3. Required parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of candidate manufacturing/remanufacturing facilities</td>
<td>4</td>
</tr>
<tr>
<td>No. of candidate collection facilities</td>
<td>4</td>
</tr>
<tr>
<td>No. of markets</td>
<td>5</td>
</tr>
<tr>
<td>No. of candidate distribution facilities</td>
<td>1</td>
</tr>
<tr>
<td>Demand of each market</td>
<td></td>
</tr>
<tr>
<td>Fixed cost of each manufacturing/remanufacturing facility</td>
<td>5,000,000</td>
</tr>
<tr>
<td>Fixed cost of each collection facility</td>
<td>500,000</td>
</tr>
<tr>
<td>Unit cost of transiting product</td>
<td>20</td>
</tr>
<tr>
<td>CO2 emission per Km (Kg)</td>
<td>0.18</td>
</tr>
<tr>
<td>Risk level</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Finally, based on the risk-averse model, we obtain the optimal network structure and report on the total costs and CO\(_2\) emissions in Table 4. The results show that opening 2 manufacturing/remanufacturing facilities and 1 collection facility is the best solution for controlling total cost and CO\(_2\) emissions simultaneously. We see that the solution from the risk-averse model is the same as that from the risk-neutral model. However, this still implies great differences in performance.

Table 4. Performance of proposed algorithm.

<table>
<thead>
<tr>
<th></th>
<th>Total Costs</th>
<th>Penalty Costs</th>
<th>CO(_2) Emissions (tons)</th>
<th>Opened Manufacturing/Remanufacturing Facilities</th>
<th>Opened Collection Facilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-averse model</td>
<td>−108</td>
<td>−106</td>
<td>291.3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Risk-neutral model</td>
<td>2.894</td>
<td>5.992</td>
<td>299.2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

To find out the impacts of risk-awareness, we compare the results to that of the risk-neutral model. We see that the total costs obtained by the risk-averse model are larger than the traditional risk-neutral model, however, note that CO\(_2\) emissions by the risk-averse model are less than that of the risk-neutral model. Moreover, the penalty costs of the risk-averse model are less than that of the risk-neutral model, which is because of the risk mitigation by CVaR on uncertain demand. In addition, we particularly take a risk control measure to reduce the CO\(_2\) emissions under a certain level. Using this method, the results of location and assignment are sub-optimal compared to the risk-neutral approach, namely, there is an increase in total costs. This is meaningful if the decision-maker is responsible to society for making the environment better. In a word, the proposed risk-averse model is more reliable than the risk-neutral model, both in reducing penalty costs and CO\(_2\) emissions.

To see the impacts of risk level on the optimal solution, we further make a sensitivity analysis on \(\alpha\) and \(\beta\). We let each one take a value from 0.5 to 0.99, and the results are reported in Figure 3. As illustrated in Figure 3, the total costs become greater as either \(\alpha\) or \(\beta\) increases. Specifically, under a certain \(\alpha\), the risk level under random demand, the higher risk level \(\beta\) taken to control CO\(_2\) emissions will generate more costs because of the sub-optimal solutions preferred to green solutions. That’s to say, the more attention paid to reduce CO\(_2\) emissions, the higher the total cost that will be incurred. Conversely, with respect to a certain \(\beta\), the risk level under random CO\(_2\) emissions, we can similarly remark that the more the risks on total costs reduce, the higher level of CO\(_2\) emissions will be generated.

Based on above analysis, we find that the risk-averse model can cause great differences in results compared to the risk-neutral models. Under uncertainty, it’s more reliable than the risk-neutral model, either in ensuring supply or reducing CO\(_2\) emissions. Thus, decision-makers can benefit from the risk-averse approach for designing more reliable green logistics networks.
we can similarly remark that the more the risks on total costs reduce, the higher level of CO2 emissions will be generated.

Figure 3. Sensitivity analysis of risk level on (a) total costs and (b) CO2 emissions.

Based on above analysis, we find that the risk-averse model can cause great differences in results compared to the risk-neutral models. Under uncertainty, it’s more reliable than the risk-neutral model, either in ensuring supply or reducing CO2 emissions. Thus, decision-makers can benefit from the risk-averse approach for designing more reliable green logistics networks.

6. Conclusions

This paper considered the risk-averse facility location problem for green CLSC design. We used a coherent risk measure, CVaR, to represent the underlying risk with uncertain customer demands and CO2 emissions. The resulting model is a 0-1 mixed integer bi-objective nonlinear programming, which is hard to solve. To this end, we develop an alternative formulation for obtaining a single objective 0-1 mixed integer bilinear programming. Then, we propose an improved RLT by decomposed piecewise McCormick envelopes to generate the lower bound efficiently. We show that our proposed risk-averse model can generate a more reliable solution than the risk-neutral model, both in reducing penalty costs and CO2 emissions. Moreover, the proposed IRLT algorithm outperforms CPLEX and the classic RLT in convergence rate and gaps.

Generally speaking, the proposed risk-averse model can contribute to reducing more CO2 emissions than the classic risk-neutral model under uncertainty. That means this approach is able to generate a more environmentally friendly green closed loop supply chain network.
Since the distribution of the uncertain element is usually difficult to obtain exactly, the stochastic programming model is still less feasible in practice. To do this, we aim to address this issue in future research by incorporating robust optimization.

**Author Contributions:** Y.G. conceived and designed the basic and risk-averse model; Z.X. performed the experiments; W.L. and X.X. analyzed the data; Y.G. contributed reagents/materials/analysis tools; Z.X. wrote the paper.

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